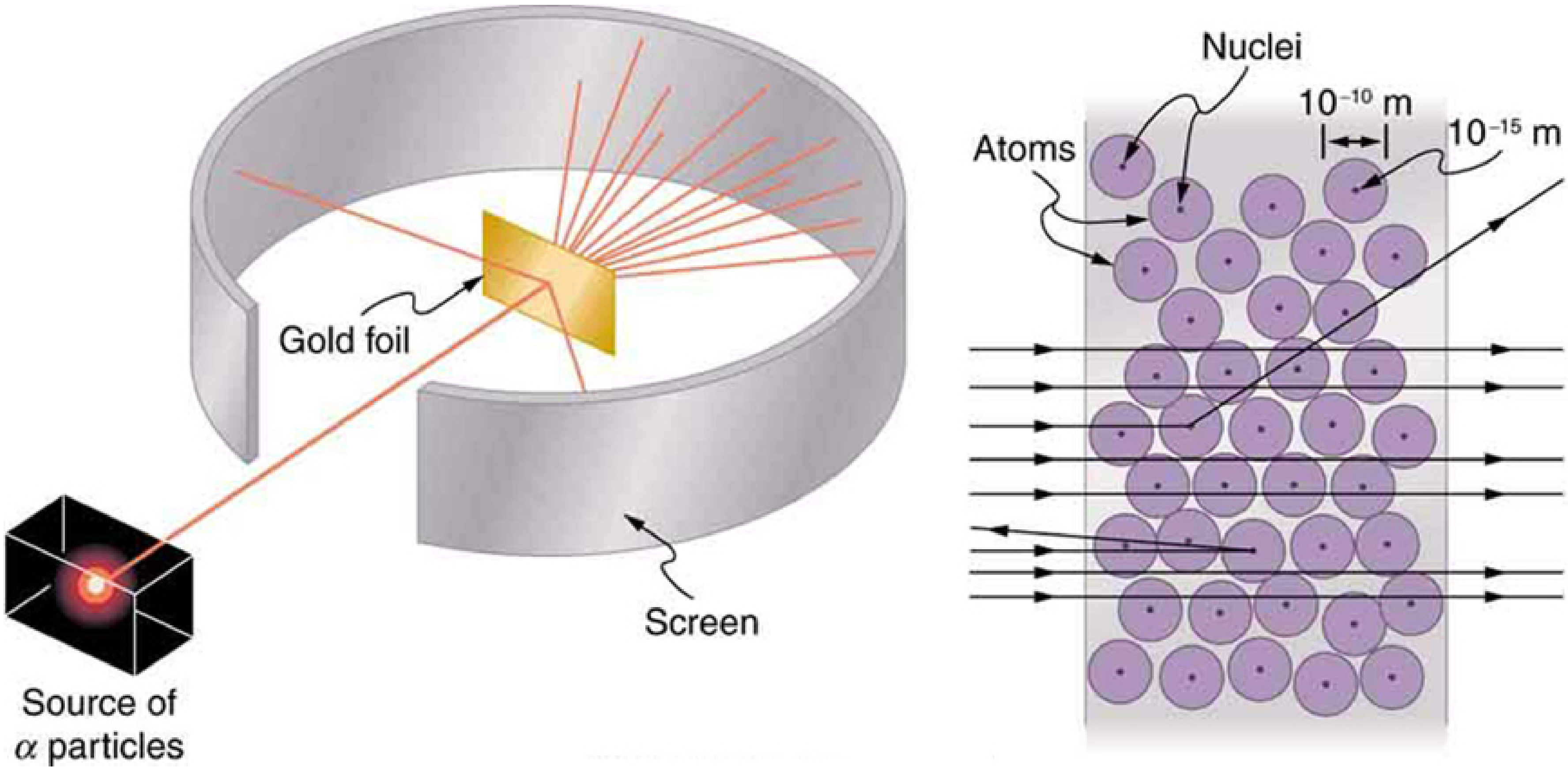
Rutherford scattering

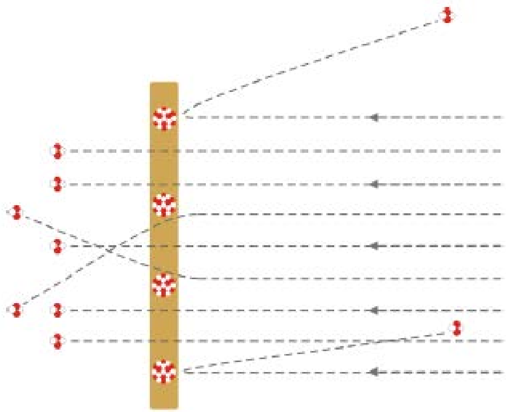
discovery of nucleus

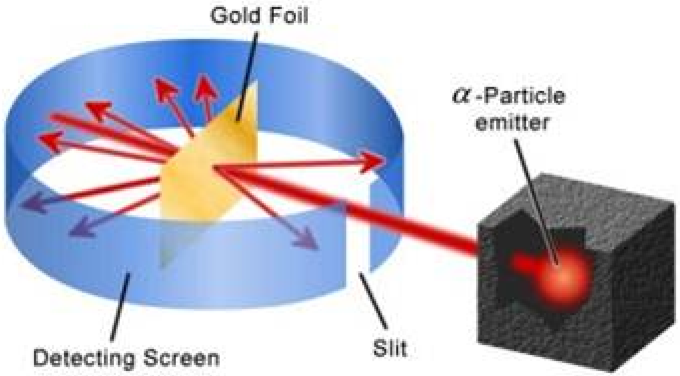


1909: Rutherford, Geiger and Marsden studied in Manchester the scattering of α-particles on thin gold foils.

