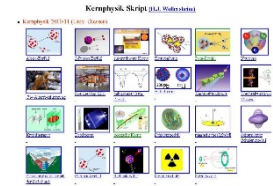


Outline: Linac-Synchrotron

Lecturer: Hans-Jürgen Wollersheim

e-mail: h.j.wollersheim@gsi.de

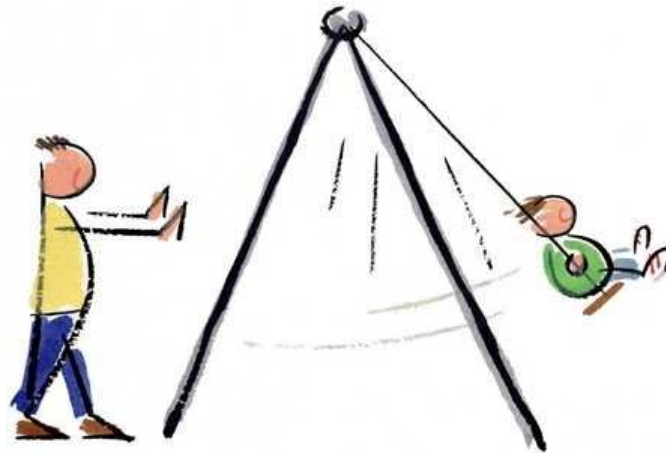
web-page: <https://web-docs.gsi.de/~wolle/> and click on



1. linear and circular accelerator
2. Wideroe structure
3. Alvarez structure
4. Linac and synchrotron
5. synchrotron: beam extraction

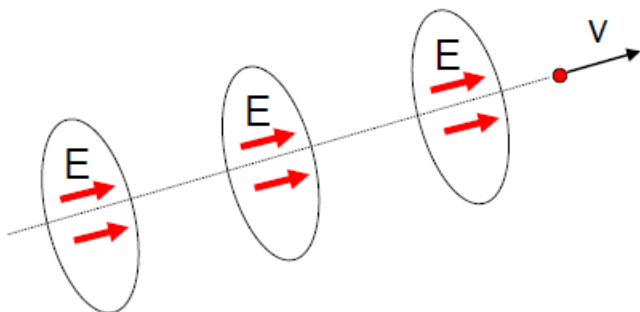
Acceleration to higher energies

- ❖ While terminal voltages of 20 MV provide sufficient beam energy for nuclear structure research, most applications nowadays require beam energies > 1 GeV
- ❖ How do we attain higher beam energies?
- ❖ Analogy: How to swing a child?
 - Pull up to maximum height and let go: difficult and tiring (electrostatic accelerator)
 - Repeatedly push in synchronism with the period of the motion



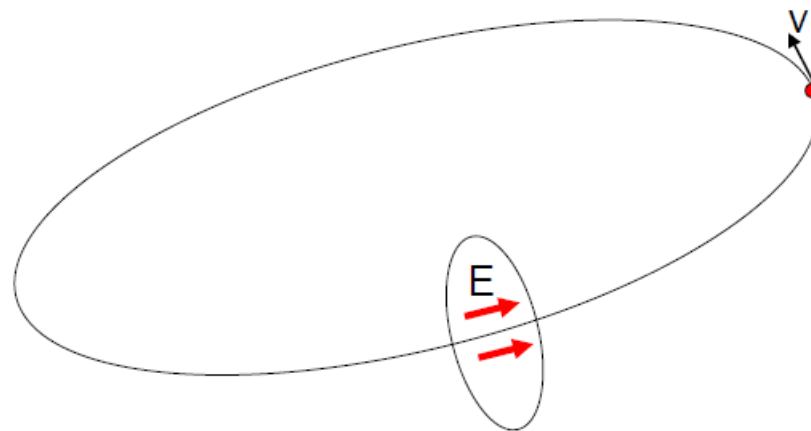
Acceleration by repeated application of time-varying fields

- ❖ Two approaches for accelerating with time-varying fields
- ❖ Make an electric field along the direction of particle motion with Radio-Frequency (RF) cavities



Linear Accelerators

Use many accelerating cavities through which the particle beam passes once.



