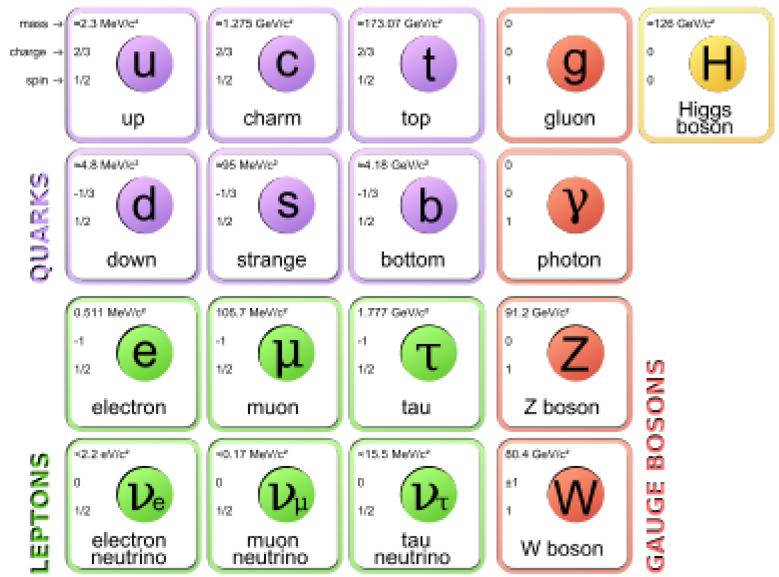
γ-ray spectroscopy

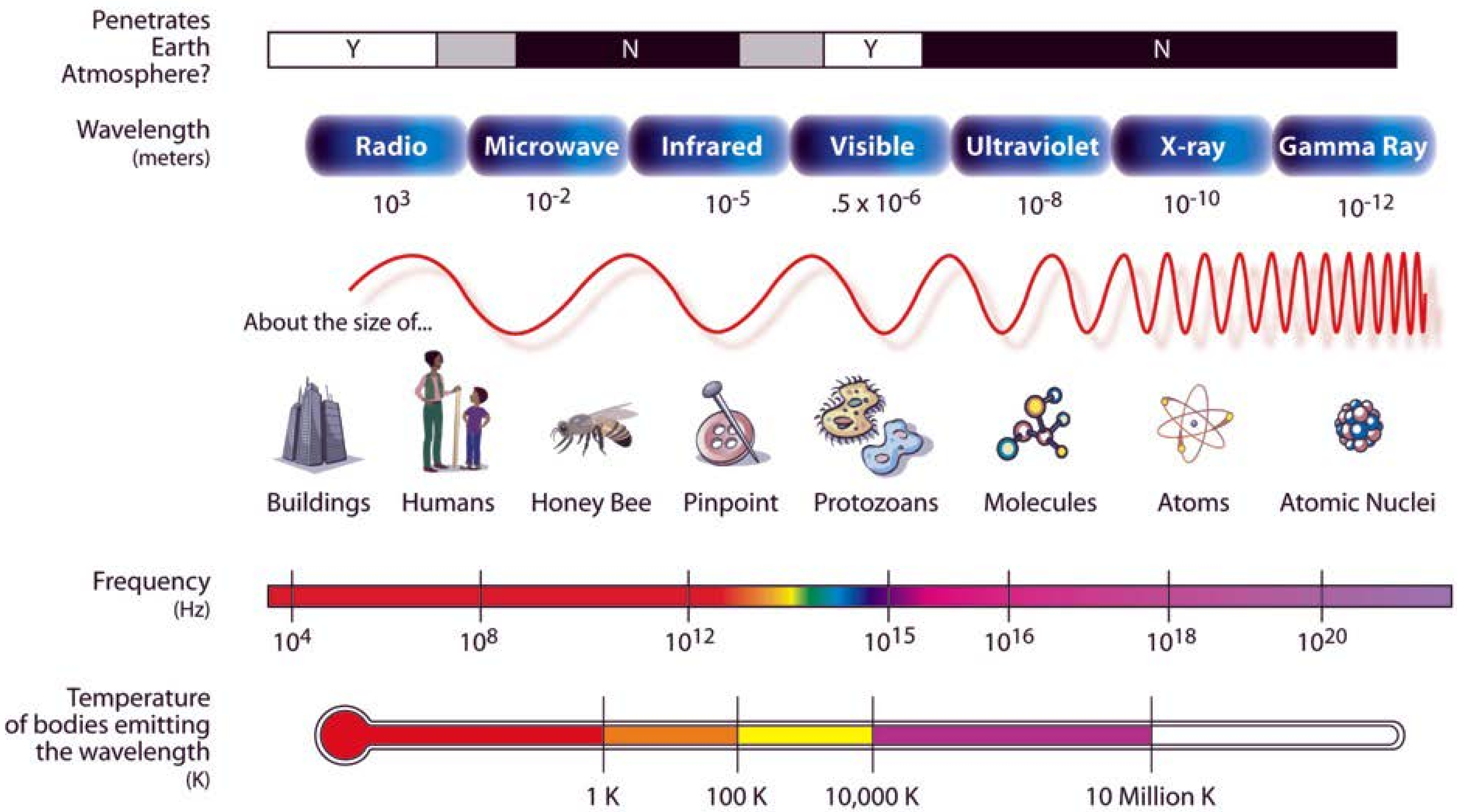
❖ γ-decay is an *electromagnetic process* where the nucleus decreases in excitation energy, but does not change proton or neutron numbers

❖ This decay process only involves the emission of photons (γ-rays carry spin 1)

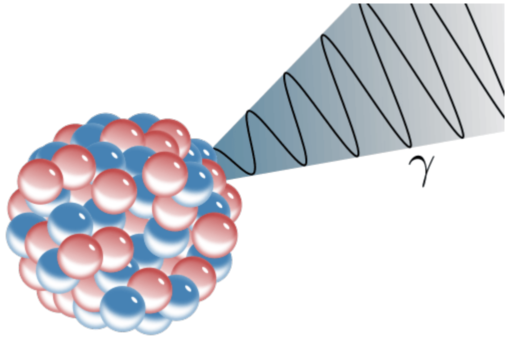


- ❖ Basic γ-ray properties, observables
- ❖ γ-ray interactions in matter
- ❖ Detector types
- ❖ Measurement techniques

THE ELECTROMAGNETIC SPECTRUM

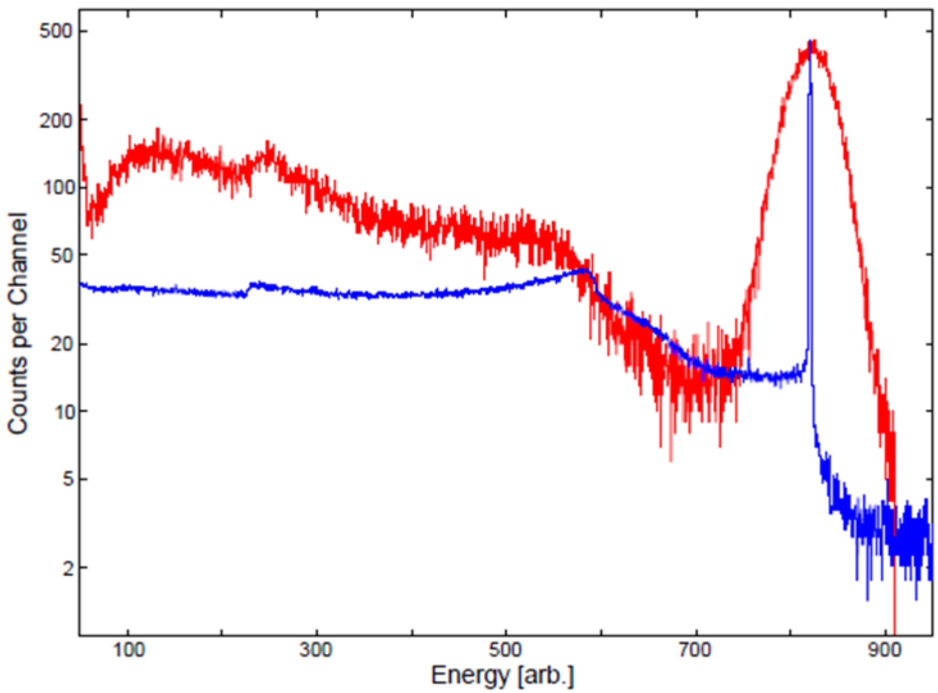


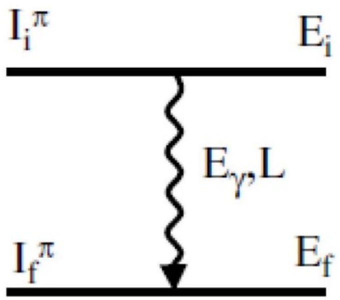
❖ Gamma-ray emission is usually the dominant decay mode



Measurements of γ-rays let us deduce:
Energy, Spin (angular distr. / correl.), Parity (polarization), magnetic moment, lifetime (recoil distance, Doppler shift), …
of the involved nuclear levels.

^{137}Cs detected in red: NaI scintillator
blue: HPGe (high purity Ge semiconductor)

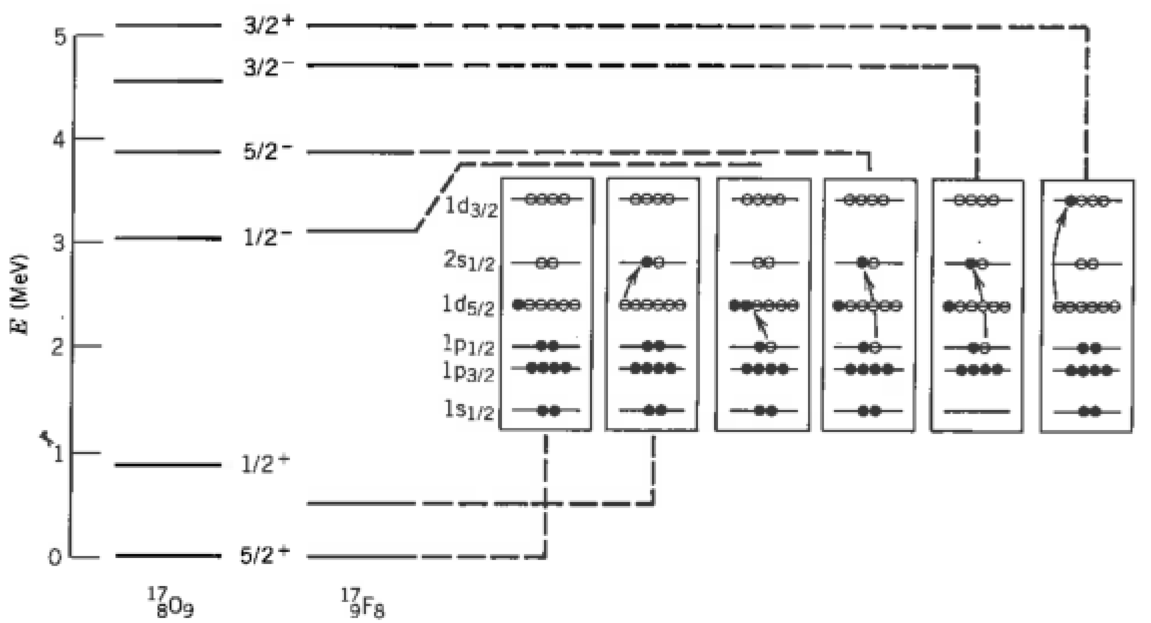




$$^{A}_{Z}X^{*}_{N} \rightarrow {}^{A}_{Z}X^{(*)}_{N}$$

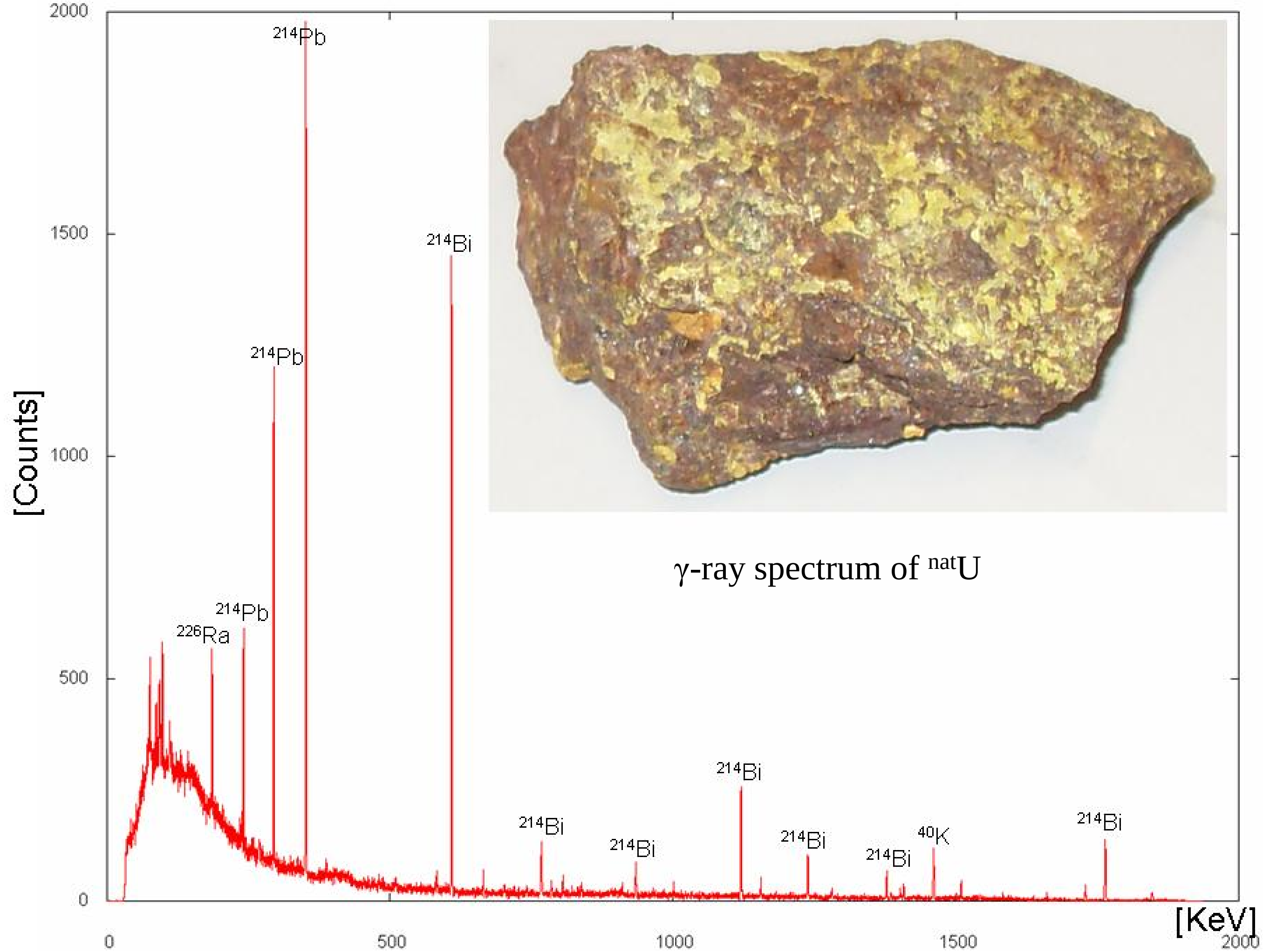
γ-decay in a Nutshell

- The photon emission of the nucleus essentially results from a re-ordering of nucleons within the shells.
- This re-ordering often follows α or β decay, and moves the system into a more energetically favorable state.

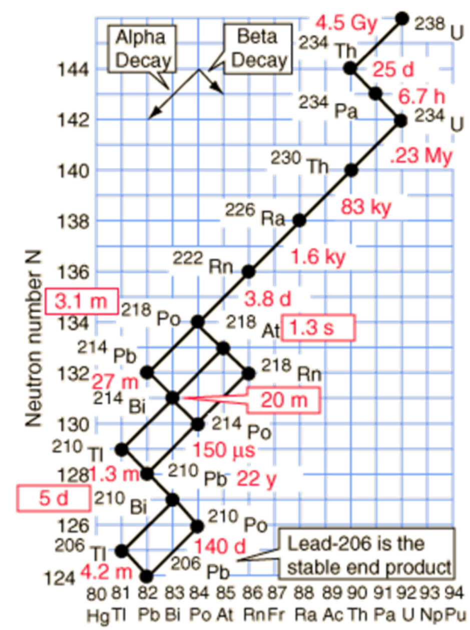


Source: Krane, Fig. 5.11

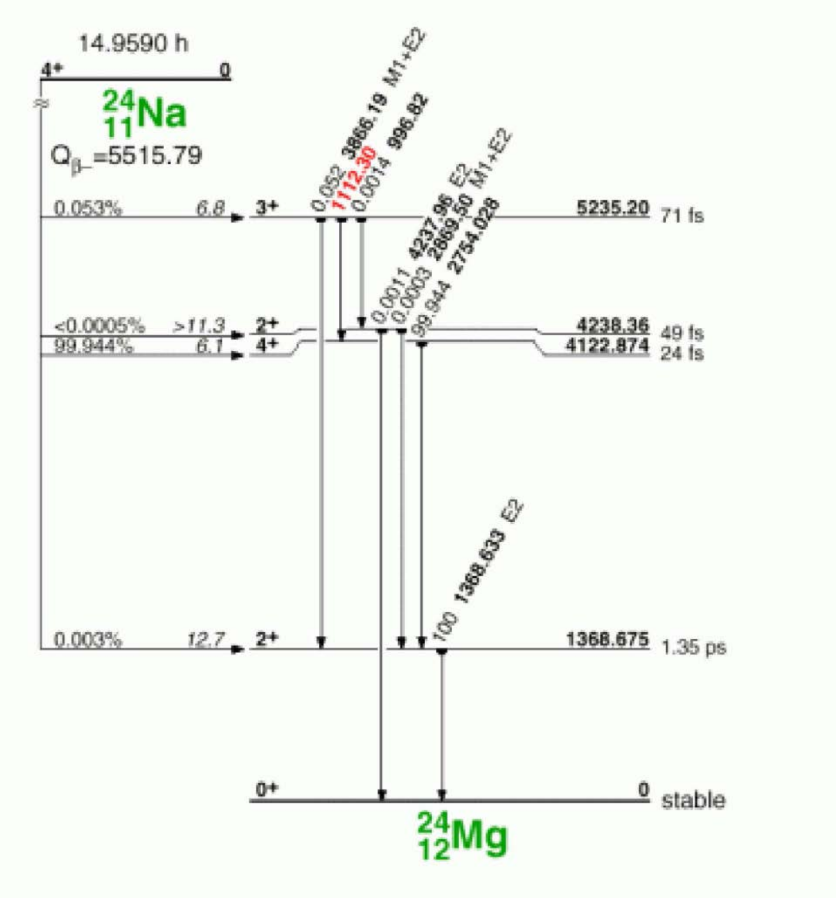
γ-decay



γ-ray spectrum of ^{nat}U



Most β-decay transitions are followed by γ-decay.



Classical Electrodynamics

- The nucleus is a collection of moving charges, which can induce magnetic/electric fields
- The power radiated into a small area element is proportional to $sin^2(\theta)$
- The average power radiated for an electric dipole is:

$$P = \frac{1}{12\pi\epsilon_0}\frac{\omega^4}{c^3}d^2$$

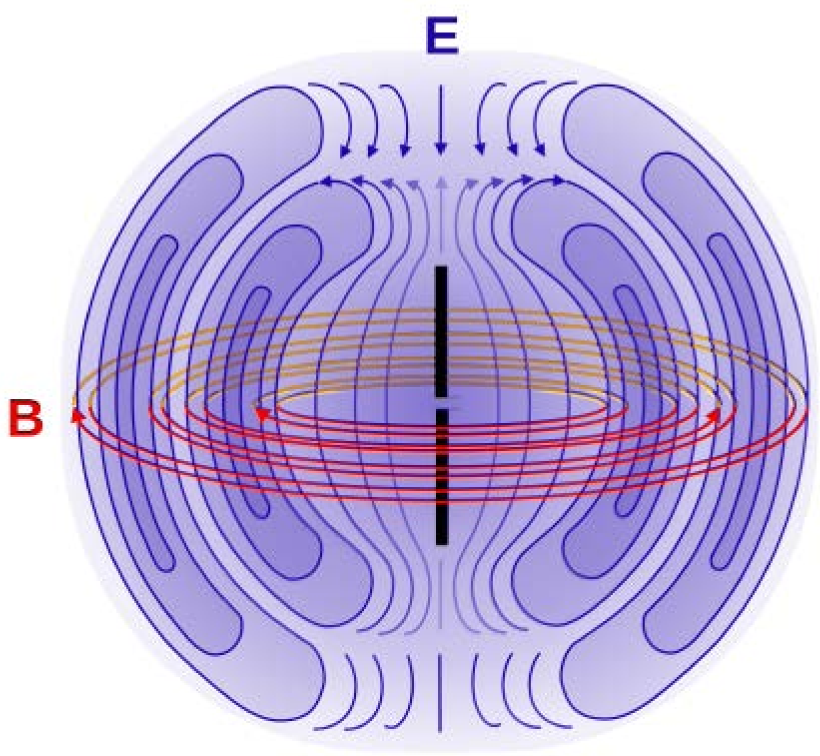
- For a magnetic dipole is

$$P = \frac{1}{12\pi\epsilon_0}\frac{\omega^4}{c^5}\mu^2$$

Electric/Magnetic Dipoles

Electric and magnetic dipole fields have opposite parity:
Magnetic dipoles have even parity and electric dipole fields have odd parity.

$$\Rightarrow \ \pi(M\ell) = (-1)^{\ell+1} \quad and \quad \pi(E\ell) = (-1)^{\ell}$$



Higher Order Multipoles

It is possible to describe the angular distribution of the radiation field as a function of the *multipole order* using Legendre polynomials.