Aim: from the angular distribution of the scattered α-particles they wanted to gain information on the structure of the scattering center.

Experimental set-up: Ra-source with $E_{kin}(\alpha) = 4.78$ MeV
thin Au-foils (Z = 79, d = 2000 atomic layers)
detection of scattered α‘s with ZnS scintillator



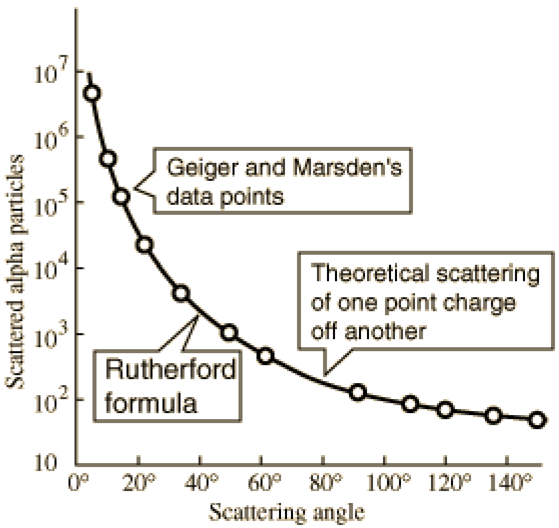


1908: Nobel price for chemistry

Ernest Rutherford (1871-1937)

Rutherford scattering

discovery of nucleus

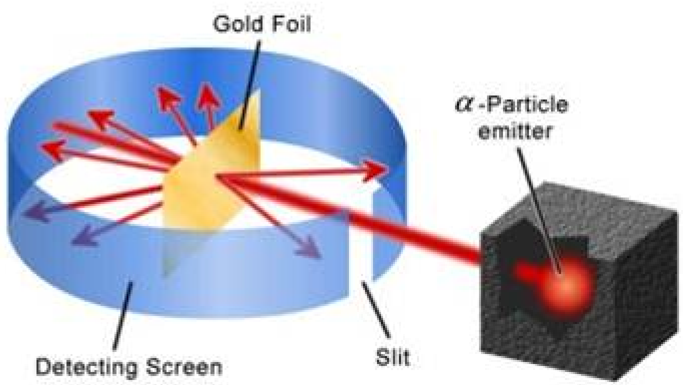


The Scattering of α and β Particles by Matter and the Structure of the Atom

E. Rutherford, F.R.S.*

Philosophical Magazine
Series 6, vol. 21
May 1911, p. 669-688

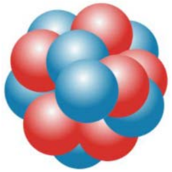
§ 1. It is well known that the α and the β particles suffer deflexions from their rectilinear paths by encounters with atoms of matter. This scattering is far more marked for the β than for the α particle on account of the much smaller momentum and energy of the former particle. There seems to be no doubt that such swiftly moving particles pass through the atoms in their path, and that the deflexions observed are due to the strong electric field traversed within the atomic system. It has generally been supposed that the scattering of a pencil of α or β rays in passing through a thin plate of matter is the result of a multitude of small scatterings by the atoms of matter traversed. The observations, however, of Geiger and Marsden** on the scattering of α rays indicate that some of the α particles, about 1 in 20,000 were turned through an average angle of 90 degrees in passing though a layer of gold-foil about 0.00004 cm. thick, which was equivalent in stopping-power of the α particle to 1.6 millimetres of air. Geiger*** showed later that the most probable angle of deflexion for a pencil of α particles being deflected through 90 degrees is vanishingly small. In addition, it will be seen later that the distribution of the α particles for various angles of large deflexion does not follow the probability law to be expected if such



observation:

$$event\ rate \sim 1/\sin^4\left(\frac{\theta}{2}\right)$$

scattering on a point-like atomic nucleus



Kinematic of elastic scattered α-particles

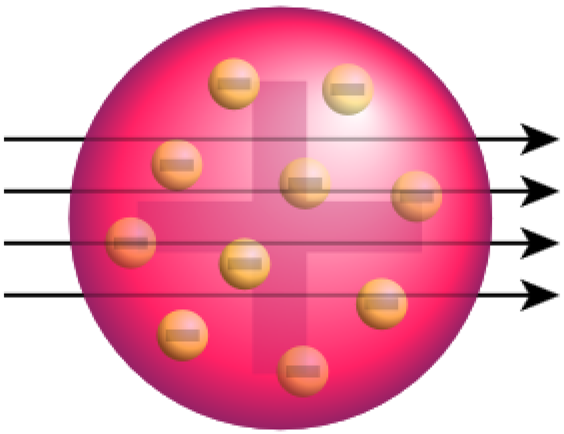
⇨ energy and momentum conservation

α-particles on electrons (Thomson model)

