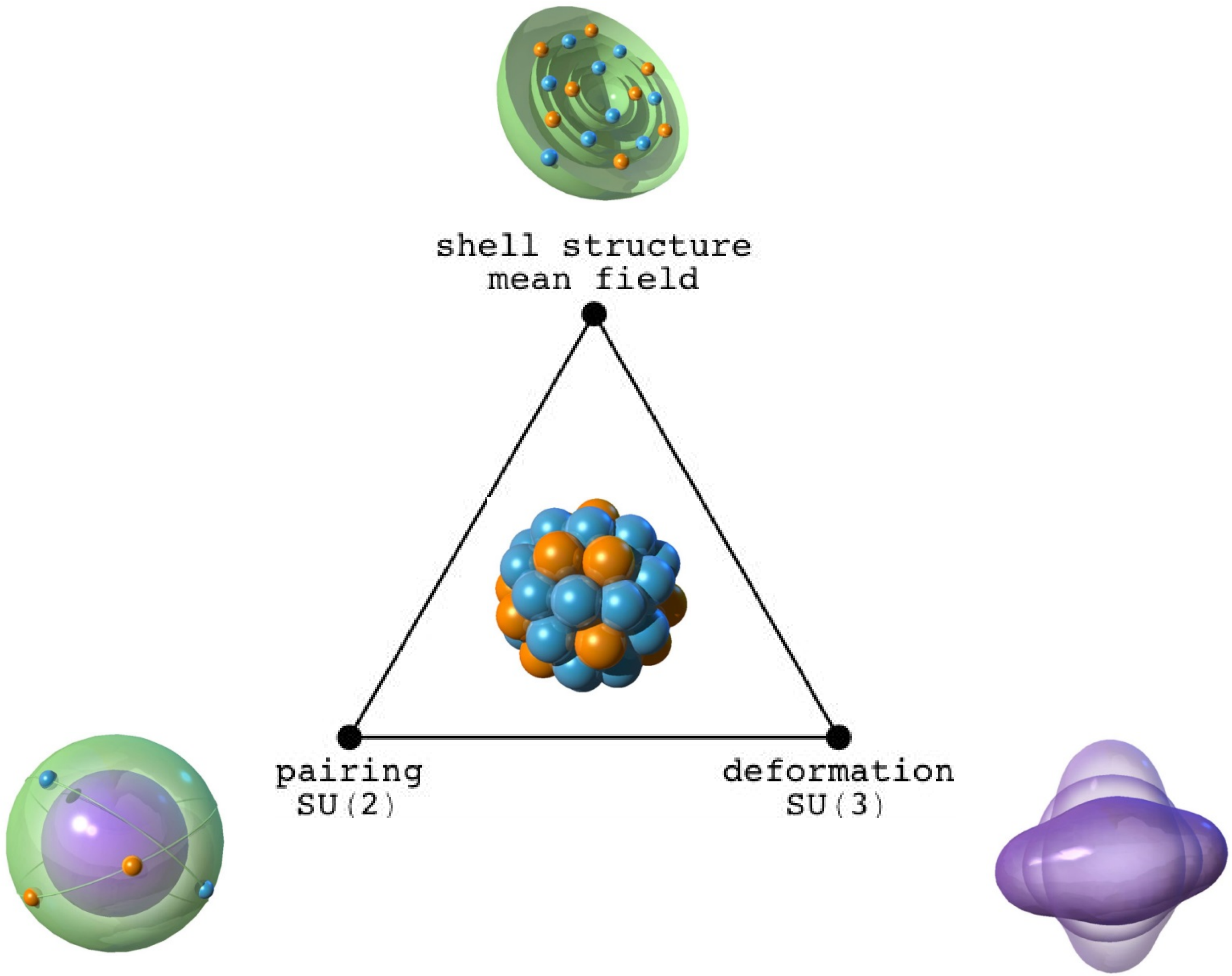
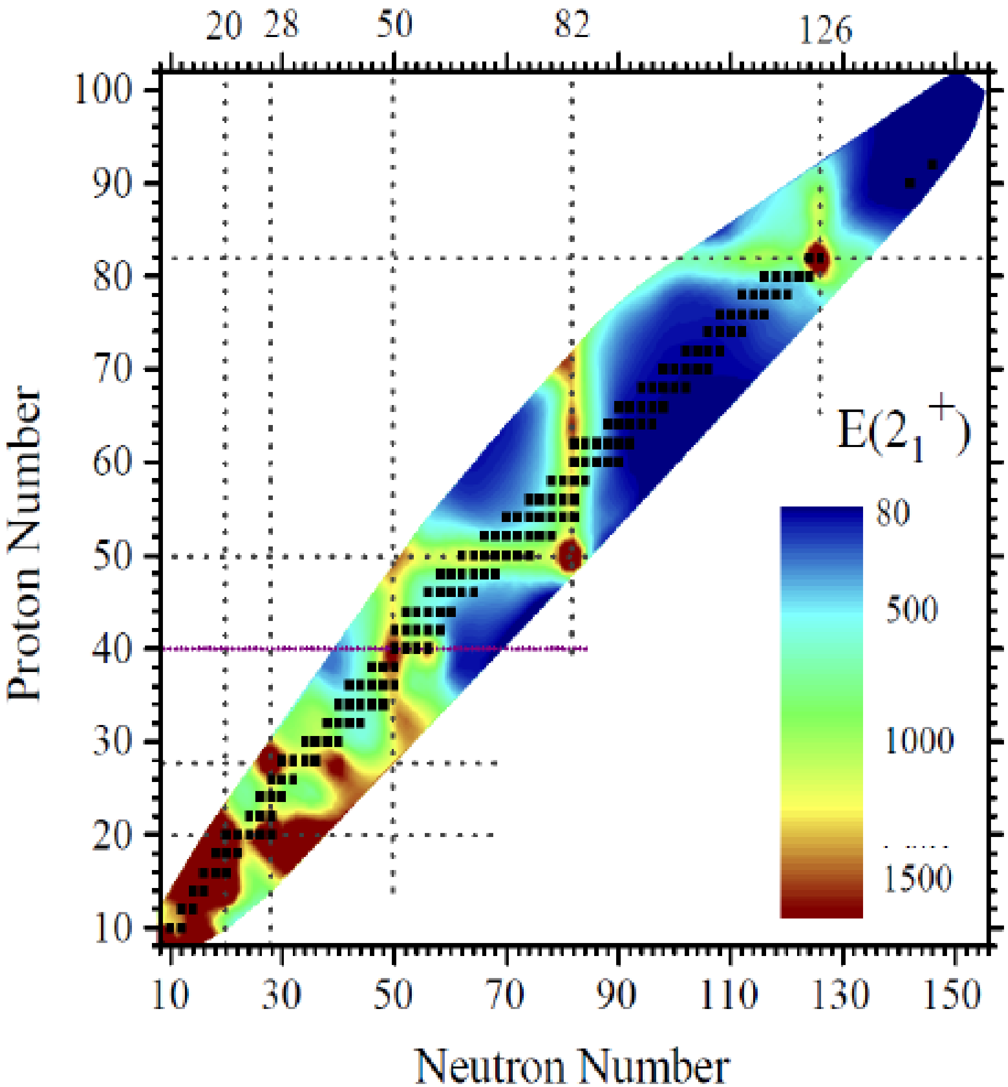
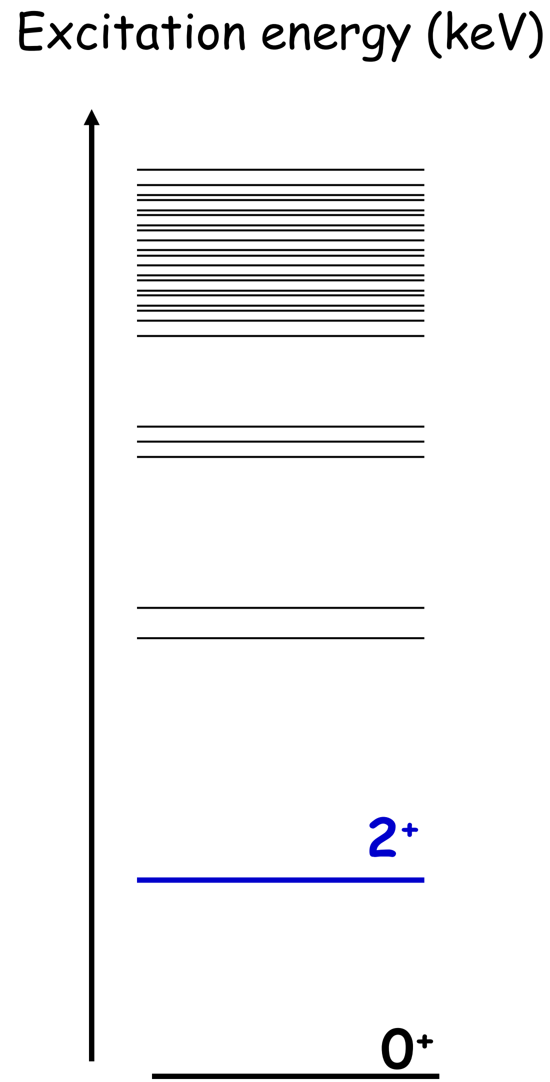
# PHL424: Nuclear surface vibration