Circular Accelerators

Use one or a small number of RF accelerating cavities and make use of repeated passage through them: This approach leads to circular accelerators:

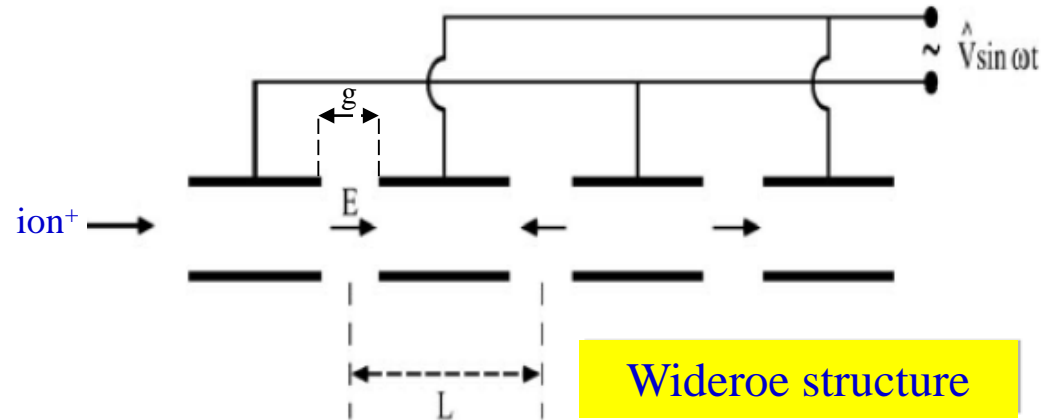
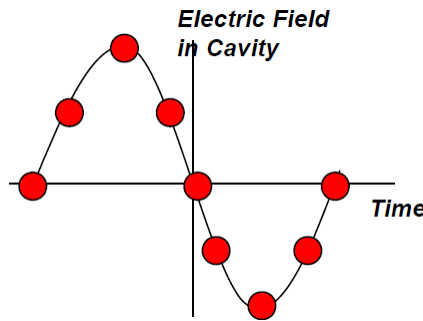
Cyclotrons, synchrotrons and their variants.

Radio-Frequency Accelerators

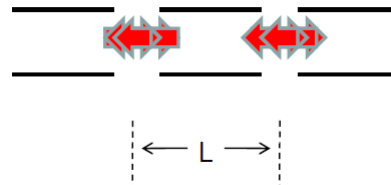
- ❖ The electric field is no longer static but sinusoidal alternating half periods of acceleration and deceleration.

$$V(t) = V_0 \cdot \sin \omega t$$

$$E(t) = (V_0/g) \cdot \sin \omega t$$



Wideroe structure

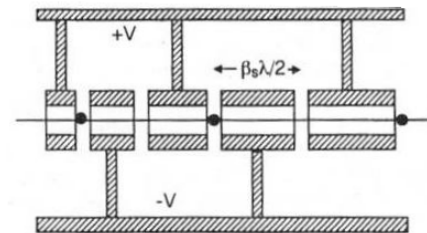


Rolf Wideroe
(1902-1996)

- ❖ Three important aspects of an RF linear accelerator

- Particles must arrive bunched in time in order for efficient acceleration
- Acceleration gaps must be spaced, so that the particle “bunches” arrive at the **acceleration phase**:

$$L = v \cdot \frac{T}{2} = \beta c \frac{1}{2} \frac{\lambda}{c} = \beta \frac{\lambda}{2}$$



- The acceleration field is varying while the particle is in the gap; energy gain is more complicated than in the static case.

Acceleration in the Wideroe structure

Energy gained after n acceleration gaps:

$$E_n = n \cdot q \cdot U_0 \cdot \sin \Psi_s$$

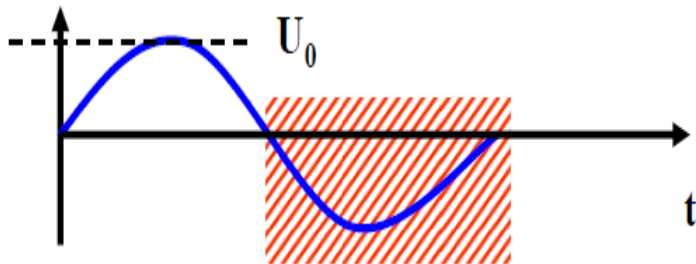
Kinetic energy of the particles:

$$E_n = \frac{1}{2} m \cdot v_n^2 \quad (\text{valid for non-relativistic particles})$$

