

Hot fusion (1961 – 1974)

successful up to element 106 (Seaborgium)

- **Coulomb barrier V_C** between projectile and target nucleus has to be exceeded

$$V_C = \frac{Z_p \cdot Z_t \cdot e^2}{R_{int}} = 126.2 \text{ MeV} \quad ({}^{26}\text{Mg} + {}^{248}\text{Cm})$$

- reaction: $a + A \rightarrow C^* \rightarrow B + b$

$$\Delta m = m_a + m_A - m_{CN}$$

$$\Delta m = (25.983 + 248.072 - 274.143) \cdot 931.478 \text{ MeV}/c^2 = -82.153 \text{ MeV}/c^2$$



<http://nuclear.lu.se/database/masses/>

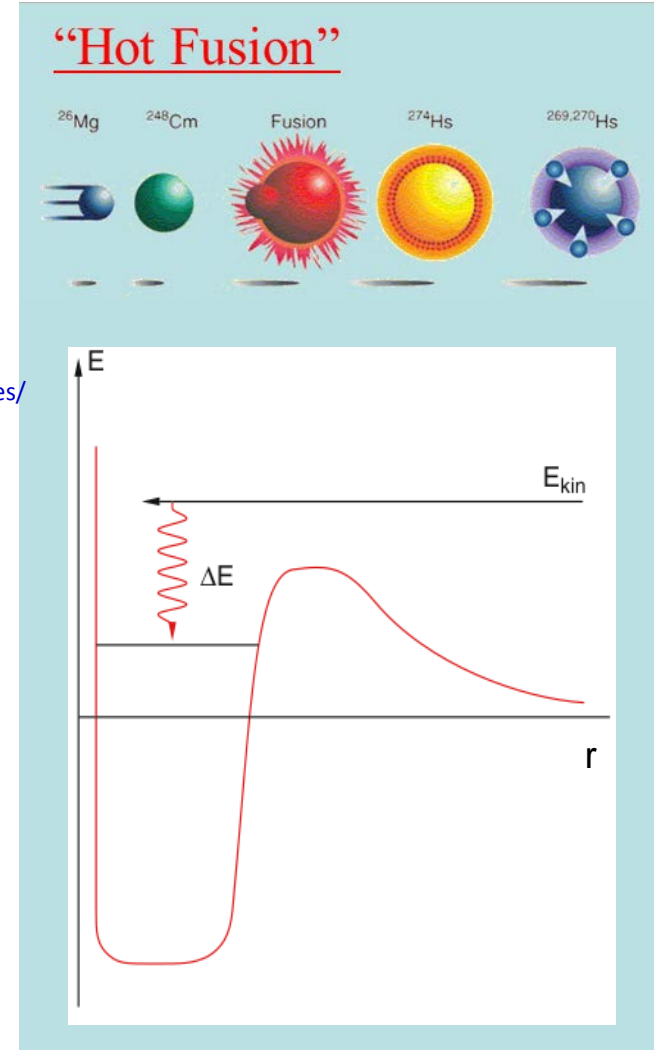
- excitation energy of compound nucleus

$$E^* = E_{kin} + \Delta m \cdot c^2$$

$$= 126.2 \text{ MeV} - 82.2 \text{ MeV}$$

$$= \mathbf{44.0 \text{ MeV}}$$

- approximate 4 neutrons will be evaporated to avoid fission



Cold fusion (1981 – 1996)

- **Coulomb barrier V_C** between projectile and target nucleus has to be exceeded

$$V_C = \frac{Z_p \cdot Z_t \cdot e^2}{R_{int}} = 223.3 \text{ MeV} \quad ({}^{58}\text{Fe} + {}^{208}\text{Pb})$$

- reaction: $a + A \rightarrow C^* \rightarrow B + b$

$$\Delta m = m_a + m_A - m_{CN}$$

$$\begin{aligned} \Delta m &= (57.933 + 207.977 - 266.130) * 931.478 \text{ MeV}/c^2 \\ &= -205.092 \text{ MeV}/c^2 \end{aligned}$$

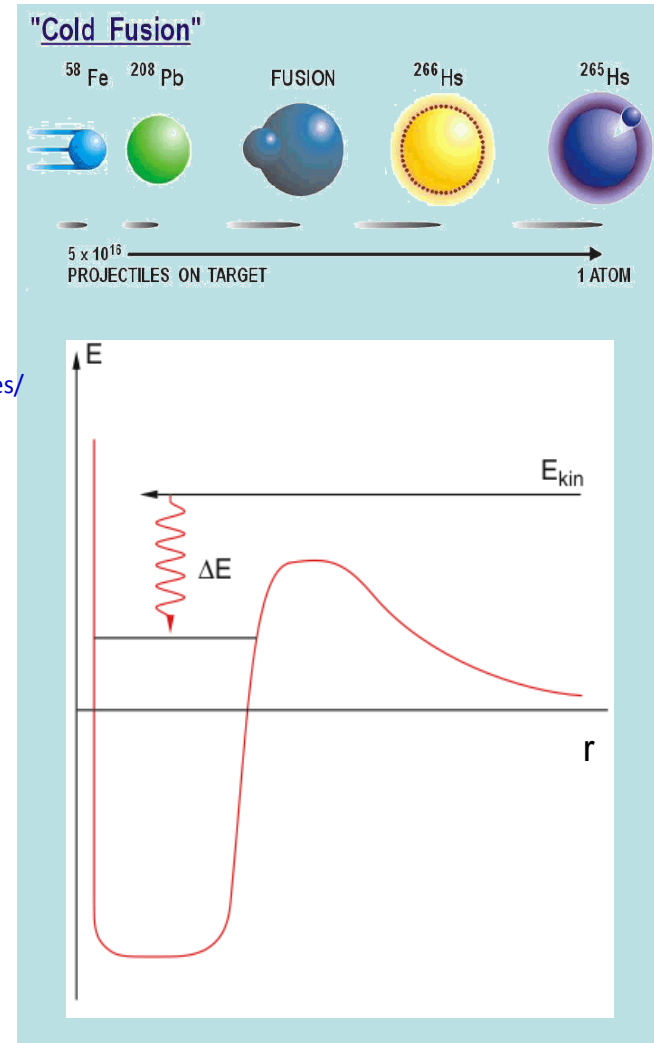


<http://nuclear.lu.se/database/masses/>

- excitation energy of compound nucleus

$$\begin{aligned} E^* &= E_{kin} + \Delta m \cdot c^2 \\ &= 223.3 \text{ MeV} - 205.1 \text{ MeV} \\ &= \mathbf{18.2 \text{ MeV}} \end{aligned}$$

- approximate 1-2 neutrons will be evaporated to avoid fission

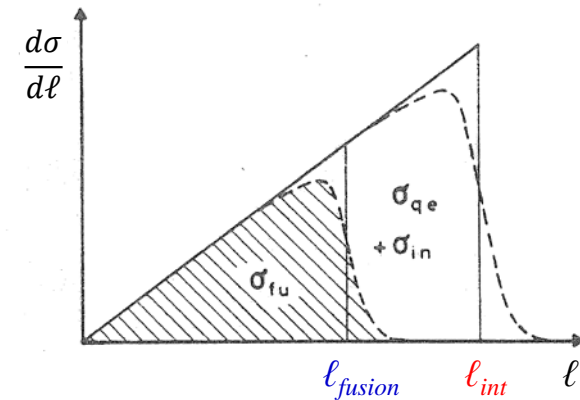


Fusion cross section

Radius for fusion barrier:

$$R_{fusion} = R_{int} - \begin{cases} 0.3117 \cdot (Z_p \cdot Z_t)^{0.2122} & Z_p \cdot Z_t < 500 \\ 1.096 + 1.391 \cdot Z_p \cdot Z_t / 1000 & Z_p \cdot Z_t \geq 500 \end{cases} \text{ [fm]}$$

	R_i [fm]	C_i [fm]	R_{int} [fm]	$V_C(R_{int})$ [MeV]	R_{fusion} [fm]	$V_C(R_{fusion})$ [MeV]
^{58}Fe	4.40	4.17	13.75	223.3	12.36	248.4
^{208}Pb	6.96	6.82				



Total cross section for fusion:

$$\sigma_{fusion} = \pi R_{fusion}^2 \cdot \left[1 - \frac{V_C(R_{fusion})}{E_{cm}} \right]$$

$$\text{with } E_{cm} = \frac{A_t}{A_t + A_p} \cdot E_{lab}$$

$$\sigma_{fusion} = \frac{\pi}{k_\infty^2} \cdot \ell_{fusion} \cdot (\ell_{fusion} + 1)$$

$$\text{with } k_\infty = 0.2187 \cdot \frac{A_t}{A_t + A_p} \cdot \sqrt{A_p \cdot E_{lab}} \text{ [fm}^{-1}\text{]}$$

Interaction potential

The potential between projectile and target nucleus is given by a function of the relative distance between them

$$V(r) = V_N(r) + V_C(r)$$

nuclear potential + Coulomb potential

$$V_C(r) = \begin{cases} \frac{Z_1 Z_2 e^2}{2 \cdot R_C} \left(3 - \frac{r^2}{R_C^2} \right) & r < R_C \\ \frac{Z_1 Z_2 e^2}{r} & r \geq R_C \end{cases}$$

$$V_N(r) = 4\pi \cdot \gamma \cdot \frac{C_p \cdot C_t}{C_p + C_t} \cdot b \cdot \Phi(\xi)$$

$$\Phi(\xi) = \begin{cases} -0.5 \cdot (\xi - 2.54)^2 - 0.0852 \cdot (\xi - 2.54)^3 & \xi \leq 1.2511 \\ -3.437 \cdot \exp(-\xi/0.75) & \xi \geq 1.2511 \end{cases}$$

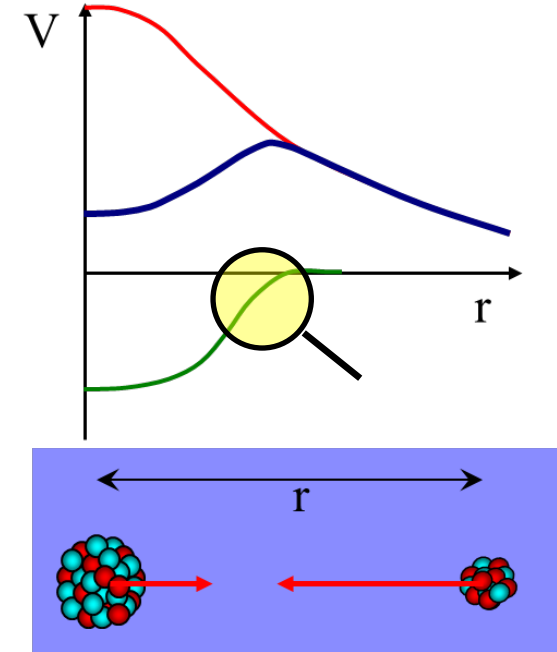
$$\xi = (r - C_p - C_t)/b$$

$$b = \frac{\pi}{\sqrt{3}} \cdot a \cong 1 \text{ fm} \quad \text{with } a = 0.55 \text{ fm}$$

$$\gamma = 0.9517 \cdot \left\{ 1 - 1.7826 \cdot \left(\frac{N_c - Z_c}{A_c} \right)^2 \right\} \frac{\text{MeV}}{\text{fm}^2}$$