- ℓ: The index of radiation
  2^{ℓ}: The multipole order of the radiation

- $\ell = 1 \quad \rightarrow \quad Dipole$
  $\ell = 2 \quad \rightarrow \quad Quadrupole$
  $\ell = 3 \quad \rightarrow \quad Octupole$

- The associated Legendre polynomials $P_{2\ell}\big(cos(\theta)\big)$ are:
  For $\ell = 1$: $P_2 = \frac{1}{2}(3 \cdot cos^2(\theta) - 1)$
  For $\ell = 2$: $P_4 = \frac{1}{8}(35cos^4(\theta) - 30cos^2(\theta) + 3)$

Angular Momentum in γ-Decay

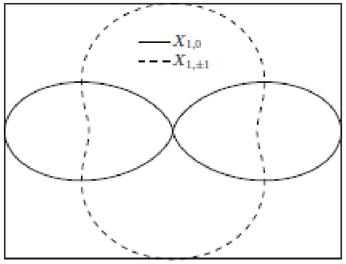
- *The photon is a spin-1 boson*
- Like α-decay and β-decay the emitted γ-ray can carry away units of *angular momentum* ℓ, which has given us different multipolarities for transitions.
- For orbital angular momentum, we can have values $\ell = 0,1,2,3,\cdots$ that correspond to our multipolarity.
- Therefore, our selection rule is:

$$|J_i - J_f| \leq \ell \leq |J_i + J_f|$$

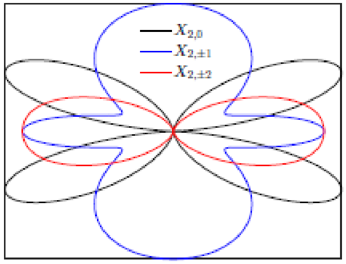
Characteristics of multipolarity

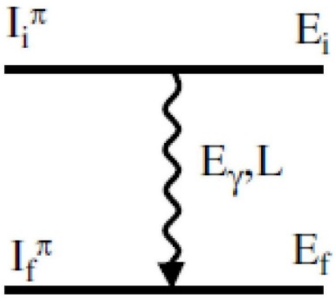
L	multipolarity	π(Eℓ) / π(Mℓ)	angular distribution
1	dipole	-1 / +1	
2	quadrupole	+1 / -1	
3	octupole	-1 / +1	
4	hexadecapole	+1 / -1	
⋮			

ℓ = 1



ℓ =2





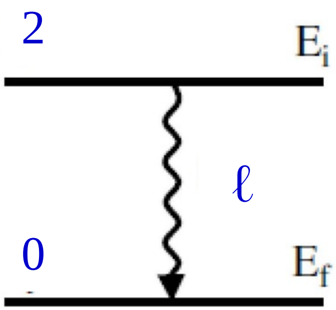
$$E_\gamma = E_i - E_f$$

$$|I_i - I_f| \leq \ell \leq I_i + I_f$$

$$\Delta\pi(E\ell) = (-1)^\ell$$

$$\Delta\pi(M\ell) = (-1)^{\ell+1}$$

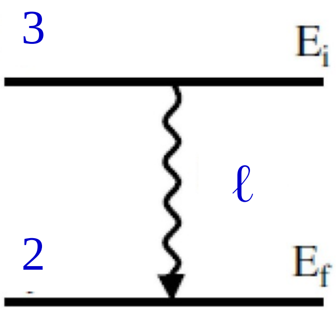
The basics of the situation



$$|2-0| \leq \ell \leq 2+0$$

Here $\Delta J = 2$ and $\ell = 2$
this is a stretched transition

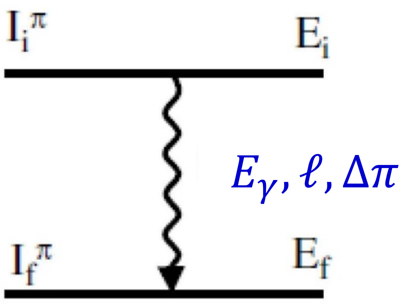
The basics of the situation



$$|3-2| \leq \ell \leq 3+2$$

Here $\Delta J = 1$ but $\ell = 1,2,3,4,5$
and the transition can be a mix of 5 multipolarities

The basics of the situation



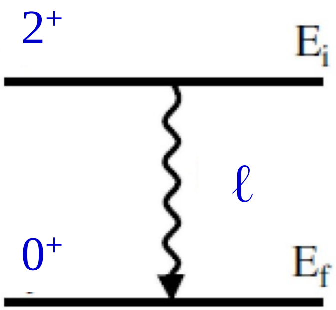
Electromagnetic transitions:

$$\Delta\pi\ (electric) = (-1)^{\ell}$$

$$\Delta\pi\ (magnetic) = (-1)^{\ell+1}$$

$\Delta\pi$	*yes*	E1	M2	E3	M4
	no	M1	E2	M3	E4

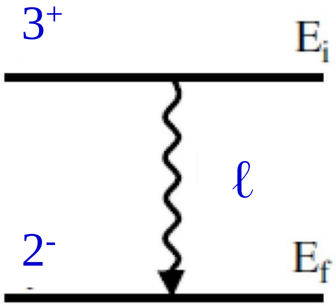
The basics of the situation



$$|2-0| \leq \ell \leq 2+0$$

$\ell = 2$ and no change in parity

$\Delta\pi$					
	no	M1	E2	M3	E4

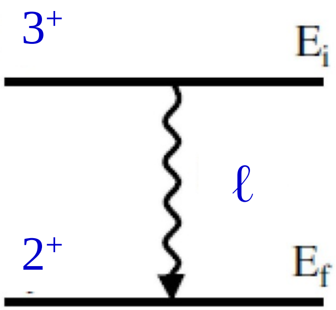


$$|3-2| \leq \ell \leq 3+2$$

Here $\Delta J = 1$ but $\ell = 1,2,3,4,5$

$\Delta\pi$	*yes*	E1	M2	E3	M4

mixed E1,M2,E3,M4,E5



$$|3-2| \leq \ell \leq 3+2$$

Here $\Delta J = 1$ but $\ell = 1,2,3,4,5$

$\Delta\pi$					
	no	M1	E2	M3	E4

mixed M1,E2,M3,E4,M5

The basics of the situation

$3^+ \rightarrow 2^-$: mixed M1,E2,M3,E4,M5

$3^+ \rightarrow 2^+$: mixed E1,M2,E3,M4,E5

In general only the lowest 2 multipoles compete

and (for reasons we will see later)

$\ell + 1$ multipole generally only competes if it is electric:

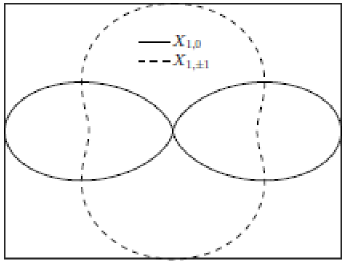
$3^+ \rightarrow 2^+$: mixed M1/E2

$3^+ \rightarrow 2^-$: almost pure E1 (very little M2 admixture)

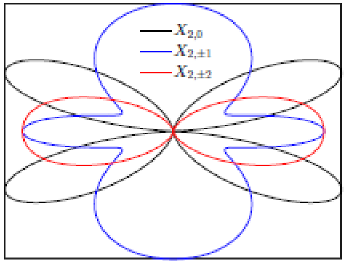
Characteristics of multipolarity

L	multipolarity	π(Eℓ) / π(Mℓ)	angular distribution
1	dipole	-1 / +1	
2	quadrupole	+1 / -1	
3	octupole	-1 / +1	
4	hexadecapole	+1 / -1	
⋮			

ℓ = 1



ℓ =2



parity: electric multipoles $\pi(E\ell) = (-1)^{\ell}$, magnetic multipoles $\pi(M\ell) = (-1)^{\ell+1}$

The power radiated is proportional to:

$$P(\sigma\ell) \propto \frac{2(\ell+1)\cdot c}{\varepsilon_0 \cdot \ell \cdot [(2\ell+1)!!]^2} \left(\frac{\omega}{c}\right)^{2\ell+2} |\mathcal{M}(\sigma\ell)|^2$$

where σ means either E or M and $\mathcal{M}(\sigma\ell)$ is the E or M multipole moment of the appropriate kind.

Emission of electromagnetic radiation

$$T(E1; I_i \to I_f) = 1.590\ 10^{17}\ E_\gamma^3\ B(E1; I_i \to I_f)$$

$$T(E2; I_i \to I_f) = 1.225\ 10^{13}\ E_\gamma^5\ B(E2; I_i \to I_f)$$

$$T(E3; I_i \to I_f) = 5.709\ 10^{8}\ E_\gamma^7\ B(E3; I_i \to I_f)$$

$$T(E4; I_i \to I_f) = 1.697\ 10^{4}\ E_\gamma^9\ B(E4; I_i \to I_f)$$

$$T(M1; I_i \to I_f) = 1.758\ 10^{13}\ E_\gamma^3\ B(M1; I_i \to I_f)$$

$$T(M2; I_i \to I_f) = 1.355\ 10^{7}\ E_\gamma^5\ B(M2; I_i \to I_f)$$

$$T(M3; I_i \to I_f) = 6.313\ 10^{0}\ E_\gamma^7\ B(M3; I_i \to I_f)$$

$$T(M4; I_i \to I_f) = 1.877\ 10^{-6}\ E_\gamma^9\ B(M4; I_i \to I_f)$$

where $E_\gamma = E_i - E_f$ is the energy of the emitted γ quantum in MeV (E_i, E_f are the nuclear level energies, respectively), and the reduced transition probabilities B(Eℓ) in units of $e^2(\mathrm{barn})^\ell$ and B(Mℓ) in units of $\mu_N^2 = (e\hbar/2m_N c)^2\ \ (fm)^{2\ell-2}$

Single particle transition (Weisskopf estimate)

$$B(E\lambda; I_i \to I_{gs}) = \frac{(1.2)^{2\lambda}}{4\pi}\left(\frac{3}{\lambda+3}\right)^2 A^{2\lambda/3} \quad e^2(fm)^{2\lambda}$$

$$B(M\lambda; I_i \to I_{gs}) = \frac{10}{\pi}(1.2)^{2\lambda-2}\left(\frac{3}{\lambda+3}\right)^2 A^{(2\lambda-2)/3} \quad \mu_N^2(fm)^{2\lambda-2}$$

For the first few values of λ, the Weisskopf estimates are

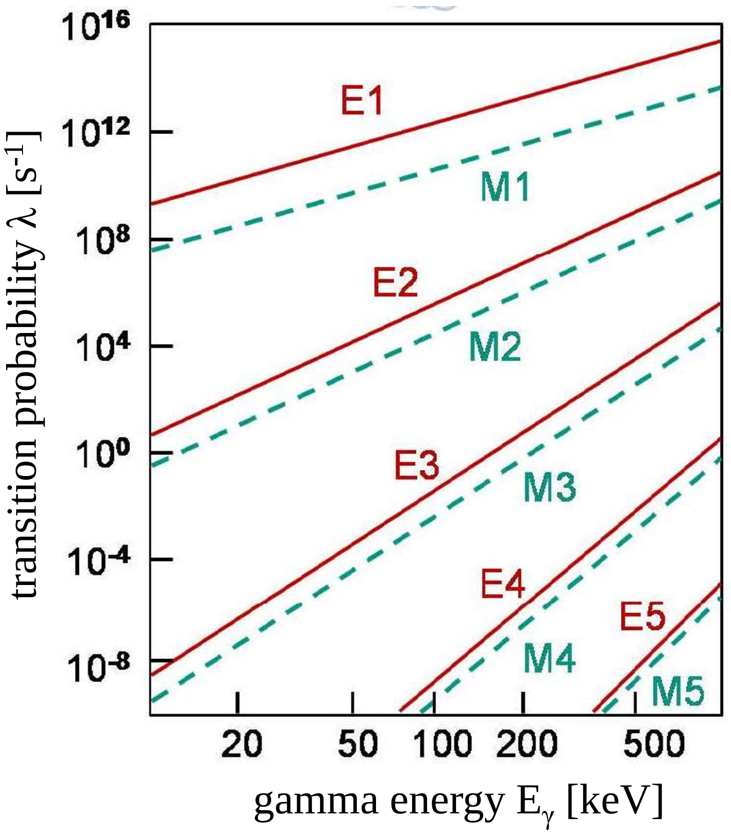
$$B(E1; I_i \to I_{gs}) = 6.446\ 10^{-4}\ A^{2/3} \quad e^2(barn)$$

$$B(E2; I_i \to I_{gs}) = 5.940\ 10^{-6}\ A^{4/3} \quad e^2(barn)^2$$

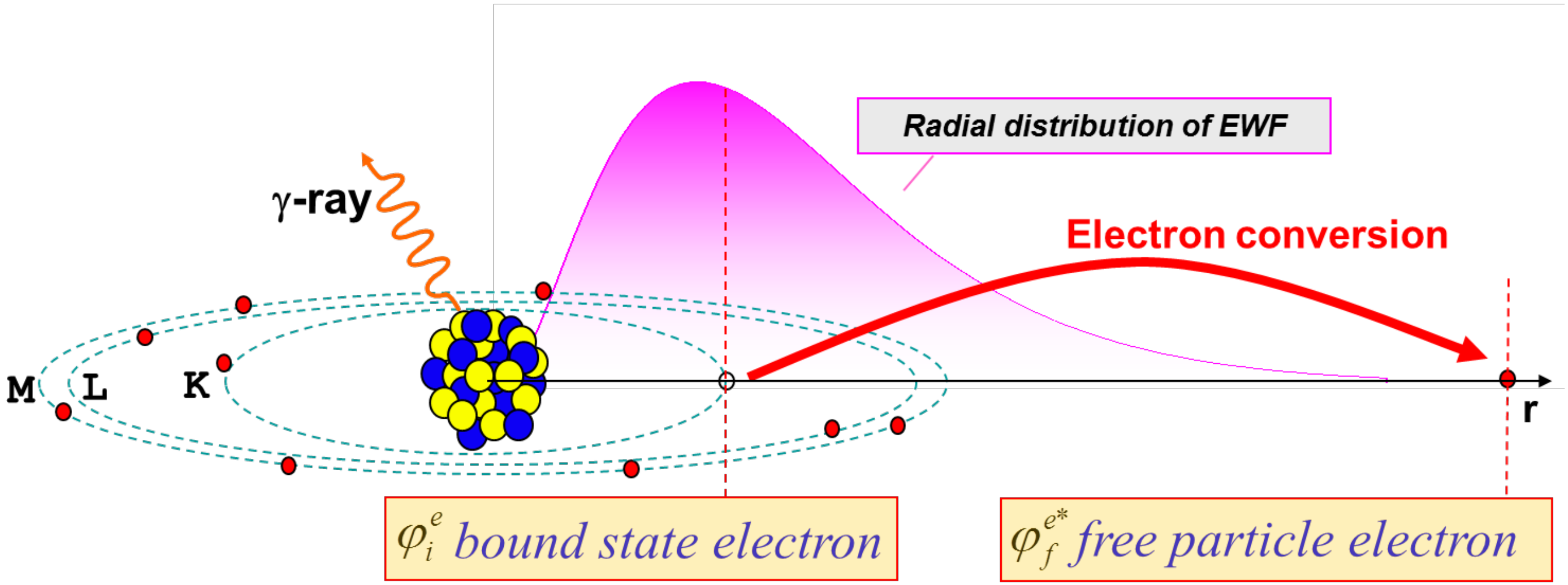
$$B(E3; I_i \to I_{gs}) = 5.940\ 10^{-8}\ A^{2} \quad e^2(barn)^3$$

$$B(E4; I_i \to I_{gs}) = 6.285\ 10^{-10}\ A^{8/3} \quad e^2(barn)^4$$

$$B(M1; I_i \to I_{gs}) = 1.790\ \left(\frac{e\hbar}{2Mc}\right)^2$$



Conversion electrons



Energetics of CE-decay (i=K, L, M,….)

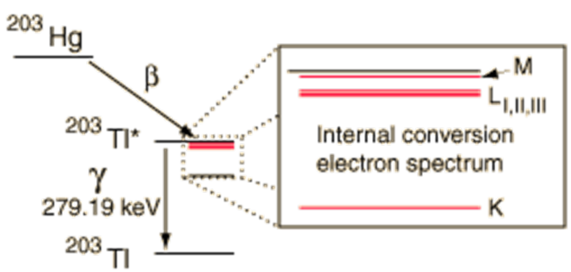
$$E_i = E_f + E_{ce,i} + E_{BE,i}$$

γ- and CE-decays are independent; transition probability (λ ~ Intensity)

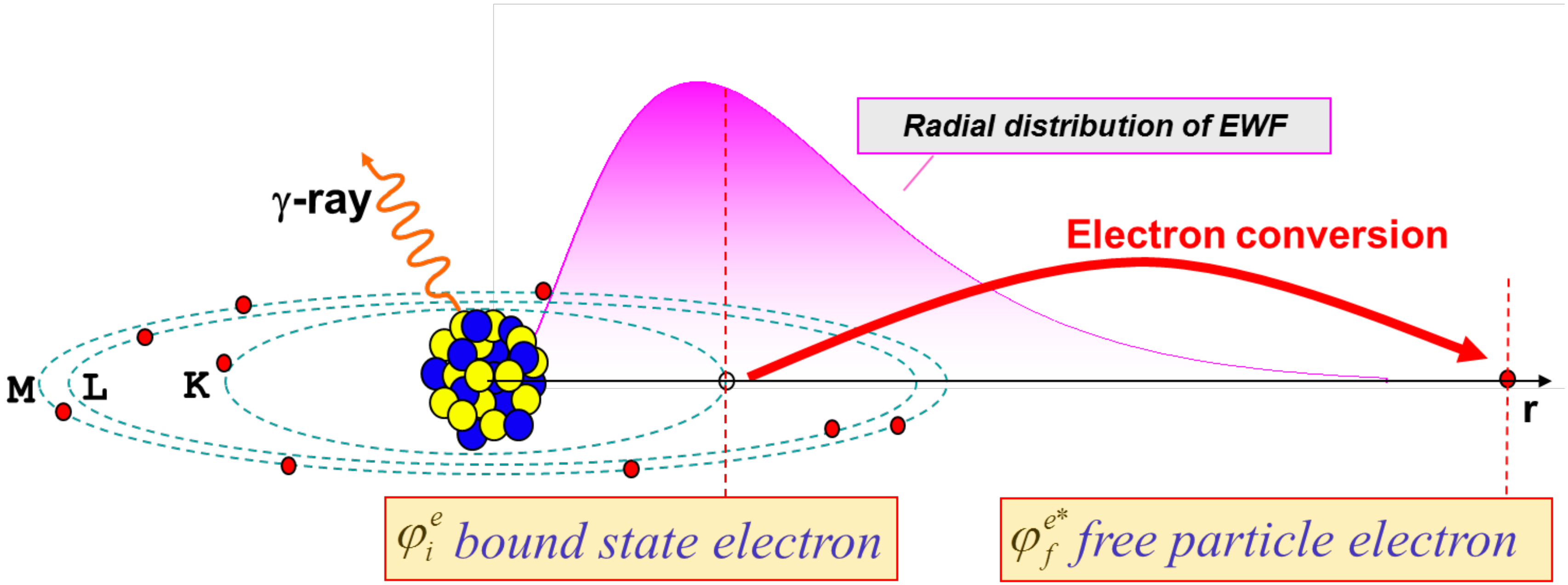
$$\lambda_T = \lambda_\gamma + \lambda_{CE} = \lambda_\gamma + \lambda_K + \lambda_L + \lambda_M \ldots\ldots$$

Conversion coefficient

$$\alpha_i = \frac{\lambda_{CE,i}}{\lambda_\gamma}$$



Internal conversion



- For an electromagnetic transition internal conversion can occur instead of emission of gamma radiation. In this case the transition energy $Q = E_\gamma$ will be transferred to an electron of the atomic shell.

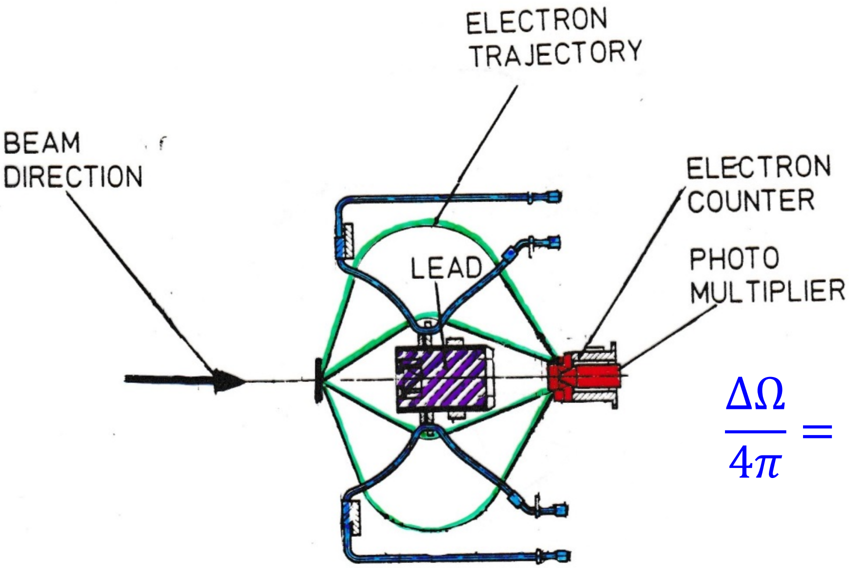
$$T_e = E_\gamma - B_e$$

T_e: kinetic energy of the electron
B_e: binding energy of the electron

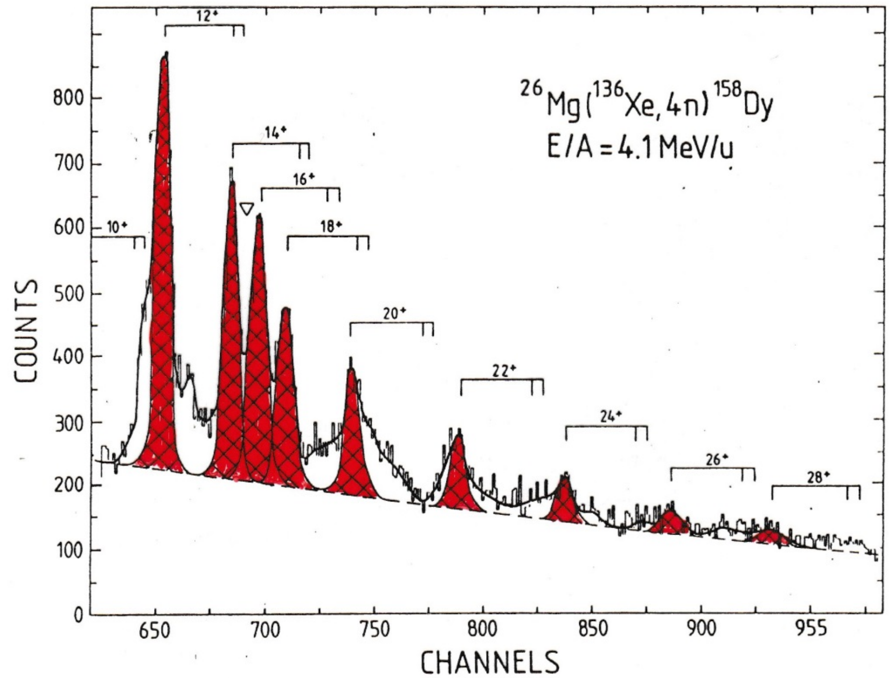
internal conversion is important for:
- heavy nuclei ~ Z^3
- high multipolarities Eℓ or Mℓ
- small transition energies

$$\alpha_k(El) \propto Z^3 \left(\frac{L}{L+1}\right)\left(\frac{2m_e c^2}{E}\right)^{L+5/2}$$

Electron spectroscopy



$$\frac{\Delta\Omega}{4\pi} = 26\%$$



Doppler shift correction for projectile:

$$T_e^* = \gamma \cdot T_e \cdot \left\{1 - \beta_1 \cdot \sqrt{1 + 2m_e\,c^2/T_e} \cdot cos\theta_{e1}\right\} + m_e c^2 \cdot (\gamma - 1)$$

$$cos\theta_{e1} = cos\vartheta_1 cos\vartheta_e + sin\vartheta_1 sin\vartheta_e cos(\varphi_e - \varphi_1)$$

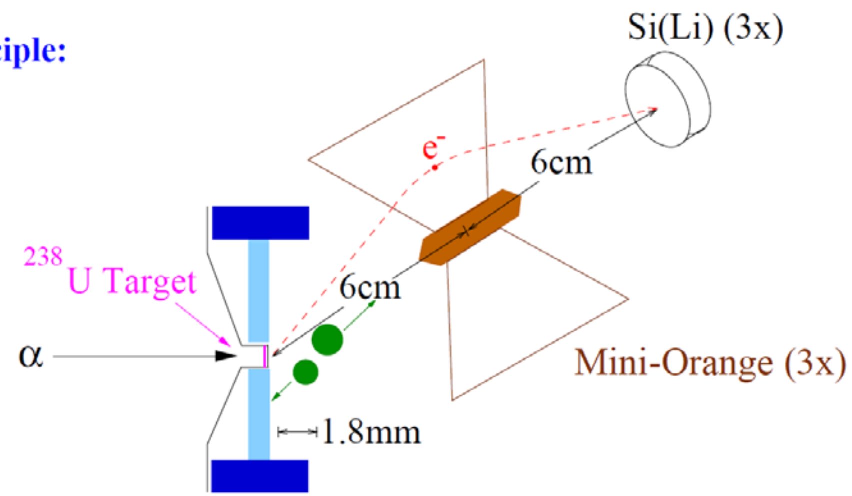
		$(\frac{\Delta p}{p})_e$ / %
resolution of the spectrometer including Doppler correction as calculated for a point source		0.4
scattering in the target	(i)	0.004
beam optics	(ii)	0.11
evaporation of neutrons	(iii)	0.09
energy loss in the target	(iv)	0.31
energy straggling of the projectiles	(v)	0.006
quadratic sum		0.53
experimental resolution		0.56

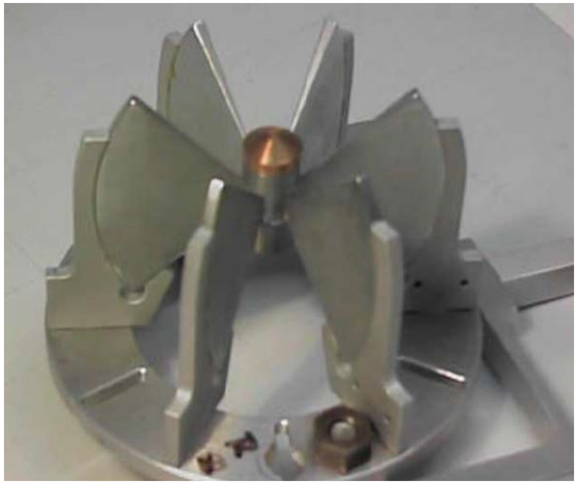
%

GSI

Mini Orange setup for conversion electron spectroscopy

Principle:





Comparison of α-decay, β-decay and γ-decay

de Broglie wavelength: $\lambda = \frac{h}{p} = \frac{h \cdot c}{\sqrt{E_{kin} \cdot (E_{kin} + 2mc^2)}} = \frac{1239.84[MeV\ fm]}{\sqrt{E_{kin} \cdot (E_{kin} + 2mc^2)}}$

decay	Energy [MeV]	de Broglie λ [fm]
α-particle, m_α = 3727 MeV/c²	5	6.42
β-particle, m_e = 0.511 MeV/c²	1	871.92
γ-photon	1	$\lambda = {}^{h \cdot c}/_{E} = {}^{1240}/_{E}$

For α-particles this dimension is somewhat smaller than the nucleus and this is why a semi-classical treatment of α-decay is successful.
The typical β-particle has a large wavelength λ in comparison to the nuclear size and a quantum mechanical is dictated and wave analysis is called for.
For γ-decay the wavelength λ ranges from 12400 – 1240 fm (0.1 – 1 MeV). Clearly, only a quantum mechanical approach has a chance of success.

