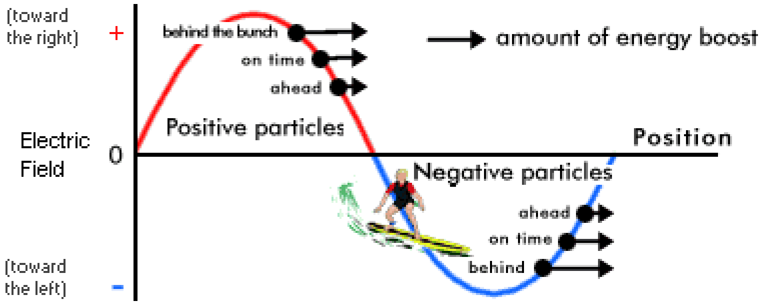
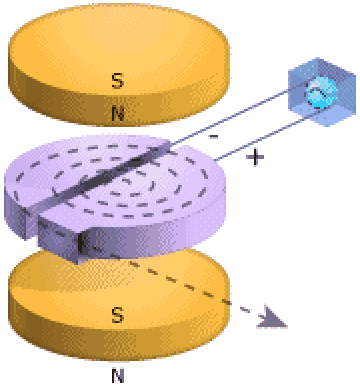
Radio-frequency (RF) accelerators

- Key idea: using rapidly changing high frequency voltages instead of electrostatic voltages avoids corona formation and discharge
  → much higher accelerating voltages possible
- But: particles must have the correct phase relation to the accelerating voltage
- But: need high power RF sources!



The Cyclotron



- 1930: Lawrence proposed the cyclotron
  (before he developed a workable color TV screen)
- 1931: Lawrence and Livingston built first cyclotron (80 keV)
- 1932: Lawrence and Livingston used a cyclotron for 1.25 MeV protons and mentioned longitudinal (phase) focusing

Ernest O. Lawrence
(1901-1958)

M. Stanley Livingston
(1905-1986)

Electromagnetic forces on charged particles

- Lorentz force equation gives the force in response to electric and magnetic fields:

$$\vec{F} = q \cdot (\vec{E} + \vec{v} \times \vec{B})$$

- The equation of motion becomes:

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}(m_o \gamma v) = q \cdot (\vec{E} + \vec{v} \times \vec{B})$$

- The kinetic energy of a charged particle increases by an amount equal to the work done (Work-Energy Theorem)

$$\Delta W = \int \vec{F} \cdot d\vec{\ell} = q \int \vec{E} \cdot d\vec{\ell} + q \int (\vec{v} \times \vec{B}) \cdot d\vec{\ell}$$

$$\Delta W = q \int \vec{E} \cdot d\vec{\ell} + q \int \cancel{(\vec{v} \times \vec{B})} \cdot \vec{v} \cdot dt = q \int \vec{E} \cdot d\vec{\ell}$$

Motion in E and B fields

- Governed by Lorentz force: $\frac{d\vec{p}}{dt} = q \cdot [\vec{E} + \vec{v} \times \vec{B}]$