Velocity of the particles:

$$v_n = \sqrt{\frac{2E_n}{m}} = \sqrt{\frac{2 \cdot n \cdot q \cdot U_0 \cdot \sin \Psi_s}{m}}$$

Shielding of the particles during the negative half wave of the RF

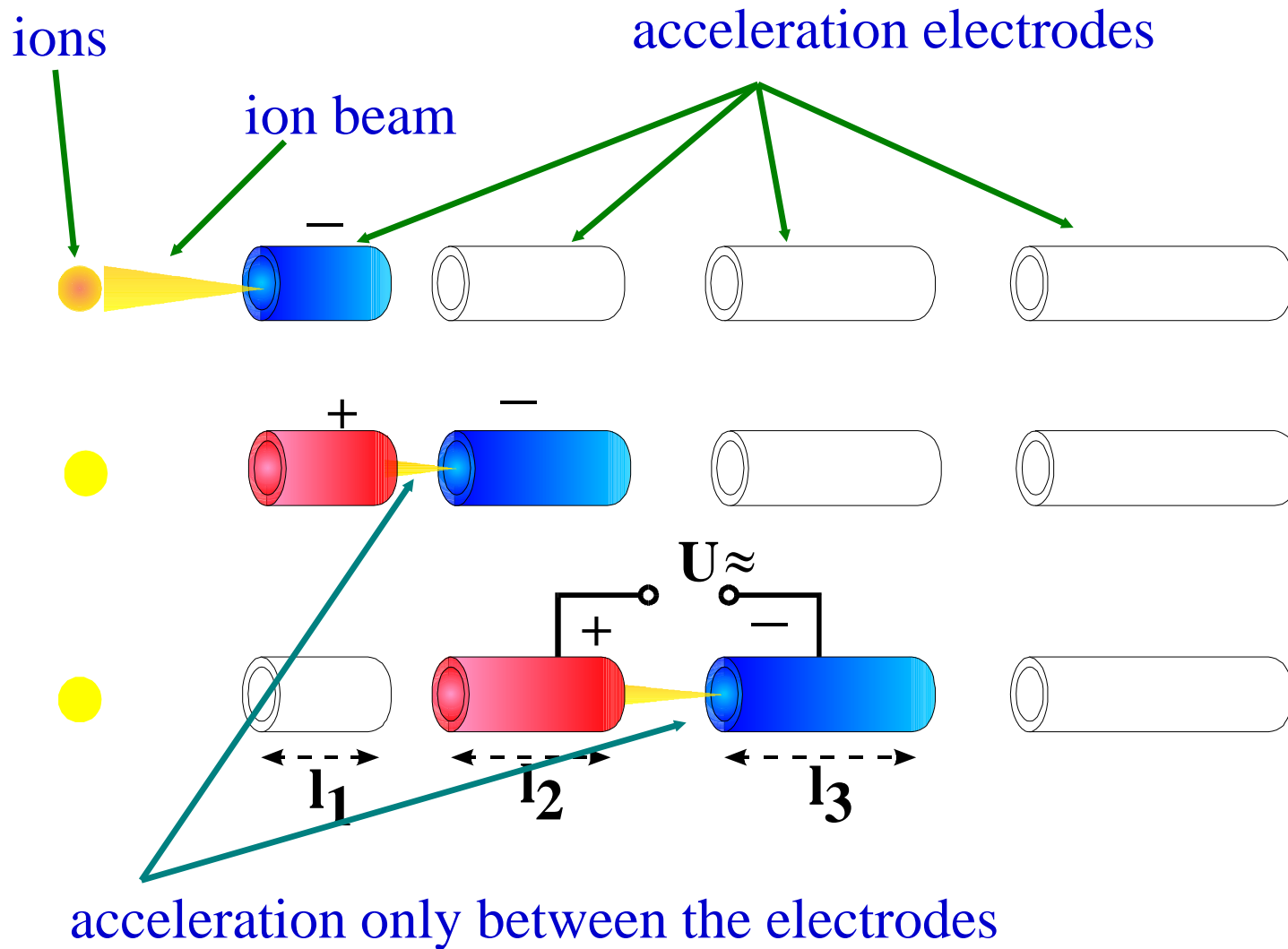


Length of the n -th drift tube:

$$l_n = v_n \cdot \frac{\tau_{RF}}{2} = v_n \cdot \frac{1}{2 \cdot \nu_{RF}}$$

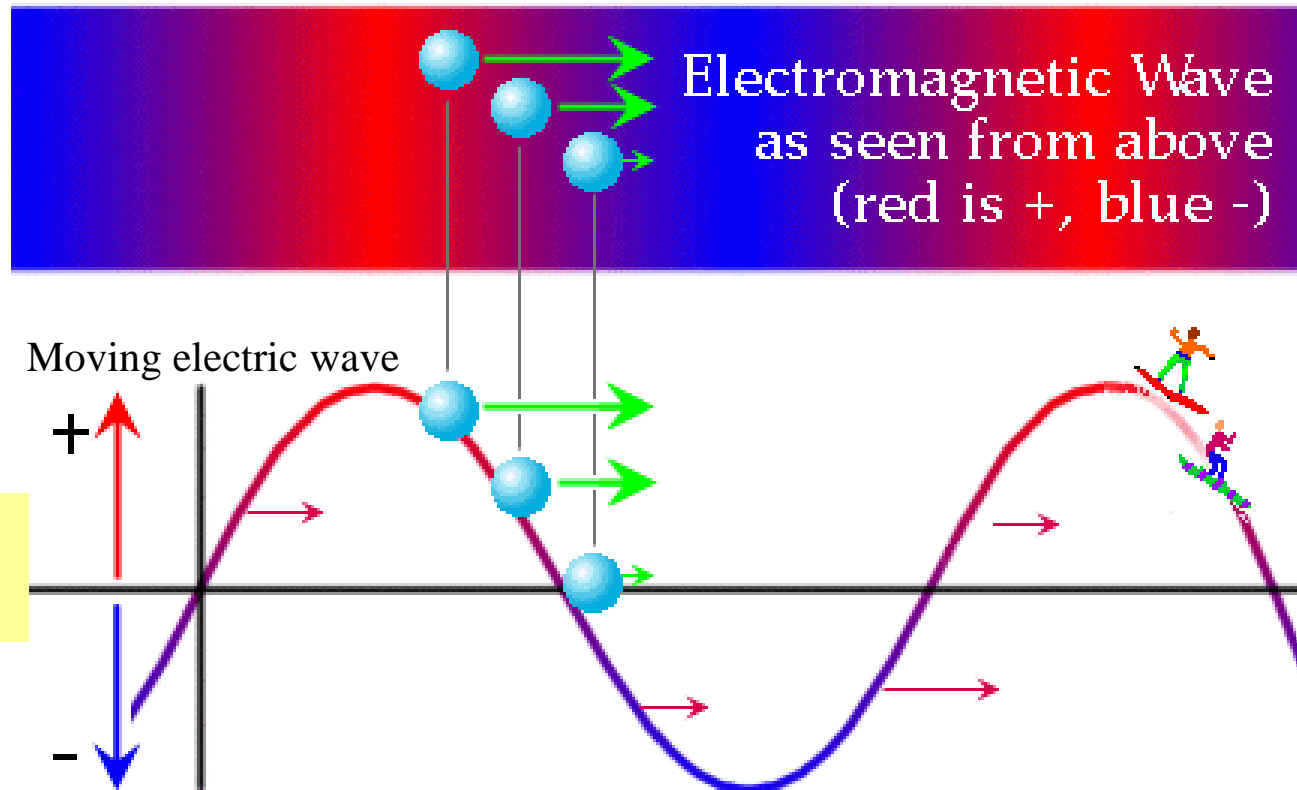
n	number of gaps between the drift tubes
q	charge of the particles
U_0	peak voltage of the RF system
Ψ_s	synchronous phase of the particles