γ-decay

γ-spectroscopy yields some of the most precise knowledge of nuclear structure, as spin, parity and ΔE are all measurable.

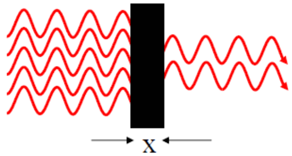
Transition rates between initial Ψ_N^* and final $\Psi_N^{´}$ nuclear states, resulting from electromagnetic decay producing a photon with energy E_γ can be described by Fermi´s Golden rule:

$$\lambda = \frac{2\pi}{\hbar} \left| \langle \Psi_N^{´} \psi_\gamma | \mathcal{M}_{em} | \Psi_N^* \rangle \right|^2 \frac{dn_\gamma}{dE_\gamma}$$

where $\mathcal{M}_{em}$ is the electromagnetic transition operator and dn_γ/dE_γ is the density of final states. The photon wave function ψ_γ and $\mathcal{M}_{em}$ are well known, therefore measurements of λ provide detailed knowledge of nuclear structure.

A γ-decay lifetime is typically 10^{-12} [s] and sometimes even as short as 10^{-19} [s]. However, this time span is an eternity in the life of an excited nucleon. It takes about $4\cdot10^{-22}$ [s] for a nucleon to cross the nucleus.

Interaction of gamma rays with matter

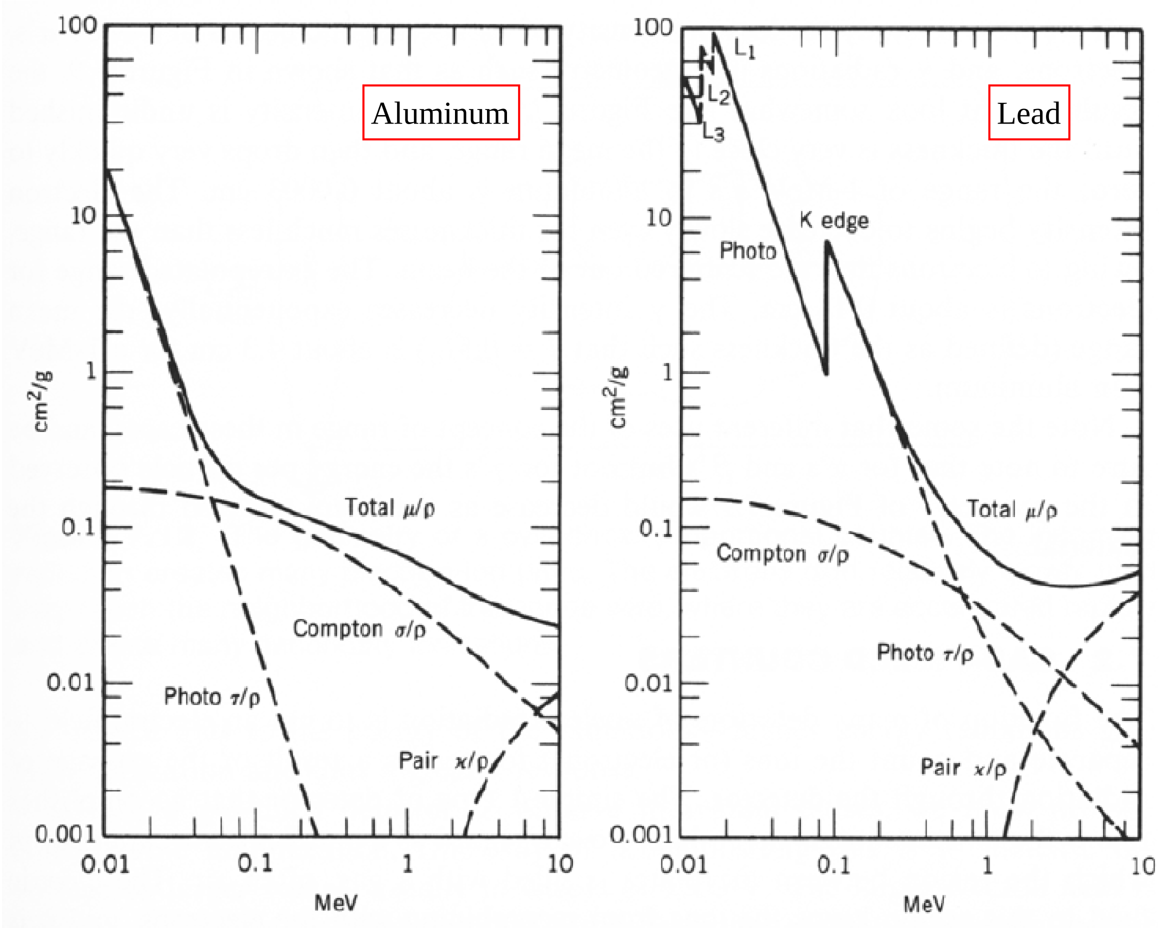


$$I(x) = I_0(\lambda) \cdot e^{-\frac{\mu(\lambda,Z)}{\rho}\rho \cdot x}$$

total absorption coefficient: **μ/ρ** [cm²/g]

$$\frac{\mu_{total}}{\rho} = \sum_{i=1}^{3} \sigma_i$$

i=1 photoelectric effect
i=2 Compton scattering
i=3 pair production

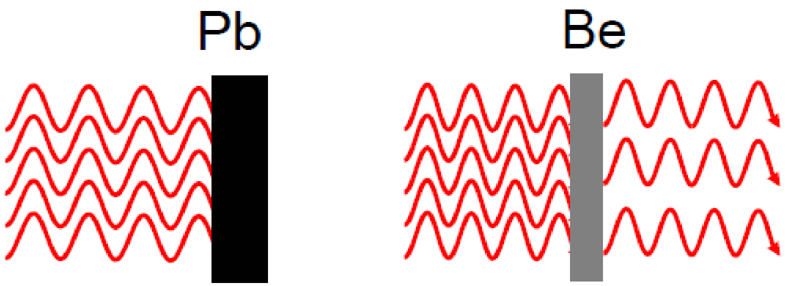


Mass dependence of X-ray absorption

For X-ray radiation the photoelectric effect is the most important interaction.

$$(\mu/\rho)_{Photo} \approx \lambda^3 \cdot Z^5$$

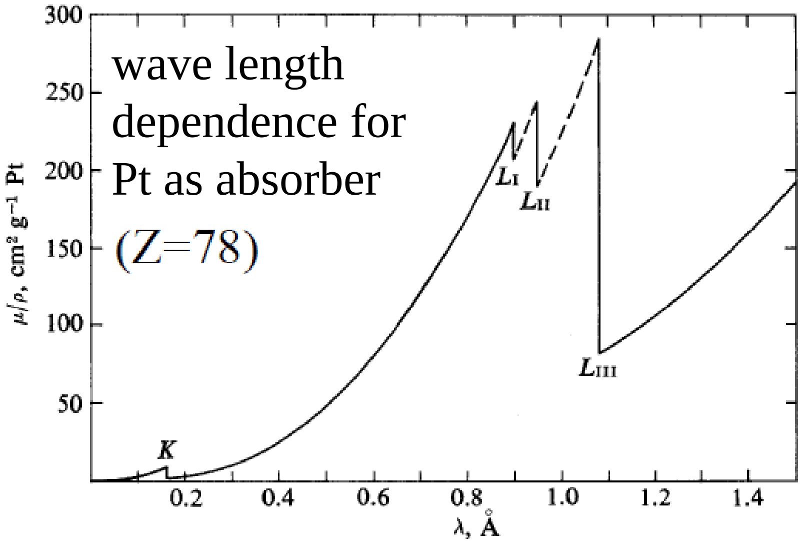
Lead absorbs more than Beryllium!

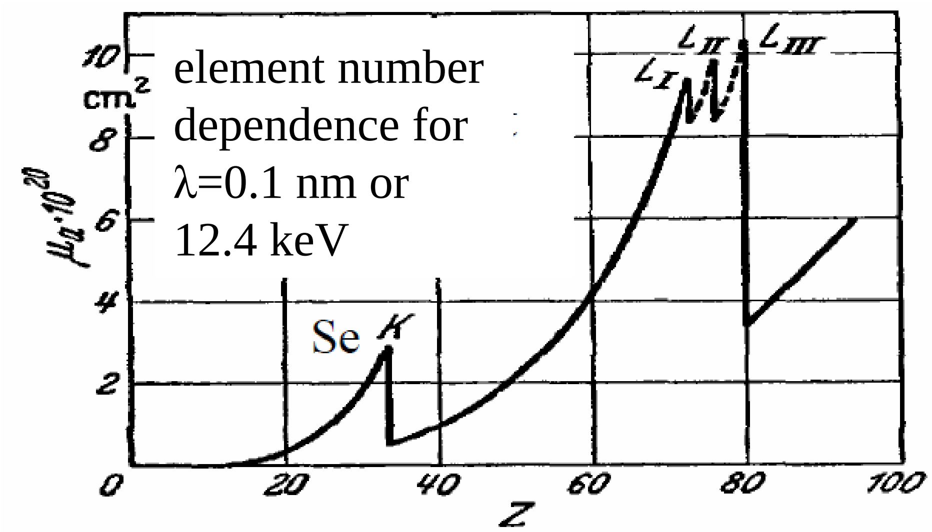


$_{82}$Pb serves as shielding for X-ray and γ-ray radiation; lead vests are used by medical staff people who are exposed to X-ray radiation. Co-sources are transported in thick lead container.

On the contrary:
$_4$Be is often used as windows in X-ray tubes to allow for almost undisturbed transmission of X-ray radiation.

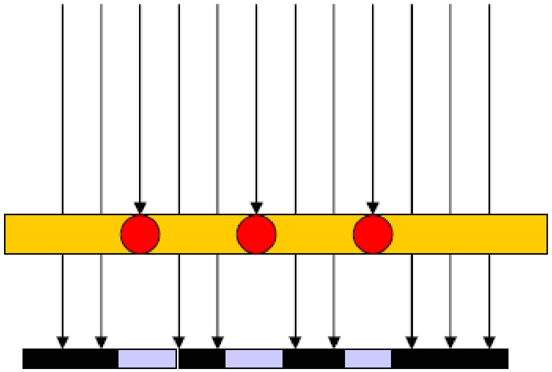
Mass dependence μ/ρ of X-ray absorption

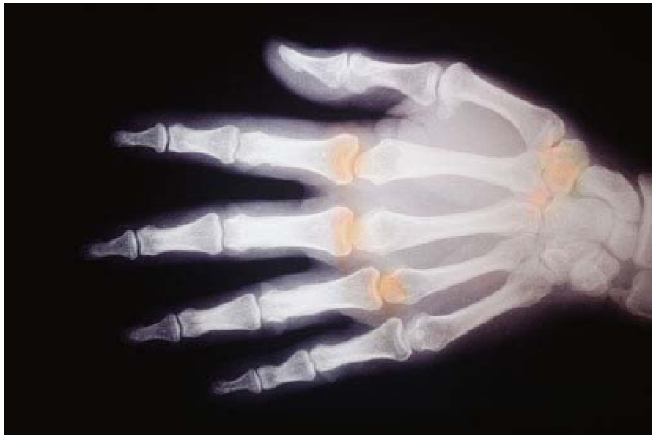




X-ray image shows the effect of different absorptions

Bones absorb more radiation as tissues because of their higher $_{20}$Ca content

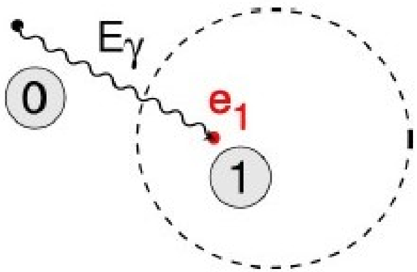




Interaction of gamma rays with matter

~ 100 keV | ~1 MeV | ~ 10 MeV | γ-ray energy

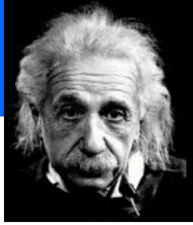
Photoelectric



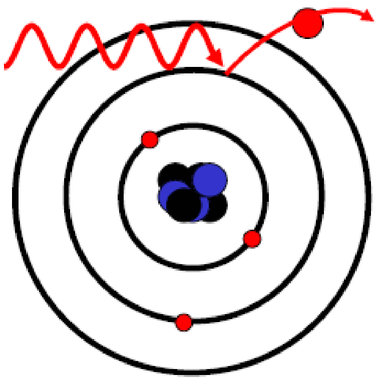
Isolated hits

Probability of
interaction depth

Interaction of gamma rays with matter



$$E_{e,kin} = h \cdot \nu - E_{Bindung}$$



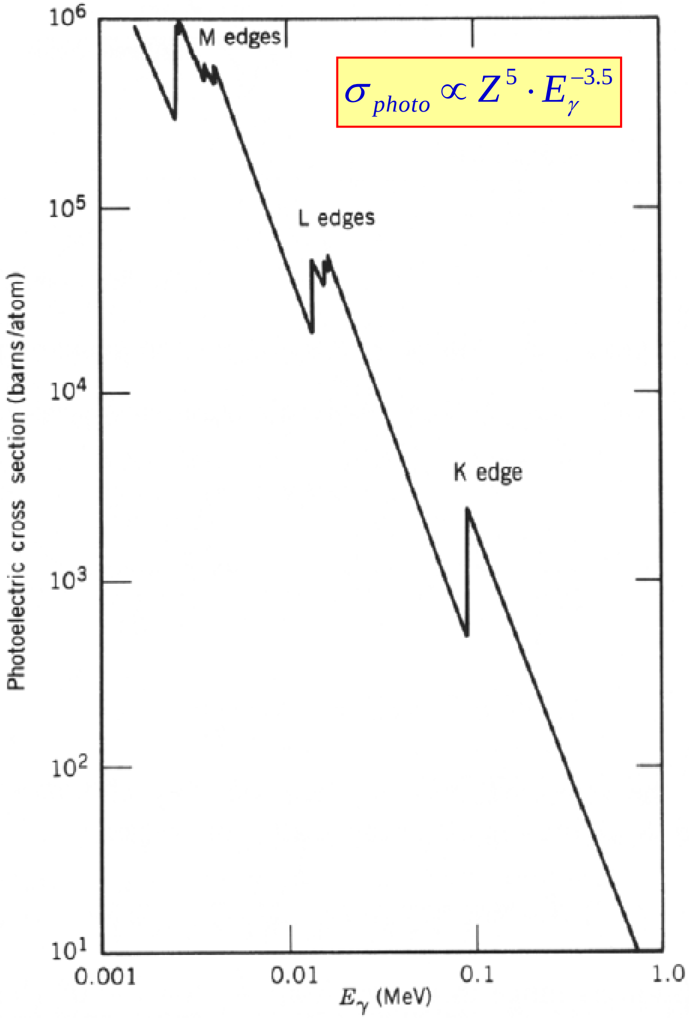
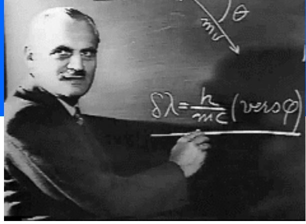


Photo effect:

Absorption of a photon by a bound electron and conversion of the γ-energy in potential and kinetical energy of the ejected electron.

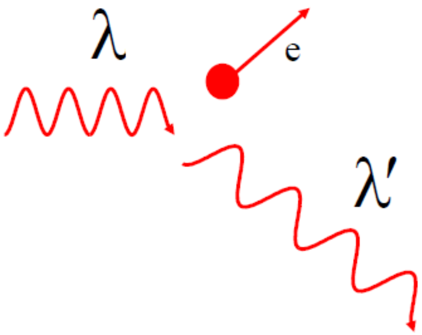
(Nucleus preserves the momentum conservation.)

Interaction of gamma rays with matter



relativistic $E^2 = (pc)^2 + (m_0c^2)^2$ $\quad$ $photons: \; m_0 = m_\gamma = 0$

$$\rightarrow E_\gamma = p_\gamma c$$



Momentum balance:

$$\overrightarrow{p_e} = \overrightarrow{p_\gamma} - \overrightarrow{p'_\gamma} \quad \rightarrow \quad |\overrightarrow{p_e}c|^2 = \left|\left(\overrightarrow{p_\gamma} - \overrightarrow{p'_\gamma}\right) c\right|^2$$

$$p_e^2c^2 = E_\gamma^2 + E_{\gamma'}^2 - 2E_\gamma E_{\gamma'} \cdot cos\theta$$

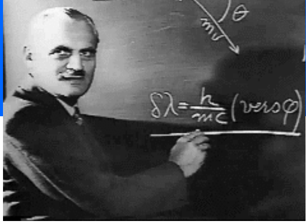
Energy balance:

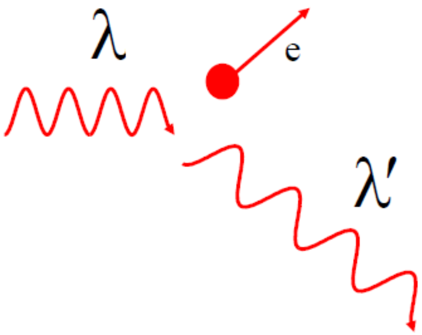
$$E_\gamma + m_ec^2 = E_{\gamma'} + \sqrt{(p_ec)^2 + (m_ec^2)^2}$$

$$E_{\gamma'} = \frac{E_\gamma}{1 + \left(E_\gamma/m_ec^2\right)(1 - cos\theta)}$$

Compton scattering:
Elastic scattering of a γ-ray on a free electron. A fraction of the γ-ray energy is transferred to the Compton electron. The wave length of the scattered γ-ray is increased: λ‘ > λ.