$$E^2 = \vec{p}^2 c^2 + m_0{}^2 c^4$$

$$\Rightarrow \quad E\frac{dE}{dt} = c^2 \vec{p} \cdot \frac{d\vec{p}}{dt}$$

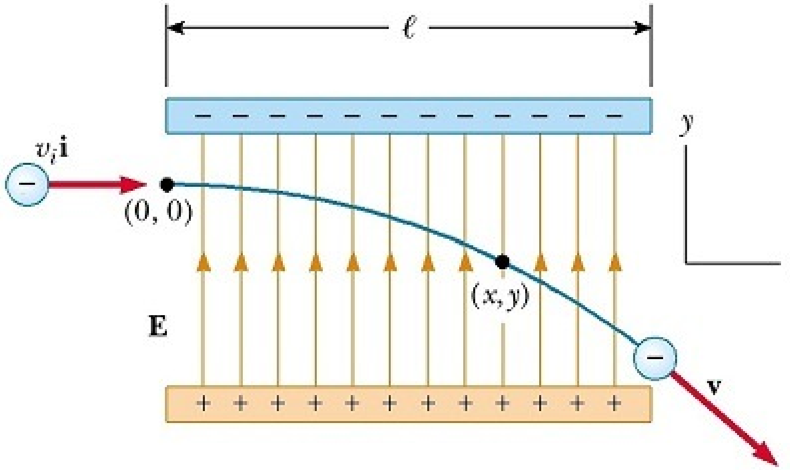
$$\Rightarrow \quad \frac{dE}{dt} = \frac{c^2 \vec{p}}{E} \cdot q \cdot (\vec{E} + \vec{v} \times \vec{B}) = \frac{qc^2}{E} \cdot \vec{p} \cdot \vec{E}$$

A magnetic field does not alter a particle's energy. Only an electric field can do this.

- Acceleration along a uniform electric field (B = 0):

$$\left.\begin{aligned} x &= v \cdot t \\ y &= \frac{1}{2} a \cdot t^2 = -\frac{1}{2}\frac{eE}{m} t^2 \end{aligned}\right\}$$

parabolic path for v « c



Electromagnetic forces on charged particles

- We therefore reach the important conclusion that
  - magnetic fields cannot be used to change the kinetic energy of a particle
- We must rely on electric fields for particle acceleration
  - acceleration occurs along the direction of the electric field
  - energy gain is independent of the particle velocity
- In accelerators:
  - longitudinal electric fields (along the direction of the particle motion) are used for acceleration
  - magnetic fields are used to bend particles for guidance and focusing

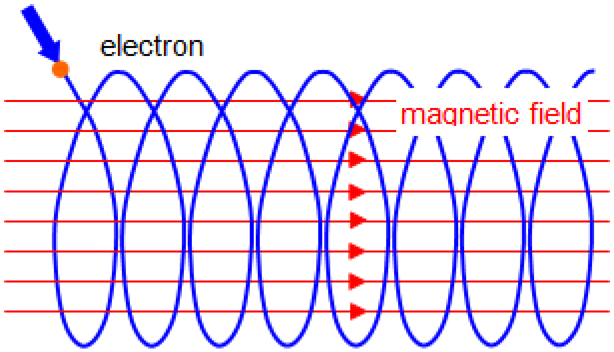
- There are many possibilities, depending on existence and time-dependence of $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ fields.
  For example, if there is no magnetic field $\vec{B} = 0$ and a time-independent electric field along the z-axis, then electrostatic accelerator. If the electric field is time-dependent, then LINAC.
- If $\vec{E} = 0$ and $B_\theta = B_r = 0$ and $B_z \perp \vec{v}$ $then$ $circular$ $motion$ $with$ $\omega_c = \dot{\theta} = \frac{q \cdot B_z}{m}$ ω_c = cyclotron frequency
- If ρ radius of curvature, then $p = q \cdot B_z \cdot \rho$ or $p[MeV/c] = 300[MeV/c] \cdot B_z[T] \cdot \rho[m]$

Behavior under constant B-field

- Motion in a uniform, constant magnetic field

Constant energy with spiraling along a uniform magnetic field

$$\frac{m_0 \cdot \gamma \cdot v^2}{\rho} = q \cdot v \cdot B \Rightarrow$$



$$(a) \quad \rho = \frac{m_0 \cdot \gamma \cdot v}{q \cdot B} \qquad \rho = \left| \frac{p}{q \cdot B} \right|$$

$$(b) \quad \omega = \frac{v}{\rho} = \frac{q \cdot B}{m_0 \cdot \gamma} \qquad \omega = \frac{q \cdot B \cdot c^2}{E} = \frac{v}{\rho}$$

