Nuclear vibrations and rotations

Introduction to Nuclear Science

Simon Fraser University
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Outline

1. Significance of collective excitations

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2. The monopole vibrations
3. The dipole vibrations
4. Rotation and moments of inertia
5. Pair alignment

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Review of collective excitations

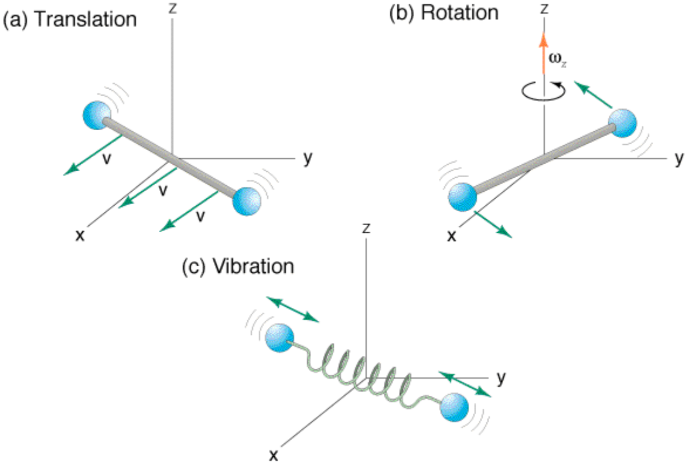
- In the previous lecture we discussed two types of collective excitations
  1. collective vibrations,
  2. collective rotation
- We discussed collective vibration of various multipolarities
- We also discussed the impact of symmetries of the axially symmetric quadrupole rotor on the rotational spectra
  1. Excitation energy of the rotational states come from the rotation about the axis perpendicular of the symmetry axis.
  2. The symmetry of the rotor with respect to the rotation about $180\circ$ around the axis perpendicular to the symmetry axis resulted in even spin only in the excitation spectrum.

Why are the collective excitations significant?

- The answer to the question of significance of the collective excitation is quite simple:
    1. collective excitation can be conclusively identified as such from the pattern of excited states
    2. collective excitations provide direct insight into the structure of nuclei, nuclear potential, nuclear binding, and ultimately, the nuclear force.
- Collective excitations, therefore, provide a probe which allows us to see what we can not see directly.
- There are not many other probes of similar utility, therefore a significant effort of nuclear science community is concentrated on studies of nuclear collective excitations.
- Today we are going to analyze a few examples of collective excitation and information they provide.

Collective excitations of di-atomic molecules

- A good analogy is provided by collective excitation of di-atomic molecules which yield information on bond length and strength from rotational and vibrational spectra.

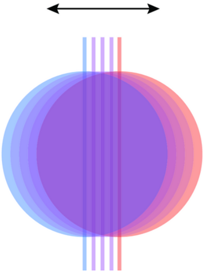


Nuclear equation of state and the breathing mode

- You are aware of the role the ideal gas law plays in science.
- The ideal gas law was first formulated empirically based on the laws of gas transformations derived from experiments (and further derived from ideal gas model supporting the atomic theory of matter).
- One of these laws of transformation was the Boyle-Mariotte law giving the relation between the pressure (force) and volume at a constant temperature and mass.
- A similar equation of state, or relation between pressure and volume, for nuclear matter is clearly of interest to nuclear scientists.
- The breathing (monopole) nuclear vibration mode is equivalent to the Boyle's transformation of the ideal gas.
- Thus the information on the berating mode provides insight into nuclear equation of state and gives information on nuclear incompressibility.

The Giant Dipole Resonance and the symmetry energy

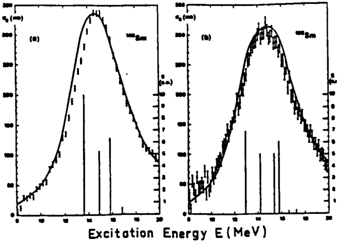
- The Giant Dipole Resonance (GDR) is a vibration of proton and neutron mass/density distributions around the common centre of mass.



- The restoring for for the GDR has the same origin as the symmetry energy in the Liquid Drop model.
- The information on frequency of the GDR provides insight into the symmetry/asymmetry properties on the nuclear equation of state.

The Giant Dipole Resonance

- Giant Dipole Resonance was discovered as a prominent increase in the probability of the photo-dissociation of a neutron, the (γ, n) reaction at nuclear excitation energy $\sim$15 MeV.