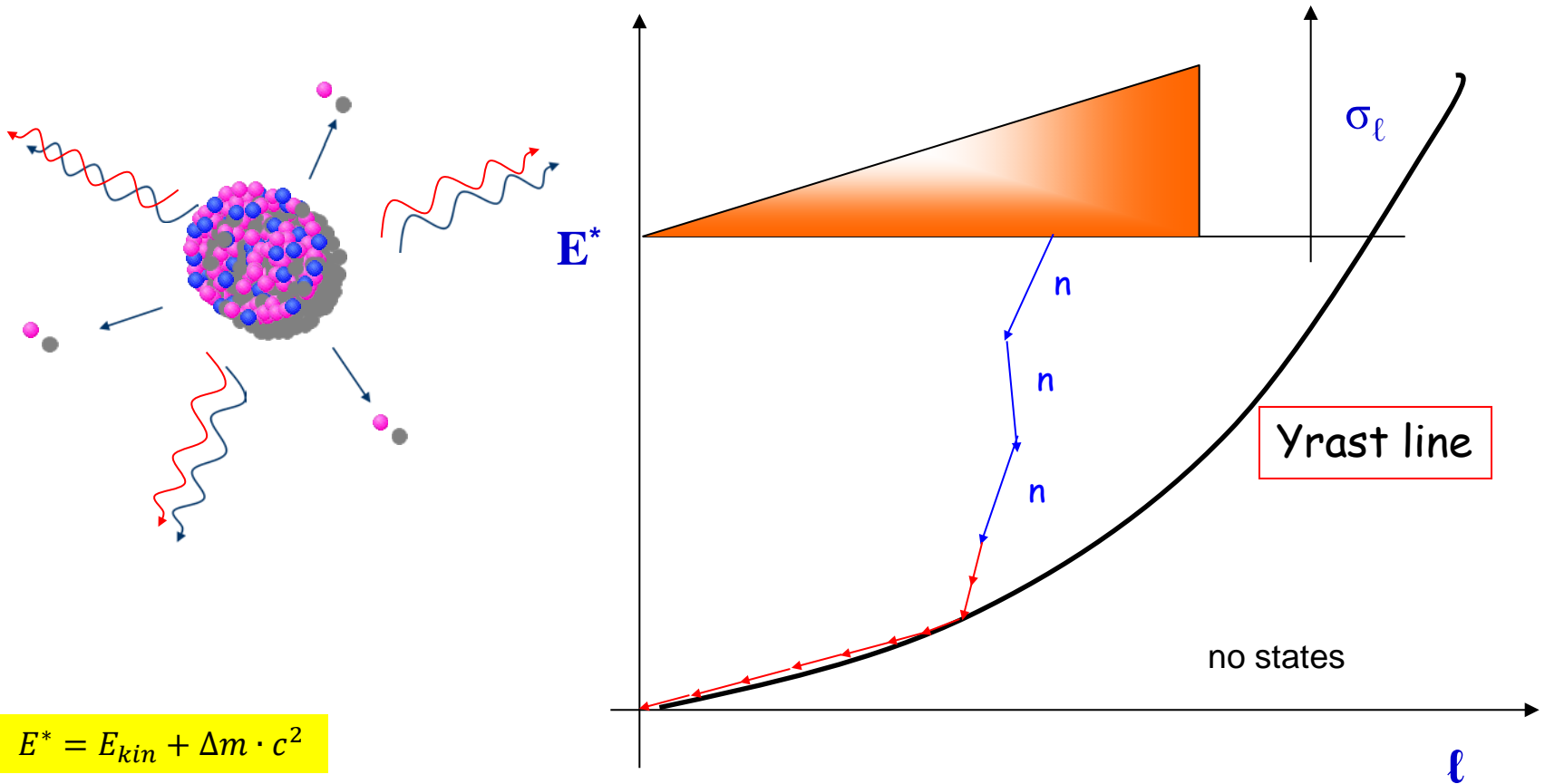
$$C_i = R_i \cdot (1 - R_i^{-2}) \quad [\text{fm}]$$

$$R_i = 1.28 \cdot A_i^{1/3} - 0.76 + 0.8 \cdot A_i^{-1/3} \quad [\text{fm}]$$



The Statistical Model

de-excitation of the hot compound system



$$E^* = E_{kin} + \Delta m \cdot c^2$$

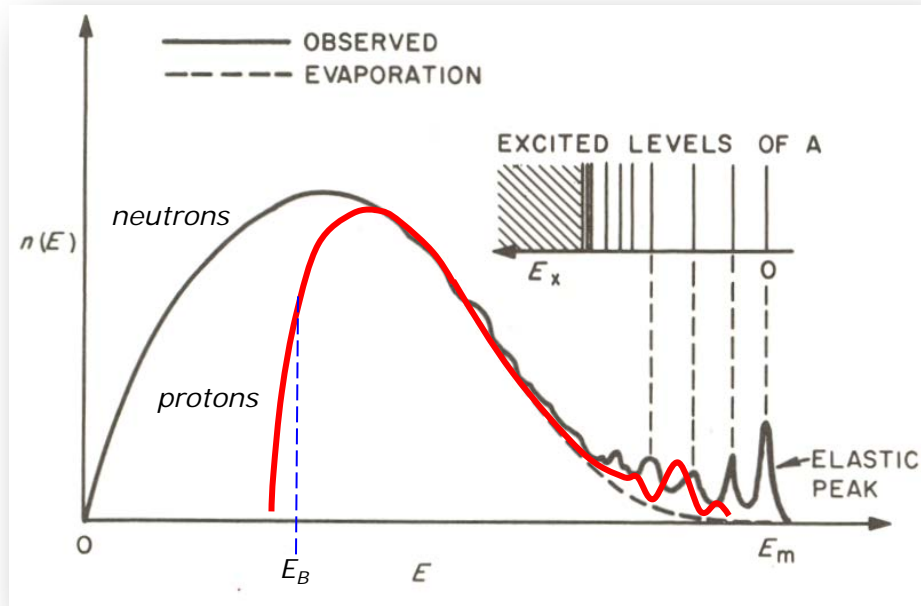
$$E_{kin} > V_C = \frac{Z_a \cdot Z_A \cdot e^2}{R_{int}}$$

$$\Delta m = m_a + m_A - m_{CN}$$



<http://nuclear.lu.se/database/masses/>

Evaporation particles



cm-spectra of particles statistically emitted from CN (evaporation) are of Maxwell Boltzmann type

$$\frac{dN}{dE} \propto (E - E_B) \cdot e^{-E/T}$$

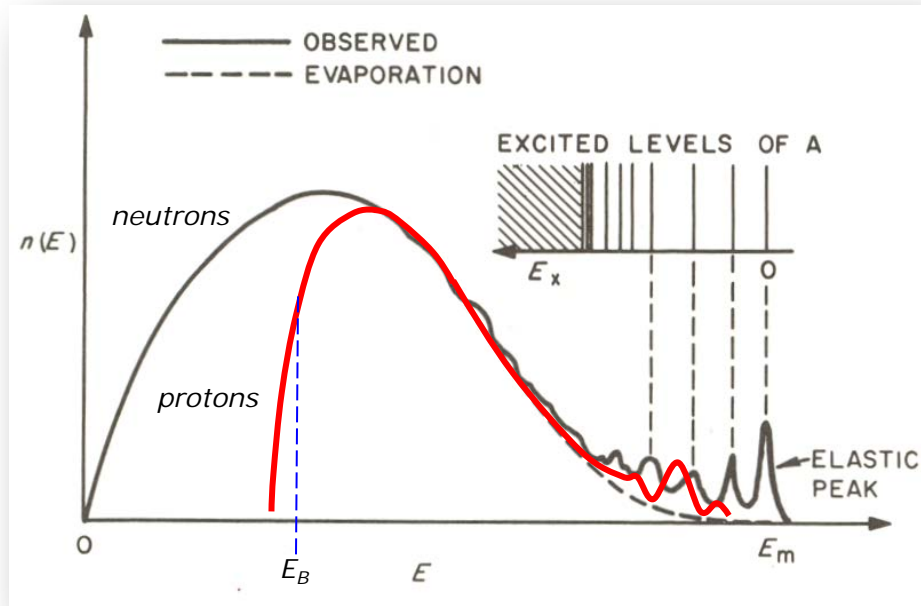
E_B = Coulomb barrier
 T = effective nuclear temperature

compound nucleus reactions

direct reactions

Typical energy spectrum of nucleons emitted at a fixed angle in inelastic nucleon-nucleon reactions.

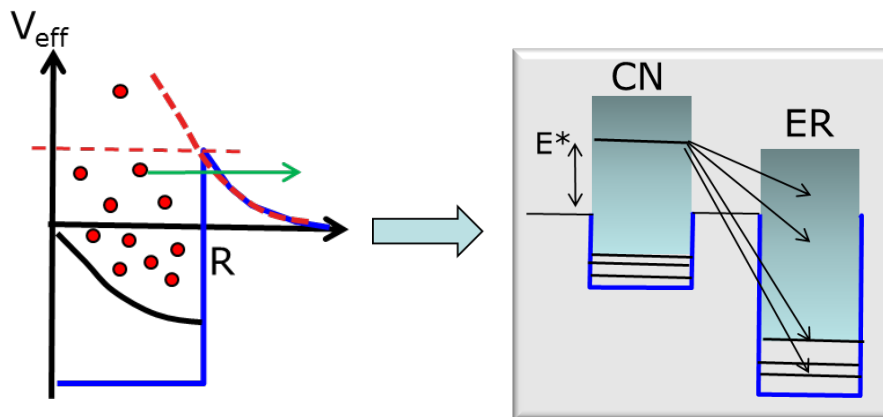
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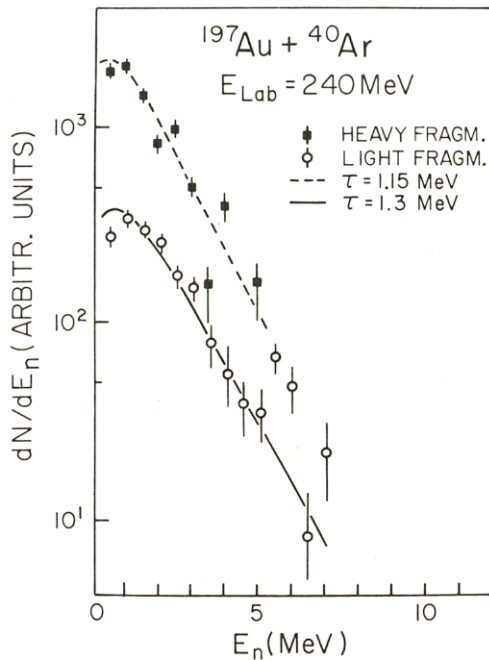
E_B = Coulomb barrier
 T = effective nuclear temperature



Even for fixed E^* the particle spectrum is continuous (Maxwell Boltzmann), except for transitions to discrete spectrum at low E_{ER}^*

Nuclear temperatures

de-excitation of the hot compound system



$$\frac{dN}{dE_n} \propto E_n \cdot e^{-E_n/T} \quad \text{spectrum of single neutron}$$

$$\langle E_n \rangle = 2T \quad \max \frac{dN}{dE_n} @ E_n = T$$

$$\frac{dN}{dE_n} \propto \sqrt{E_n} \cdot e^{-E_n/T_{eff}} \quad \text{spectrum of cascade of neutrons}$$

$$\langle E_n \rangle = 1.5T \quad T_{eff} \approx 0.92 \cdot T \quad (1^{st} \text{ daughter})$$