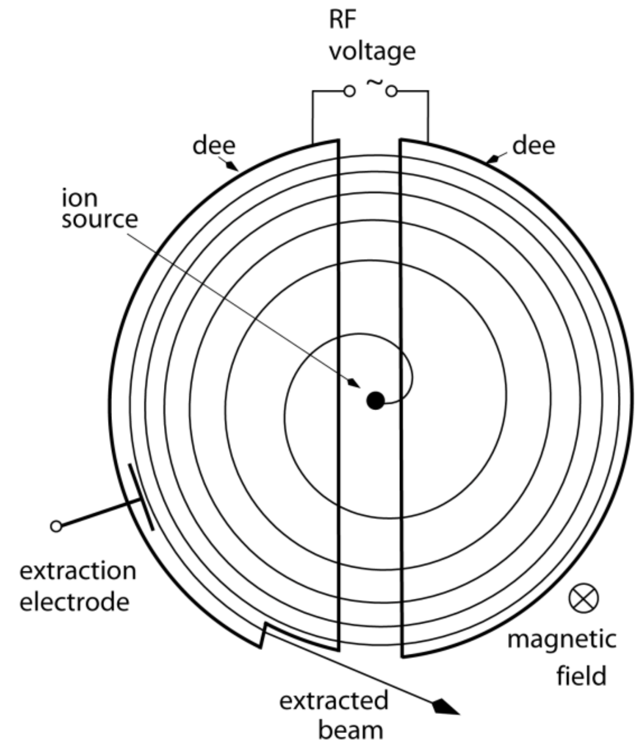
The Cyclotron

- This is a constant frequency orbital accelerator, but one in which the orbit radius increases. Cyclotron angular frequency given by:

  $\omega_c = q \cdot B/m$ independent of particle velocity

- Acceleration occurs, provided the synchronism condition, RF frequency of the source matches the cyclotron frequency $\omega_{RF} \sim \omega_c$, is met.
- Continues gaining velocity until is spirals out $r = m \cdot v/e \cdot B$
  Radius increment per turn decreases with increasing energy because the revolution time must stay constant.
- Correct for low energy $(\gamma \sim 1)$ independent of $\vec{p}$ - earlier ones were proton accelerators for a few MeV!
- As the mass increases $(m_0\gamma)$, orbital frequency changes and resonance condition is no longer fulfilled. To overcome this, either
  - Modulate the frequency → 'synchrocyclotron' or
  - Allow B_z to increase with R, to keep ω_c = constant. However, we will see this is unstable (n < 0 and axial motion is unstable). This can be restored by abandoning cylindrical symmetry of B field, i.e. magnet is now split into segments, and using the focusing of the magnet edges → 'sector focused cyclotron'.

Typical parameters: B = 1.5 T, ω = 50 MHz, U = 200-500 kV, I [mA]
E_{proton} = 20-30 MeV



First successful cyclotron, 4-5 inch model built by Lawrence and Livingston, 1929

Lawrence on the cover of *Time magazine*, 1937

Basics – Cyclotron frequency and K-value

- Cyclotron frequency (homogenous) B-field

$$\omega_c = \frac{e \cdot B}{\gamma \cdot m_0}$$

- Cyclotron K-value
  - K is the relativistic **kinetic energy reached** for protons **from bending strength**:

$$p^2 = {m_0}^2 c^2 (\gamma^2 - 1) = m_0 \cdot m_0 c^2 (\gamma - 1)(\gamma + 1) = m_0 T_{kin}(\gamma + 1) \quad \rightarrow T_{kin} = \frac{p^2}{m_0(\gamma + 1)}$$