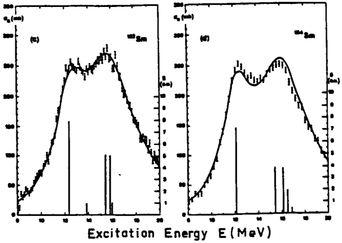


- The name resonance comes from the Lorentz-peak structure of the excitation probability function which is centred on the resonance frequency of the proton-neutron vibrational mode.

- The name giant comes from the large amplitude.

The split Giant Dipole Resonance and the deformation

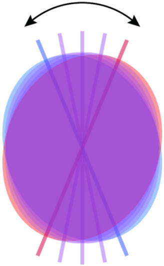
- Studies of the GDR indicated double-peak structure in nuclei far from the magic gap.



- This observation has been interpreted as an evidence of two different frequencies of the proton-neutron oscillation in deformed nuclei.
- Thus, the GDR provides also information on nuclear deformation.

The scissors mode

- The scissors mode is an oscillation of the axes of deformed proton and neutron distribution with respect to the common axis.



- It is distinct from GDR in deformed nuclei where axes stay parallel.
- Scissors mode probes asymmetry of nuclear matter an deformation of nuclei but differently than the GDR.

Phonon excitations

- We discussed the phonon model and phonon excitations of different multipolarities (rank).
- The phonons are identified from the data by a comparison between predicted and observed pattern of excitations.
- In particular, the first excited state has the spin and parity of the phonon, the next group of excited states has twice the energy of the first one, positive parity, and spins and degeneracy defined by the two-phonon coupling.
- Excitation energy of the phonon provides information on frequency of the vibrations at the given multipolarity, which in turn provides information on nuclear potential, nuclear states near the Fermi level, and coupling between valence nucleons outside closed shells.

Quantum quadrupole axial rotor

- Excitation energies of a quantum quadrupole axial rotor are

$$E_I = \frac{\hbar^2}{2J} I(I+1) \tag{1}$$

- I is the angular momentum (spin) of the state
- J is the moment of inertia.
- Symmetries allow only even values of I and positive parity
- Consequently the energy levels are

Spin/parity I^π	0^+	2^+	4^+	6^+	8^+
Energy E	0	$6\frac{\hbar^2}{2J}$	$20\frac{\hbar^2}{2J}$	$42\frac{\hbar^2}{2J}$	$72\frac{\hbar^2}{2J}$
E_{I^π}/E_{2^+}	0	1	3.33	7	12

Quantum quadrupole axial rotor: ^{178}Hf

- Let us look into the lowest energy excitations in ^{178}Hf

Spin/parity I^π	0^+	2^+	4^+	6^+	8^+
Energy E [keV]	0	93.2	306.6	632.2	1058.6
E_{I^π}/E_{2^+}	0.00	1.00	3.29	6.78	11.36

- If we compare with the prediction of the rotor model we see a pretty good agreement (and small deviations to be discussed later).

Spin/parity I^π	0^+	2^+	4^+	6^+	8^+
Energy E	0	$6\frac{\hbar^2}{2J}$	$20\frac{\hbar^2}{2J}$	$42\frac{\hbar^2}{2J}$	$72\frac{\hbar^2}{2J}$
E_{I^π}/E_{2^+}	0.00	1.00	3.33	7.00	12.00

Moment of inertia

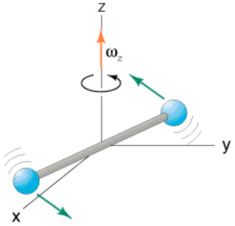
- The comparison of the experimental excitation energies of ^{178}Hf and axially symmetric quadrupole rotor yields reasonably good agreement.
- From this comparison we can extract the moment of inertia

$$E_{2^+} = \frac{\hbar^2}{2J} 2 * 3 = 3\frac{\hbar^2}{J} \implies J = 3\frac{\hbar^2}{E_{2^+}} \quad (2)$$

- For ^{178}Hf we get $J = 32.2\ \hbar^2/\mathrm{MeV}$, and the same can be done for a large number of know rotational nuclei.
- If we know quadrupole moments and radii of these rotational nuclei we could calculate moments of inertia for the corresponding deformed shapes and compare to these extracted from rotational energies.

Rigid body moment of inertia

- The question becomes how to calculate the moment of inertia.
- The first answer which may come to mind is to assume rigid shape of the nuclear distribution.