Linear accelerator



Principle of the acceleration

Electromagnetic wave is traveling, pushing particles along with it



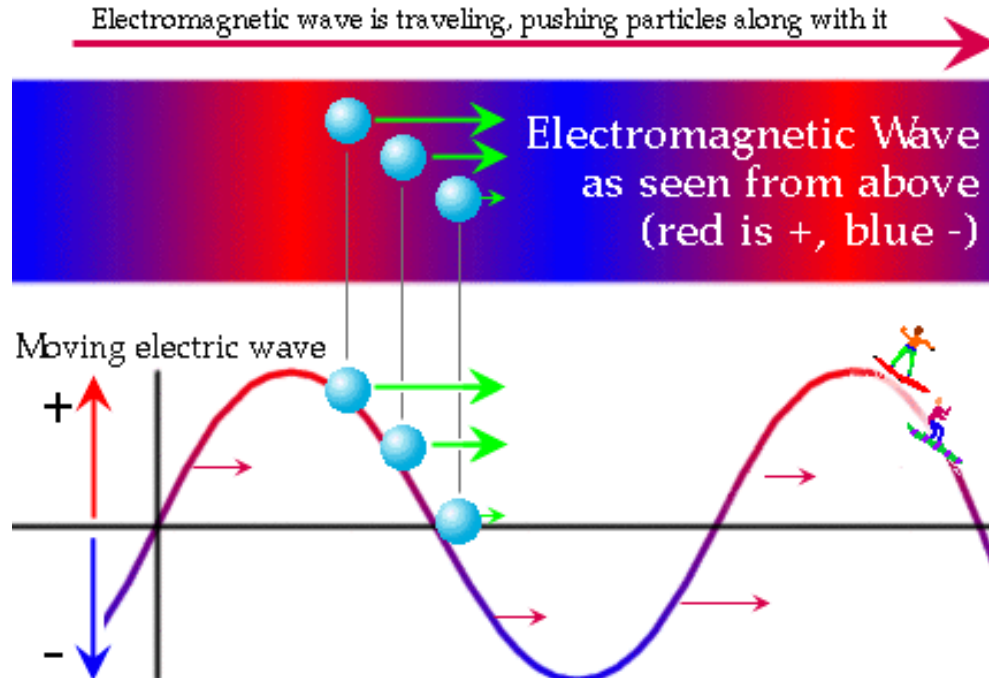
Positively charged particles (blue sphere) close to the crest of the E-M wave experience the most force forward; those closer to the centre experience less of a force. The result is that the particles tend to move together with the wave.