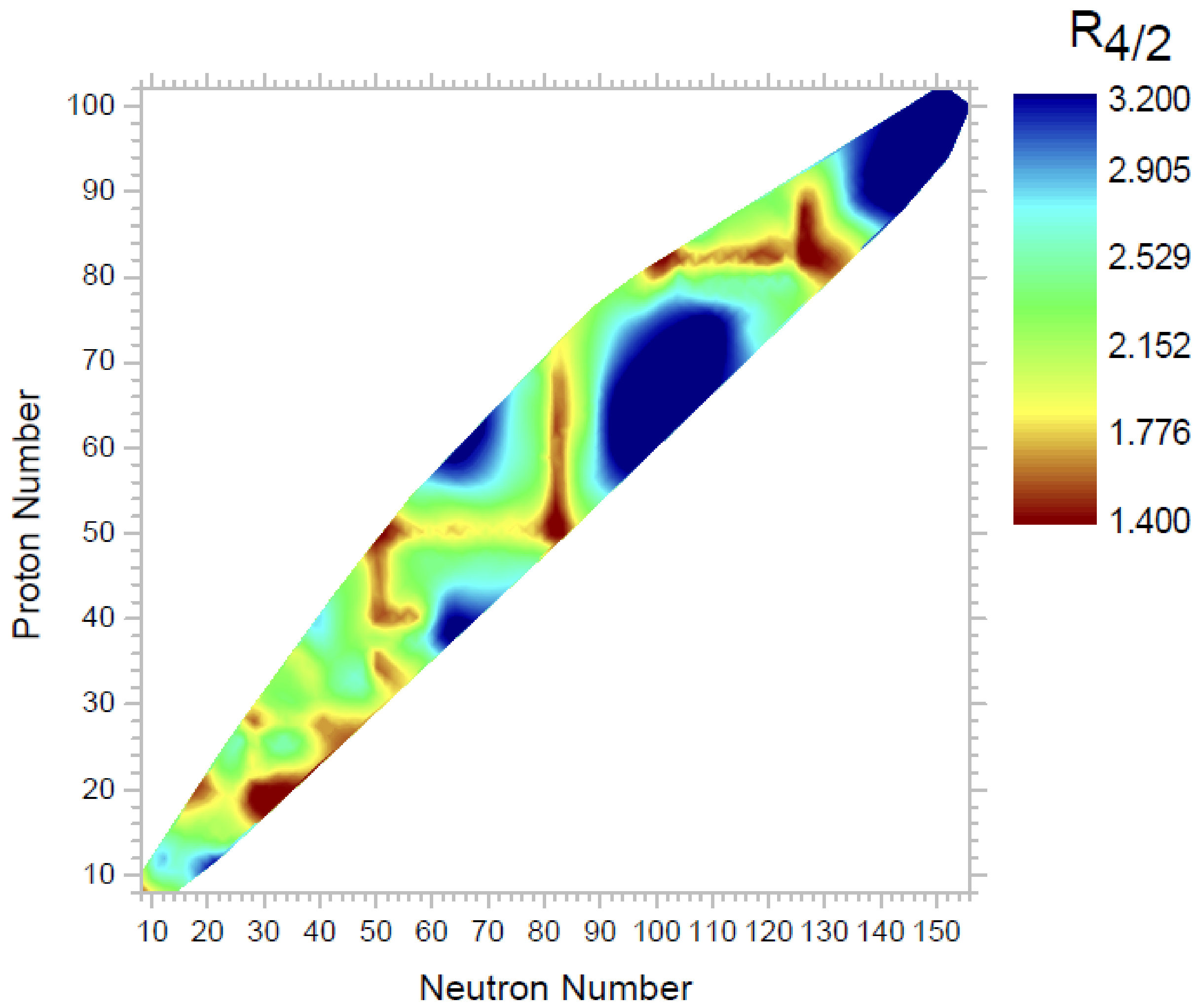
# $2^+$ Systematics

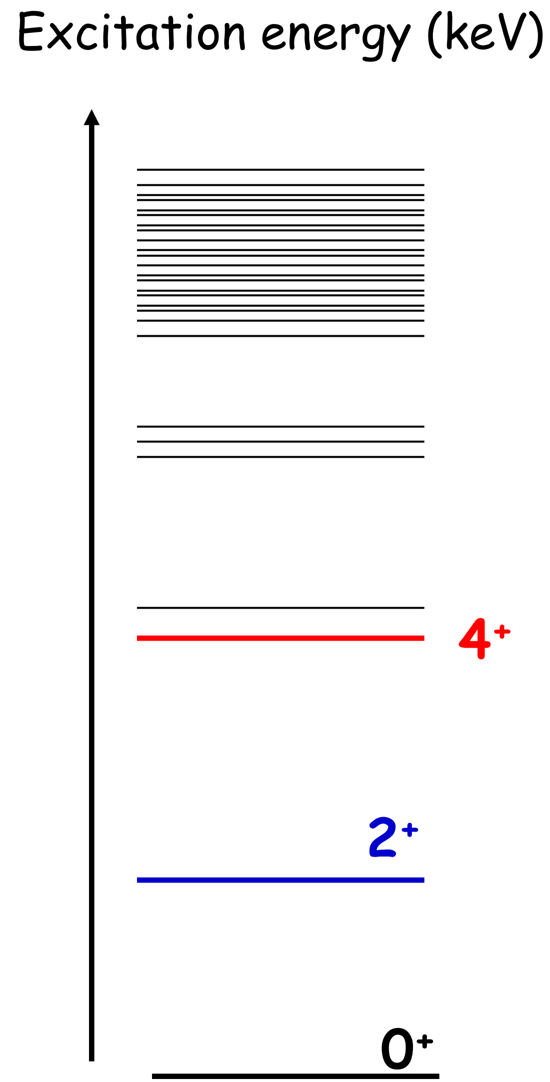



Ground state Configuration.
Spin/parity $I^\pi = 0^+$ ; $E_x$ = 0 keV

# $4^+/2^+$ Energy ratio: mirrors $2^+$ systematics

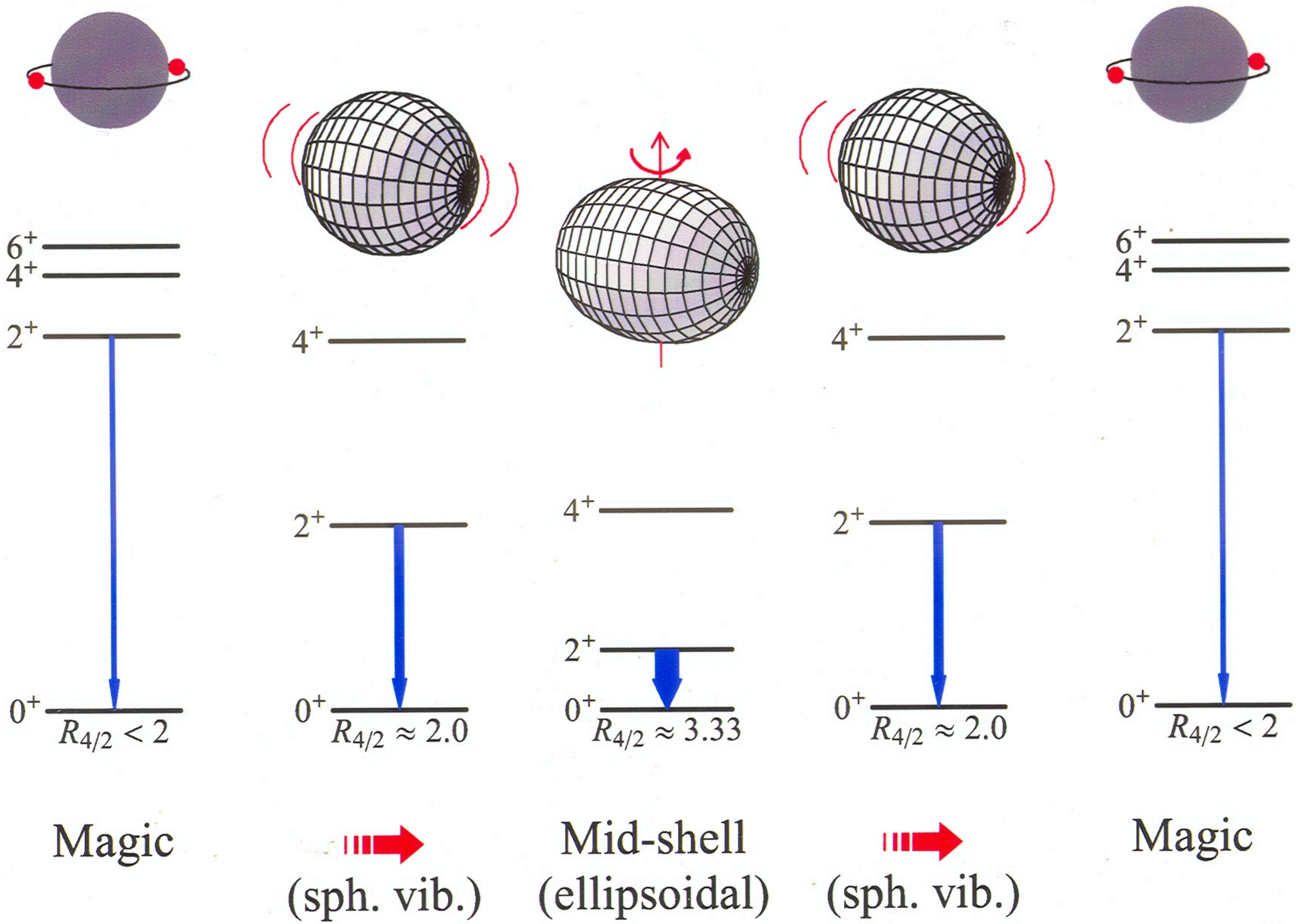



Ground state Configuration.
Spin/parity $I^\pi=0^+$ ; $E_x$ = 0 keV

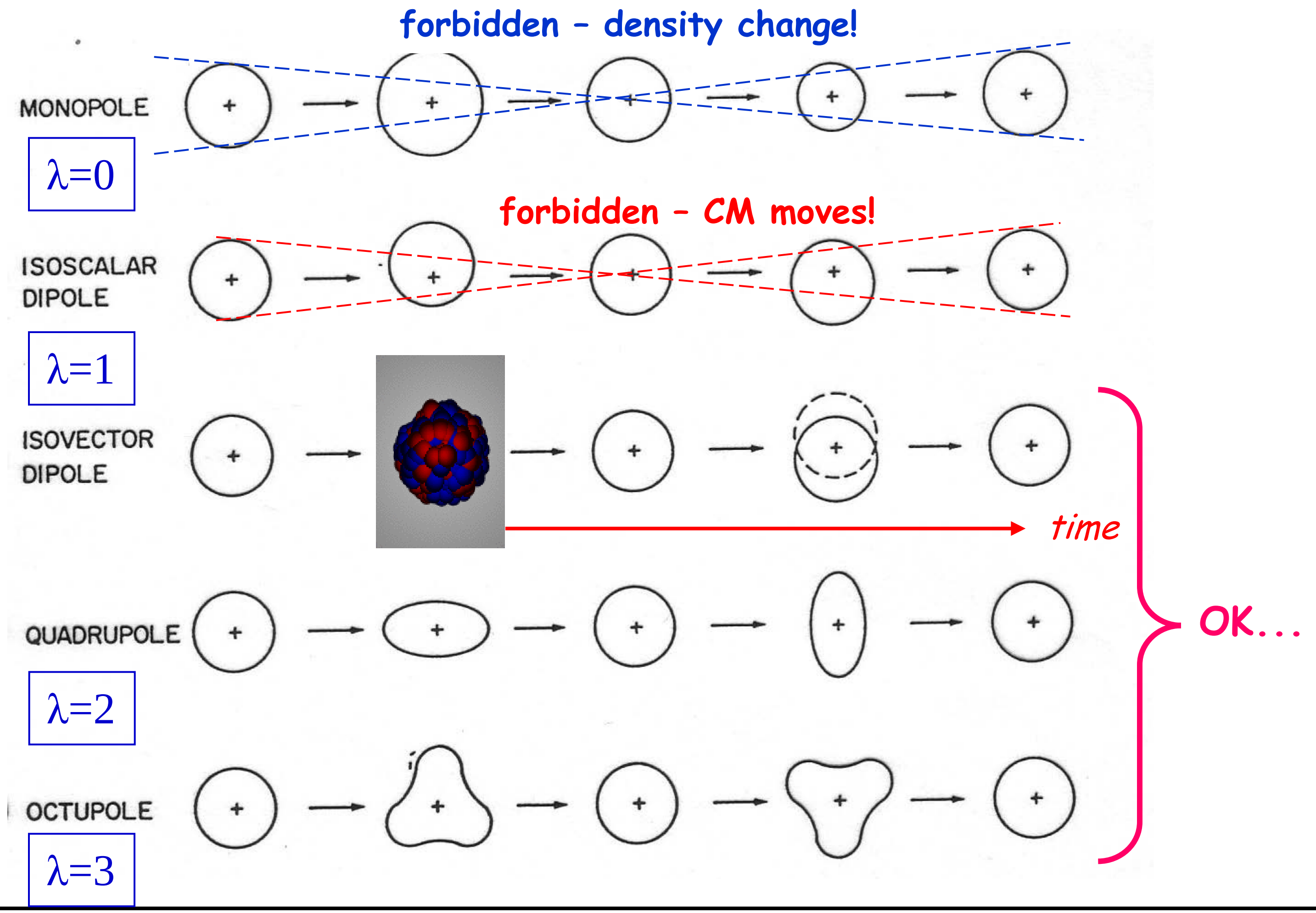
# Evolution of nuclear structure

as a function of nucleon number



# Collective vibration

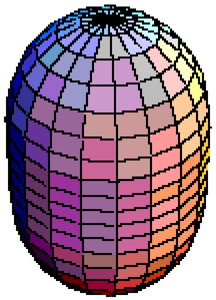