Interaction of gamma rays with matter





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Maximum energy of the scattered electron:

$$T\left(e^-\right)_{max} = E_\gamma \cdot \frac{2 \cdot E_\gamma}{m_e c^2 + 2 \cdot E_\gamma}$$

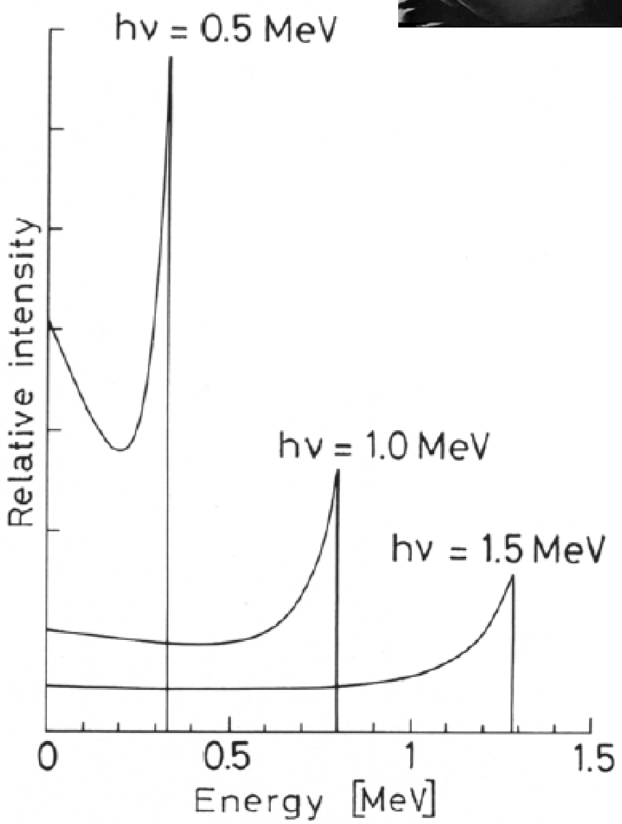
Energy of the scattered γ-photon:

$$E_\gamma' = \frac{E_\gamma \cdot m_e c^2}{m_e c^2 + E_\gamma \cdot (1 - \cos\theta)}$$

$$\cos\theta = 1 + \frac{m_e c^2}{E_\gamma} - \frac{m_e c^2}{E_\gamma'}$$

Special case for E>>$m_e c^2$:
γ-ray energy after 180^0 scatter is approximately

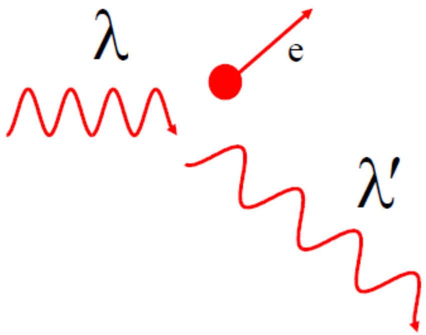
$$E_\gamma' = \frac{m_e c^2}{2} = 256\,keV$$

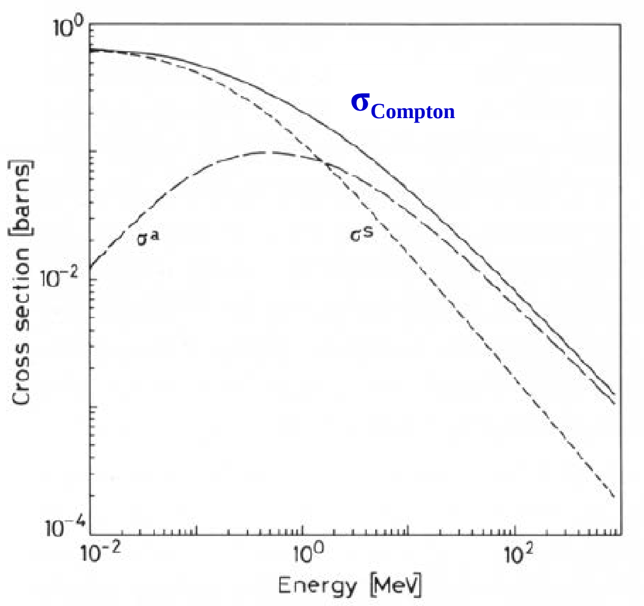


Gap between the incoming γ-ray and the maximum electron energy.

$$E_{kin}^{max} = E_\gamma - E_\gamma' = E_\gamma \cdot \frac{2 \cdot E_\gamma / m_e c^2}{1 + 2 \cdot E_\gamma / m_e c^2}$$

Interaction of gamma rays with matter





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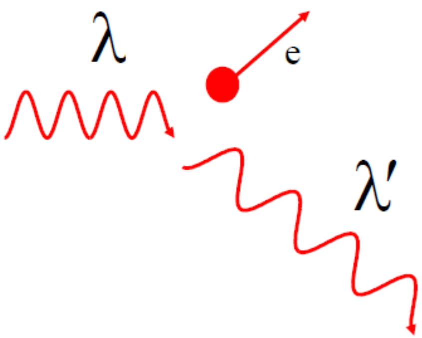
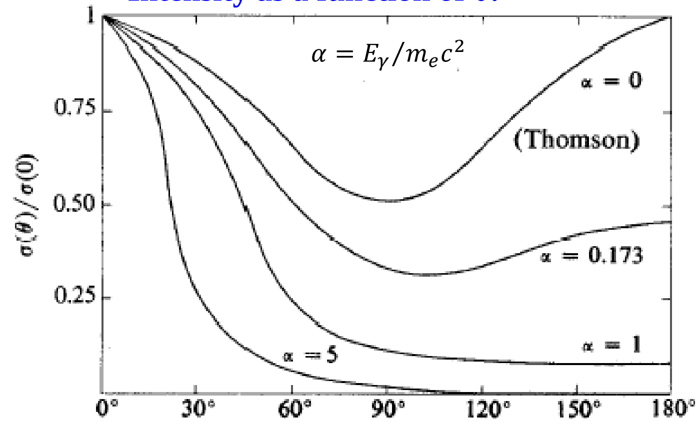
Elastic scattering of a γ-ray on a free electron. A fraction of the γ-ray energy is transferred to the Compton electron. The wave length of the scattered γ-ray is increased: λ‘ > λ.

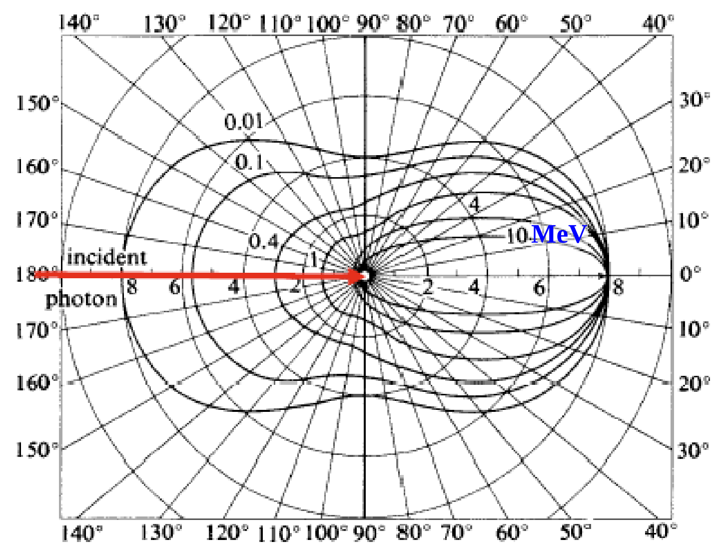
Interaction of gamma rays with matter



Intensity as a function of θ:



Angular distribution:



Compton scattering:

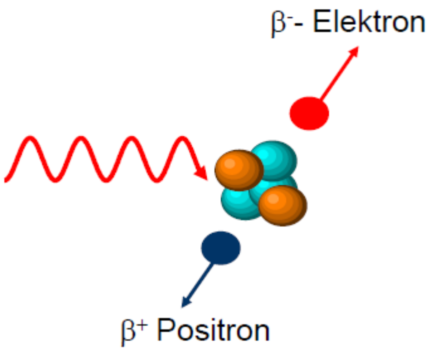
Elastic scattering of a γ-ray on a free electron.
The angle dependence is expressed by the

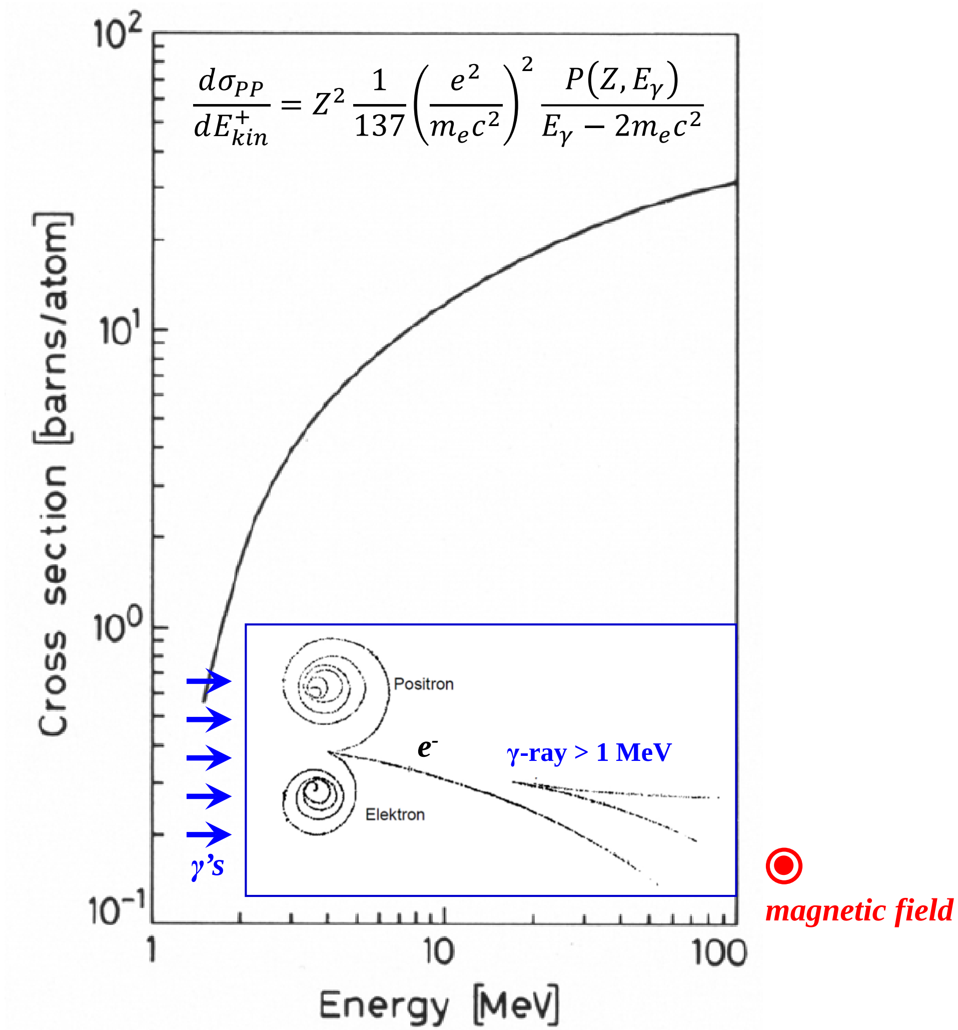
Klein-Nishina-Formula:

$$\frac{d\sigma_c}{d\Omega} = \frac{r_0^2}{2}\left(\frac{E_{\gamma'}}{E_\gamma}\right)^2 \cdot \left\{\frac{E_\gamma}{E_{\gamma'}} + \frac{E_{\gamma'}}{E_\gamma} - 2\sin^2\theta \cdot \cos^2\phi\right\}$$

As shown in the plot **forward scattering** (θ small) is dominant for E_γ>100 keV.

r_0=2.818 fm (classical electron radius)





Pair production:

If γ-ray energy is >> $2m_0c^2$ (electron rest mass 511 keV), a positron-electron pair can be formed in the strong Coulomb field of a nucleus. This pair carries the γ-ray energy minus $2m_0c^2$.

Pair production for $E_\gamma > 2m_ec^2 = 1.022$MeV

picture of a bubble chamber

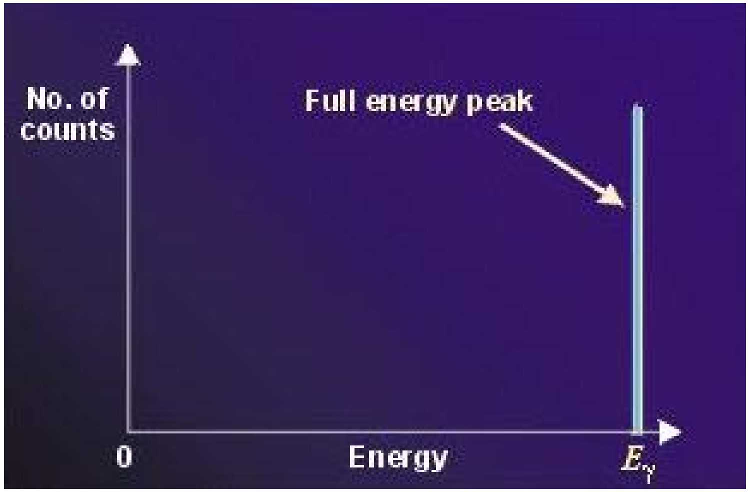
Interaction of gamma rays with matter

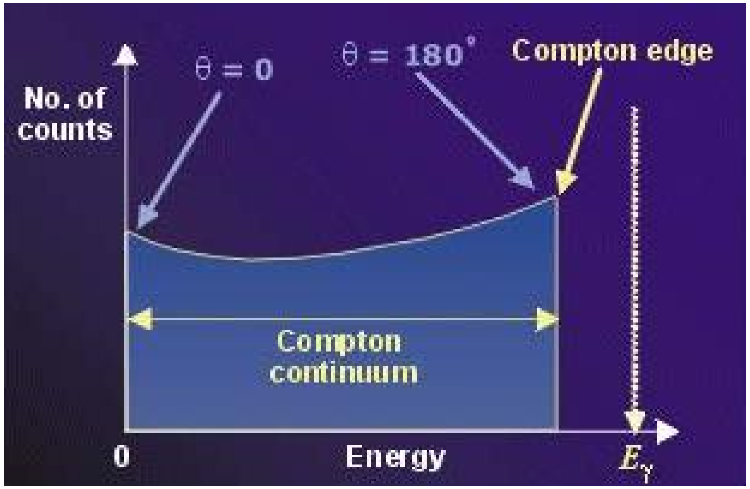
γ-rays interaction with matter via three main reaction mechanisms:

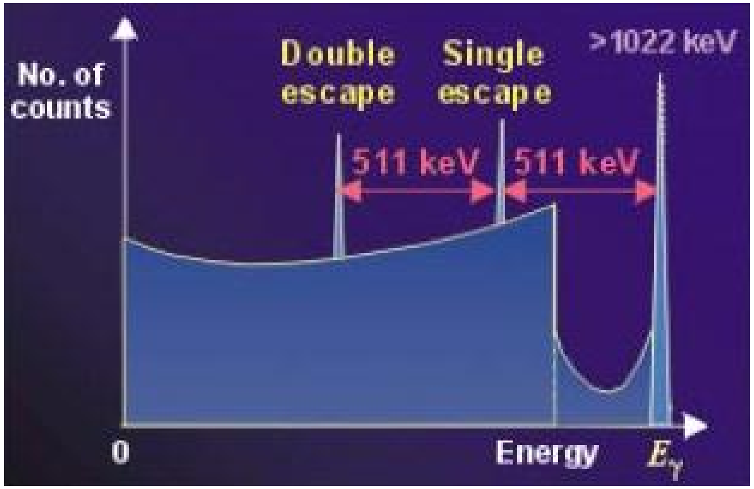
Photoelectric absorption

Compton scattering

Pair production



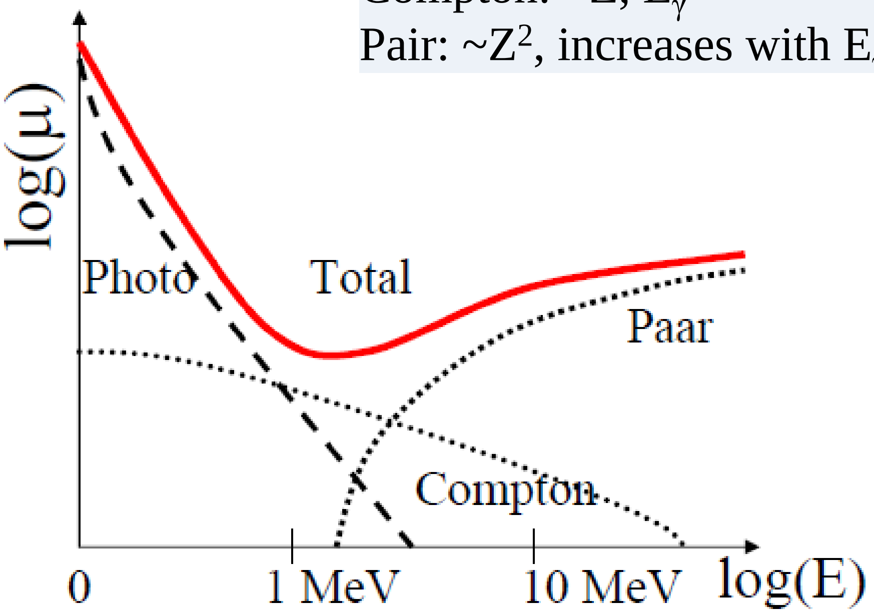


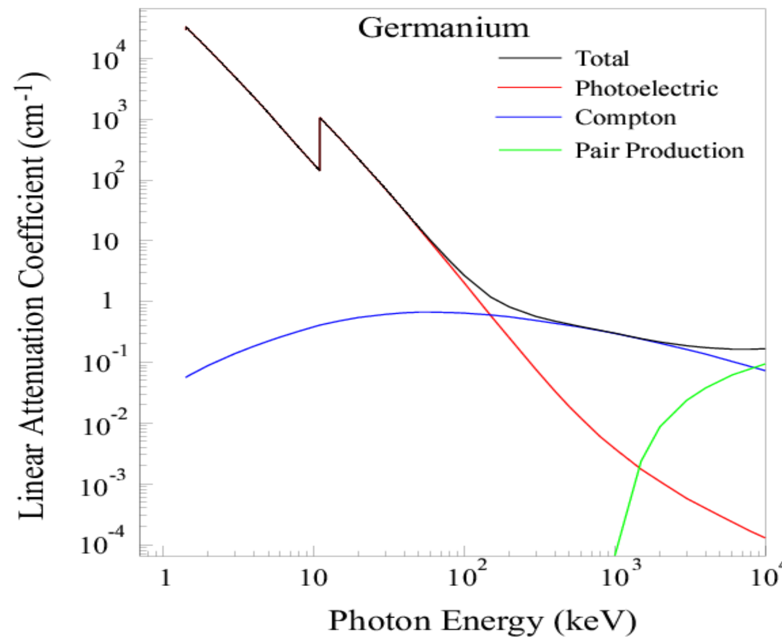


Gamma-ray interaction cross section

All three interaction (photo effect, Compton scattering and pair production) lead to an attenuation of the γ-ray or X-ray radiation when passing through matter. The particular contribution depends on the γ-ray energy:

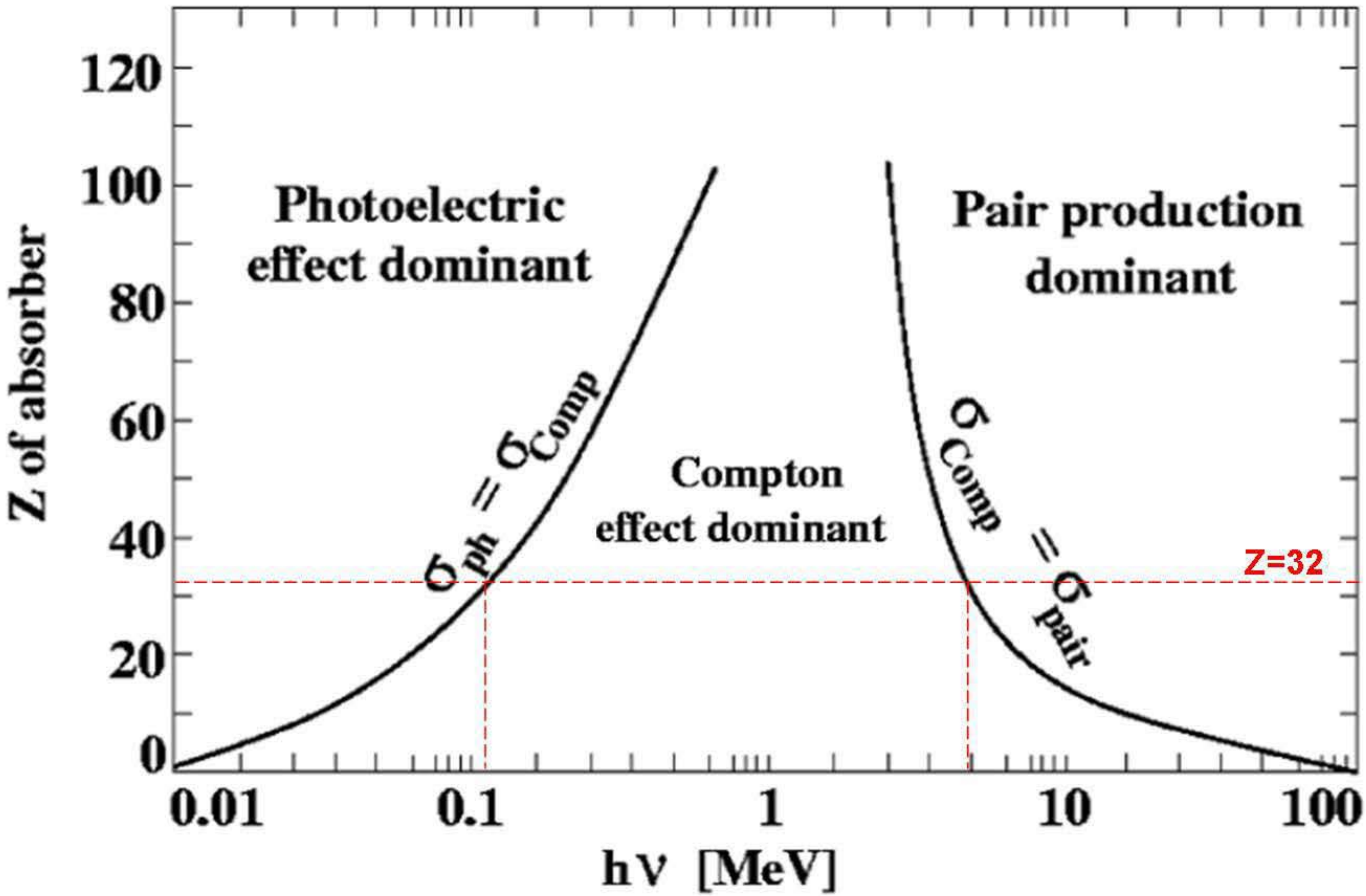
Photo effect: $\sim Z^{4\text{-}5}$, $E_\gamma^{-3.5}$
Compton: $\sim Z$, E_γ^{-1}
Pair: $\sim Z^2$, increases with E_γ





The absorption attenuates the intensity, but the energy and the frequency of the γ-ray and X-ray radiation is preserved!