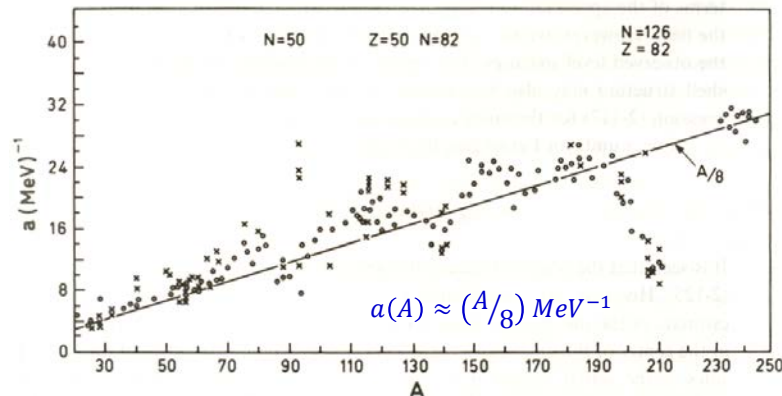
Fermi gas relations:

$$E^* = a \cdot T^2 \quad \text{"little - a"}$$

$$S = \int \frac{dE^*}{E^*} = 2\sqrt{a \cdot E^*}$$

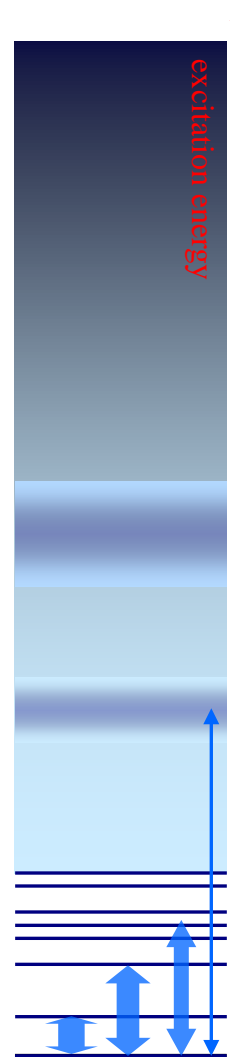
$$\rho(E^*) = \rho_0 \cdot e^{2\sqrt{a \cdot E^*}}$$

deviations at shell closures

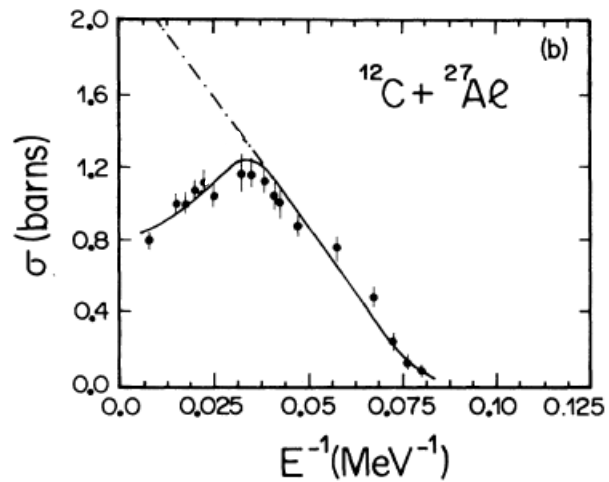
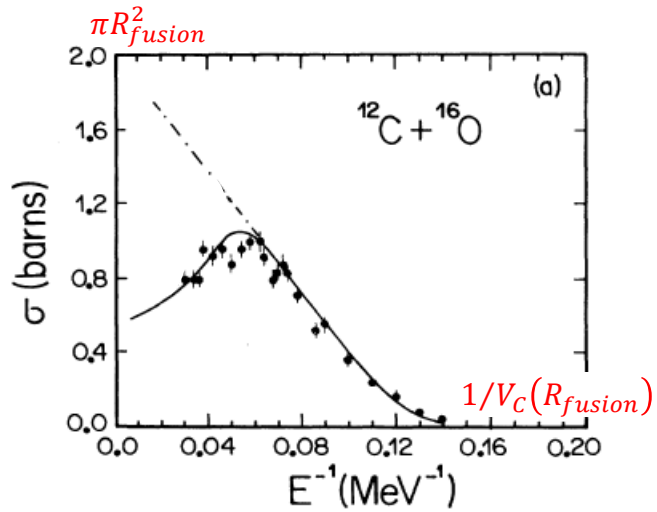


collective states

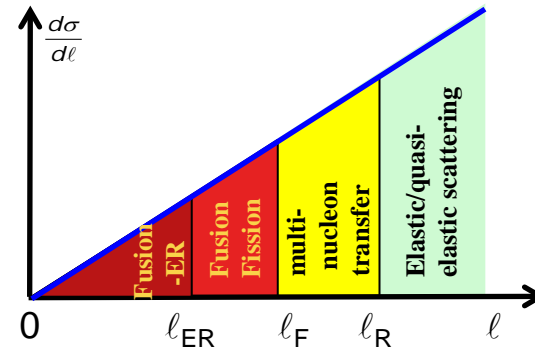
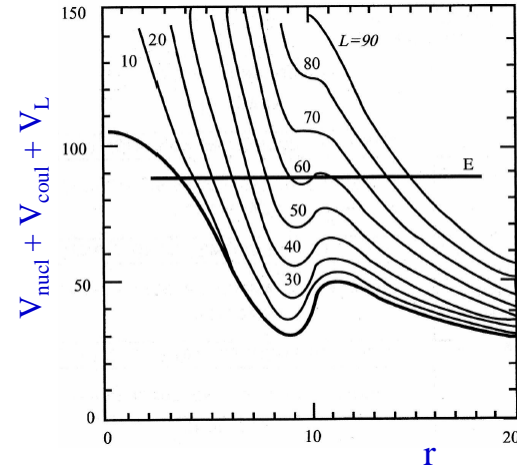
ground state



Fusion excitation function



maximum ℓ_{fusion} due to nuclear centrifugal stability



A limiting nuclear angular momentum

rotating charged liquid drop

surface energy:

$$E_S^{(0)} = 17.9439 \cdot \left[1 - 1.7826 \cdot \left(\frac{N-Z}{A} \right)^2 \right] \cdot A^{2/3} \quad [\text{MeV}]$$

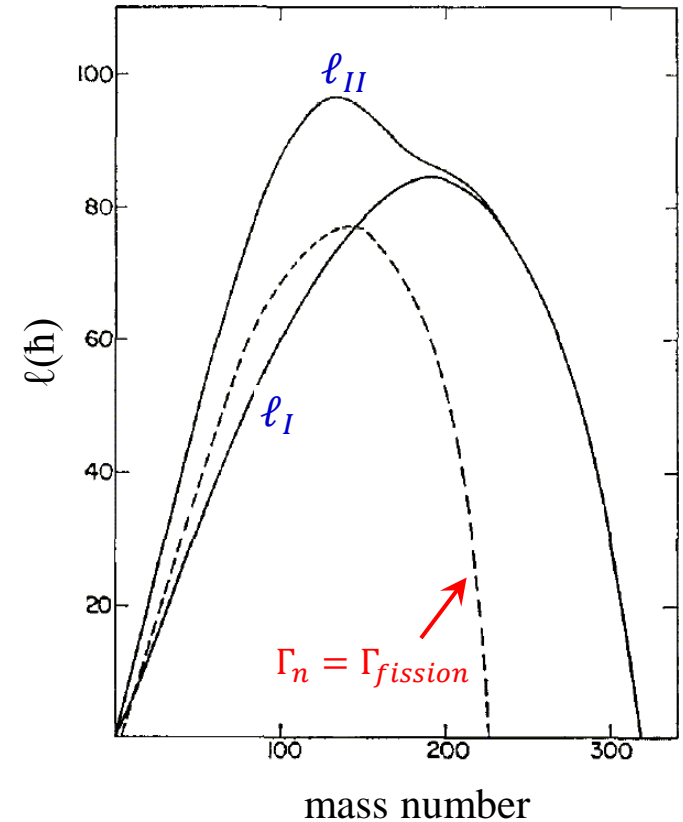
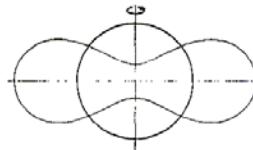
Coulomb energy:

$$E_{Coul}^{(0)} = 0.7053 \cdot (Z^2/A^{1/3}) \quad [\text{MeV}]$$

rotational energy:

$$E_{Rot}^{(0)} = \frac{1}{2} \frac{\hbar^2 \cdot \ell^2}{(2/5) \cdot A \cdot m \cdot R^2} = 34.54 \cdot \frac{\ell^2}{A^{5/3}} \quad [\text{MeV}]$$

change of the nuclear shape

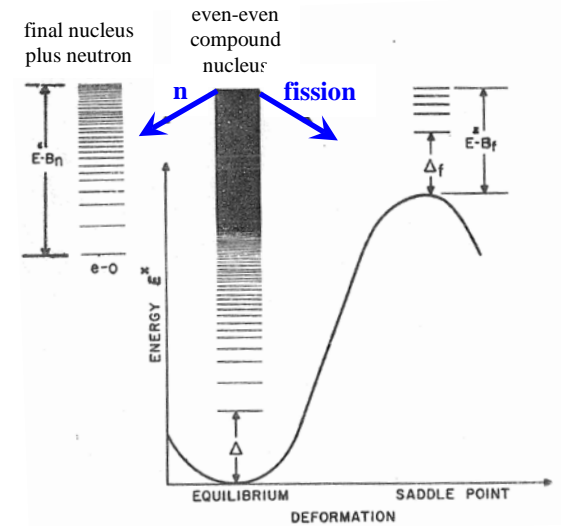
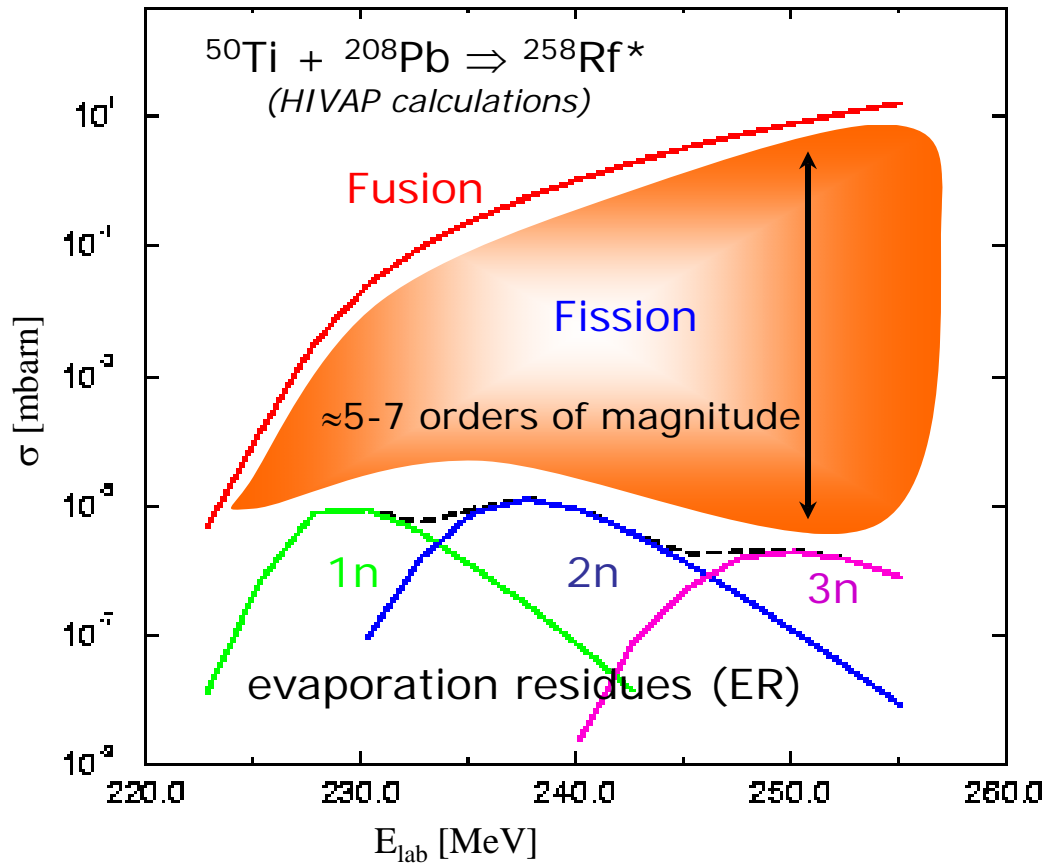


$$\frac{E_{Rot}^{(0)}}{E_S^{(0)}} = \begin{cases} 0.2829 - 0.3475 \cdot X - 0.0016 \cdot X^2 + 0.0501 \cdot X^3 & 0 \leq X \leq 0.75 \\ (7/5) \cdot (1-X)^2 - 4.5660 \cdot (1-X)^3 + 6.7443 \cdot (1-X)^4 & 0.75 \leq X \leq 1.0 \end{cases} \quad \text{with } X = \frac{E_{Coul}^{(0)}}{2 \cdot E_S^{(0)}} \quad \text{“fissility parameter”}$$

example: ${}^{127}_{57}\text{La}$ $E_S^{(0)} = 444.9 \text{ [MeV]}$ $E_{Coul}^{(0)} = 455.9 \text{ [MeV]}$ $X = 0.512$ $E_{Rot}^{(0)}/E_S^{(0)} = 0.1112$ $E_{Rot}^{(0)} = 49.48 \text{ [MeV]}$ $\ell_I = 67.8 \text{ [}\hbar\text{]}$