In general, $$R(\theta,\phi) = R_0 \cdot \left[1 + \sum_{\lambda} \sum_{\mu=-\lambda}^{+\lambda} \alpha_{\lambda\mu} \cdot Y^*_{\lambda\mu}(\theta,\phi)\right]$$

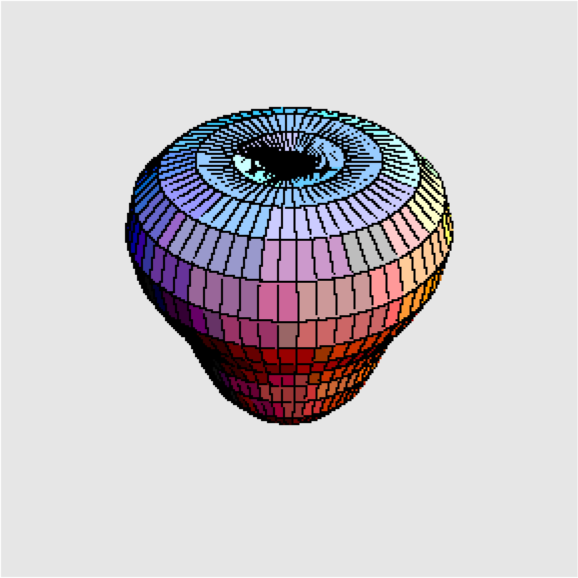


# Collective vibration

In general, $R(\theta,\phi) = R_0 \cdot \left[1 + \sum_{\lambda} \sum_{\mu=-\lambda}^{+\lambda} \alpha_{\lambda\mu} \cdot Y_{\lambda\mu}^{*}(\theta,\phi)\right]$

$V = \frac{1}{2} \sum_{\lambda,\mu} C_{\lambda} \cdot |\alpha_{\lambda\mu}|^2$ harmonic vibration





λ=2: quadrupole vibration

λ=3: octupole vibration

# Collective vibration

classical Hamiltonian

$$E = \frac{1}{2} \cdot \sum_{\lambda\mu} B_\lambda \cdot \left|\dot{\alpha}_{\lambda\mu}\right|^2 + \frac{1}{2} \cdot \sum_{\lambda\mu} C_\lambda \cdot \left|\alpha_{\lambda\mu}\right|^2$$

constants:

$$B_\lambda = \frac{3}{4 \cdot \pi} \frac{M \cdot A \cdot R_0^2}{\lambda}$$

$$C_\lambda = \frac{a_S \cdot A^{2/3}}{4 \cdot \pi} \cdot (\lambda - 1) \cdot (\lambda + 2) \cdot \left\{1 - \frac{6 \cdot (Z \cdot e)^2}{a_S \cdot r_0 \cdot A} \cdot \frac{1}{(2 \cdot \lambda + 1) \cdot (\lambda + 2)}\right\}$$