$$\boxed{\frac{m_e}{m_\alpha} \approx 10^{-4}}$$

max. momentum transfer $\Delta p \sim 10^{-4} \cdot p_i$

only small scattering angles $\theta \sim 0^0$



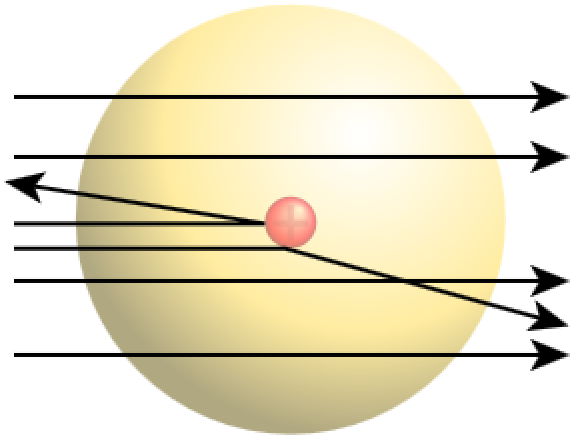
α-particles on Au-nuclei (Rutherford model)

$$\boxed{\frac{m_{Au-197}}{m_\alpha} \approx 50}$$

max. momentum transfer $\Delta p \sim 2 \cdot p_i$

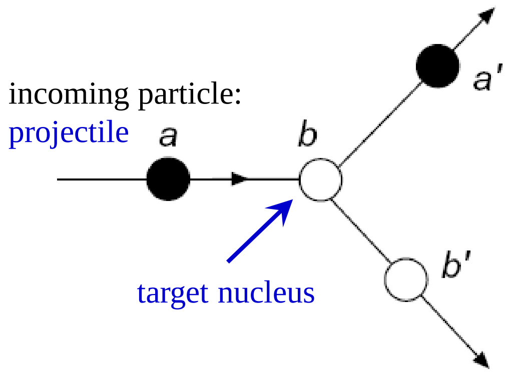
scattering angles up to $\theta_{max} \sim 180^0$

(backscattered α-particles)



$m_\alpha = 4\ GeV/c^2$ $m_e = 0.511\ MeV/c^2$ $m_{AU-197} = 197\ GeV/c^2$

Elastic scattering

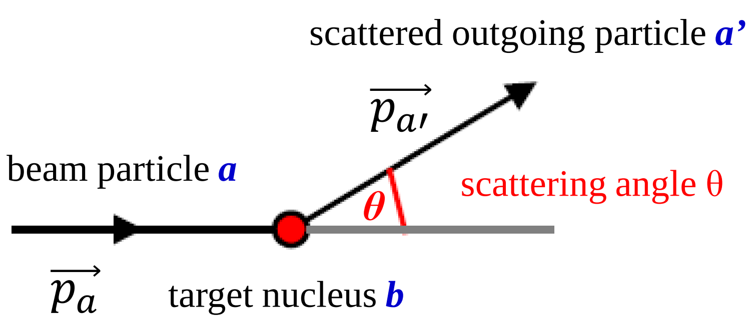


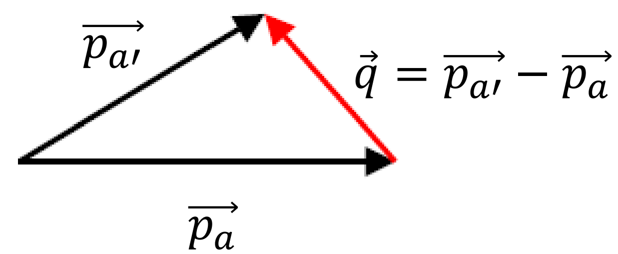
In an elastic process

$$a + b \longrightarrow a' + b'$$

the same particles are present both before and after the scattering, i.e. the initial and the final state are identical (including quantum numbers) up to momenta and energy.

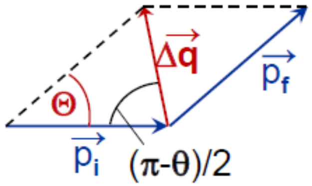
The ***target b*** remains in its ground state, absorbing merely the recoil momentum and hence changing its kinetic energy.
The scattering angle and the energy of the ***projectile a*** and the recoil energy and energy of the ***target b*** are unambiguously correlated.