Fusion and evaporation

cold fusion



Both decay processes are determined by the level density, either from the residual nucleus or at the saddle point.

level density: $\rho(E^*) = \text{const} \cdot \exp(E^*/T)$

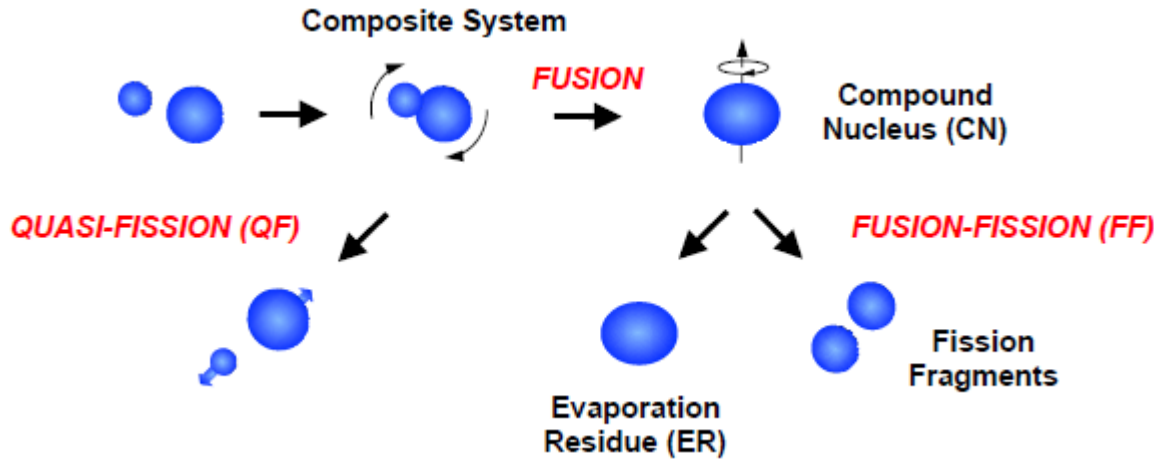
$$\frac{\Gamma_n}{\Gamma_f} = \frac{2 \cdot T \cdot A_{CN}^{2/3}}{K_0} \cdot \exp[(B_f - B_n)/T]$$

$$K_0 = \hbar^2 / 2 \cdot m \cdot r_0^2 \approx 11.4 \text{ MeV}$$

$$T = \sqrt{8 \cdot E^* / A_{CN}}$$

Fusion / Fission competition

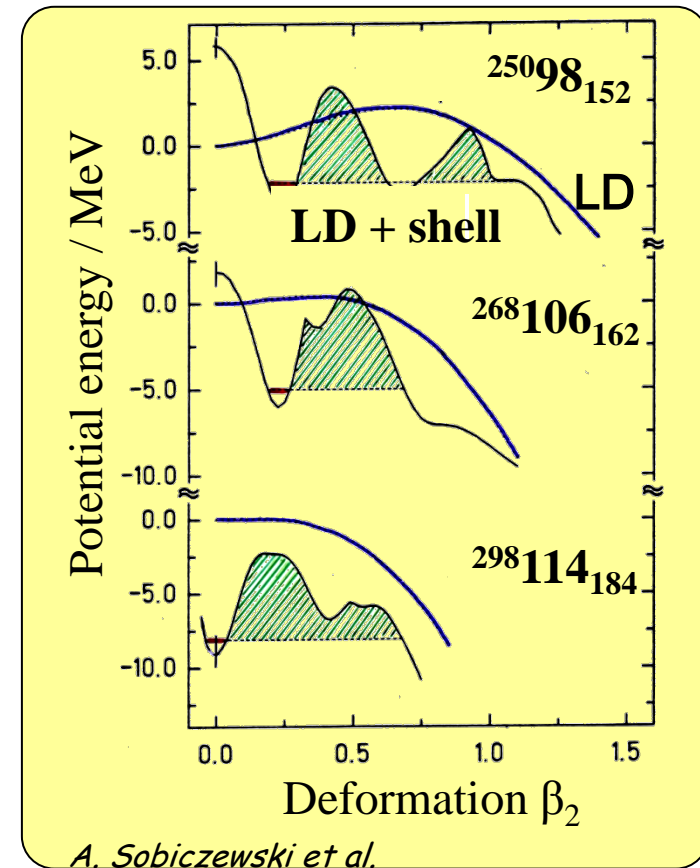
liquid drop + shell corrections



$$\sigma_{ER} = \sigma_{capture} \cdot P_{CN} \cdot P_{survival}$$

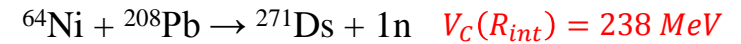
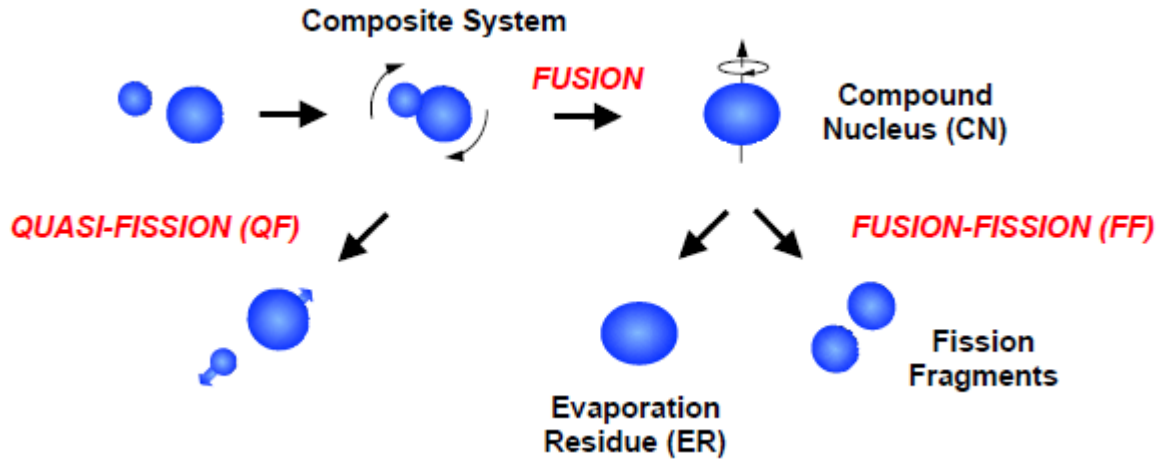
$$\sigma_{ER}^{xn}(E) = \frac{\pi}{k^2} \sum_{\ell=0}^{\ell_{max}} (2\ell + 1) \cdot \underbrace{P_{CN}(E^*, \ell)}_{\text{formation}} \cdot \underbrace{P_{xn}(E^*, \ell)}_{\text{survival}}$$

Superheavy system: $\sigma_{ER} \ll \sigma_{capture}$

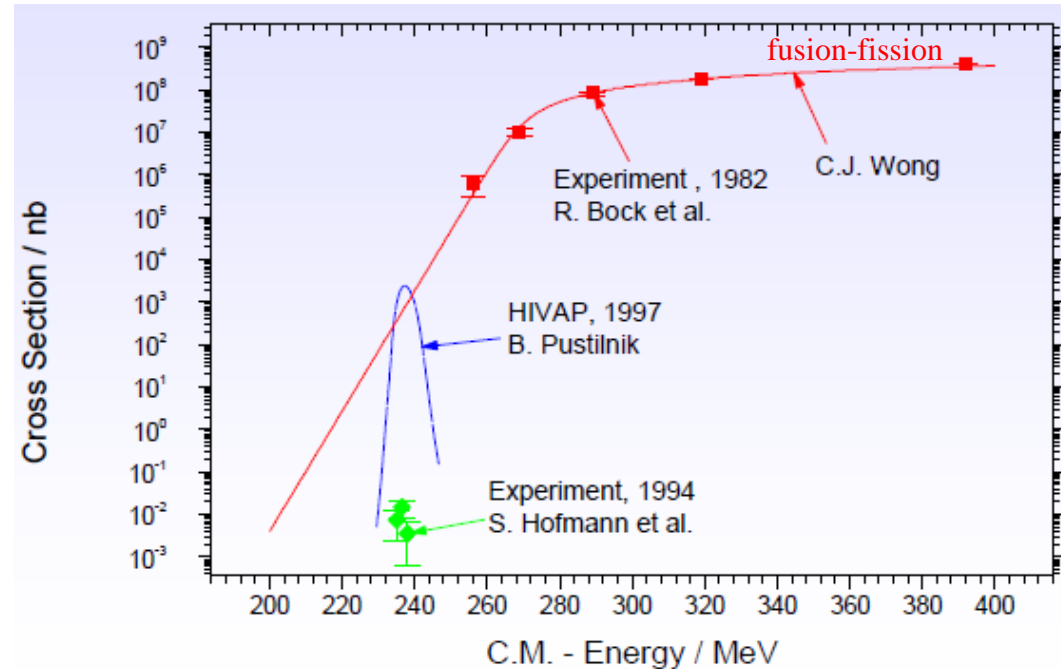


Fusion / Fission competition

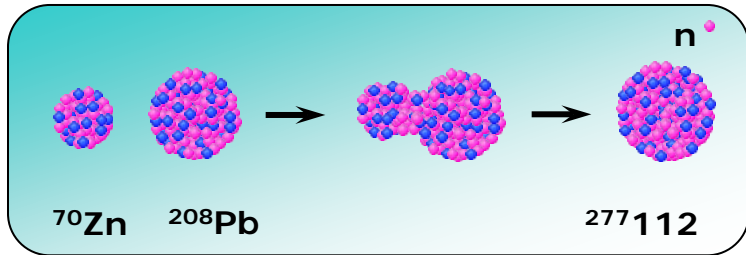
liquid drop + shell corrections



$$\sigma_{ER} = \sigma_{capture} \cdot P_{CN} \cdot P_{survival}$$

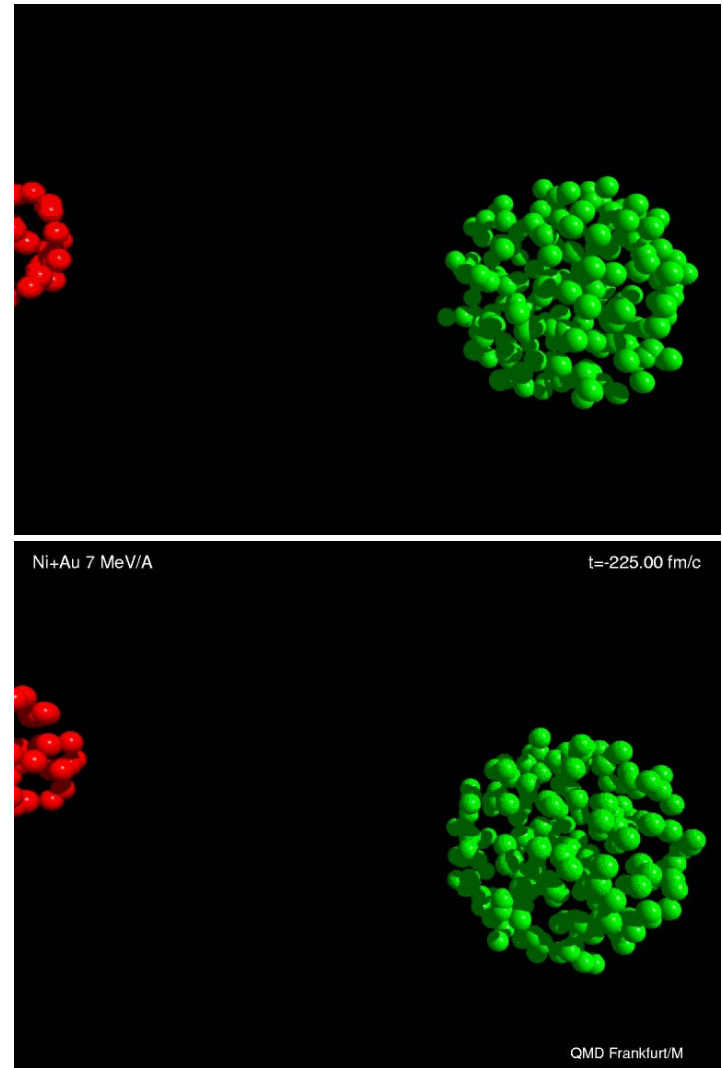


Synthesis of heavy elements



Fusion

$$-\frac{1}{10^{12}}$$



The production cross section

fusion cross section and survival probability

Nucleus:

1 barn = $10^{-24} \text{ cm}^2 = 10^{-28} \text{ m}^2$

fusion cross section:

< 1 barn

Production cross section

$^{277}112$:

$\approx 1 \text{ pbarn} = 10^{-12} \text{ barn}$

$1:10^{12}$

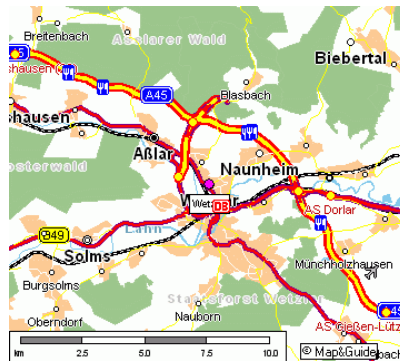


Earth:

\emptyset -Area $1.3 \times 10^8 \text{ km}^2$

$1.3 \times 10^{14} \text{ m}^2$

$1:10^7$



Wetzlar:

Area 75.67 km^2

$\approx 1.3 \times 10^7 \text{ m}^2 / 2$

$1:10^5$



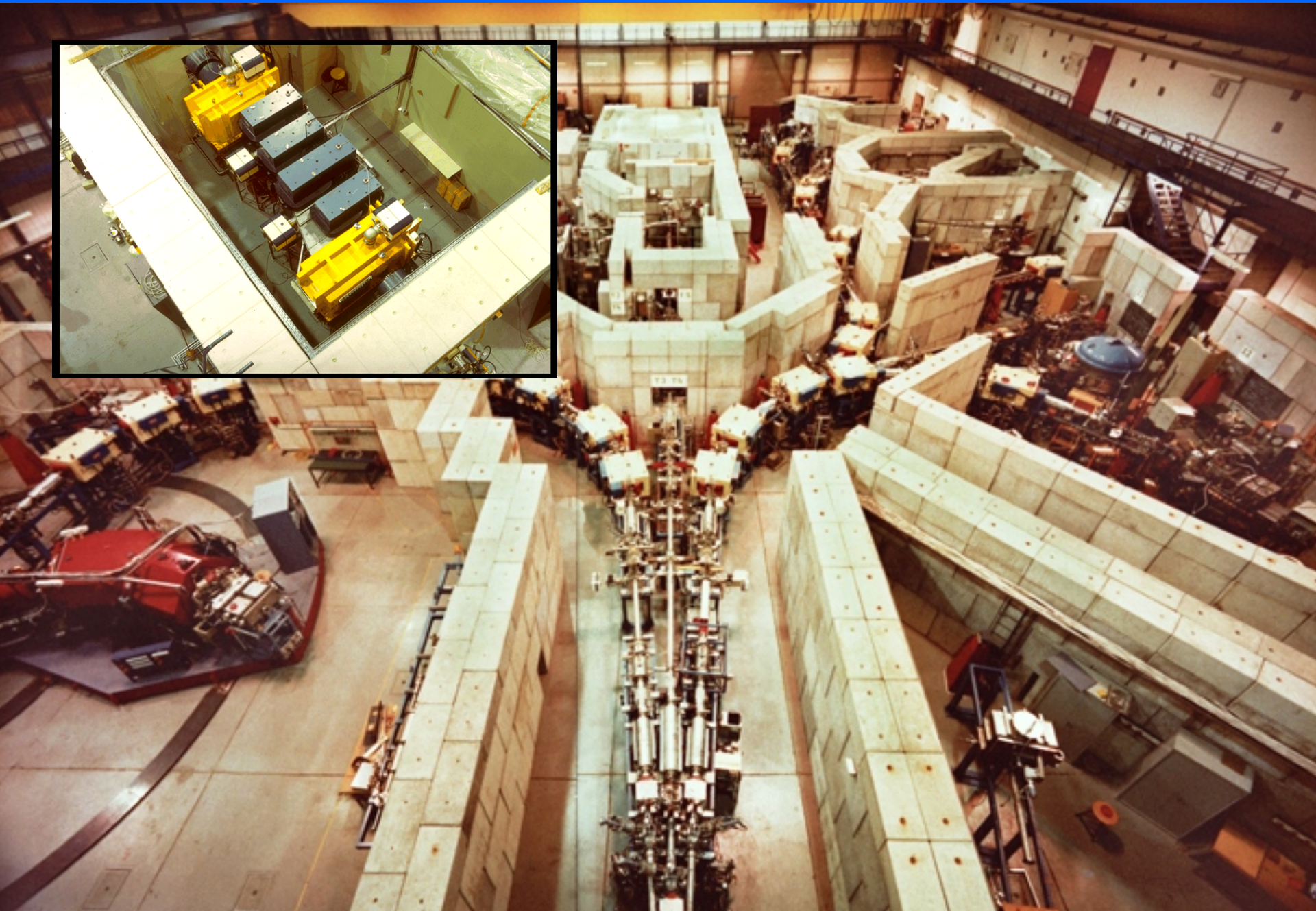
Charlotte Buff's house:

Area $\times 130 \text{ m}^2$

$1.3 \times 10^2 \text{ m}^2$

$1:10^{12}$

Separator for Heavy Ion Products (SHIP)



Separator for Heavy Ion Products (SHIP)

- Fusion products are slower than scattered or transfer particles

$$v_{CN} = [m_p / (m_p + m_t)] \cdot v_p$$

$$e.g. v_p \approx 10.3\% \rightarrow v_{CN} \approx 2.2\%$$

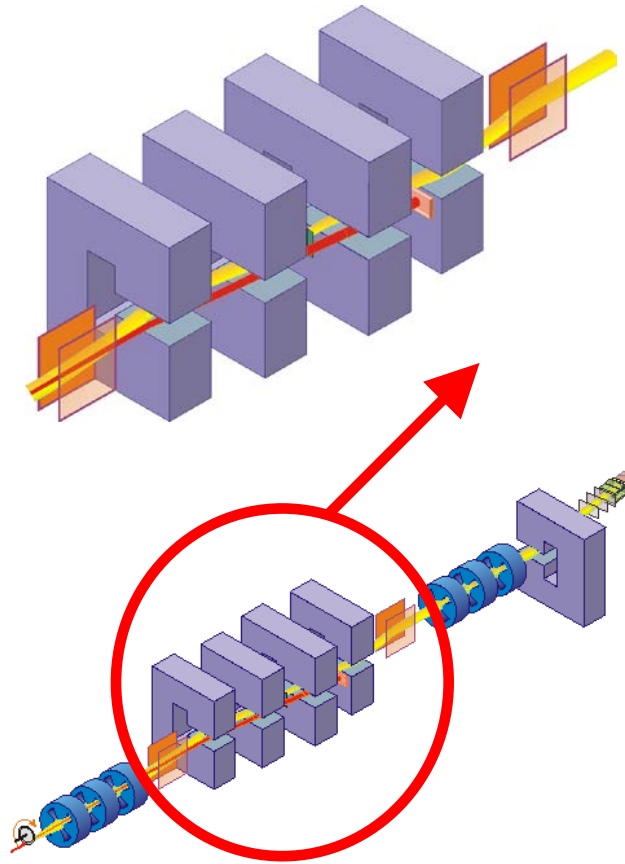
- E- and B-field are perpendicular to each other

$$B \cdot \rho = \frac{m \cdot v}{e \cdot q}$$

$$E \cdot \rho = \frac{m \cdot v^2}{e \cdot q}$$

$$F_{mag} = F_{el} \Rightarrow F_{tot} = 0$$

electric deflectors: ± 330 kV dipole magnets: 0.7 T max

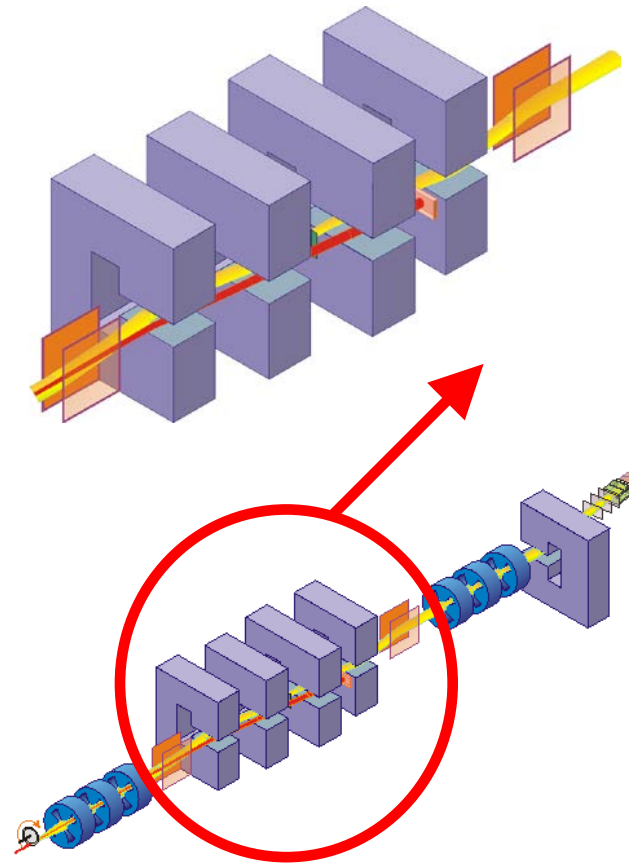


Separator for Heavy Ion Products (SHIP)