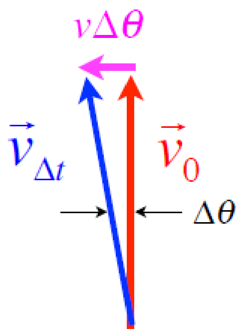
$$\frac{T_{kin}}{A} = \frac{p^2}{(\gamma + 1) m_u} \cdot \frac{1}{A^2} = \frac{B^2 \cdot \rho^2 \cdot q^2}{(\gamma + 1) m_u} \cdot \frac{1}{A^2}$$

$$\frac{T_{kin}}{A} = \frac{(B \cdot \rho)^2 \cdot e^2}{(\gamma + 1) m_u} \left(\frac{q}{A}\right)^2 = K \cdot \left(\frac{q}{A}\right)^2$$

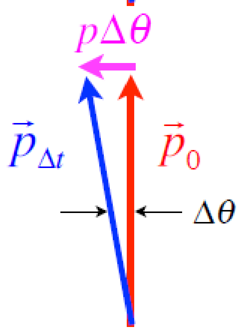
  - K can be used to rescale the energy reach of protons to other charge-to-mass ratios (q/A)
  - K in [MeV] is often used for naming cyclotrons

example: K-130 cyclotron, Jyväskylä

Orbit in uniform magnetic field