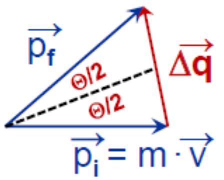


Rutherford scattering

In the repulsive Coulomb potential $V(r) \sim \frac{z \cdot Z \cdot e^2}{r}$ the α-particle experiences a momentum change $\Delta\vec{q} = \vec{p_f} - \vec{p_i}$



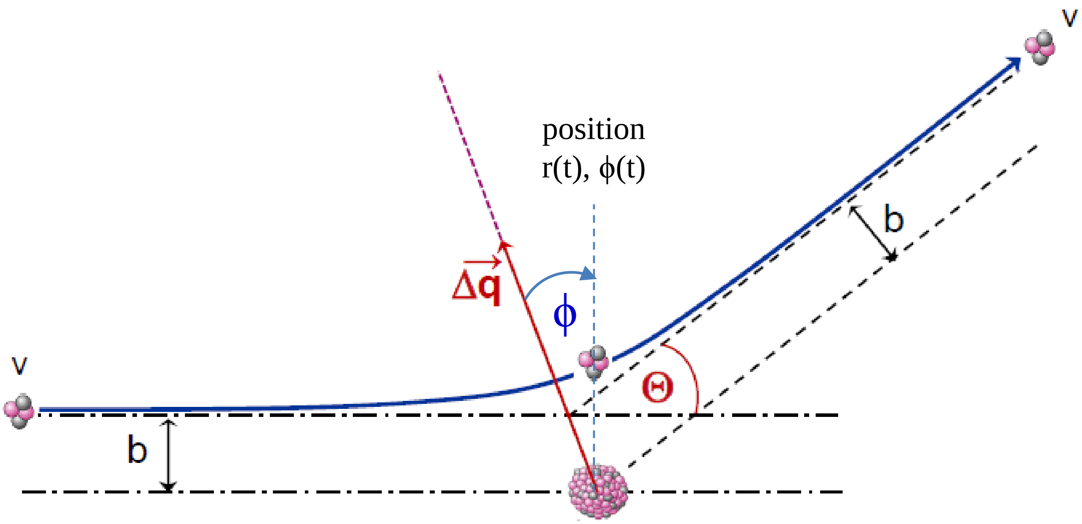
$$\boxed{\Delta q = 2 \cdot m \cdot v \cdot sin\frac{\theta}{2}}$$



$$\Delta q = \int F_{\Delta q} \cdot dt = \int_{-\infty}^{+\infty} \frac{1}{4\pi\varepsilon_0} \cdot \frac{z \cdot Z \cdot e^2}{r^2} \cdot cos\phi \cdot dt$$

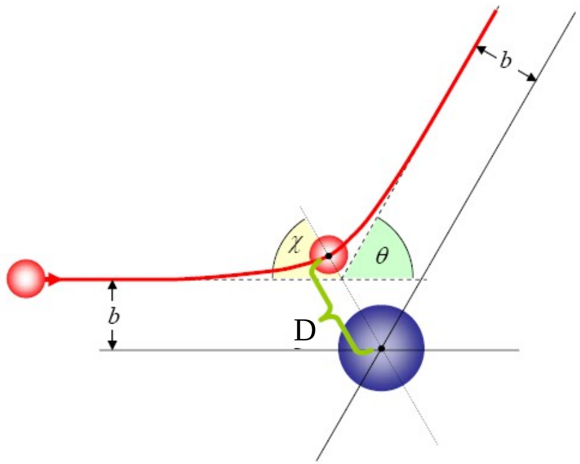
angular momentum: $\vec{L} = |\vec{r} \times m \cdot \vec{v}| = m \cdot v \cdot b = m \cdot \frac{d\phi}{dt} \cdot r^2$

$$\Delta q = \int_{-(\pi-\theta)/2}^{(\pi-\theta)/2} \frac{1}{4\pi\varepsilon_0} \frac{zZe^2}{v \cdot b} \cdot cos\phi \cdot d\phi = \frac{1}{4\pi\varepsilon_0} \frac{zZe^2}{v \cdot b} \cdot 2 \cdot cos\frac{\theta}{2}$$



relation between impact parameter **b** and scattering angle **θ**:

$$tan\frac{\theta}{2} = \frac{1}{4\pi\varepsilon_0} \frac{z \cdot Z \cdot e^2}{m \cdot v^2} \cdot \frac{1}{b}$$



$$\theta = \pi - 2\chi$$

impact parameter:

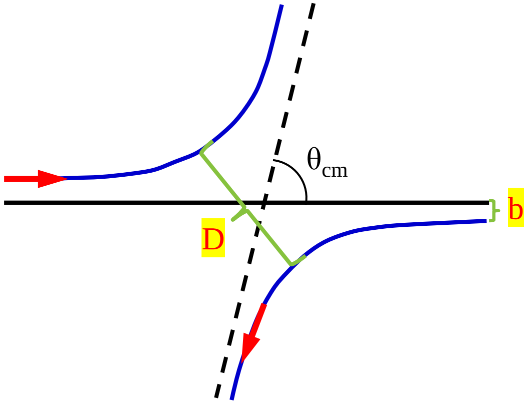
$$b = a \cdot cot\frac{\theta_{cm}}{2}$$