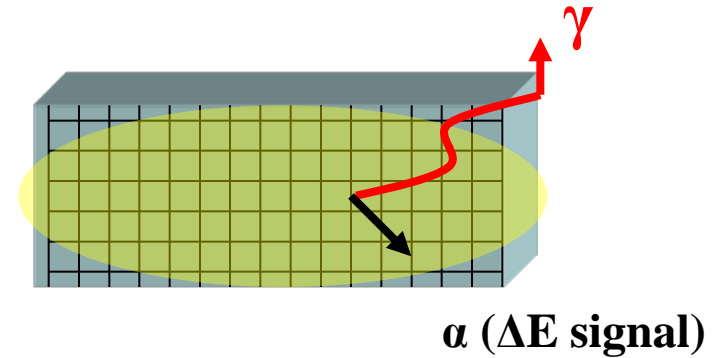
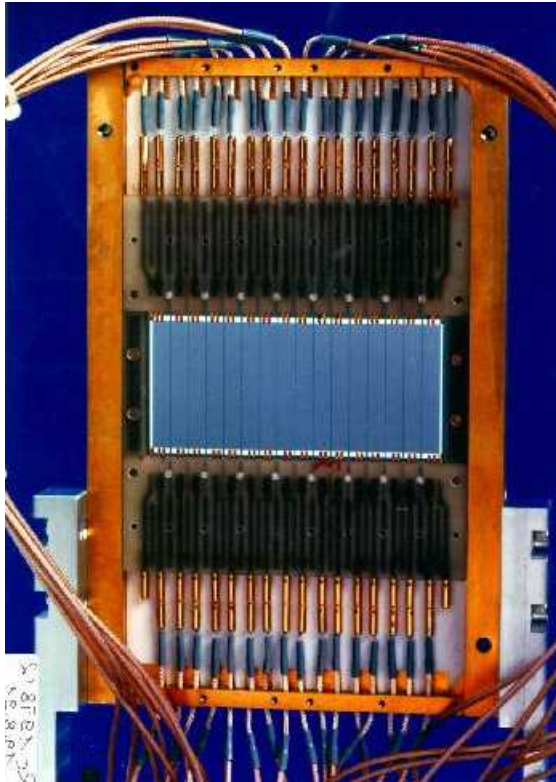
- The choice of E and B determines the transmitted velocity

$$v = \frac{E}{B}$$

- The rejected beam will be stopped on a cooled Cu plate



SHIP – stop detector



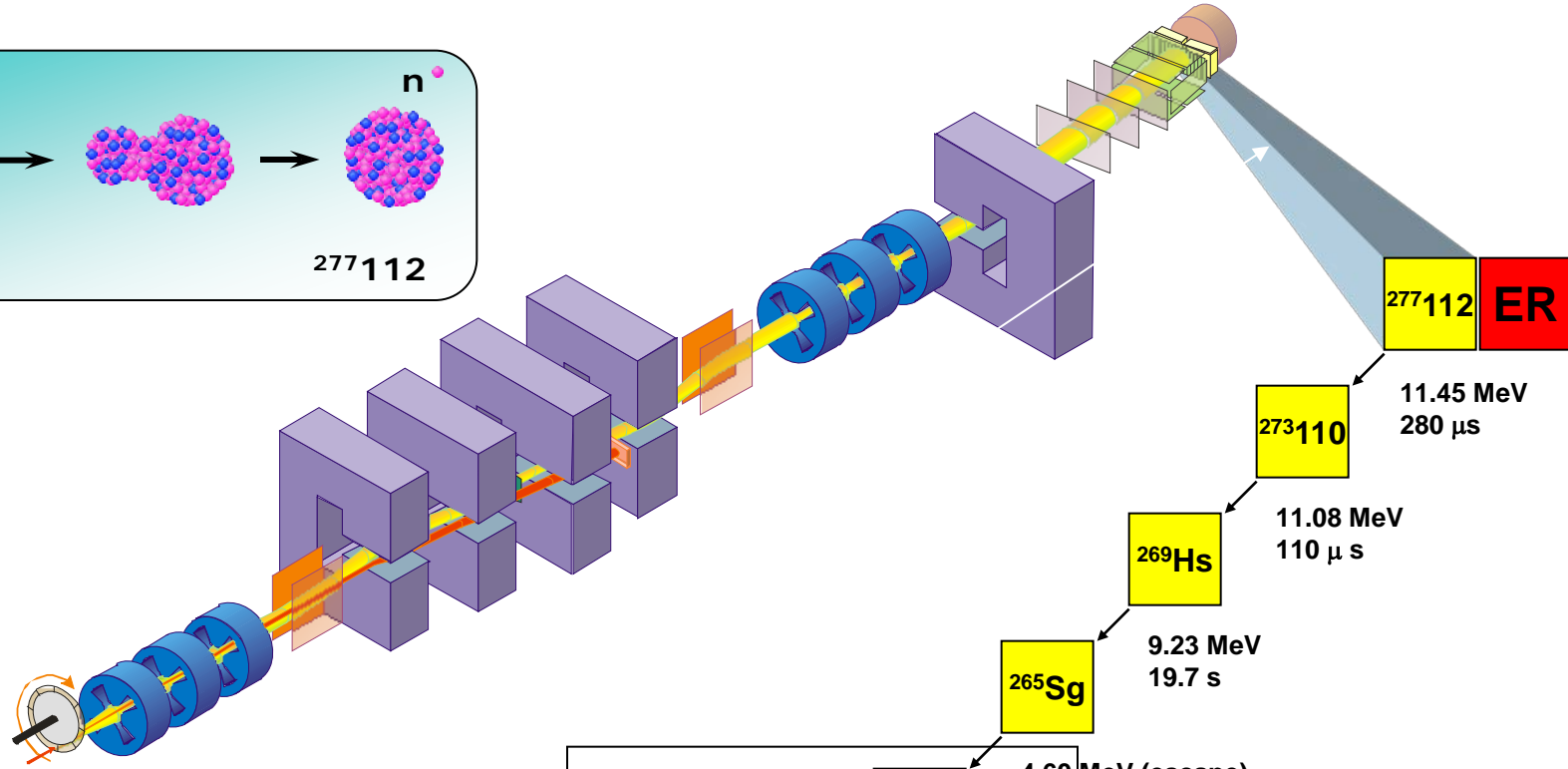
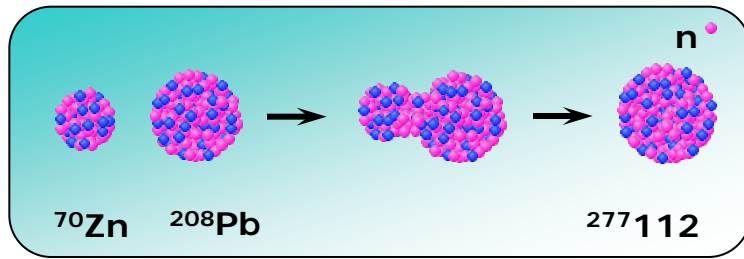
SHE will be measured in a pixel

Wait for the emission of an α -particle
(or β -particle)
**correlation method: implantation and
decay event in the same pixel**

➤ position sensitive Silicon detector determines
the position and energy of SHE and α , β , ...

area: $27 \times 87 \text{ mm}^2$, thickness: 0.3mm, 16 strips
energy resolution $\Delta E = 18\text{-}20 \text{ keV}$ @ $E_\alpha > 6 \text{ MeV}$ (cooling 260K)
position resolution $\Delta x = 0.3 \text{ mm}$ (FWHM)

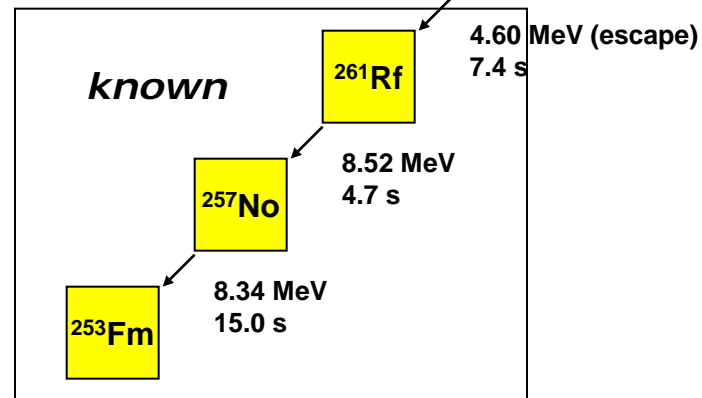
Synthesis and identification of heavy elements with SHIP



kinematical separation (in flight)

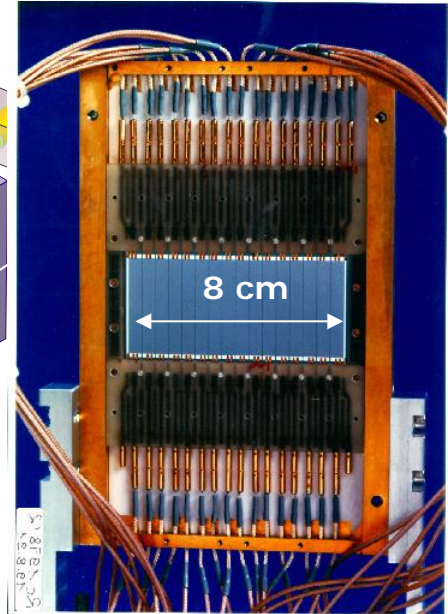
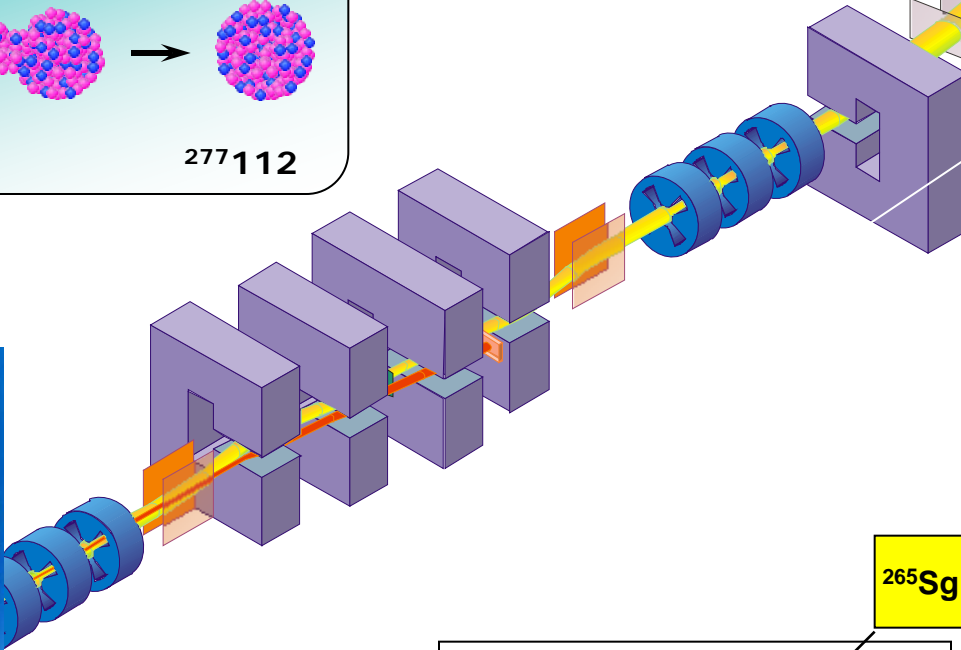
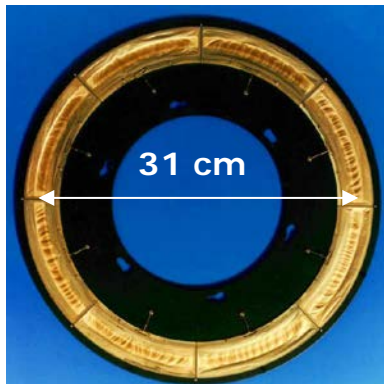
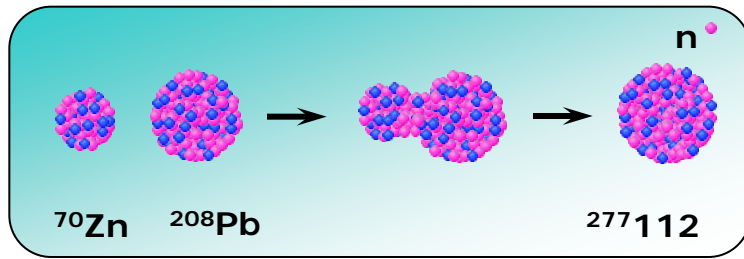
*using electric deflectors
and dipole magnets*