Z dependence of interaction probabilities



Detector types

Solid state semiconductor detectors: Ge

Electron-hole pairs are collected as charge

knock-on effect → an avalanche arrives at the electrode

lots of electrons → good energy resolution

cooled to liquid N_2 temperature (77K) to reduce noise

Advantage: good energy resolution (~0.15% FWHM at 1.3 MeV)

Disadvantage: relative low efficiency, cryogenic operation, limited size of crystal/detector

Scintillation detectors: e.g. NaI, BGO, $LaBr_3$(Ce)

Recoiling electrons excite atoms, which then de-excite by emitting visible light

Light is collected in photomultiplier tubes (PMT) where it generates a pulse proportional to the light collected

Advantage: good time resolution

detector can be made relative large e.g. NaI detector 14"Ø x 10"

no need for cryogenics

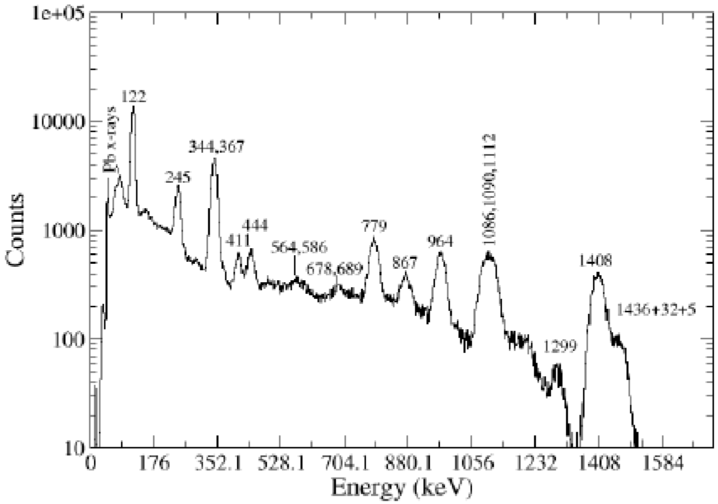
Disadvantage: poor energy resolution (~5% FWHM at 1.3 MeV

$LaBr_3(Ce)$

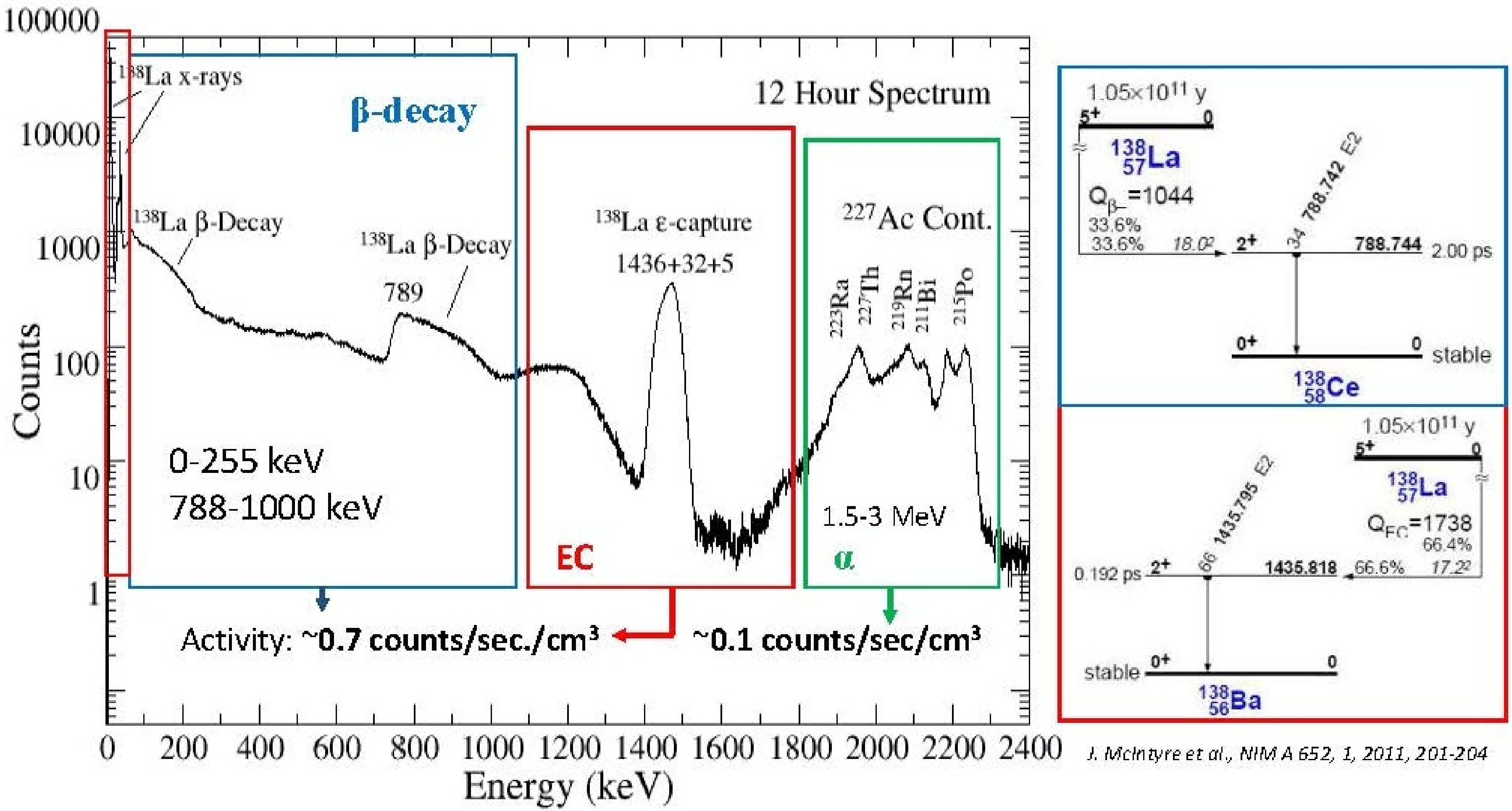
- $LaBr_3(Ce)$ timing properties:
  - ~ 25 ns decay time
  - Timing Resolution FWHM of 130-150 ps with ^{60}Co for a Ø1"x1" crystal.

- High energy resolution, 3 % FWHM at 662 keV.

- Peak Emission wavelength in Blue/UV part of EM spectrum (380 nm), compatible with PMTs.

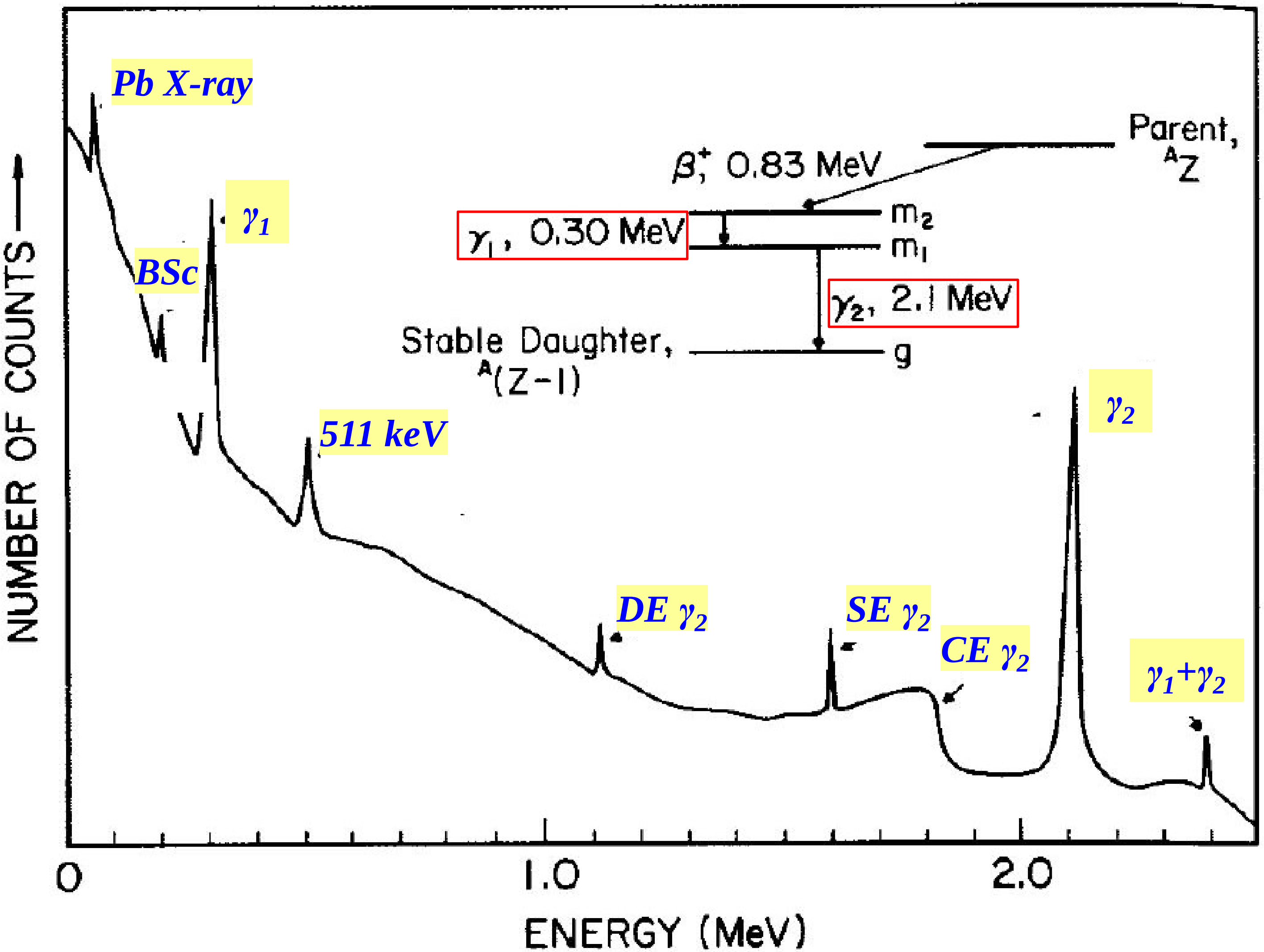


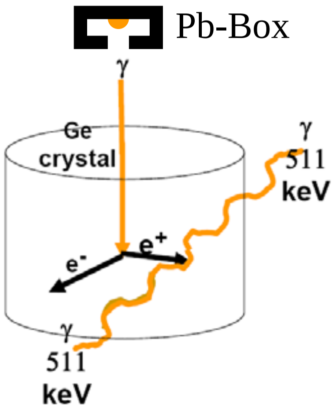
Detector characterization



J. McIntyre et al., NIM A 652, 1, 2011, 201-204

Gamma-ray spectrum of a radioactive decay





Spins and parities

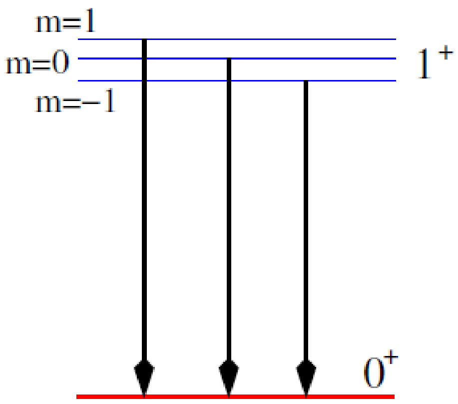
Two distinct types of measurements:

Angular correlation : can be done with a non-aligned source but need γ-γ coincidence information.

Angular distribution: need an aligned source but can be done with singles data.

...note that these cannot measure parity but you can usually infer something about the transition

The basics of the situation

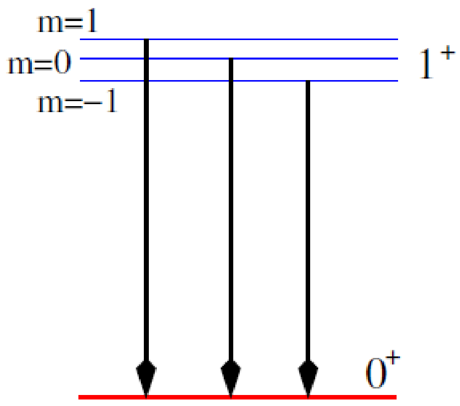


Imagine the situation of an M1 decay between two states, the initial one has J^π value of 1^+ and the final one a J^π of 0^+

The initial $J^\pi=1^+$ state has 3 degenerate magnetic substates which differ by the magnetic quantum numbers m of ± 1 and 0.

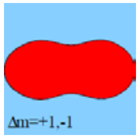
The final $J^\pi=0^+$ state has a single magnetic substate with m=0.

When the substates of $J^\pi=1^+$ state decay, the γ-rays emitted have different angular patterns.

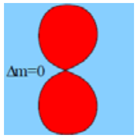


For the M1 case the angular distributions W(θ) are:

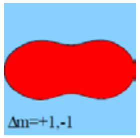
$$W_{M1,\Delta m=1}(\theta) = \frac{3}{16\pi}(1+cos^2\theta)$$



$$W_{M1,\Delta m=0}(\theta) = \frac{3}{8\pi}sin^2\theta$$

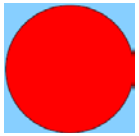


$$W_{M1,\Delta m=-1}(\theta) = \frac{3}{16\pi}(1+cos^2\theta)$$



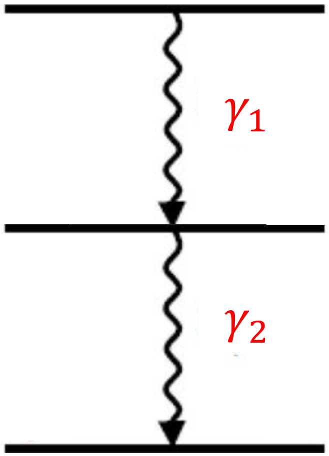
So the total distribution is $W_{M1} = \frac{1}{3}W_{M1,\Delta m=1} + \frac{1}{3}W_{M1,\Delta m=0} + \frac{1}{3}W_{M1,\Delta m=-1}$

$$= \frac{1}{8\pi}(1+cos^2\theta+sin^2\theta) = \frac{1}{4\pi}$$



no angular dependence

Angular correlation – non-oriented source

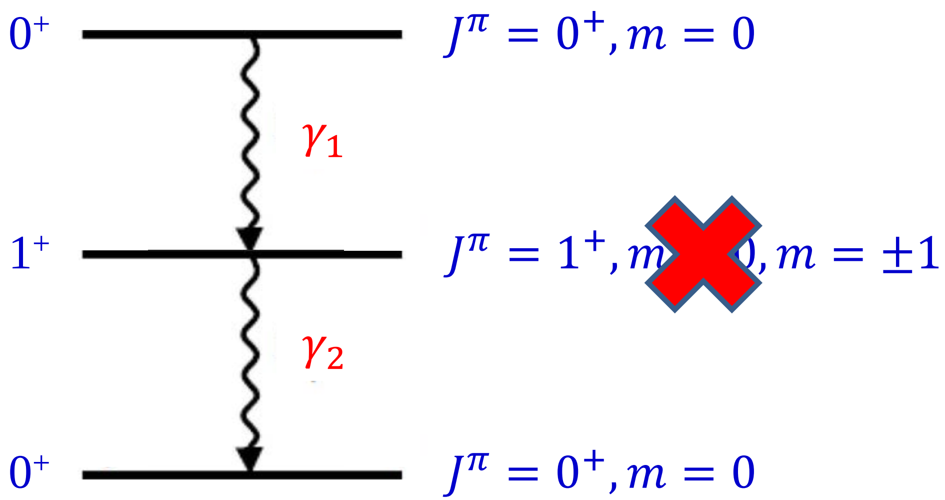


Let's imagine we have two γ-rays which follow immediately after each other in the level scheme.

If we measure γ_1 or γ_2 in singles, then the distribution will be isotropic (same intensity at all angles) ... there is no preferred direction of emission

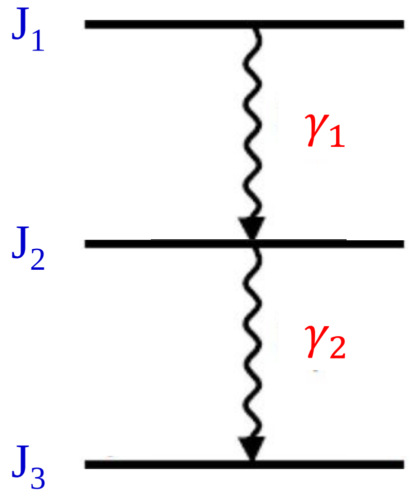
Now imagine that we measure γ_1 and γ_2 in coincidence. We say that measuring γ_1 causes the intermediate state to be aligned. We define the z-direction as the direction of γ_1

The angular distribution of the emission of γ_2 then depends on the spin/parities of the states involved and on the multipolarity of the transition.



Hence, for γ_2 we only see the m=±1 to m=0 part of the distribution i.e. we see that the intensity measured as a function of angle (relative to γ_1) follows a $1 + cos^2\theta$ distribution.

General formula



I_1 (ℓ_1)	I_2 (ℓ_2)	I_3	a_2	a_4
0 (1)	1 (1)	0	1	0
1 (1)	1 (1)	0	-1/3	0
1 (2)	1 (1)	0	-1/3	0
2 (1)	1 (1)	0	1/13	0
3 (2)	1 (1)	0	-3/29	0
0 (2)	2 (2)	0	-3	4
1 (1)	2 (2)	0	-1/3	0
2 (1)	2 (2)	0	3/7	0
2 (2)	2 (2)	0	-15/13	16/13
3 (2)	2 (2)	0	-3/29	0
4 (2)	2 (2)	0	1/8	1/24

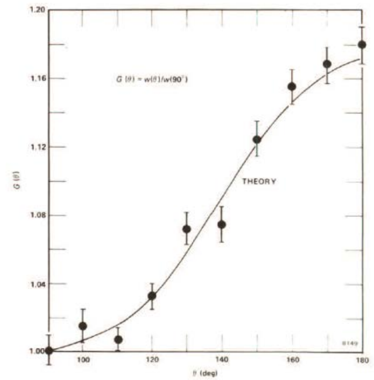
In general, the γ-ray intensity varies as:

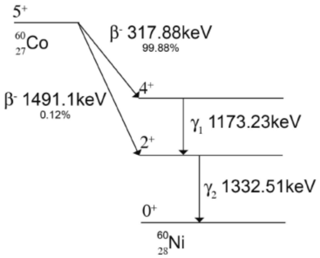
$$W(\theta) = \sum_{k_{even}} A_k(\gamma_1)A_k(\gamma_2)Q_k(\gamma_1)Q_k(\gamma_2)P_k(cos\theta)$$

where
θ is the relative angle between the two γ-rays
Q_k accounts for the fact that we do not have point detectors
A_k depends on the details of the transition and the spins of the level

$$P_0 = 1 \quad P_2 = \frac{1}{2}(3 \cdot cos^2(\theta) - 1) \quad P_4 = \frac{1}{8}(35cos^4(\theta) - 30cos^2(\theta) + 3)$$

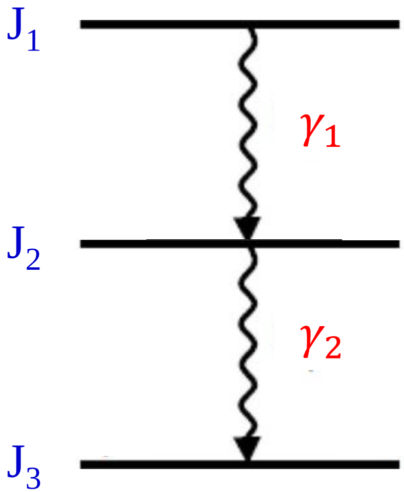
$$W(\theta) = 1 + a_2 cos^2\theta + a_4 cos^4\theta$$





R.D. Evans, The Atomic Nucleus

General formula



In general, the γ-ray intensity varies as:

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$$A_k(\gamma_1) = \frac{F_k(J_2J_1\ell,\ell) - 2\cdot\delta\cdot F_k\,(J_2J_1\ell,\ell+1) + \delta^2\cdot F_k\,(J_2J_1\ell+1,\ell+1)}{1+\delta^2}$$

$$A_k(\gamma_2) = \frac{F_k(J_2J_3L,L) - 2\cdot\delta\cdot F_k\,(J_2J_3L,L+1) + \delta^2\cdot F_k\,(J_2J_3L+1,L+1)}{1+\delta^2}$$

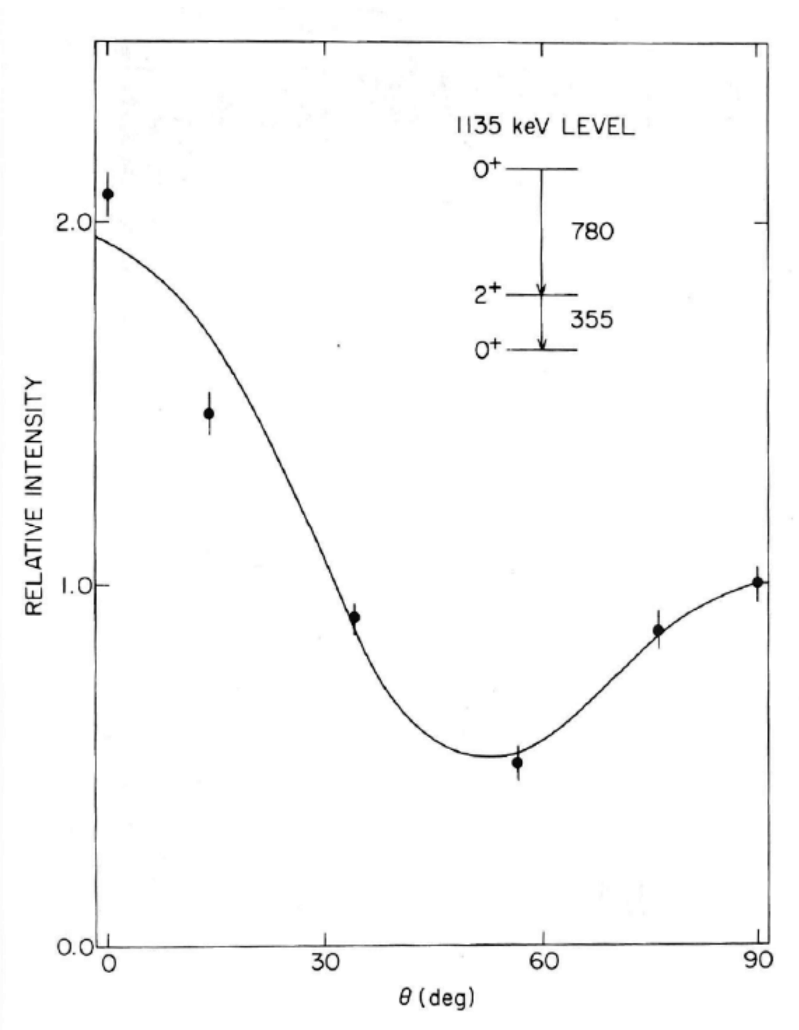
Ferentz-Rosenzweig coefficients

$$F_k(LL'I_1I_2) = (-1)^{I_1+I_2+1}\sqrt{2k+1}\sqrt{2L+1}\sqrt{2L'+1}\sqrt{2I_2+1}\begin{pmatrix} L & L' & k \\ 1 & -1 & 0 \end{pmatrix}\begin{Bmatrix} L & L' & k \\ I_1 & I_1 & I_2 \end{Bmatrix}$$

https://griffincollaboration.github.io/AngularCorrelationUtility/

A special case:

$^{195}_{78}Pt(n,\gamma)^{196}_{78}Pt$



Angular correlations with arrays

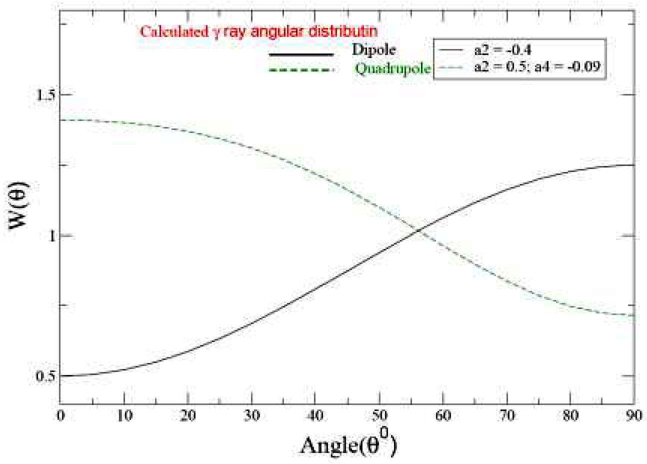
Many arrays are designed symmetrically, so the range of possible angles is reduced.