Binding energy of a nucleus:

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(N-Z)^2}{A} \pm \delta + shell\ corr.$$

$a_V$ = 15.560 MeV $a_S$ = 17.230MeV $a_C$ = 0.6970 MeV $a_A$ = 23.385 MeV $a_P$ = 12.000 MeV

# Collective vibration

classical Hamiltonian

$$E = \frac{1}{2} \cdot \sum_{\lambda\mu} B_\lambda \cdot \left|\dot{\alpha}_{\lambda\mu}\right|^2 + \frac{1}{2} \cdot \sum_{\lambda\mu} C_\lambda \cdot \left|\alpha_{\lambda\mu}\right|^2$$

$$H = \frac{1}{2} \cdot \sum_{\lambda\mu} \frac{1}{B_\lambda} \cdot \left|\pi_{\alpha_{\lambda\mu}}\right|^2 + C_\lambda \cdot \left|\alpha_{\lambda\mu}\right|^2$$

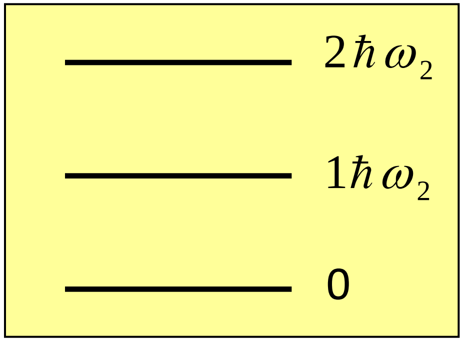
quantization

$$\left[\pi_{\alpha_{\lambda\mu}}, \alpha_{\lambda'\mu'}\right] = -i \cdot \hbar \cdot \delta_{\lambda\lambda'} \cdot \delta_{\mu\mu'}$$

$$H = \frac{1}{2} \cdot \sum_{\lambda\mu} -\frac{\hbar^2}{B_\lambda} \frac{\partial^2}{\partial x_{\lambda\mu}^2} + C_\lambda \cdot x_{\lambda\mu}^2$$

energy eigenvalue

$$E = \sum_{\lambda\mu} \hbar \cdot \omega_\lambda \cdot \left(n_{\lambda\mu} + \frac{1}{2}\right)$$




wave function

$$\psi_n = N_n \cdot H_n(x) \cdot e^{-x^2/2}$$

Hermite polynomials

# Second quantization

Hamilton operator

$$H = \frac{1}{2} \cdot \sum_{\lambda\mu} \frac{1}{B_\lambda} \cdot \left|\pi_{\alpha_{\lambda\mu}}\right|^2 + C_\lambda \cdot \left|\alpha_{\lambda\mu}\right|^2$$

$$H = \sum_{\lambda\mu} \hbar \cdot \omega_\lambda \cdot \left( \beta^+_{\lambda\mu} \, \beta_{\lambda\mu} + \frac{1}{2} \right)$$

$$\alpha_{\lambda\mu} = \sqrt{\frac{\hbar}{2B_\lambda \omega_\lambda}} \cdot \left( \beta^+_{\lambda\mu} + (-)^\mu \beta_{\lambda\mu} \right)$$

$$\pi_{\alpha_{\lambda\mu}} = \sqrt{\frac{\hbar B_\lambda \omega_\lambda}{2}} \cdot \left( (-)^\mu \beta^+_{\lambda-\mu} - \beta_{\lambda\mu} \right)$$

rule for boson operators: $\beta_{\lambda\mu} \cdot \beta^+_{\lambda'\mu'} - \beta^+_{\lambda\mu} \cdot \beta_{\lambda'\mu'} = \delta_{\lambda\lambda'} \cdot \delta_{\mu\mu'}$

# Second quantization

Hamilton operator $$H = \sum_{\lambda\mu} \hbar \cdot \omega_\lambda \cdot \left( \beta^+_{\lambda\mu} \, \beta_{\lambda\mu} + \frac{1}{2} \right)$$

rule for boson operators: $$\beta_{\lambda\mu} \cdot \beta^+_{\lambda'\mu'} - \beta^+_{\lambda\mu} \cdot \beta_{\lambda'\mu'} = \delta_{\lambda\lambda'} \cdot \delta_{\mu\mu'}$$

creation & annihilation operators
increase or decrease the number of phonons in a wave function

ground state $|0>$ (vacuum)

1-phonon state $\beta^+_{\lambda\mu} \, |0> = |1>$

2-phonon state $\sim \beta^+_{\lambda\mu} \, \beta^+_{\lambda'\mu'} \, |0> = |2>$

# Second quantization

Hamilton operator
$$H = \sum_{\lambda\mu} \hbar \cdot \omega_\lambda \cdot \left( \beta^+_{\lambda\mu}\, \beta_{\lambda\mu} + \frac{1}{2} \right)$$

rule for boson operators:
$$\beta_{\lambda\mu} \cdot \beta^+_{\lambda'\mu'} - \beta^+_{\lambda\mu} \cdot \beta_{\lambda'\mu'} = \delta_{\lambda\lambda'} \cdot \delta_{\mu\mu'}$$

1-phonon energy:

$$<1|H|1> = \hbar \cdot \omega \cdot \left( <0|\beta\beta^+\beta\beta^+|0> + \frac{1}{2} <0|\beta\beta^+|0> \right)$$

$$<1|H|1> = \hbar \cdot \omega \cdot \left( <0|\beta\beta^+\overbrace{(1+\beta^+\beta)}|0> + \frac{1}{2} <0|\overbrace{(1+\beta^+\beta)}|0> \right)$$

$$<1|H|1> = \hbar \cdot \omega \cdot \left( <0|\beta\beta^+|0> + \frac{1}{2} <0|1|0> \right)$$

$$<1|H|1> = \hbar \cdot \omega \cdot \left( <0|\overbrace{(1+\beta^+\beta)}|0> + \frac{1}{2} \right)$$

$$<1|H|1> = \hbar \cdot \omega \cdot \left( 1 + \frac{1}{2} \right)$$

# Second quantization

Hamilton operator $$H = \sum_{\lambda\mu} \hbar \cdot \omega_\lambda \cdot \left( \beta^+_{\lambda\mu}\, \beta_{\lambda\mu} + \frac{1}{2} \right)$$

rule for boson operators: $$\beta_{\lambda\mu} \cdot \beta^+_{\lambda'\mu'} - \beta^+_{\lambda\mu} \cdot \beta_{\lambda'\mu'} = \delta_{\lambda\lambda'} \cdot \delta_{\mu\mu'}$$