During time Δt, the ***velocity vector*** rotates through the exact same angle Δθ. The velocity magnitude doesn't change. So the change in the velocity vector is Δv = v·Δθ.



The same statements are true about the ***momentum vector***, which is parallel to the velocity vector. So the change in the momentum vector is Δp = p·θ.

Then $\vec{F} = \frac{d\vec{p}}{dt} \rightarrow q \cdot v \cdot B = \frac{\Delta p}{\Delta t} = \frac{p \cdot \Delta\theta}{\Delta t} = \frac{p \cdot (v \cdot \Delta t / R)}{\Delta t} \rightarrow q \cdot B = \frac{p}{R}$

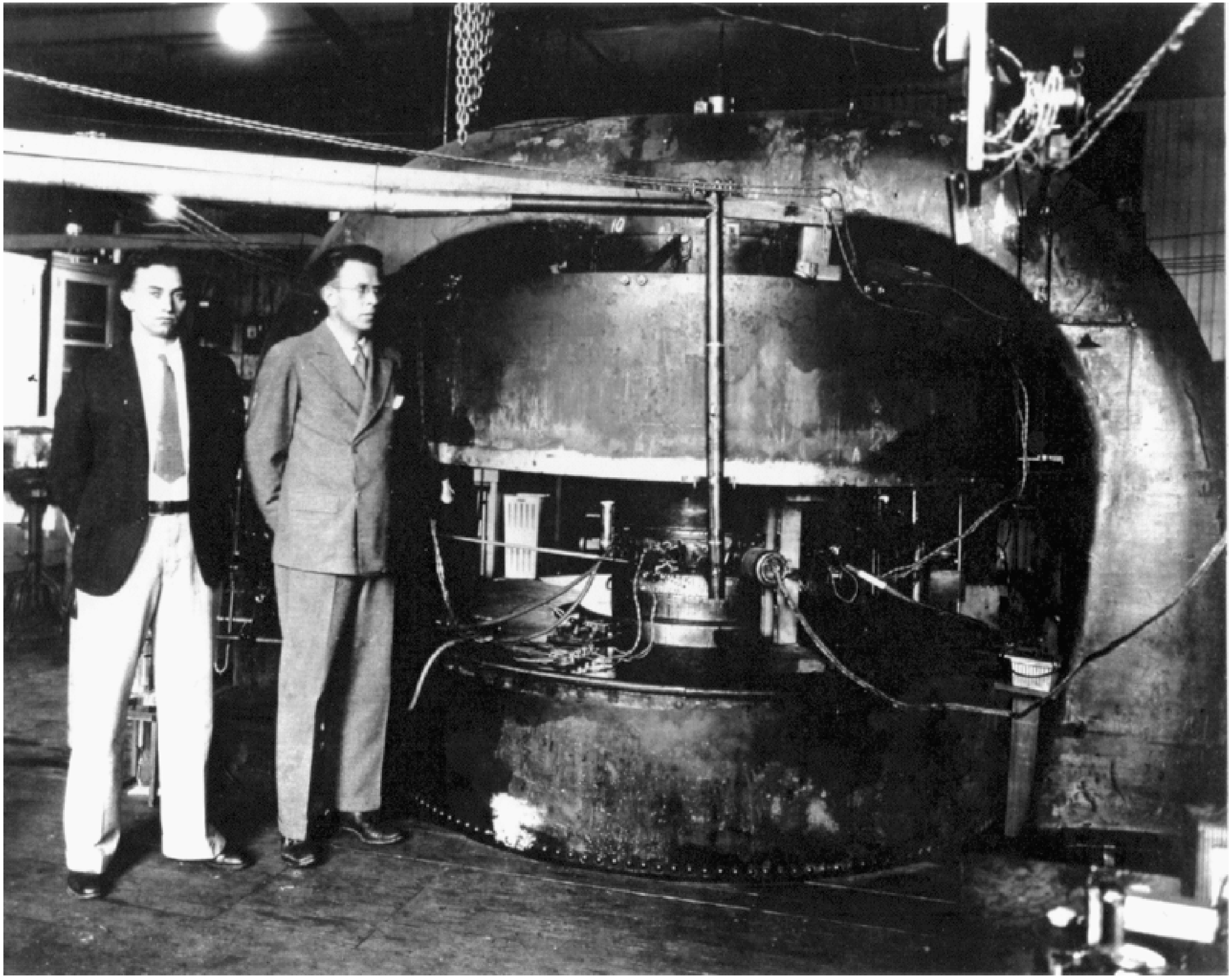
$$p = 300 \frac{MeV}{c} \cdot R_{meters} \cdot B_{Tesla}$$