Therefore one measures a Directional Correlation from Oriented Nuclei (DCO ratio)

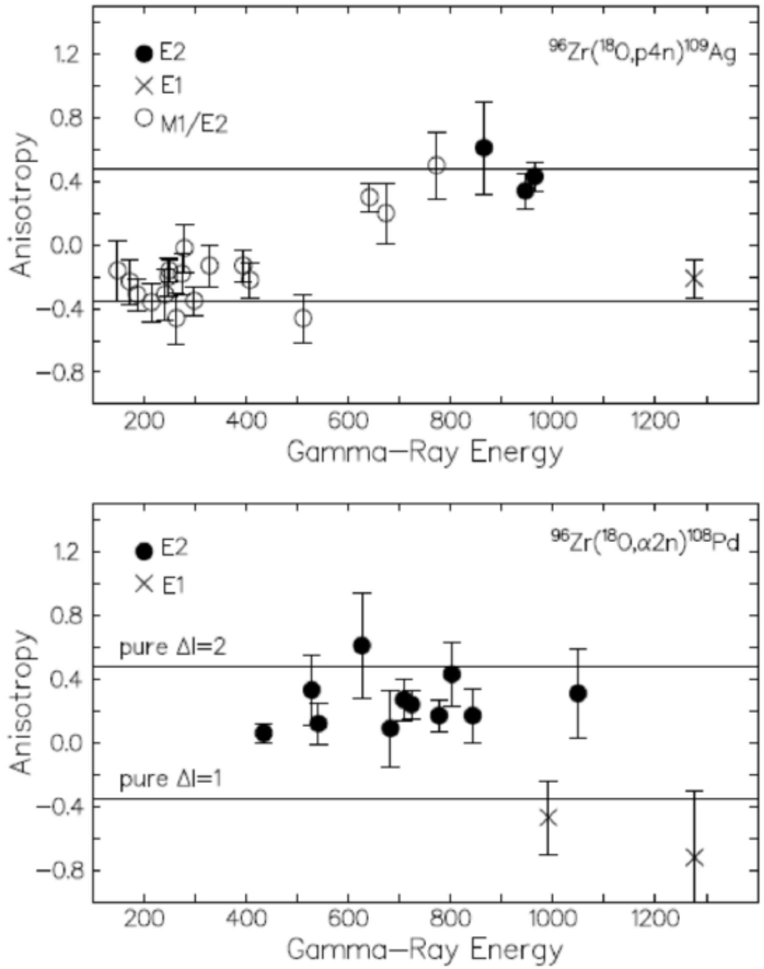
In the simplest case, if you have an array with detectors at 35^0 and 90^0.
Gate on 90^0 detector, measure coincident intensities in

- other 90^0 detectors
- 35^0 detectors

Take the ratio and compare with calculations ... can usually separate quadrupoles from dipoles but cannot measure mixing ratios



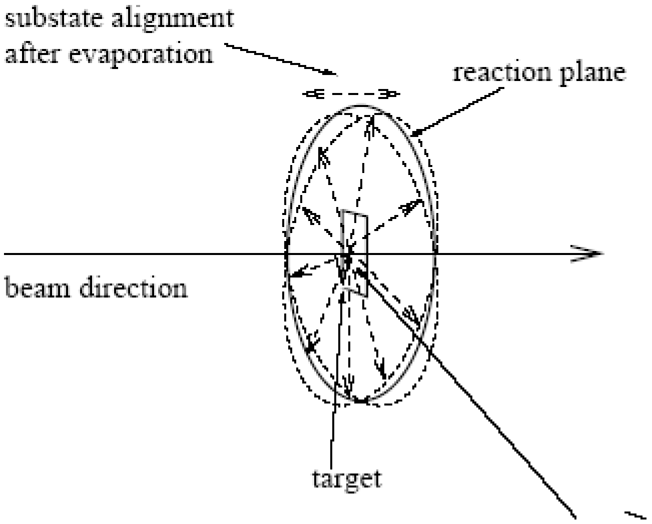
Angular correlations with arrays



K.R.Pohl et al., Phys Rev C53 (1996) 2682

Angular distribution

In heavy-ion fusion-evaporation reactions, the compound nuclei have their spin aligned in a plane perpendicular to the beam axis:

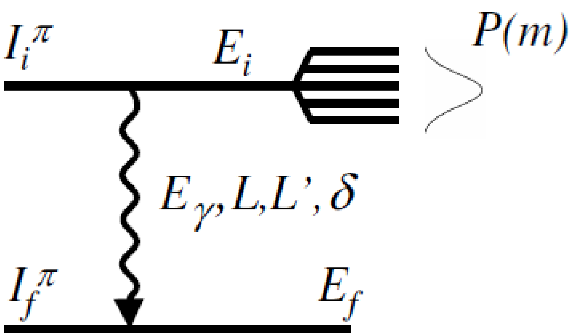


$$\vec{\ell} = \vec{r} \times \vec{p}$$

Depending on the number and type of particles 'boiled off' before a γ-ray is emitted, transitions are emitted from oriented nuclei and therefore their intensity shows an angular dependence.

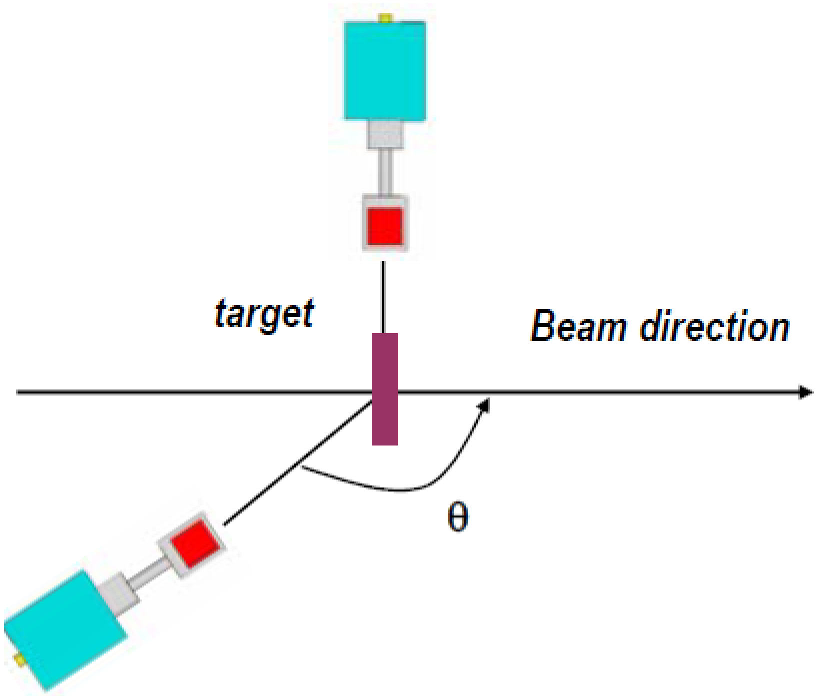
$$W(\theta) = A_0 \left(1 + \frac{A_2}{A_0} \cdot B_2 \cdot Q_2 \cdot P_2\,(cos\theta) + \frac{A_4}{A_0} \cdot B_4 \cdot Q_4 \cdot P_4(cos\theta) \right)$$

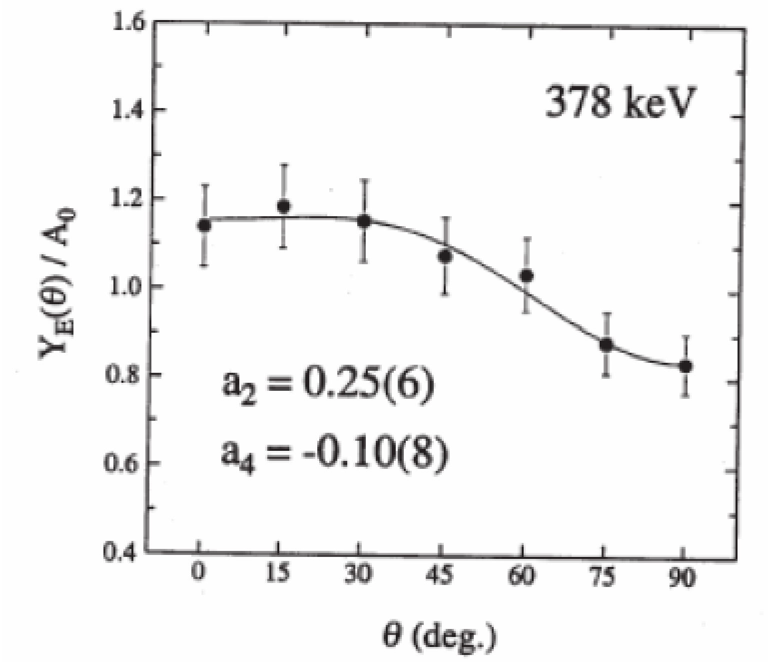
where A_k, Q_k and P_k are as before and B_k contains information about the alignment of the state



$$B_k(I_i) = \sqrt{2I_i + 1} \sum_{m=-I}^{+I} (-1)^{I_i - m} \langle I_i m I_i - m | k0 \rangle P(m) \qquad P(m) = \frac{exp\left(-\frac{m^2}{2\sigma^2}\right)}{\sum_{m'=-I}^{+I} exp\left(-\frac{m'^2}{2\sigma^2}\right)}$$

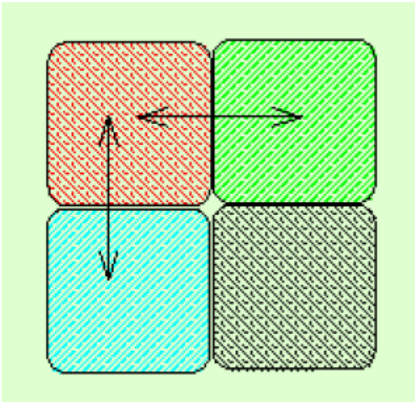
Angular distribution



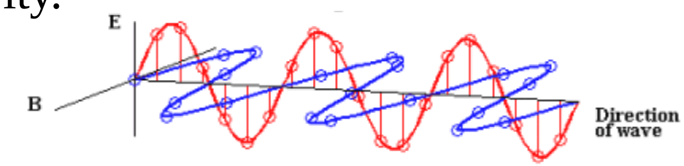


Measure: the γ-ray yield as a function of θ

Linear polarization



A segmented detector can be used to measure the linear polarization which can be used to distinguish between magnetic (M) and electric (E) character of radiation of the same multipolarity.



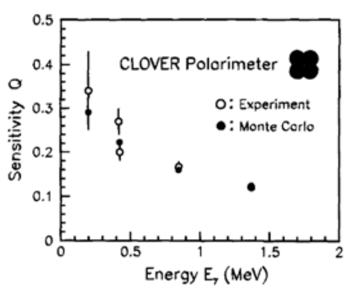
The Compton scattering cross section is larger in the direction perpendicular to the electrical field vector of the radiation.

Define experimental asymmetry as: $A = \frac{N_{90} - N_0}{N_{90} + N_0}$

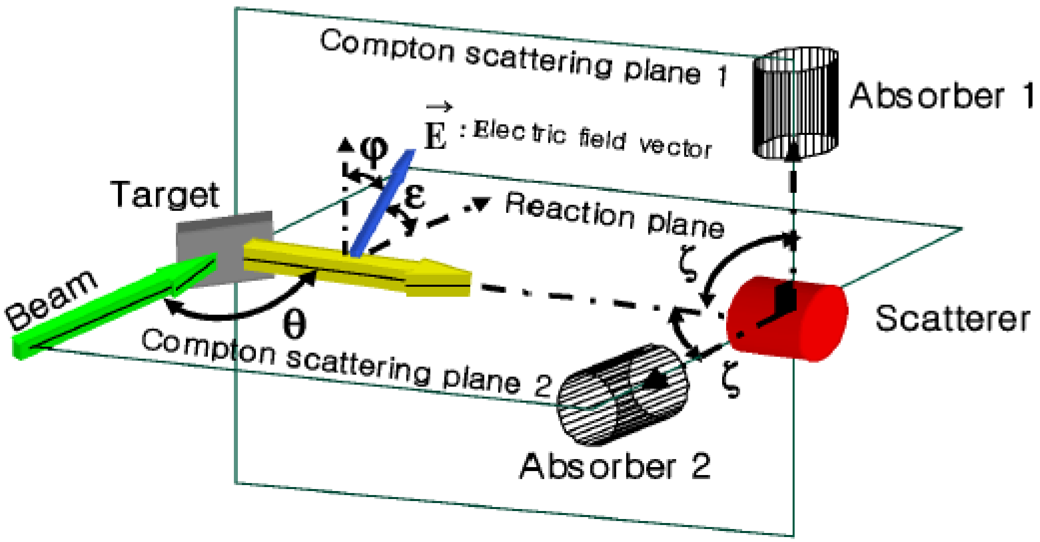
where N_{90} and N_0 are the intensities of scattered photons perpendicular and parallel to the reaction plane.

The experimental linear polarization P=A/Q where Q is the polarization sensitivity of the detector

Q~13% at 1 MeV



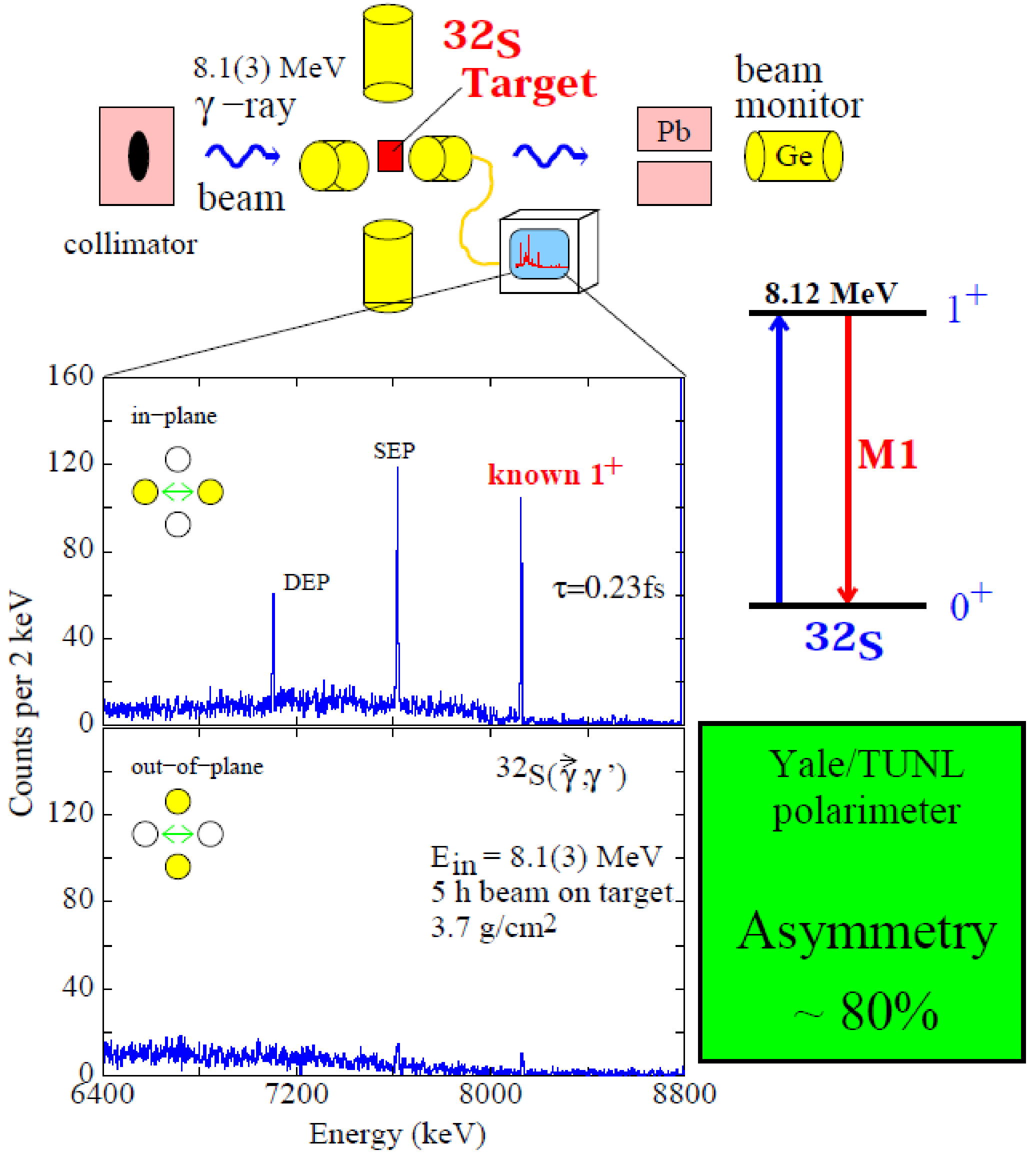
Linear polarization



Klein-Nishina formula:

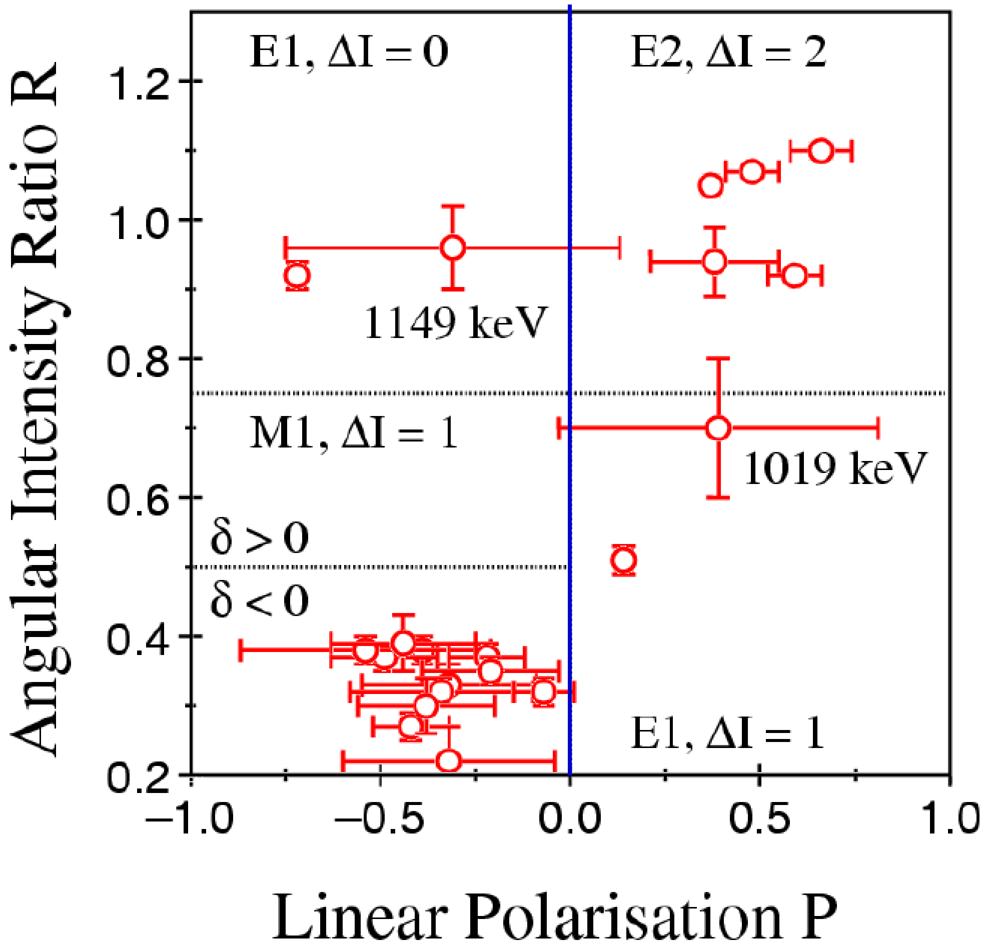
$$\frac{d\sigma_c}{d\Omega} = \frac{r_0^2}{2}\left(\frac{E_{\gamma'}}{E_\gamma}\right)^2 \cdot \left\{\frac{E_\gamma}{E_{\gamma'}} + \frac{E_{\gamma'}}{E_\gamma} - 2sin^2\theta \cdot cos^2\varphi\right\}$$

Maximum polarization at $\theta=90^0$



N. Pietralla, Nucl Instr Meth A483, 556 (2002)

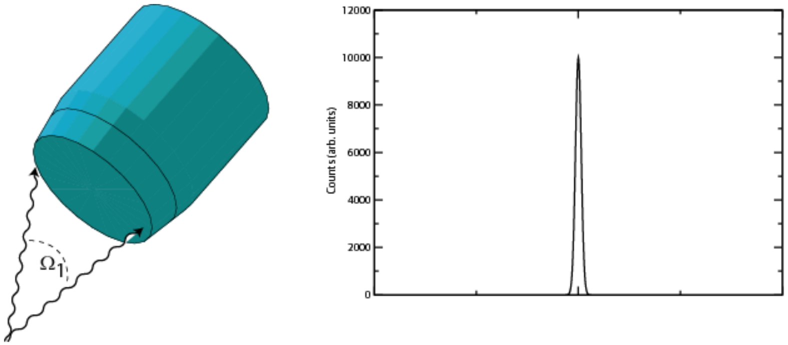
Linear polarization



Plot P against the angular distribution information to uniquely define the multipolarity.

Data from Eurogam

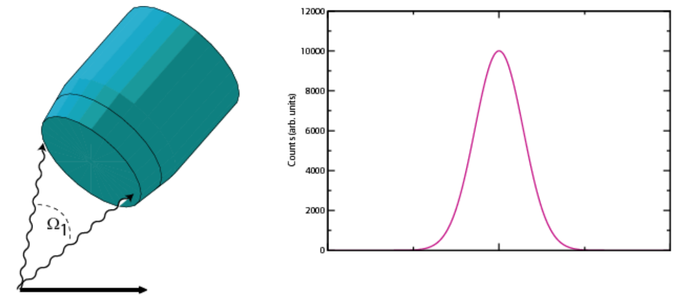
Efficiency versus resolution



With a source at rest, the intrinsic resolution of the detector can be reached;

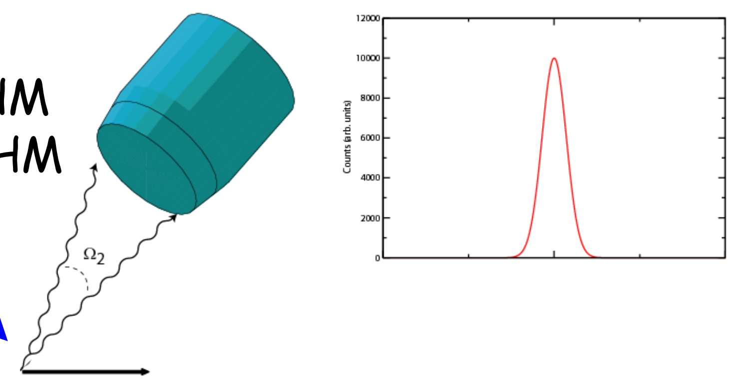
efficiency decreases with the increasing detector-source distance.

With a moving source also the effective energy resolution depends on the detector-source distance (Doppler effect)



Small d ⟷ Large Ω ⟷ High ε ⟷ Poor FWHM

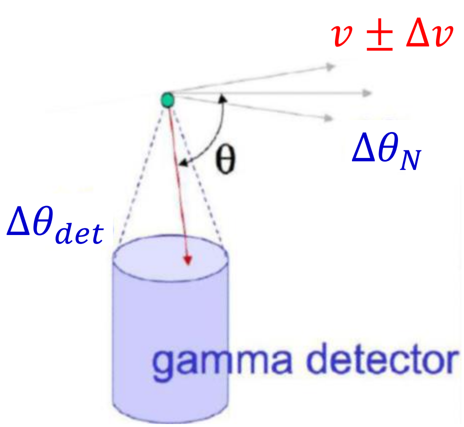
Large d ⟷ Small Ω ⟷ Low ε ⟷ Good FWHM



Energy resolution

The major factors affecting the final energy resolution (FWHM) at a particular energy are as follows:

$$\Delta E_\gamma^{final} = \left(\Delta E_{Int}^2 + \Delta\theta_{det}^2 + \Delta\theta_N^2 + \Delta v^2\right)^{1/2}$$



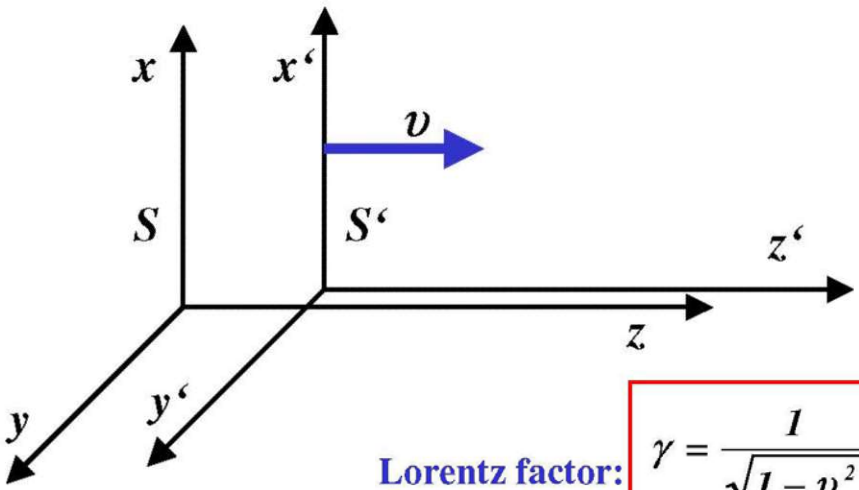
ΔE_{Int} – The intrinsic resolution of the detector system. It includes contributions from the detector itself and the electronic components used to process the signal.