2-phonon energy:

$$<2|H|2> \sim \hbar \cdot \omega \cdot \left( <0|\beta\beta\beta^+\beta\beta^+\beta^+|0> + \frac{1}{2} <0|\beta\beta\beta^+\beta^+|0> \right)$$

$$<2|H|2> \sim \hbar \cdot \omega \cdot \left( <0|\beta\beta\beta^+\left(1+\beta^+\beta\right)\beta^+|0> + \frac{1}{2} <0|\beta\left(1+\beta^+\beta\right)\beta^+|0> \right)$$

$$<2|H|2> \sim \hbar \cdot \omega \cdot \left( <0|\beta\beta\beta^+\beta^+ + \beta\beta\beta^+\beta^+\beta\beta^+|0> + \frac{1}{2} <0|\beta\beta^+ + \beta\beta^+\beta\beta^+|0> \right)$$

etc.

$$\langle 2|H|2\rangle \sim 2 \cdot \hbar \cdot \omega \cdot \left(2 + \frac{1}{2}\right)$$

normalization of wave function !!!

# Second quantization

Hamilton operator $H = \sum_{\lambda\mu} \hbar \cdot \omega_\lambda \cdot \left( \beta^+_{\lambda\mu}\, \beta_{\lambda\mu} + \frac{1}{2} \right)$

rule for boson operators: $\beta_{\lambda\mu} \cdot \beta^+_{\lambda'\mu'} - \beta^+_{\lambda\mu} \cdot \beta_{\lambda'\mu'} = \delta_{\lambda\lambda'} \cdot \delta_{\mu\mu'}$

2-phonon state: $\Psi_{IM} = N \sum_{m_1 m_2} (\ell_1 \ell_2 m_1 m_2 \mid IM) \cdot \beta^+_{\ell_1 m_1} \cdot \beta^+_{\ell_2 m_2} \mid 0 >$

normalization (approximation):

$$1 = N^2 < 0 \mid \beta\beta\beta^+\beta^+ \mid 0 >$$

$$1 = N^2 < 0 \mid \beta(1 + \beta^+\beta)\beta^+ \mid 0 >$$

$$1 = N^2 < 0 \mid \beta\beta^+ + \beta\beta^+\beta\beta^+ \mid 0 >$$

$$1 = N^2 < 0 \mid (1 + \beta^+\beta) + (1 + \beta^+\beta)(1 + \beta^+\beta) \mid 0 > = N^2 \cdot 2$$

# Second quantization

Hamilton operator $H = \sum_{\lambda\mu} \hbar \cdot \omega_\lambda \cdot \left( \beta^+_{\lambda\mu} \, \beta_{\lambda\mu} + \frac{1}{2} \right)$

rule for boson operators: $\beta_{\lambda\mu} \cdot \beta^+_{\lambda'\mu'} - \beta^+_{\lambda\mu} \cdot \beta_{\lambda'\mu'} = \delta_{\lambda\lambda'} \cdot \delta_{\mu\mu'}$

2-phonon state: $\Psi_{IM} = N \sum_{m_1 m_2} (\ell_1 \ell_2 m_1 m_2 \mid IM) \cdot \beta^+_{\ell_1 m_1} \cdot \beta^+_{\ell_2 m_2} \mid 0 >$

normalization:

$$1 = N^2 \sum_{m_1 m_2 m_1' m_2'} (\ell_1 \ell_2 m_1 m_2 \mid IM) \cdot (\ell_1 \ell_2 m_1' m_2' \mid IM) \cdot < 0 \mid \beta_{\ell_1 m_1} \beta_{\ell_2 m_2} \beta^+_{\ell_1 m_1'} \beta^+_{\ell_2 m_2'} \mid 0 >$$

$$1 = N^2 \sum_{m_1 m_2 m_1' m_2'} (\ell_1 \ell_2 m_1 m_2 \mid IM) \cdot (\ell_1 \ell_2 m_1' m_2' \mid IM) \cdot < 0 \mid \beta_{\ell_1 m_1} \left[ \delta_{\ell_1 \ell_2} \delta_{m_2 m_1'} + \beta_{\ell_1 m_1'} \beta_{\ell_2 m_2} \right] \beta^+_{\ell_2 m_2'} \mid 0 >$$

$$1 = N^2 \sum_{m_1 m_2 m_1' m_2'} (\ell_1 \ell_2 m_1 m_2 \mid IM) \cdot (\ell_1 \ell_2 m_1' m_2' \mid IM) \cdot \left[ \delta_{\ell_1 \ell_2} \delta_{m_2 m_1'} \delta_{m_1 m_2'} + \delta_{m_1 m_1'} \delta_{m_2 m_2'} \right]$$

$$1 = N^2 \sum_{m_1 m_2} (\ell_1 \ell_2 m_1 m_2 \mid IM) \cdot \left\{ (\ell_1 \ell_2 m_1 m_2 \mid IM) + \delta_{\ell_1 \ell_2} (\ell_1 \ell_2 m_2 m_1 \mid IM) \right\}$$

$$1 = N^2 \sum_{m_1 m_2} (\ell_1 \ell_2 m_1 m_2 \mid IM) \cdot (\ell_1 \ell_2 m_1 m_2 \mid IM) \left\{ 1 + (-)^I \delta_{\ell_1 \ell_2} \right\} = N^2 \left\{ 1 + (-)^I \delta_{\ell_1 \ell_2} \right\}$$