- This, after all, is done for extracting the bond length for diatomic molecules. The moment of inertia for two equal masses m separated by L rotating about the centre of mass is

$$J = \frac{1}{2} m L^2 \tag{3}$$

Moment of inertia

- Nuclear moment of inertia depends on the mass distribution.
- For a quadrupole rotor we can use the Bohr-Wheeler parametrization for the axially symmetric shape with parameter β fully defining the deformation of the shape.
- The calculations involve the volume integral over the density distribution but yields a relatively simply result

$$J_R = \int_V r^2 \rho(r) r^2 \sin\theta dr d\theta d\phi = \frac{2}{5} M R_0^2 (1 + 0.31\beta) \tag{4}$$

- You may recall the moment of inertia for a rigid sphere with radius R_0

$$J_S = \frac{2}{5} M R_0^2 \tag{5}$$

Rigid body moment of inertia for ^{178}Hf

- Let us estimate J for ^{178}Hf. First let us calculate

$$J_S = \frac{2}{5}MR_0^2 = \frac{2}{5}Au\left(r_0A^{\frac{1}{3}}\right)^2 = \frac{2}{5}\frac{uc^2r_0^2}{(\hbar c)^2}A^{\frac{5}{3}}\ [\hbar^2/\mathrm{MeV}] = \tag{6}$$
$$= 0.0138A^{\frac{5}{3}}\ [\hbar^2/\mathrm{MeV}]$$

- For ^{178}Hf this yields

$$J_s = 0.0138(178)^{\frac{5}{3}} = 77.7\ [\hbar^2/\mathrm{MeV}] \tag{7}$$

- The deformation increases the moment of inertia. For a reasonable guess of $\beta \sim 0.3$

$$J_R = 77.7(1 + 0.31 \times 0.3) \sim 85\ [\hbar^2/\mathrm{MeV}] \tag{8}$$

Rigid body moment of inertia for ^{178}Hf

- The rigid body moment of inertia for ^{178}Hf is

$$J_R = 85\ [\hbar^2/\mathrm{MeV}] \tag{9}$$

- The moment of inertia measured from the rotational spectrum of

$$J = 32\ [\hbar^2/\mathrm{MeV}] \tag{10}$$

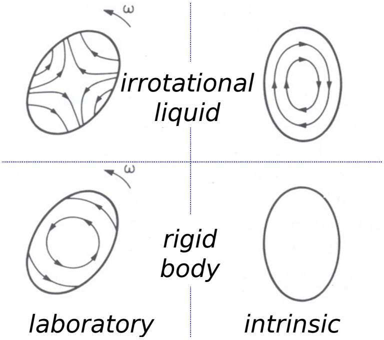
- We observe a factor of ~ 3 discrepancy.
- Which should prompt us the legitimate conclusion that nuclei do not rotate as rigid bodies.

Irrotational flow moment of inertia

- We have observed that nuclei do not rotate like rigid bodies.
- Which prompts a question, how do they rotate?
- If rigid body does not work, we could take another extreme and assume that nuclei rotate like a deformed drop of liquid.
- After all, that may be in correspondence to the Liquid Drop model.
- Before we pursue this idea, we should investigate what is a difference between rigid body and irrotational liquid rotation.
- One way to do it is to compare rotation of a fresh and hard boiled egg.
- Another way is to look into the flow lines.

Irrotational flow vs. rigid body

- The figure below shows a comparison between flow lines for irrotational flow and a rigid body in the laboratory and intrinsic reference frames.



Irrotational flow moment of inertia

- Calculations of the irrotational flow moment of inertia involve fluid dynamics and yield

$$J_I = \frac{9}{8\pi} M R_0^2 \beta^2 = \frac{45}{16\pi} J_S \beta^2 \tag{11}$$

- The irrotational flow moment of inertia is smaller than the rigid body moment of inertia (that may be good since the rigid body moment of inertia for ^{178}Hf was $\sim$ three times too large.)
- The irrotational flow moment of inertia for ^{178}Hf is

$$J_I = \frac{45}{16\pi} J_S \beta^2 = \frac{45}{16\pi} 77.7(0.3)^2 = 6.3[\hbar^2/\mathrm{MeV}] \tag{12}$$

- This is $\sim$ 5 times too small as compared to the experimental value of 32 $[\hbar^2/\mathrm{MeV}]$.
- Which should prompt us to a legitimate conclusion that nuclei do not rotate like irrotational flow either.

How do nuclei rotate?

- We know how nuclei do not rotate, and we are back to the same question on the origin of nuclear rotation.
- The experimental moment of inertia is smaller than the rigid body which may indicate that there is a part of nucleus which rotates and part which does not.
- This lead towards a macroscopic picture of a distortion wave flowing on the surface of a non-rotating spherical core.
- In the microscopic picture we should note the impact of pairing interactions which bind nucleons into pairs of zero angular momenta, which according to quantum mechanics, can not contribute to rotational excitations.
- The relation between this to pictures is still an active area of study.