$\Delta\theta_{det}$ – The Doppler broadening arising from the opening angle of the detectors

$\Delta\theta_N$ – The Doppler broadening arising from the angular spread of the recoils in the target

Δv – The Doppler broadening arising from the velocity (energy) variation of the excited nucleus

Lorentz transformation:



Lorentz factor: $\gamma = \frac{1}{\sqrt{1-v^2}}$

❑ Consider the space-time point

- in a given frame S: (t, x, y, z)
- and in a (moving) frame S': (t', x', y', z')

1) S' moves with a constant velocity v along z-axis

Space-time Lorentz transformation S←→S':

$S \Rightarrow S'$	$S' \Rightarrow S$
$x' = x$	$x = x'$
$y' = y$	$y = y'$
$z' = \gamma(z - vt)$	$z = \gamma(z' + vt)$
$t' = \gamma(t - vz)$	$t = \gamma(t' + vz)$

Note: units c=1

$$t' = \gamma(t - \frac{v}{c^2} z)$$

$$t = \gamma(t' + \frac{v}{c^2} z)$$

$$v_z = |\vec{v}| = v$$

❑ Consider the 4-momentum:

- in a given frame S: $p \equiv (E, p) = (E, p_x, p_y, p_z)$
- in the (moving) frame S': $p' \equiv (E', \vec{p}') = (E', p'_x, p'_y, p'_z)$

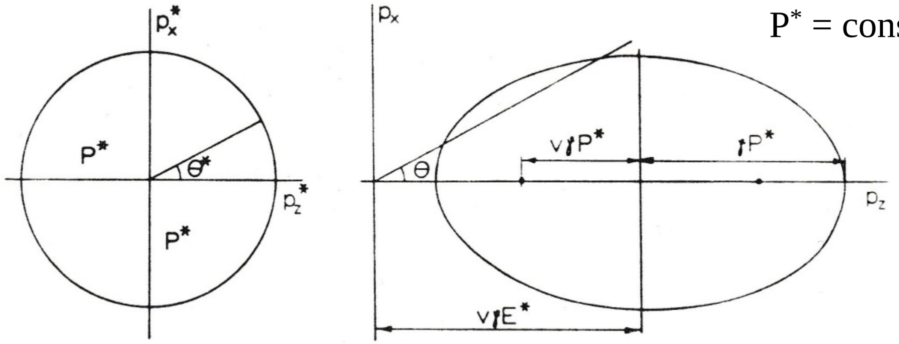
Lorentz transformation for 4-momentum S←→S':

$$p'_x = p_x, \quad p'_y = p_y$$

$$p'_z = \gamma(p_z - vE)$$

$$E' = \gamma(E - vp_z)$$

Lorentz transformation



$P^* = \text{const.}$

rest system laboratory system

total energy:

$$E^* = \gamma \cdot E - \gamma \cdot v \cdot P \cdot cos\theta$$

with

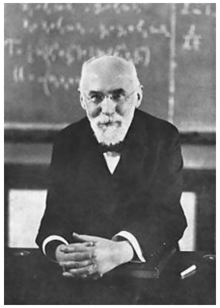
$$E = \sqrt{(mc^2)^2 + (Pc)^2}$$

E^*, P^* total energy and momentum in the rest system
E, P total energy and momentum in the laboratory system

Doppler formula for zero-mass particle (photon): E=Pc

$$E^* = \gamma \cdot E - \gamma \cdot \beta \cdot E \cdot cos\theta$$

$$\boxed{E^* = \gamma \cdot E(1 - \beta \cdot cos\theta)}$$



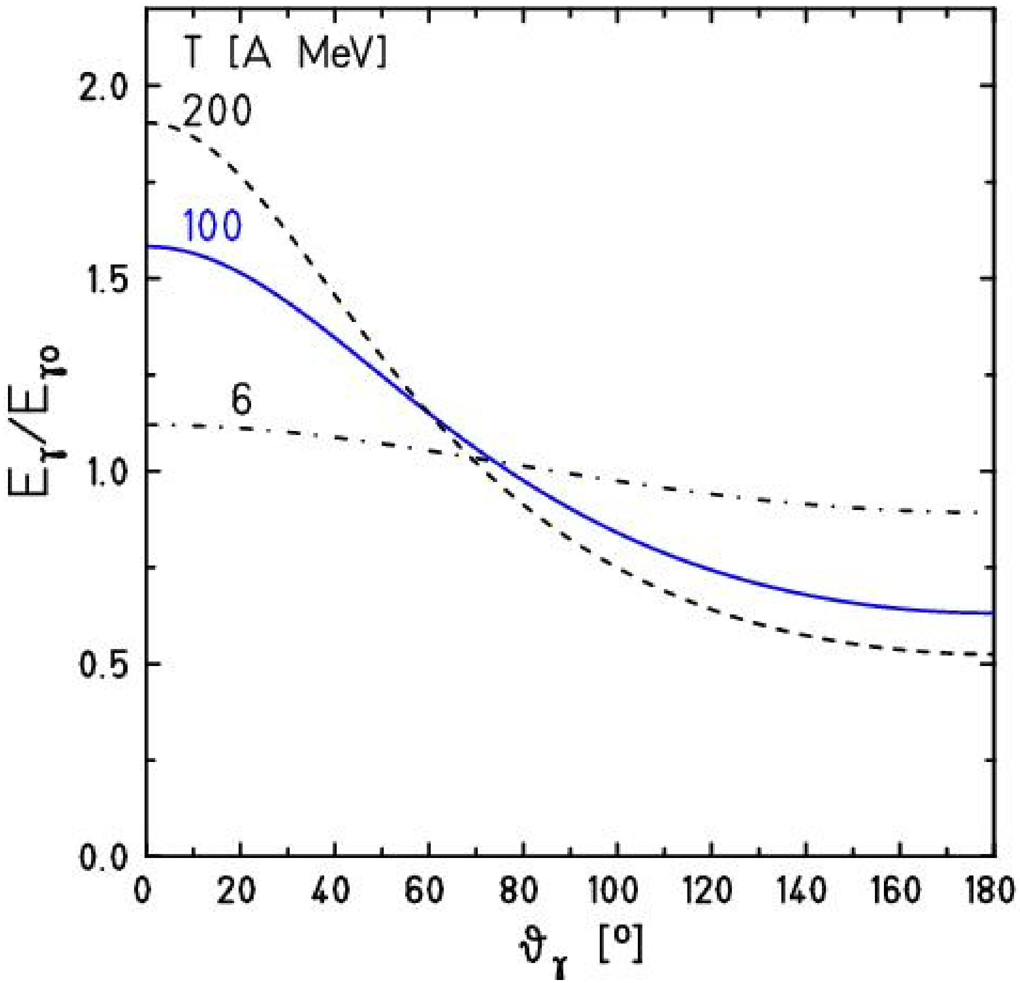
Hendrik Lorentz

E. Byckling, K. Kajantie J. Wiley & Sons London

Doppler effect

$$\frac{E_{\gamma 0}}{E_{\gamma}} = \frac{1-\beta\cdot\cos\vartheta_{\gamma}^{\ell ab}}{\sqrt{1-\beta^2}}$$

for $\vartheta_p \cong 0^0$



$$\frac{d\Omega_{rest}}{d\Omega_{lab}} = \left(\frac{E_{\gamma}}{E_{\gamma 0}}\right)^2$$

Doppler broadening and position resolution

$$E_{\gamma 0} = E_{\gamma} \frac{1-\beta\cdot\cos\vartheta_{\gamma}}{\sqrt{1-\beta^{2}}} \qquad \left(\beta,\ \vartheta_{p} = 0^{0},\ \vartheta_{\gamma} \text{ and } E_{\gamma} \text{ in lab-frame}\right)$$

$$\left(\frac{\Delta E_{\gamma 0}}{E_{\gamma 0}}\right)^{2} = \left(\frac{\beta\cdot\sin\vartheta_{\gamma}}{1-\beta\cdot\cos\vartheta_{\gamma}}\right)^{2}\left(\Delta\vartheta_{\gamma}\right)^{2} + \left(\frac{\beta-\cos\vartheta_{\gamma}}{\left(1-\beta^{2}\right)\cdot\left(1-\beta\cdot\cos\vartheta_{\gamma}\right)}\right)^{2}\left(\Delta\beta\right)^{2} + \left(\frac{1}{E_{\gamma}}\right)^{2}\left(\Delta E_{\gamma}\right)^{2}$$

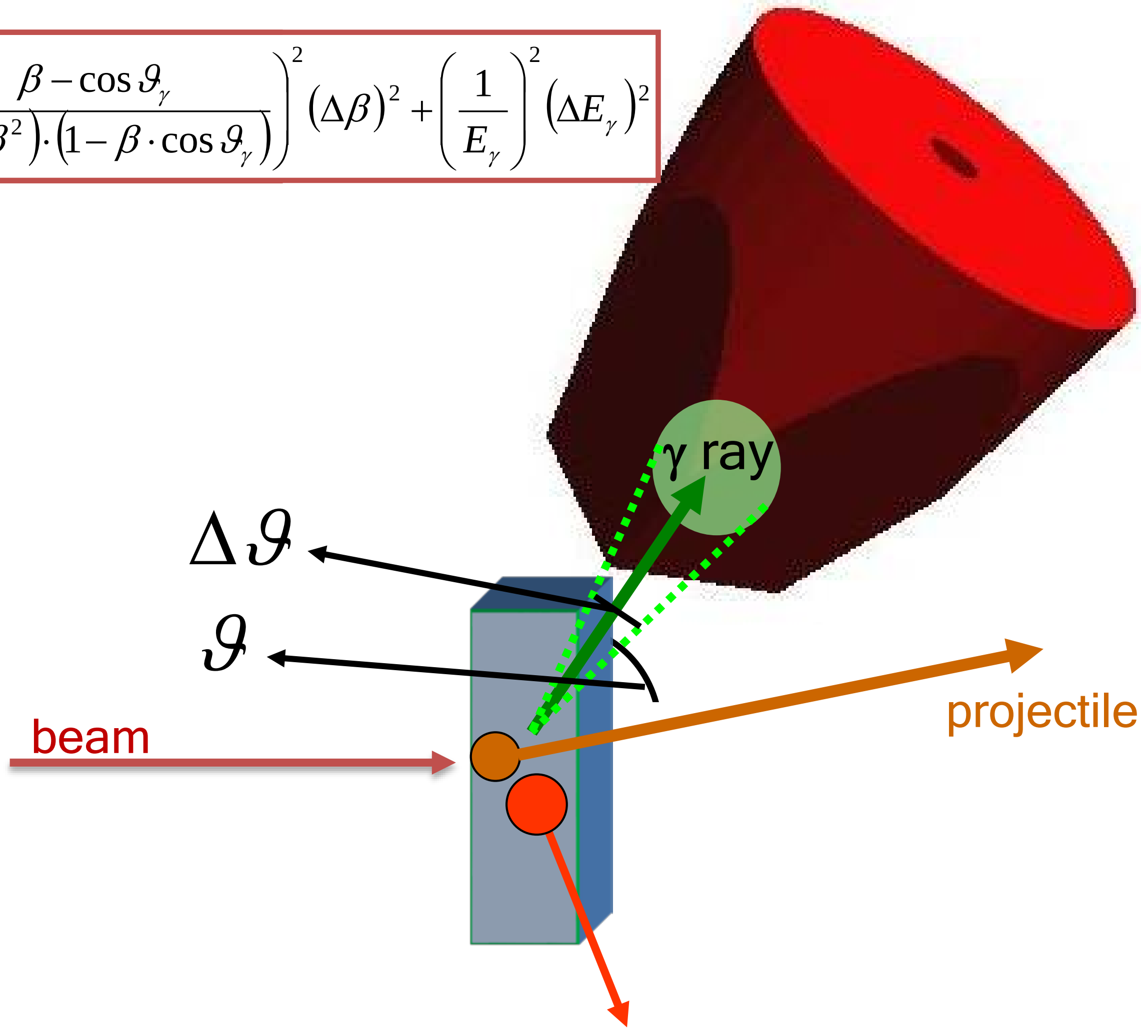
Position resolution

↓

Angular resolution

↓

Energy resolution



Doppler broadening (opening angle of detector)

$$\frac{\Delta E_{\gamma 0}}{E_{\gamma 0}} = \frac{\beta \cdot \sin \vartheta_\gamma}{1 - \beta \cdot \cos \vartheta_\gamma} \cdot \Delta \vartheta_\gamma$$

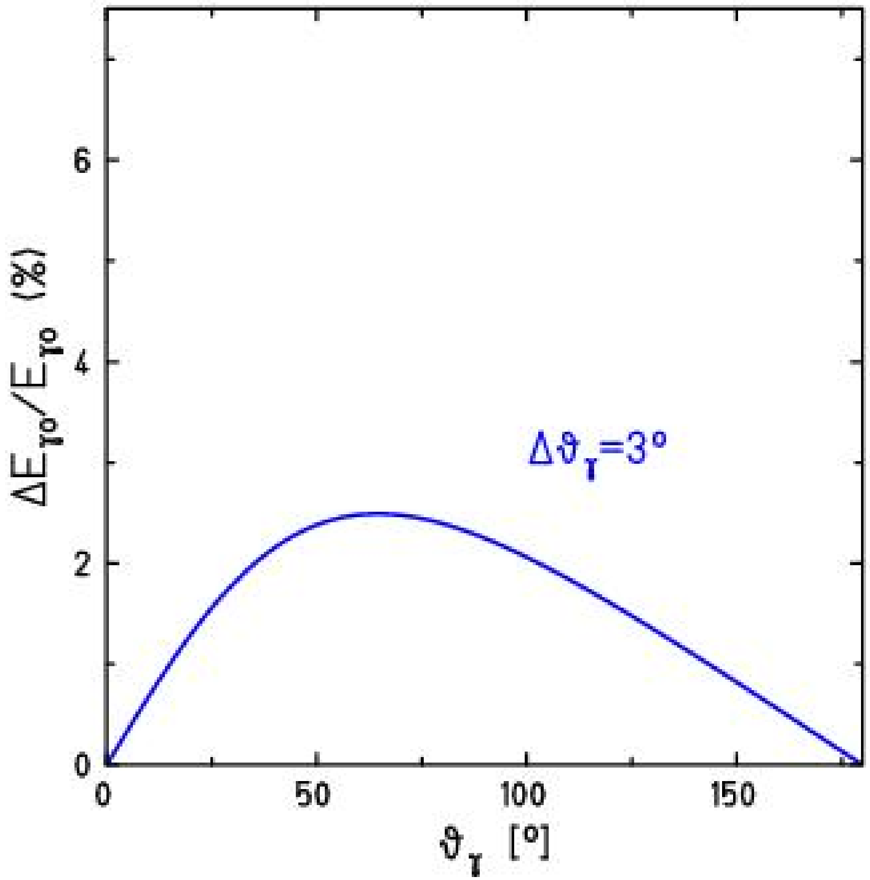
for $\vartheta_p \cong 0^0$

with

$$\Delta \vartheta_\gamma = 0.622 \cdot \arctan \frac{d[mm]}{R[mm] + 30[mm]}$$

$$R = 700[mm]$$

$$d = 59[mm]$$

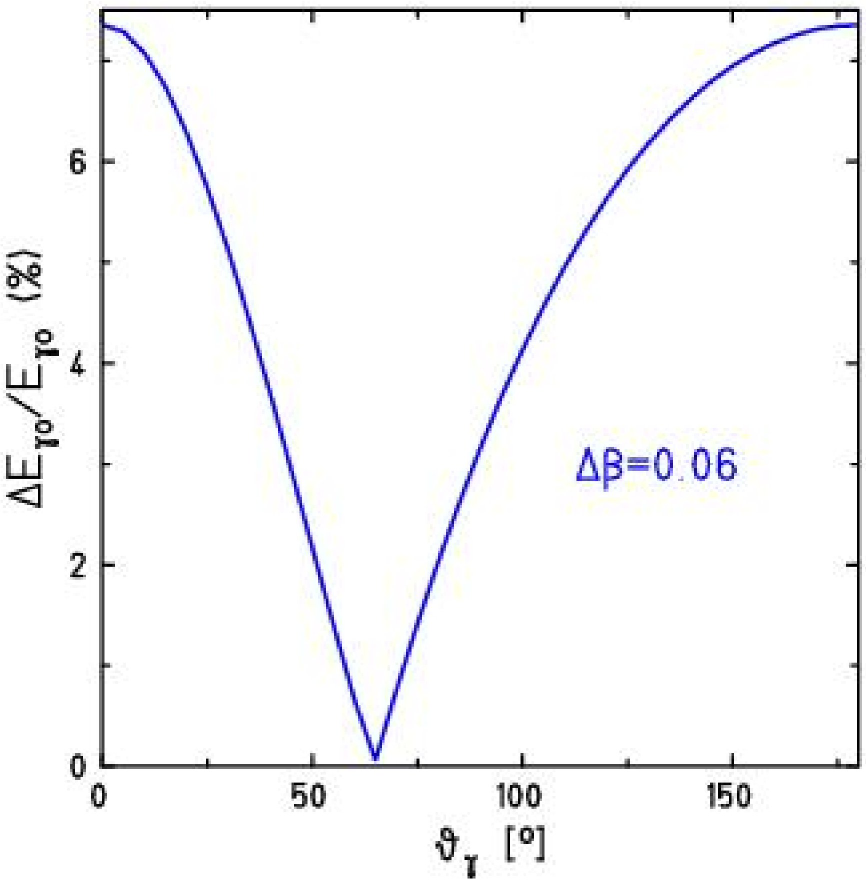


Doppler broadening (velocity variation)

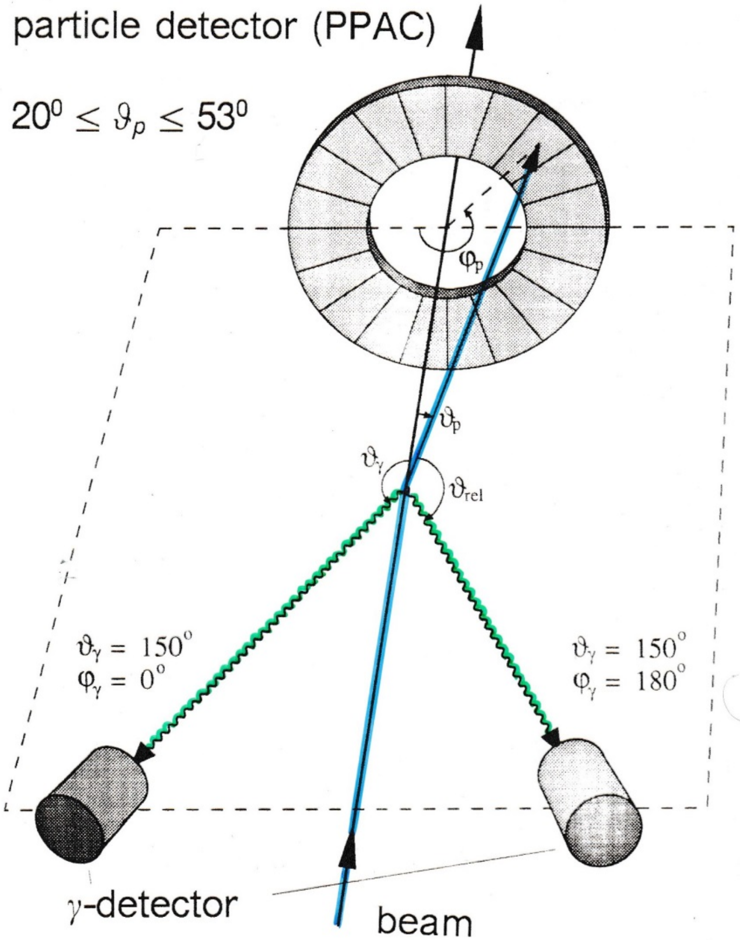
$$\frac{\Delta E_{\gamma 0}}{E_{\gamma 0}} = \frac{\beta - \cos\vartheta_\gamma}{\left(1-\beta^2\right)\cdot\left(1-\beta\cdot\cos\vartheta_\gamma\right)}\cdot\Delta\beta$$

for $\vartheta_p \cong 0^0$

with $\Delta\beta = 6\%$



Experimental arrangement



experimental problem:

Doppler broadening due to finite size of Ge-detector

$\frac{\Delta E}{E} \sim 1\%$ for $\Delta\vartheta_\gamma = 20^0 \quad \beta_1 \cong 10\%$

For projectile excitation:

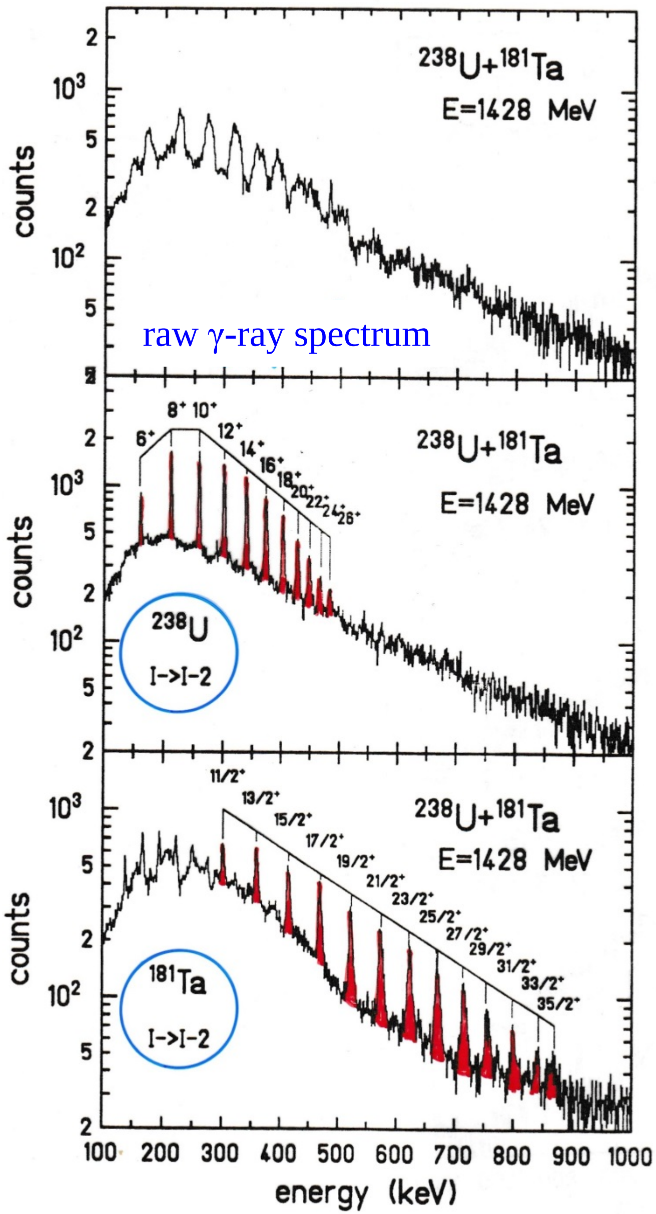
$E^* = \gamma \cdot E \cdot (1 - \beta_1 \cdot cos\theta_{\gamma 1})$ Doppler shift