# Collective vibration

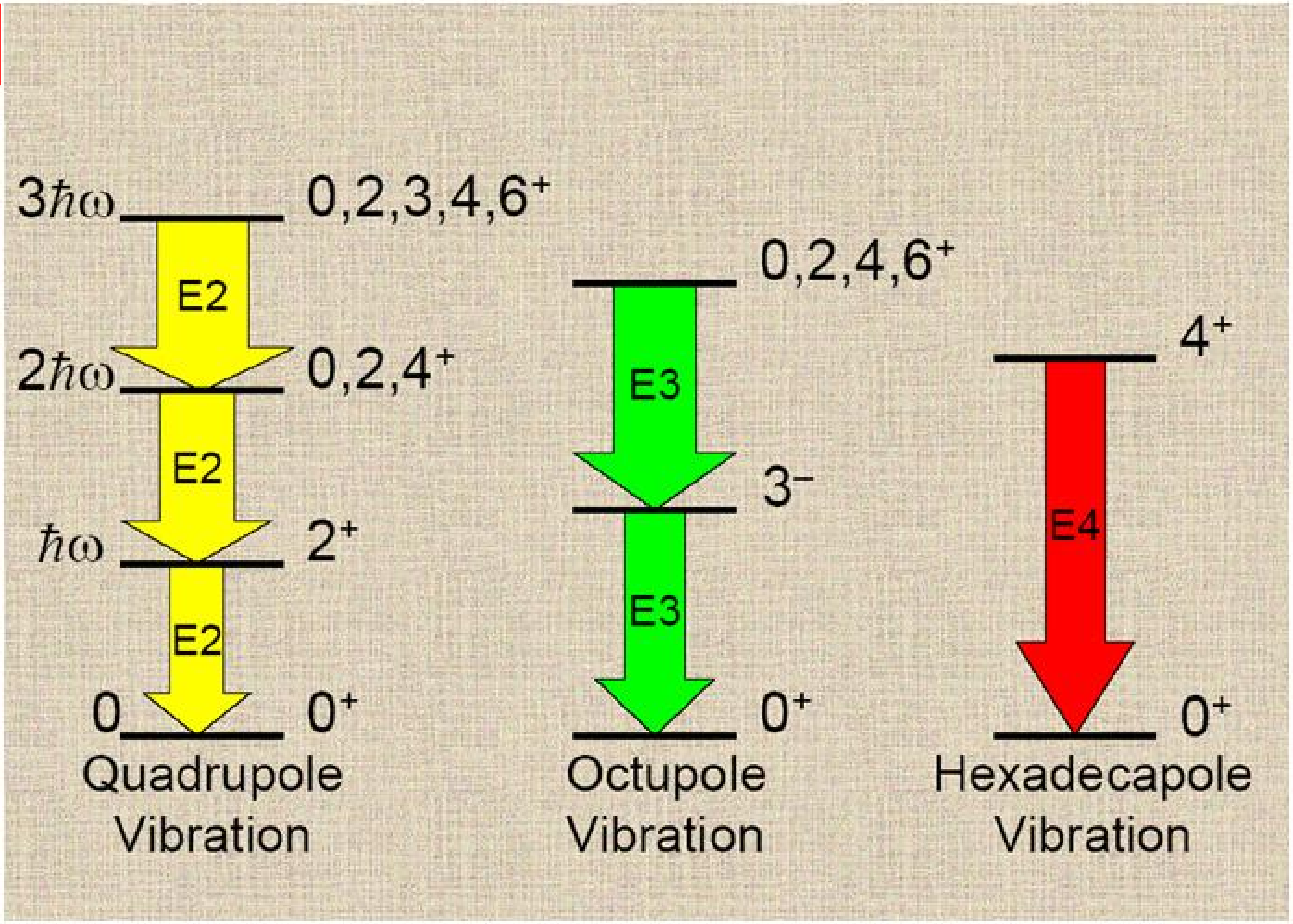
$$E_N = \hbar\omega \cdot \left(N + {}^5\!/_2\right)$$

$$E_3 = \hbar\omega \cdot \left(3 + {}^5\!/_2\right)$$

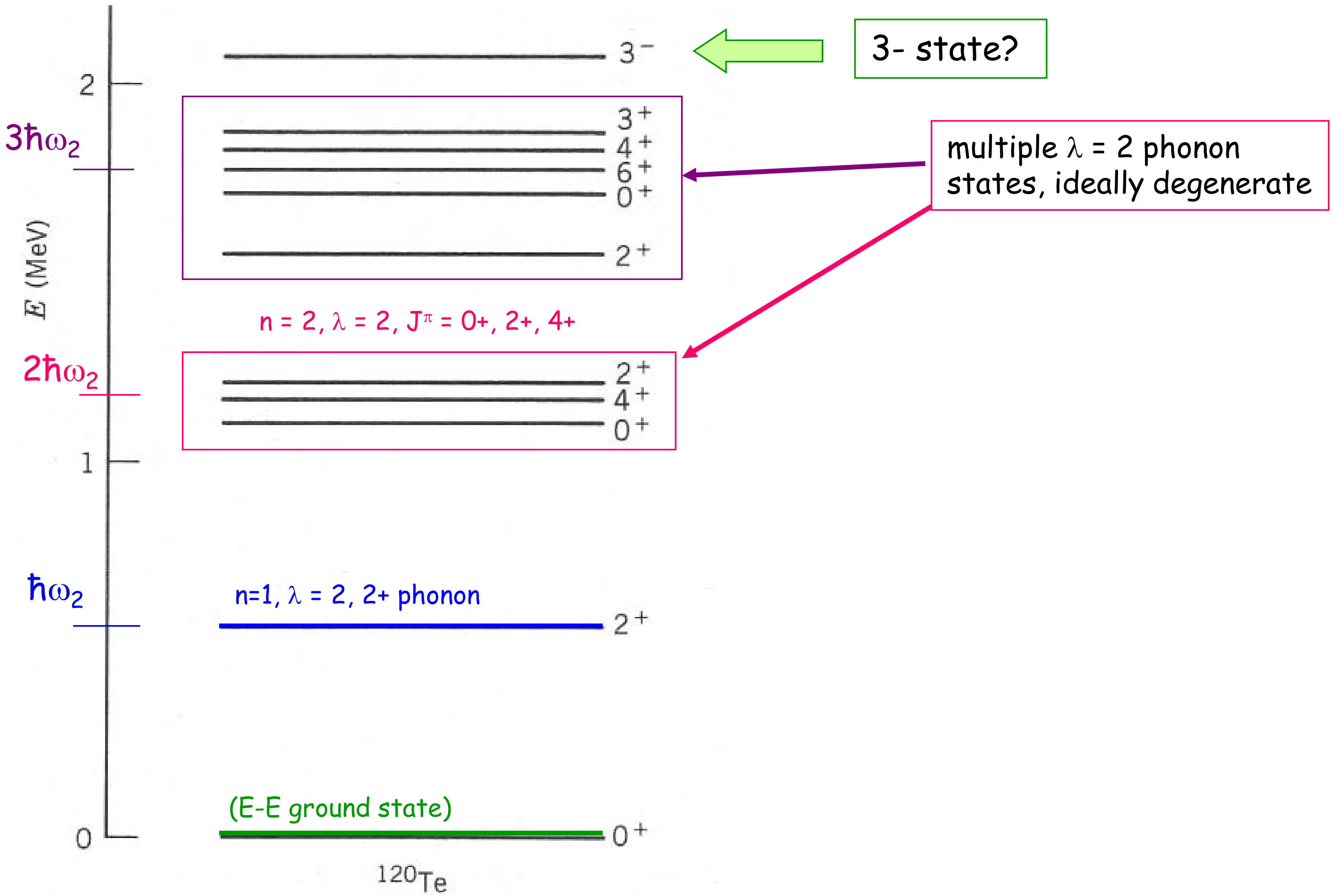
$$E_2 = \hbar\omega \cdot \left(2 + {}^5\!/_2\right)$$