distance of closest approach:

$$D = a \cdot \left[sin^{-1}\frac{\theta_{cm}}{2} + 1\right]$$

orbital angular momentum:

$$\ell = k_\infty \cdot b = \eta \cdot cot\frac{\theta_{cm}}{2}$$



center of mass system

half distance of closest approach
in a head-on collision ($\theta_{cm}=180^0$):

$$a = \frac{0.72 \cdot Z_1 Z_2}{T_{lab}} \cdot \frac{A_1 + A_2}{A_2} \quad [fm]$$

asymptotic wave number:

$$k_\infty = 0.219 \cdot \frac{A_2}{A_1 + A_2} \cdot \sqrt{A_1 \cdot T_{lab}} \quad [fm^{-1}]$$

Sommerfeld parameter:

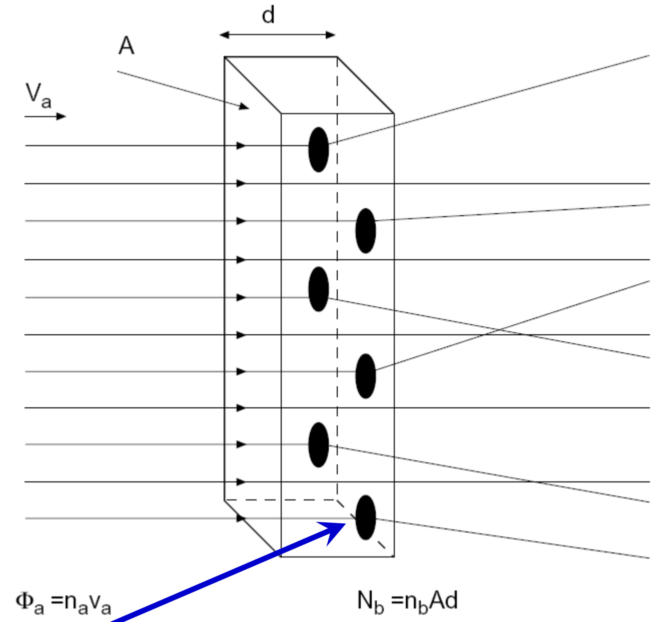
$$\eta = k_\infty \cdot a = 0.157 \cdot Z_1 Z_2 \cdot \sqrt{\frac{A_1}{T_{lab}}}$$

Cross section

The **cross section** gives the probability of a reaction between the two colliding particles

Consider an idealized experiment:

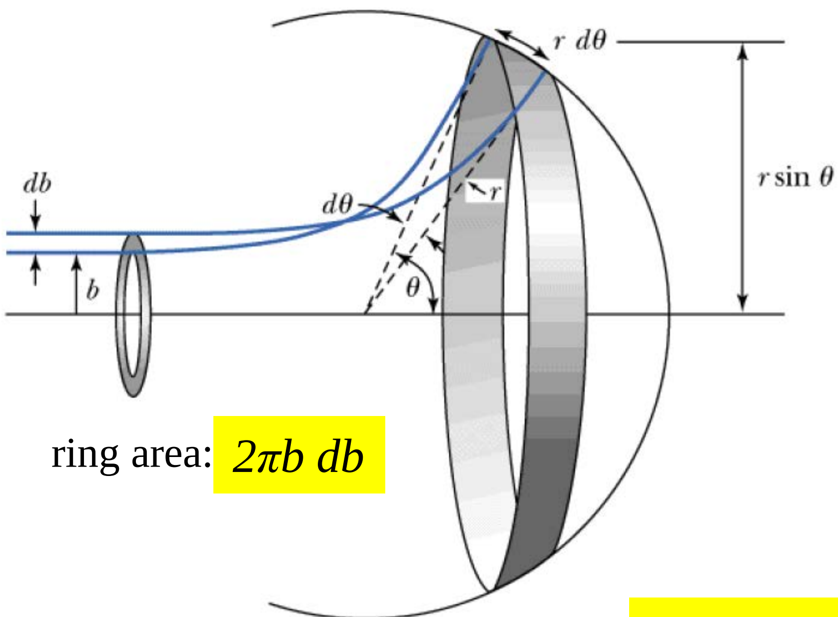
- we bombard the target with a monoenergetic beam of point-like particles $\boldsymbol{a}$ with a velocity $\boldsymbol{v_a}$.
- a thin target of thickness $\boldsymbol{d}$ and a total area $\boldsymbol{A}$ with $\boldsymbol{N_b}$ scattering centers $\boldsymbol{b}$ and with a particle density $\boldsymbol{n_b}$.
- each target particle has a cross-sectional area $\boldsymbol{\sigma_b}$, which we have to find by experiment!



→ Some beam particles are scattered by the scattering centers of the target, i.e. they are deflected from their original trajectory. The frequency of this process is a measure of the cross section area of the scattered particles $\boldsymbol{\sigma_b}$.