So to get high momentum, one needs a strong magnet.

It's hard to get the field strength more than 1.5 T, because iron saturates. So one must increase the diameter.

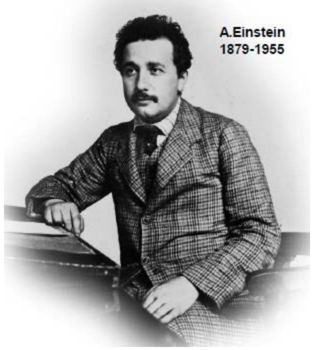
With a 2 meter diameter one obtains p = 450 MeV/c. For protons, that's about half the speed of light, where relativity starts to become noticeable.

27-inch Cyclotron with Lawrence and Livingstone



184-inch Cyclotron Magnet in Berkeley

Cyclotron Limits



energy

$E = \gamma \cdot E_0$

kinetic energy:
$E_{kin} = (\gamma - 1) \cdot E_0$

velocity

$v = \beta c$

revolution time:
$\tau = \frac{2\pi R}{\beta c}$

momentum

$p = \beta\gamma \cdot m_0 c$

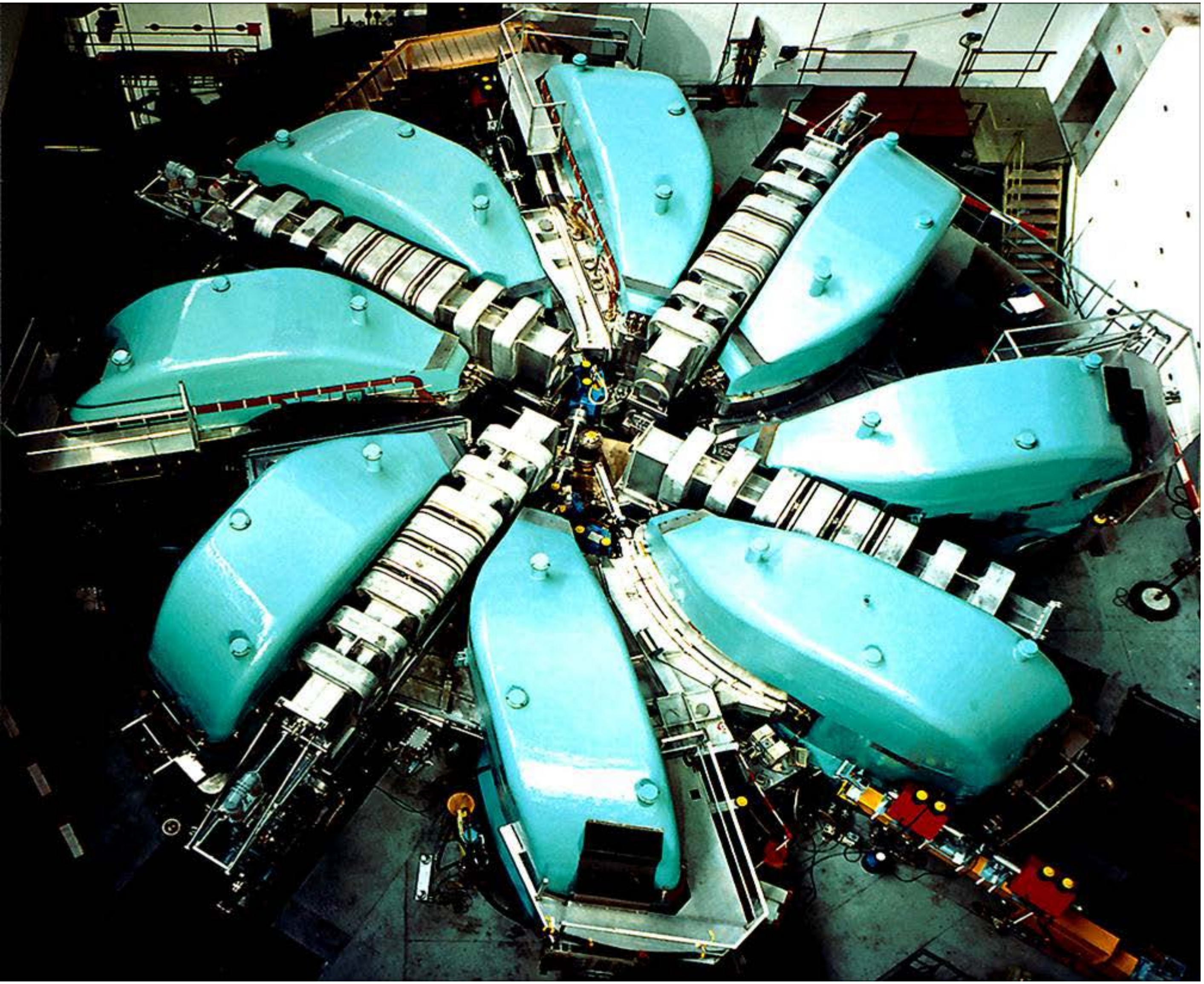
bending strength:
$B \cdot R = \beta\gamma \frac{m_0 c}{e}$

Ideally, the "184 inch" cyclotron with a field of 2.2 Tesla and orbital radius of 2.08 meters would get to p = 1373 MeV/c. Since $pc = \beta\gamma mc^2$ and $mc^2 = 938.6\ MeV$ for protons, $\beta\gamma = \frac{1373}{938.6} = 1.463$ and $\gamma = \sqrt{1 + (\beta\gamma)^2} = 1.772$

That means the kinetic energy would be 0.772 times mc^2, or 724 MeV.

But, the period would be 77% longer at the maximum radius than it was at small radius. So particles would get out of phase rapidly and the acceleration stops.