How do nuclei rotate?

- In the microscopic picture the angular momentum and excitation energy of rotational states has to originate from angular momenta and excitation energies of nucleons forming the nucleus.
- We can distinguish between nucleons well below the Fermi level which are bound to pairs of zero spin and require significant energies to be excited and nucleons near the Fermi level which can scatter to free states near the Fermi level.
- Nuclear rotation results from gradual and smooth alignment of tiny angular momentum contributions of many nucleons near the Fermi level. These contributions form the total angular momentum, while energy required for the alignment is reflected in the excitation energy of the resulting state.
- Paired nucleons well below the Fermi level form the non-rotating core of spin 0.

Pair alignment

- The qualitative picture presented above is consistent with a phenomenon of pair alignment observed for some nuclei.
- Experimentally, the pair alignment is observed as a sudden increase of the moment of inertia which happens in a narrow range of spin around $I \sim$10-12 $\hbar$ in a rotational sequence.
- The sequence remains rotational below and above the alignment.
- The microscopic mechanism for the alignment is shown by this applet.
- Pair alignment is a sudden effect impacting a single pair of nucleons in contrast to the nuclear rotation resulting from gradual alignment of a large number of pairs.

Rotating Nilsson potential

- For nuclei with odd number of either protons or neutrons two contributions to the total angular momentum I can be distinguished
  1. the collective contribution from the rotation of even-even core
  2. the single-particle contribution from the valence single nucleon.
- For a quadrupole axially symmetric rotor the Nilsson model can be used to identify the single-particle contribution.
- In so called "strong coupling limit" this contribution is equal to the projection of the single-particle angular momentum on the symmetry axis which is given by the Nilsson quantum number Ω.
- The rotational contribution R to the total angular momentum can be extracted from the vector coupling model as

$$R^2 = I(I+1) - \Omega^2 \tag{13}$$

Rotating Nilsson potential

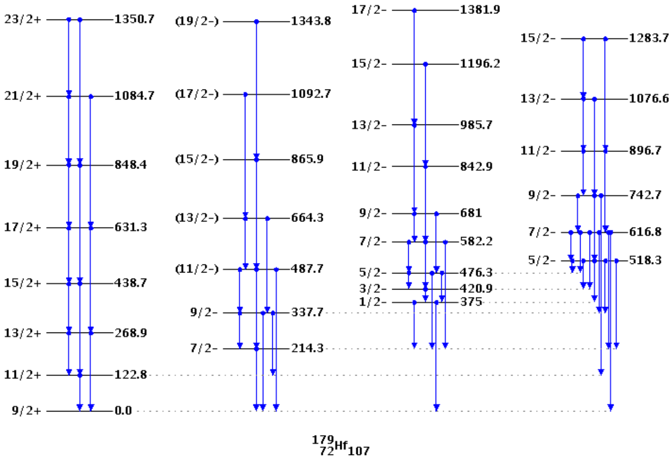
- Each Nilsson state has a rotational band built upon it.
- The symmetries of the axially symmetric rotor with an odd nucleon coupled are different than these for the axially symmetric rotor alone. Both, even and odd spins are allowed within each band.
- The smallest value of I for the band is Ω
- The lowest energy state (with spin Ω) is called the band head.
- The rotational energies are give by

$$E = E_0 + \frac{\hbar^2}{2J} R^2 = E_0 + \frac{\hbar^2}{2J_\omega} \left(I(I+1) - \Omega^2 \right) \tag{14}$$

- E_0 is the energy of the bandhead
- Each band can have different moment of inertia J_Ω.

Rotating Nilsson potential: ^{179}Hf

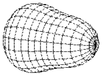
- Partial level scheme of ^{179}Hf with rotational bands built on four different Nilsson states.

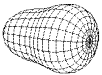


Symmetries and rotations

- We have discussed the impact of symmetries on the excitation spectrum of axially symmetric quadrupole rotor.
- The consequence of the axial symmetry was the rotation about the axis perpendicular to the symmetry axis, the symmetry with respect to rotation about this axis by 180° restricted spins to even values only.
- It turns out that impact of the symmetries on the rotational spectra is general and significant.
- Every symmetry in the intrinsic frame has an impact on the pattern of excited levels observed in the laboratory reference frame.
- Thus collective rotation provides a significant tool to study underlying symmetries in nuclei.

An axially symmetric octupole rotot

$|\Psi\rangle = |$  $\rangle$

$P|\Psi\rangle = |$  $\rangle$

$$P|\Psi\rangle \neq |\Psi\rangle$$