$$E_1 = \hbar\omega \cdot \left(1 + {}^5\!/_2\right)$$

$$E_0 = \hbar\omega \cdot {}^5\!/_2$$



# Example of vibrational excitations



# Second quantization

Hamilton operator $H=\sum_{\lambda\mu}\hbar\cdot\omega_\lambda\cdot\left(\beta^+_{\lambda\mu}\,\beta_{\lambda\mu}+\frac{1}{2}\right)$

rule for boson operators: $\beta_{\lambda\mu}\cdot\beta^+_{\lambda'\mu'}-\beta^+_{\lambda\mu}\cdot\beta_{\lambda'\mu'}=\delta_{\lambda\lambda'}\cdot\delta_{\mu\mu'}$

2-phonon state: $\Psi_{IM}=N\sum_{m_1m_2}\left(\ell_1\ell_2m_1m_2\,|\,IM\right)\cdot\beta^+_{\ell_1m_1}\cdot\beta^+_{\ell_2m_2}\,|0>$

reduced transition probability: $B\left(E\ell;i\to f\right)=\sum_{M_f m}\left|\left\langle f\left|M\left(\ell,m\right)\right|i\right\rangle\right|^2$

$$M\left(\ell,m\right)=\int\rho_p\cdot r^\ell\cdot Y^*_{\ell m}\left(\theta,\phi\right)d\tau\cong\frac{3\cdot Z}{4\cdot\pi\cdot R_0^3}\cdot R_0^{\ell+3}\cdot\alpha^*_{\ell m}$$

$$B\left(E2;2\to 0\right)=\left[\frac{3\cdot Z\cdot e\cdot R_0^2}{4\cdot\pi}\right]^2\cdot\sum_{M_f m}\left|\left\langle 0\left|\left\{\sqrt{\frac{\hbar}{2\cdot B_2\cdot\omega_2}}\cdot(-1)^m\cdot\left[\beta^+_{2-m}+(-1)^m\beta_{2m}\right]\right\}\beta^+_{2M_i}\right|0\right\rangle\right|^2$$

$$B\left(E2;2\to 0\right)=\left[\frac{3\cdot Z\cdot e\cdot R_0^2}{4\cdot\pi}\right]^2\cdot\frac{\hbar}{2\cdot B_2\cdot\omega_2}\qquad\sim\frac{1}{E_{2^+}}$$

# Reduced transition probabilities

2-phonon state $$B(E2; n_2 = 2 \rightarrow n_2 = 1) = 2 \cdot B(E2; n_2 = 1 \rightarrow n_2 = 0)$$

3-phonon state $$B(E2; n_2 = 3 \rightarrow n_2 = 2) = 3 \cdot B(E2; n_2 = 1 \rightarrow n_2 = 0)$$

$$B(E2; I_i \rightarrow I_f) = \frac{1}{2 \cdot I_i + 1} \left| \left\langle I_f \| M(E2) \| I_i \right\rangle \right|^2$$

$$Q_{vib} = \frac{3 \cdot Z \cdot R_0^2}{4 \cdot \pi} \cdot \sqrt{\frac{\hbar}{2 \cdot B_2 \cdot \omega_2}}$$

# Reduced transition probabilities

1-phonon state

$$\langle I=2, n_2=1 \| M(E2) \| I=0, n_2=0 \rangle = \sqrt{5} \cdot Q_{vib} \cdot e$$

2-phonon state

$$\langle I=4, n_2=2 \| M(E2) \| I=2, n_2=1 \rangle = \sqrt{18} \cdot Q_{vib} \cdot e$$
$$\langle I=2, n_2=2 \| M(E2) \| I=2, n_2=1 \rangle = \sqrt{10} \cdot Q_{vib} \cdot e$$
$$\langle I=0, n_2=2 \| M(E2) \| I=2, n_2=1 \rangle = \sqrt{2} \cdot Q_{vib} \cdot e$$

3-phonon state

$$\langle I=6, n_2=3 \| M(E2) \| I=4, n_2=2 \rangle = \sqrt{39} \cdot Q_{vib} \cdot e$$
$$\langle I=4, n_2=3 \| M(E2) \| I=4, n_2=2 \rangle = \sqrt{\frac{90}{7}} \cdot Q_{vib} \cdot e$$
$$\langle I=4, n_2=3 \| M(E2) \| I=2, n_2=2 \rangle = \sqrt{\frac{99}{7}} \cdot Q_{vib} \cdot e$$
$$\langle I=3, n_2=3 \| M(E2) \| I=4, n_2=2 \rangle = \sqrt{6} \cdot Q_{vib} \cdot e$$
$$\langle I=3, n_2=3 \| M(E2) \| I=2, n_2=2 \rangle = -\sqrt{15} \cdot Q_{vib} \cdot e$$
$$\langle I=2, n_2=3 \| M(E2) \| I=2, n_2=2 \rangle = \sqrt{\frac{20}{7}} \cdot Q_{vib} \cdot e$$
$$\langle I=2, n_2=3 \| M(E2) \| I=0, n_2=2 \rangle = \sqrt{7} \cdot Q_{vib} \cdot e$$
$$\langle I=0, n_2=3 \| M(E2) \| I=0, n_2=2 \rangle = \sqrt{3} \cdot Q_{vib} \cdot e$$

$$B(E2; I_i \to I_f) = \frac{1}{2 \cdot I_i + 1} \left| \langle I_f \| M(E2) \| I_i \rangle \right|^2$$

$$Q_{vib} = \frac{3 \cdot Z \cdot R_0^2}{4 \cdot \pi} \cdot \sqrt{\frac{\hbar}{2 \cdot B_2 \cdot \omega_2}}$$