Sector Focusing Cyclotron - PSI



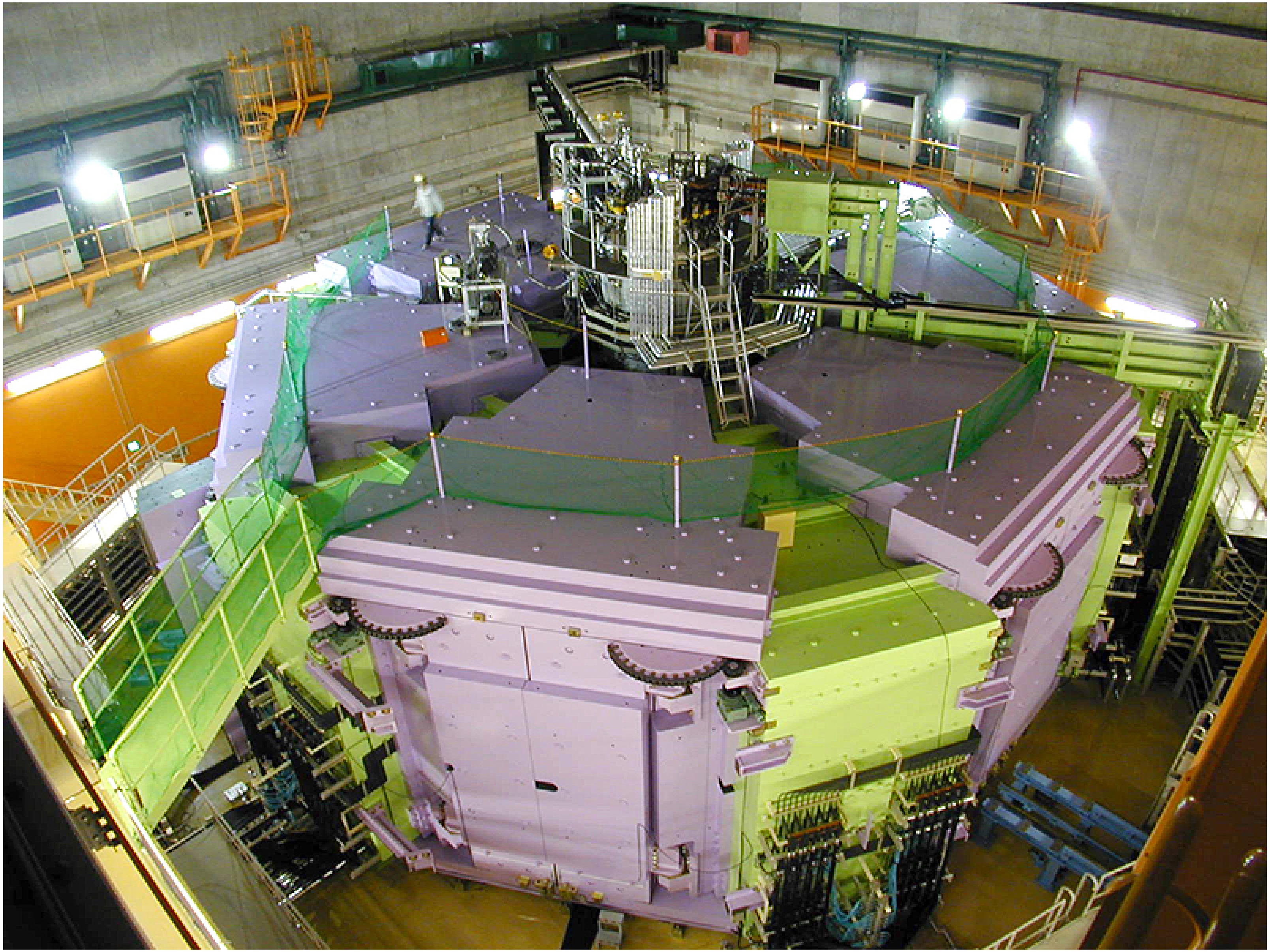
In case $\gamma > 1$:

$$\omega_c = \frac{q \cdot B_z(r)}{m_0 \cdot \gamma}$$

K = 592 MeV
I = 2 mA

1.3 MW

RIKEN – Superconducting Cyclotron (K-2600)



Acceleration Scheme of Uranium