$v = E/B \rightarrow$ *velocity filter*

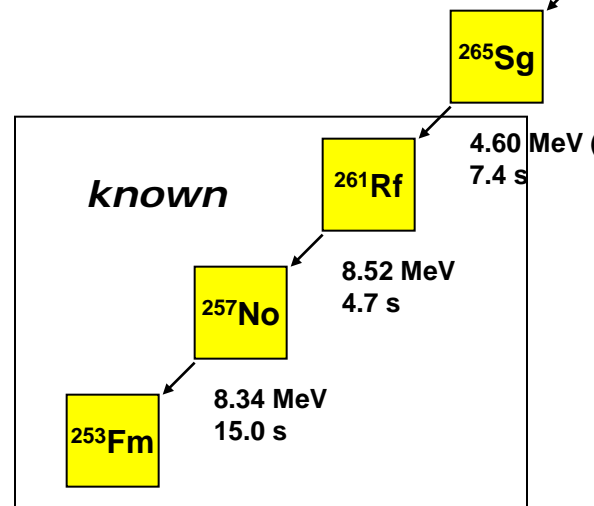


Date: 09-Feb-1996
Time: 22:37 h

Synthesis and identification of heavy elements with SHIP

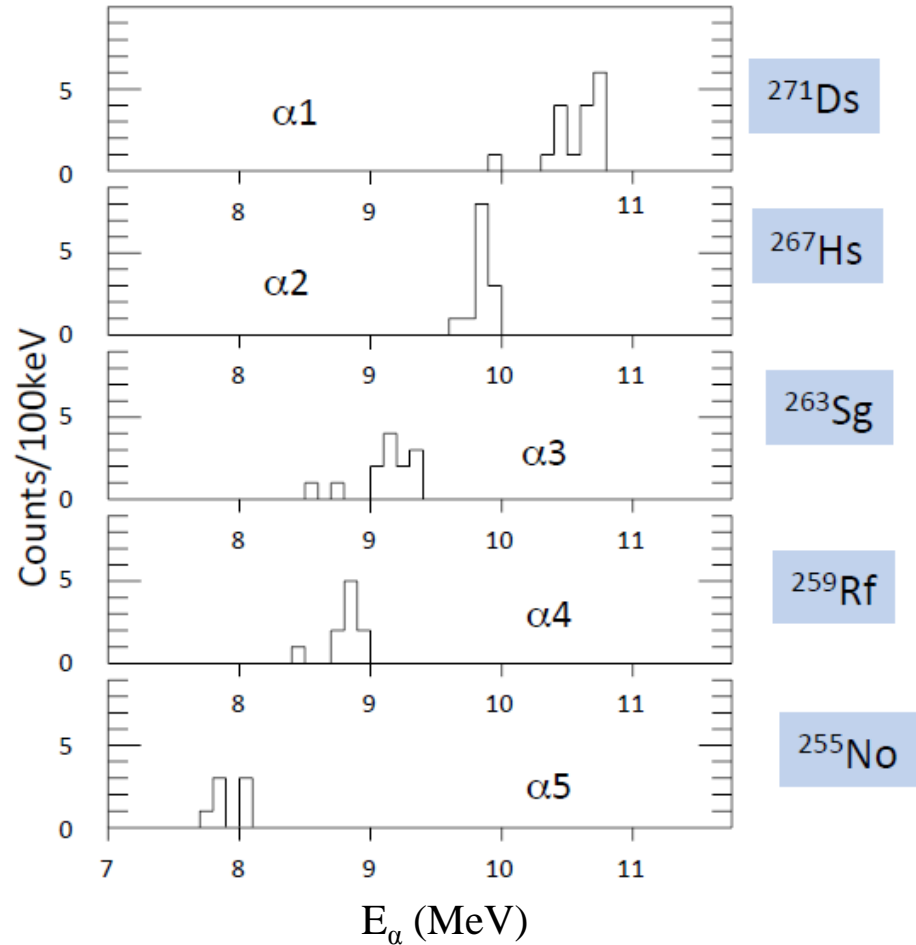
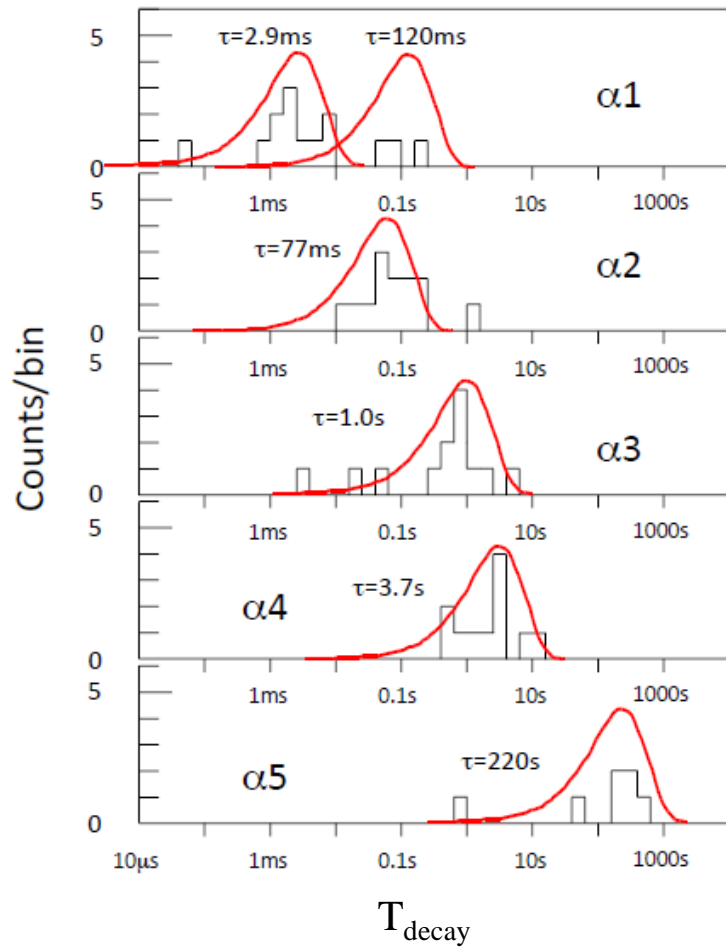


kinematical separation (in flight)
using electric deflectors
and dipole magnets
 $v = E/B \rightarrow$ *velocity filter*



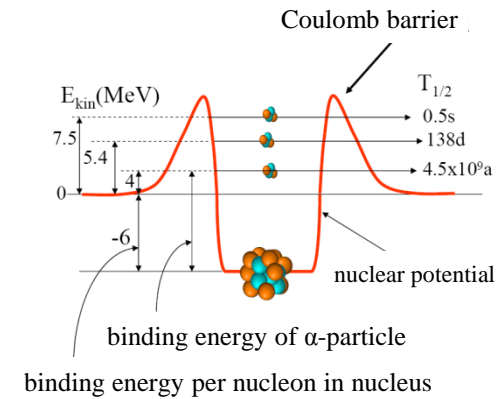
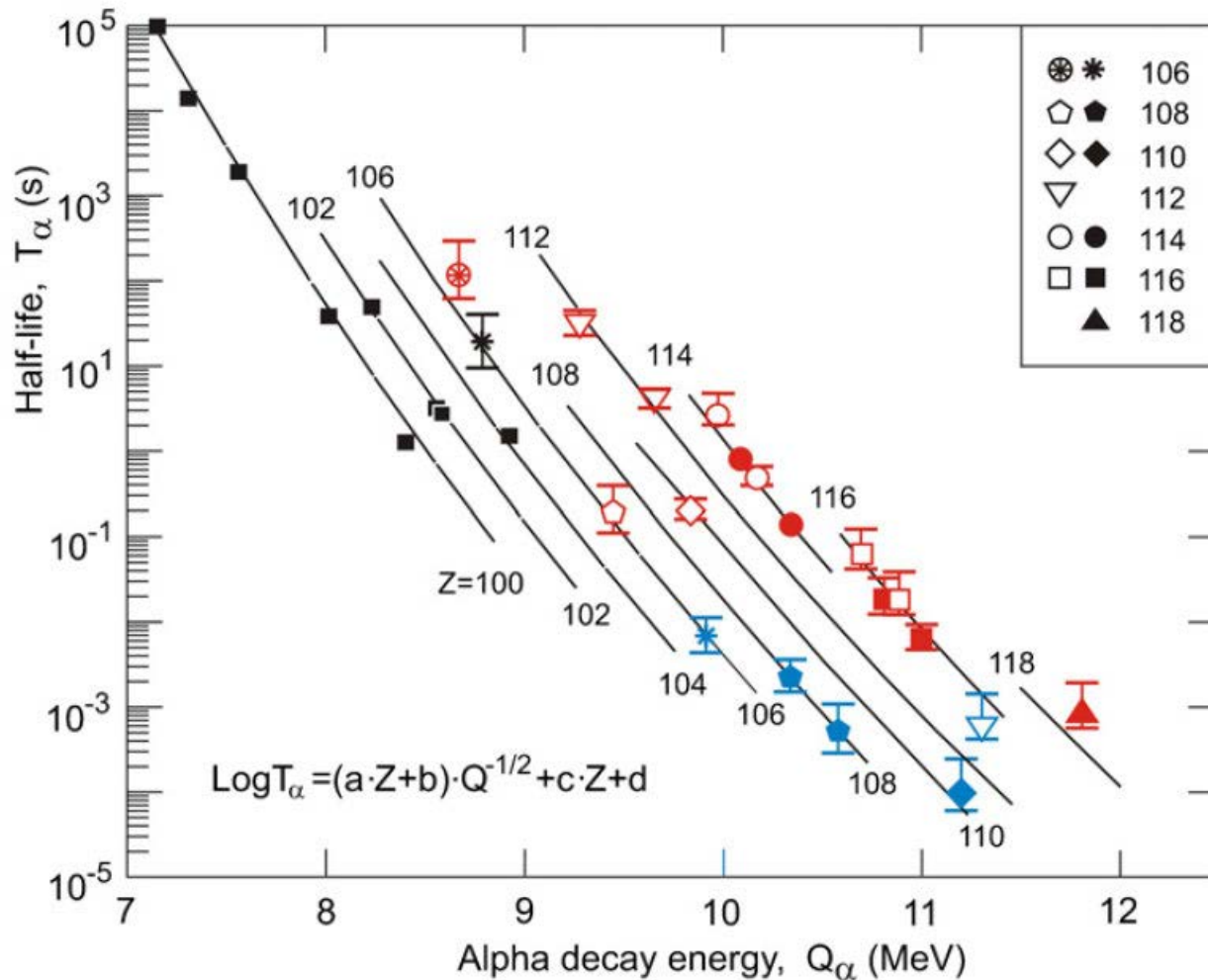
Identification by
 α - α correlations
down to known
isotopes