Rutherford cross section

❖ Particles from the ring defined by the impact parameter b and $b+db$ scatter between angle θ and $\theta+d\theta$



ring area: $2\pi b\, db$

solid angle: $2\pi \sin\theta\, d\theta$

$$j \cdot 2\pi \cdot b \cdot db = j \cdot 2\pi \cdot sin\theta \cdot \frac{d\sigma}{d\Omega}$$

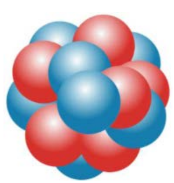
$$\frac{d\sigma}{d\Omega} = \frac{b}{sin\theta}\left|\frac{db}{d\theta}\right|$$

impact parameter: $b = a \cdot cot\frac{\theta}{2}$

$$\left|\frac{db}{d\theta}\right| = \frac{a}{2} \cdot \frac{-sin\frac{\theta}{2} \cdot sin\frac{\theta}{2} - cos\frac{\theta}{2} \cdot cos\frac{\theta}{2}}{sin^2\frac{\theta}{2}} = \frac{a}{2} \cdot \frac{1}{sin^2\frac{\theta}{2}}$$

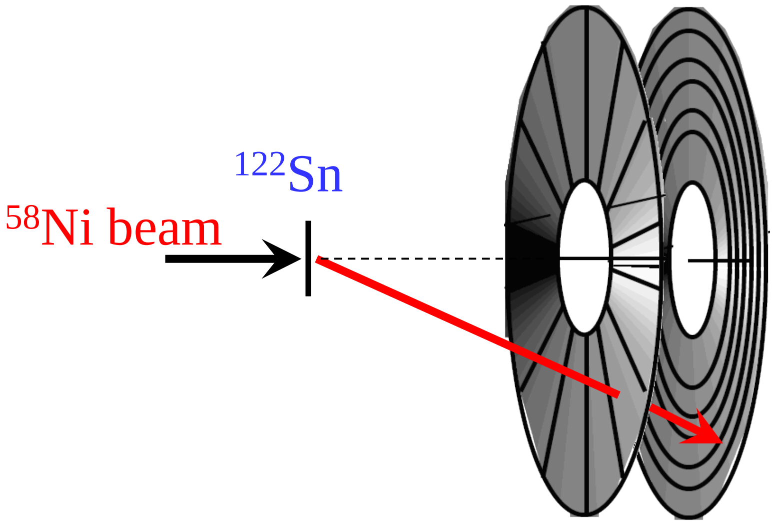
$$\frac{d\sigma}{d\Omega} = a \cdot \frac{cos\frac{\theta}{2}}{sin\frac{\theta}{2}} \cdot \frac{1}{2 \cdot cos\frac{\theta}{2} \cdot sin\frac{\theta}{2}} \cdot \frac{a}{2 \cdot sin^2\frac{\theta}{2}}$$

an atomic nucleus exist



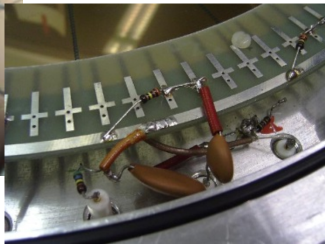
$$\boxed{\frac{d\sigma}{d\Omega} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta}{2}}$$

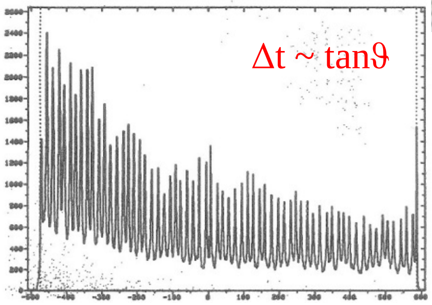
Annular gas-filled parallel-plate avalanche counter (PPAC)

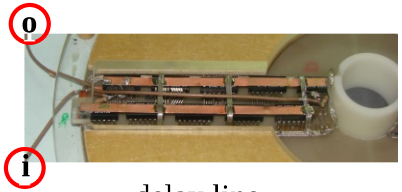


V_0 ~ 500 V
p = 5-10 Torr
gap ~ 3 mm (anode-cathode)