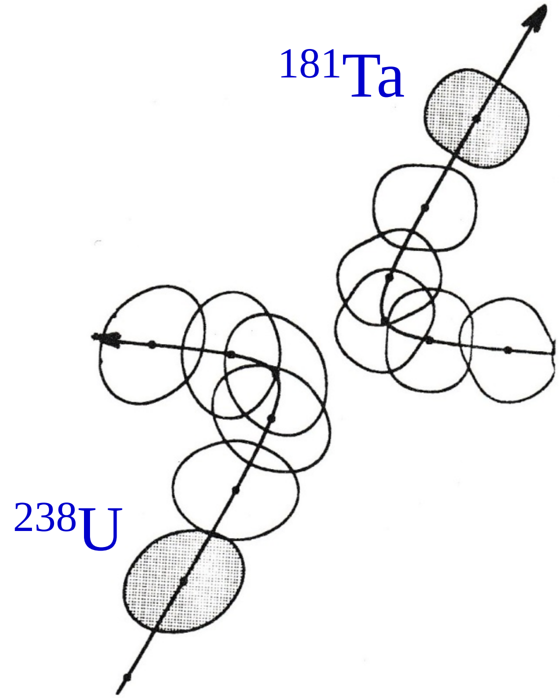
with

$cos\theta_{\gamma 1} = cos\vartheta_1 cos\vartheta_\gamma + sin\vartheta_1 sin\vartheta_\gamma cos(\varphi_\gamma - \varphi_1)$

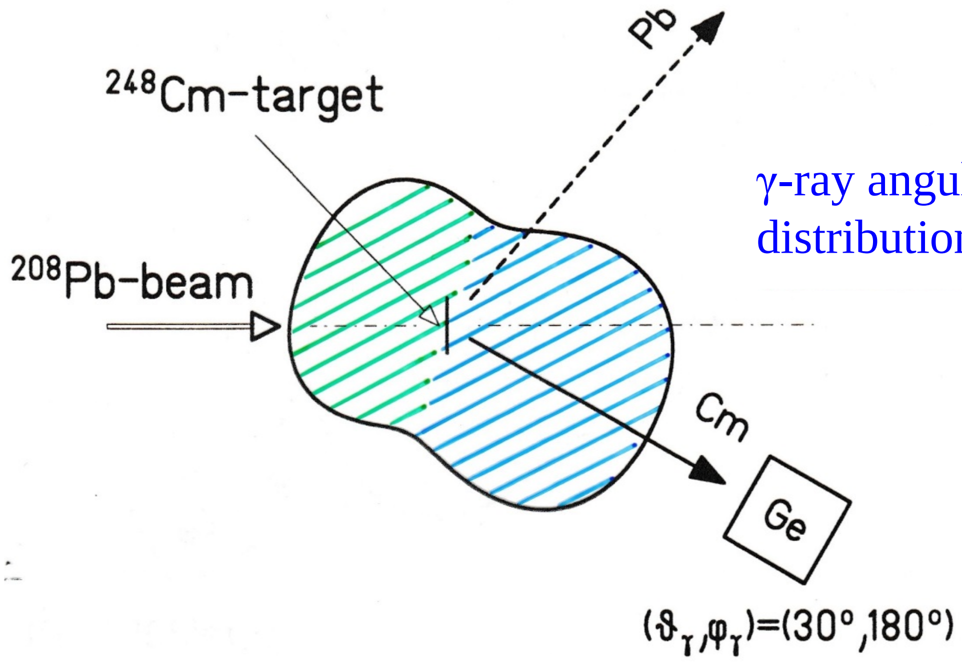
$\Delta E \cong E^* \cdot \beta_1 \cdot sin\theta_{\gamma 1} \cdot \Delta\theta_{\gamma 1}$ Doppler broadening

Inelastic heavy-ion scattering





Lorentz transformation



γ-ray angular distribution

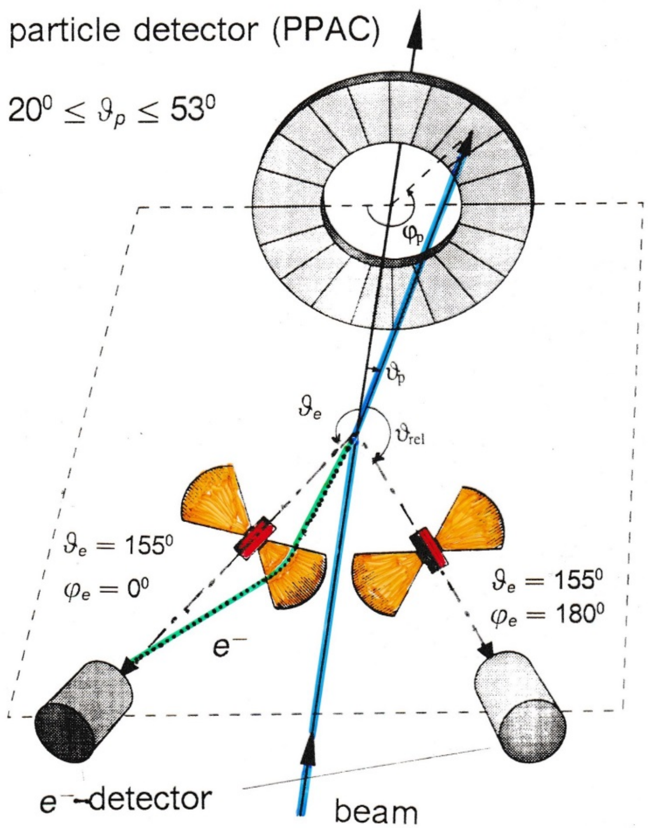
Contraction of the solid angle element in the laboratory system

$$\frac{d\Omega}{d\Omega^*} = \left\{\frac{E^*}{E}\right\}^2$$

with

$$E^* = \gamma \cdot E \cdot (1 - \beta \cdot cos\theta)$$ Doppler formula

Experimental arrangement (electron detection)



Doppler broadening

$\Delta\vartheta_e = 20^0$
target – Mini-Orange: 19 cm
Mini-Orange – Si detector: 6 cm

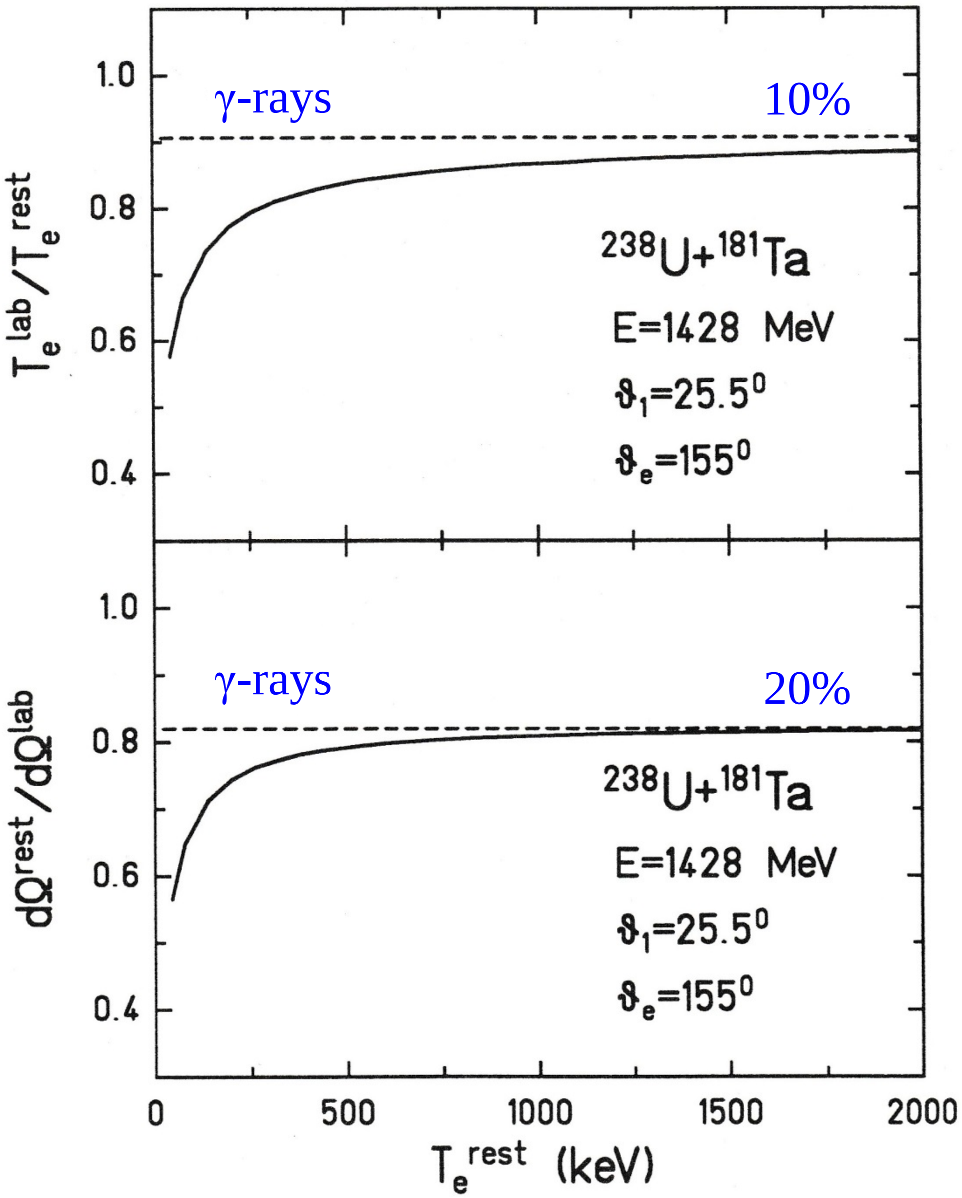
For projectile excitation:

$$T_e^* = \gamma \cdot T_e \cdot \left\{1 - \beta_1 \cdot \sqrt{1 + 2m_e\, c^2/T_e} \cdot cos\theta_{e1}\right\} + m_e c^2 \cdot (\gamma - 1)$$

with

$$cos\theta_{e1} = cos\vartheta_1 cos\vartheta_e + sin\vartheta_1 sin\vartheta_e cos(\varphi_e - \varphi_1)$$

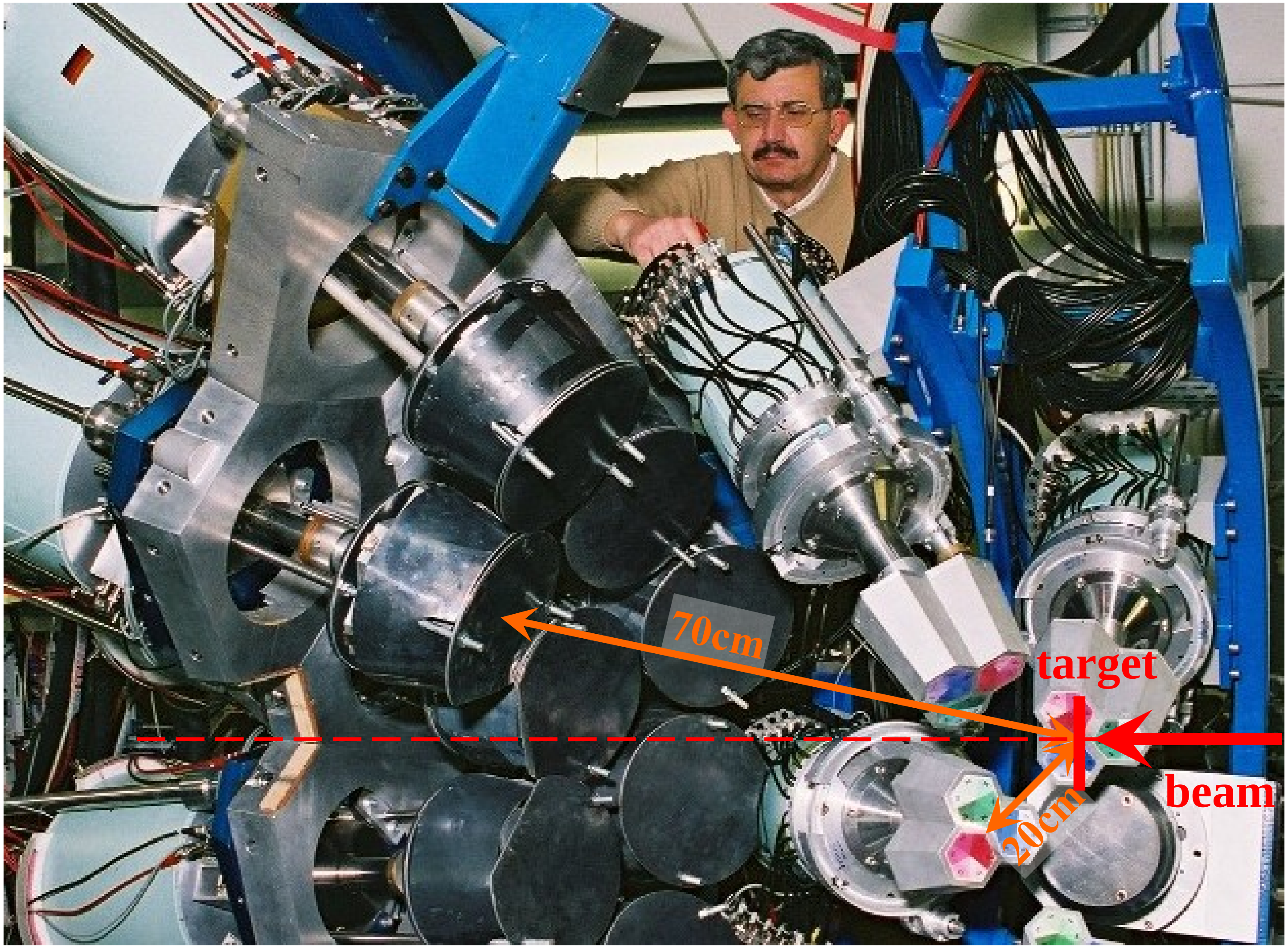
Lorentz transformation



energy shift

solid angle contraction

Segmented detectors

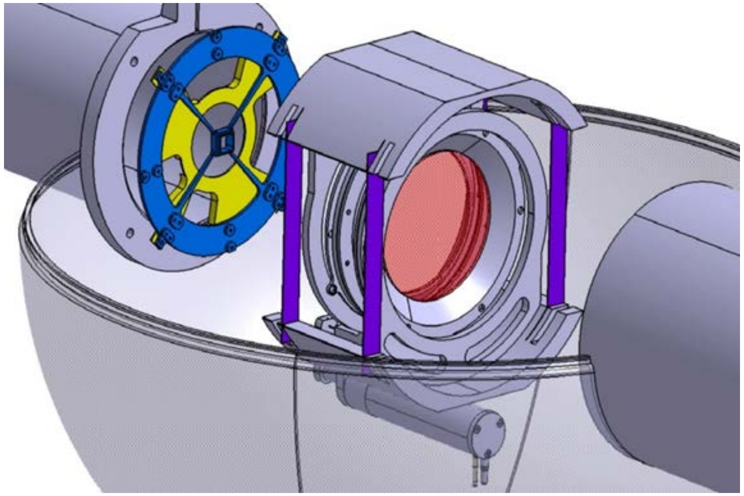


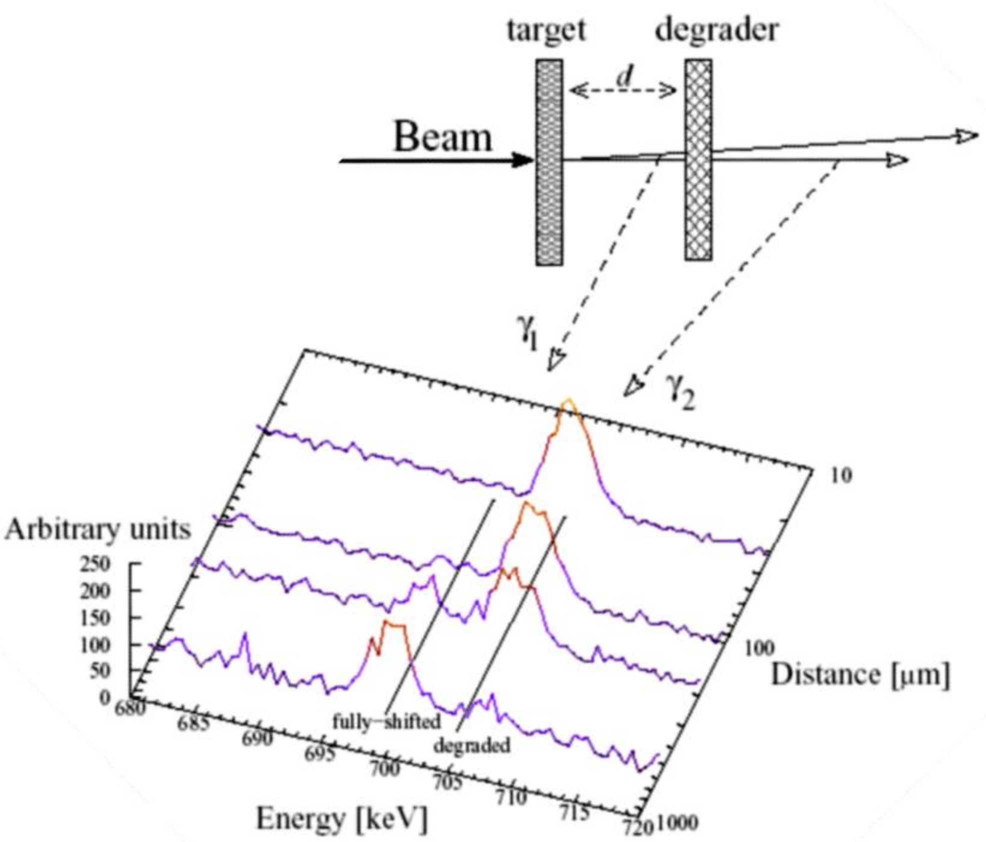
Recoil distance method

$$I_{degraded} = I \cdot e^{-d/v\tau}$$

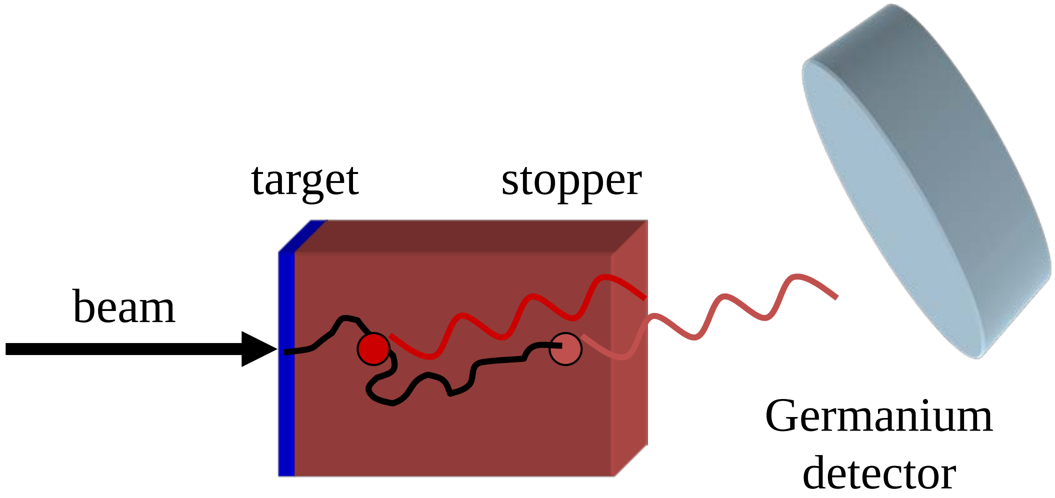
$$I_{shifted} = \left(1 - e^{-d/v\tau}\right)$$

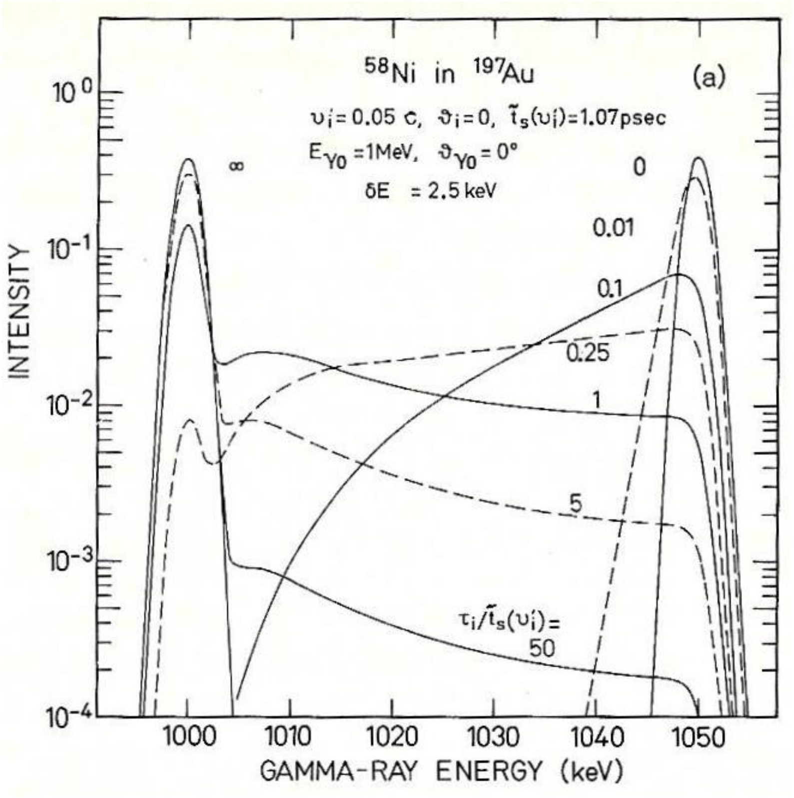
$$\frac{I_{degraded}}{I_{degraded} + I_{shifted}} = e^{-d/v\tau}$$





Doppler Shift Attenuation Method





Legendre polynomials

$$P_0(cos\theta) = 1$$

$$P_1(cos\theta) = cos\theta$$

$$P_2(cos\theta) = \frac{1}{2}(3cos^2\theta - 1)$$

$$P_3(cos\theta) = \frac{1}{2}(5cos^3\theta - 3cos\theta)$$

$$P_4(cos\theta) = \frac{1}{8}(35cos^4\theta - 30cos^2\theta + 3)$$

$$P_5(cos\theta) = \frac{1}{8}(63cos^5\theta - 70cos^3\theta + 15cos\theta)$$

$$P_6(cos\theta) = \frac{1}{16}(231cos^6\theta - 315cos^4\theta + 105cos^2\theta - 5)$$