Date: 09-Feb-1996
 Time: 22:37 h

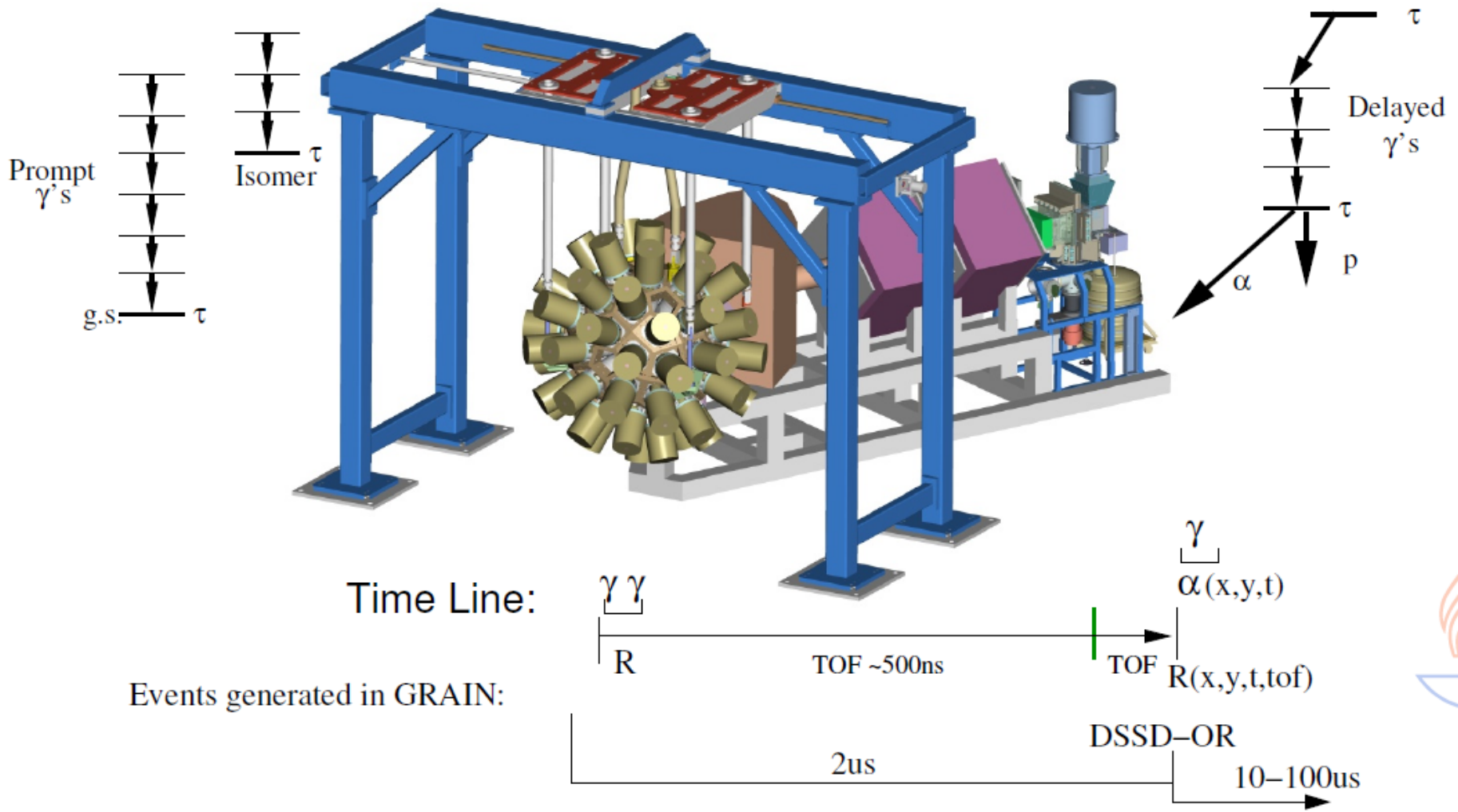


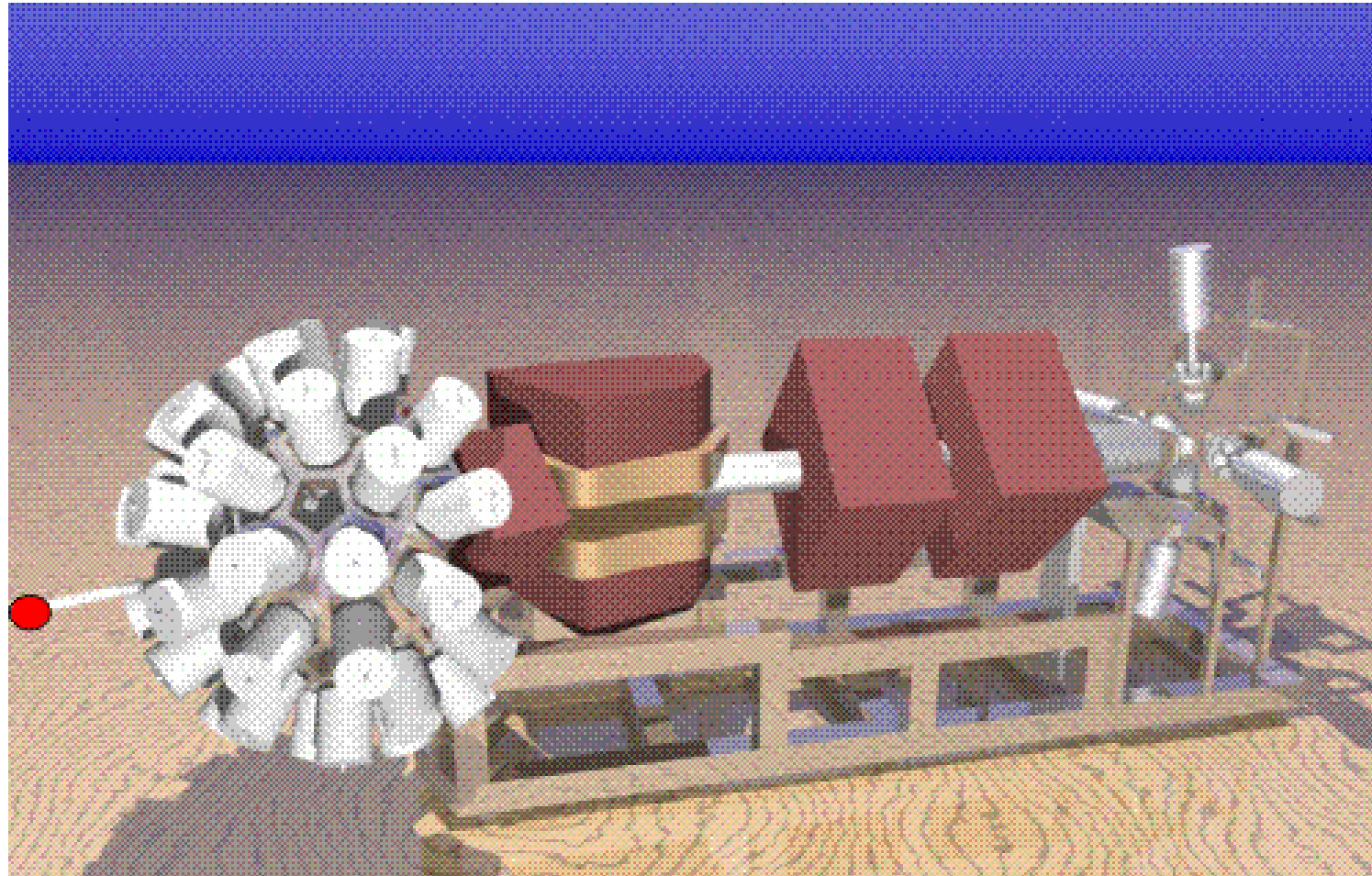
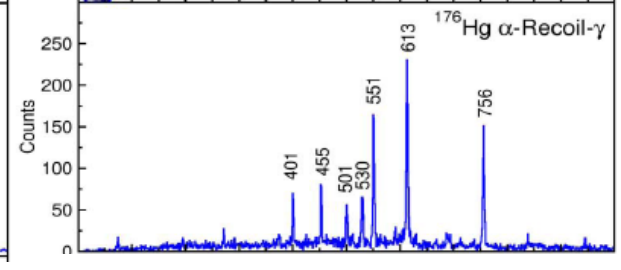
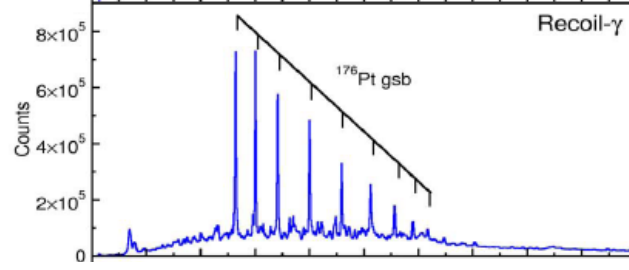
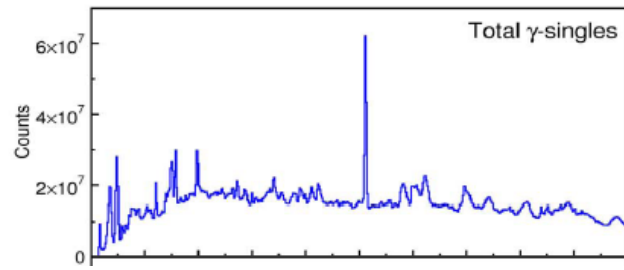
Geiger-Nuttall relationship

- ❖ The average decay properties of even mass decay chains match the Geiger-Nuttall relationship



The JUROGAM array + RITU + GREAT spectrometer

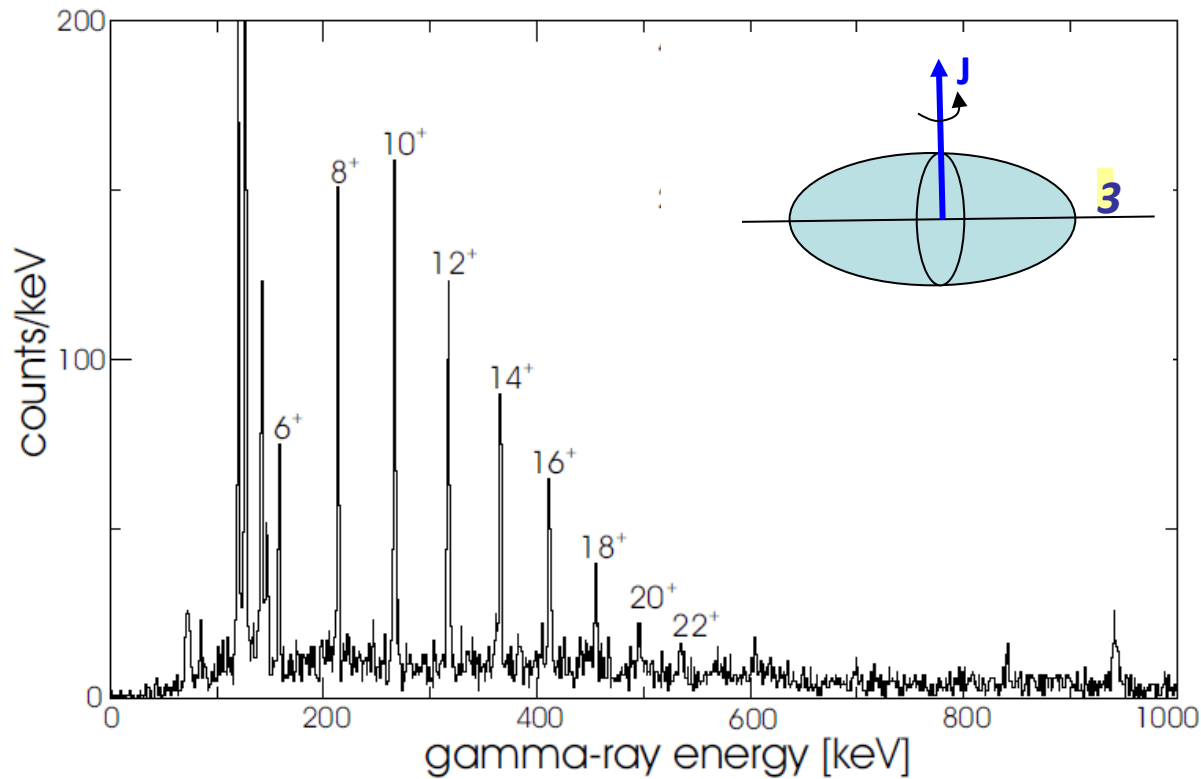
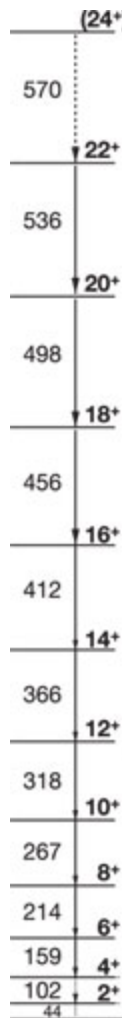




The rotational spectrum of ^{254}No

rotational energy:
$$E_J = \frac{\hbar^2}{2\mathfrak{I}} \cdot J \cdot (J + 1)$$

gamma - energy:
$$E_J - E_{J-2} = \frac{\hbar^2}{2\mathfrak{I}} \cdot (4 \cdot J - 2)$$



Chemistry of superheavy elements

Group→	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
↓Period																			
1	1 H																	2 He	
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne	
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	
6	55 Cs	56 Ba	57 La	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				*	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				*	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

- Are the new elements in the same period?
- Does e.g. **Lv** show the same chemical properties as **O, S, Se, Te** and **Po**?