$\boxed{\varphi_p}$ $\boxed{\approx \tan \vartheta_p}$



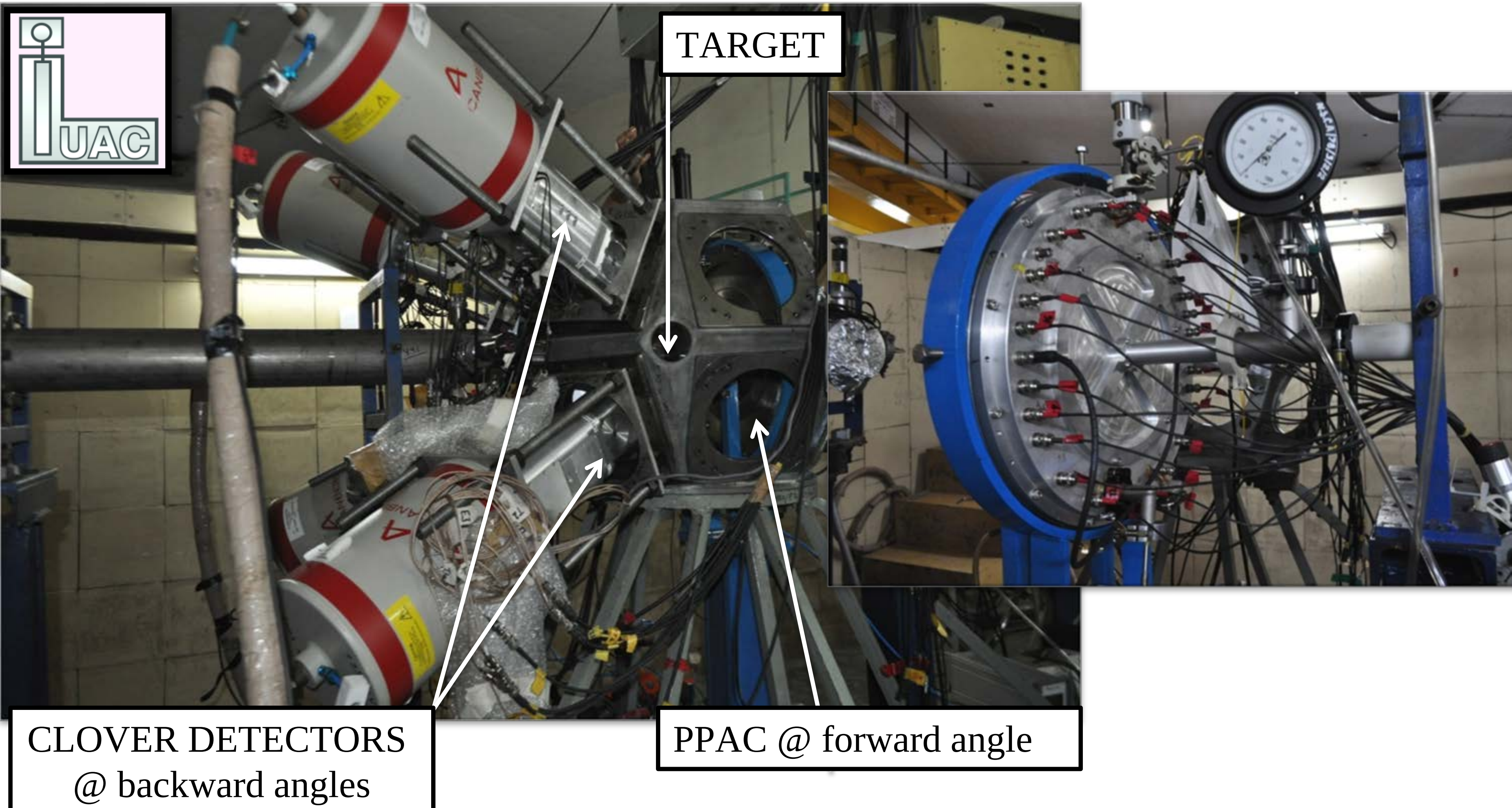




delay line

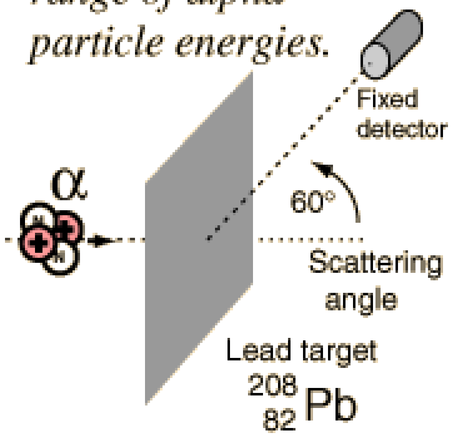
A. Jhingan detector laboratory at IUAC

Experimental set-up at IUAC

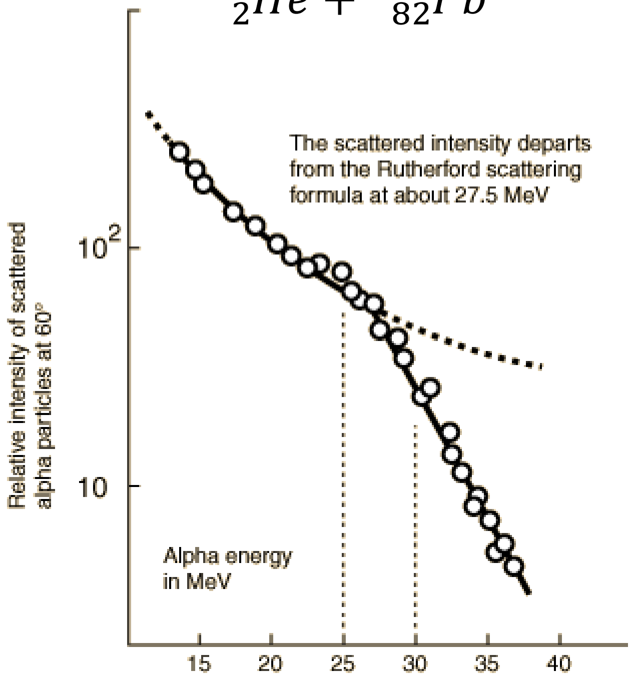


Elastic scattering and nuclear radius

Fixed scattering angle, range of alpha particle energies.



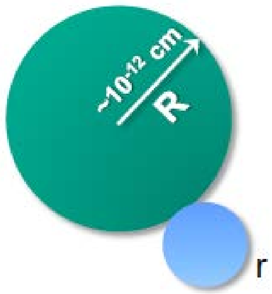
$${}^{4}_{2}He + {}^{208}_{82}Pb$$



$\theta_{1/4} = 60^0$, $E_\alpha = 30$ MeV

$\rightarrow R_{int} = 12.0$ [fm]

$$R_{int} \cong R_{He} + R_{Pb} + 3fm$$

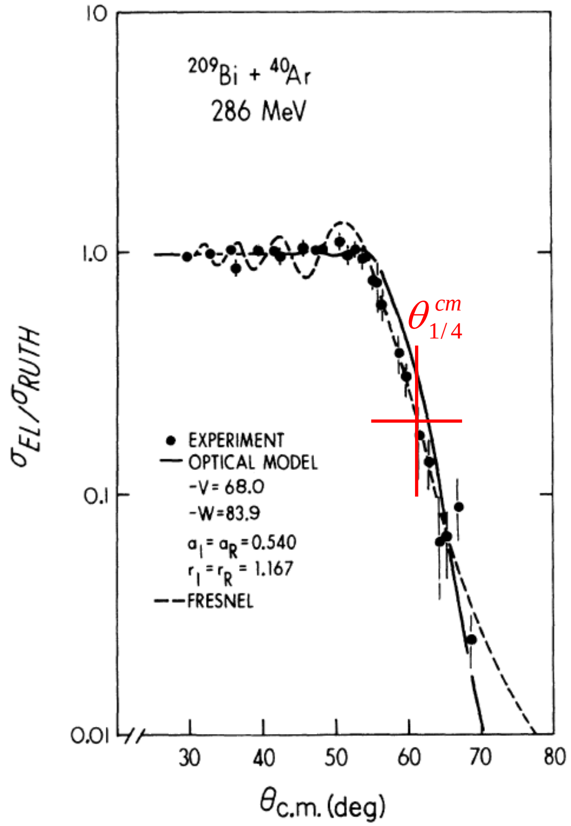


$$D = a \cdot \left[sin^{-1}\frac{\theta_{cm}}{2} + 1 \right]$$

$$a = \frac{0.72 \cdot Z_1 Z_2}{T_{lab}} \cdot \frac{A_1 + A_2}{A_2} \quad [fm]$$

R.M. Eisenberg and C.E. Porter, Rev. Mod. Phys. 33, 190 (1961)

Elastic scattering and nuclear radius



$\theta_{1/4} = 60^0 \rightarrow R_{int} = 13.4$ [fm]

$\rightarrow \ell_{gr} = 152$ [ħ]

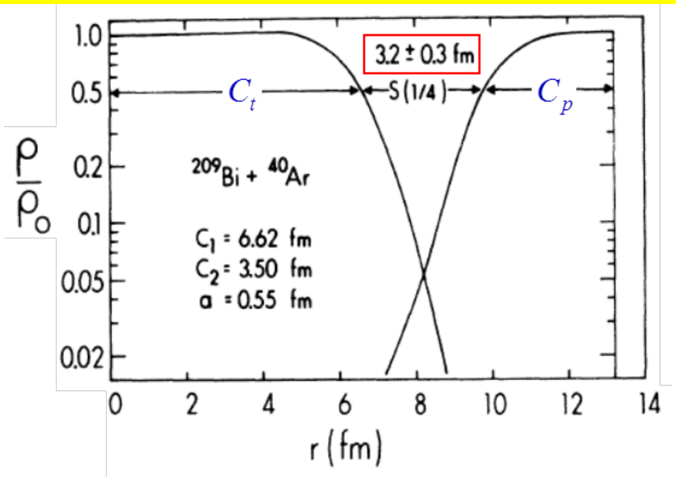
Nuclear interaction radius: (distance of closest approach)

$$R_{int} = D = a \cdot \left[sin^{-1}\frac{\theta_{1/4}}{2} + 1 \right]$$

$$\boxed{R_{int} = C_p + C_t + 4.49 - \frac{C_p + C_t}{6.35} \quad [fm]}$$

$C_i = R_i \cdot (1 - R_i^{-2}) \quad [fm]$ $\qquad R_i = 1.28 \cdot A_i^{1/3} - 0.76 + 0.8 \cdot A_i^{-1/3} \quad [fm]$

Nuclear density distributions at the nuclear interaction radius



J.R. Birkelund et al., Phys.Rev.C13 (1976), 133