Principle of the acceleration

1 Positive particles just sitting there



Electromagnetic wave is traveling, pushing particles along with it



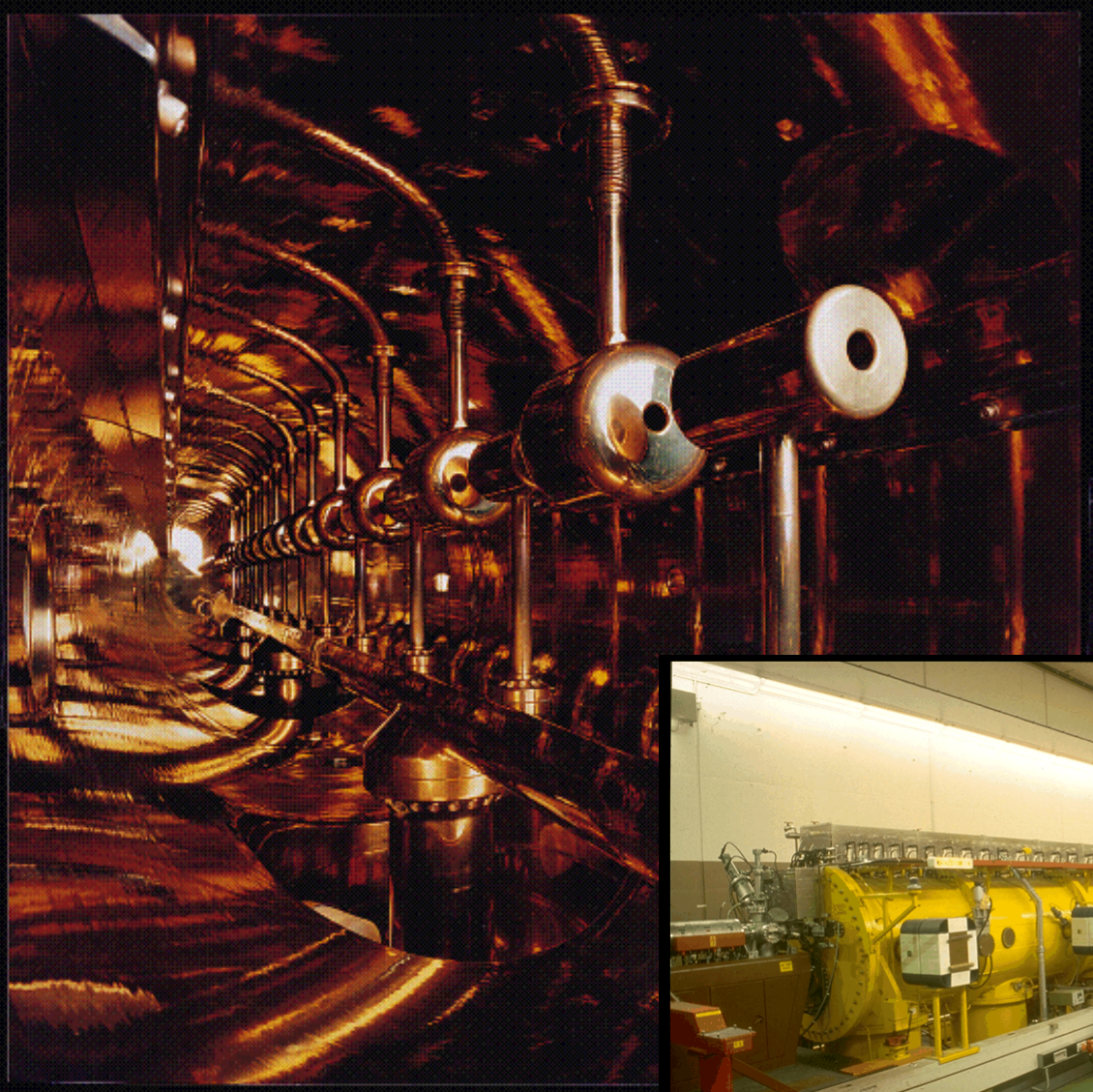
Moving electric wave

+

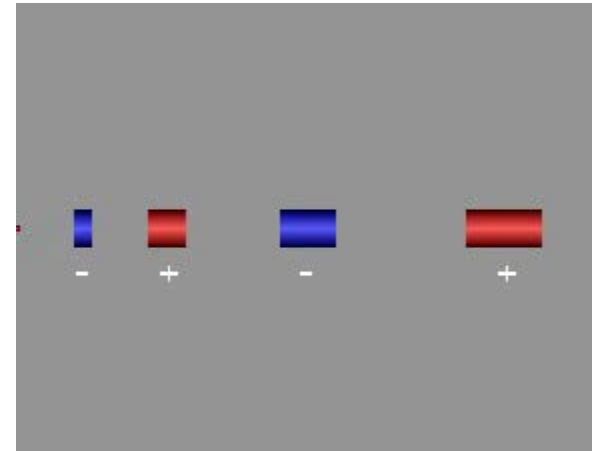
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Positively charged particles (●) close to the crest of the E-M wave experience the most force forward; those closer to the center experience less of a force. The result is that the particles tend to move together with the wave.

Wideroe structure at GSI

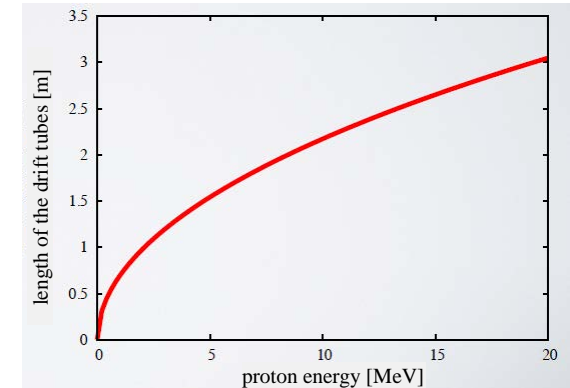
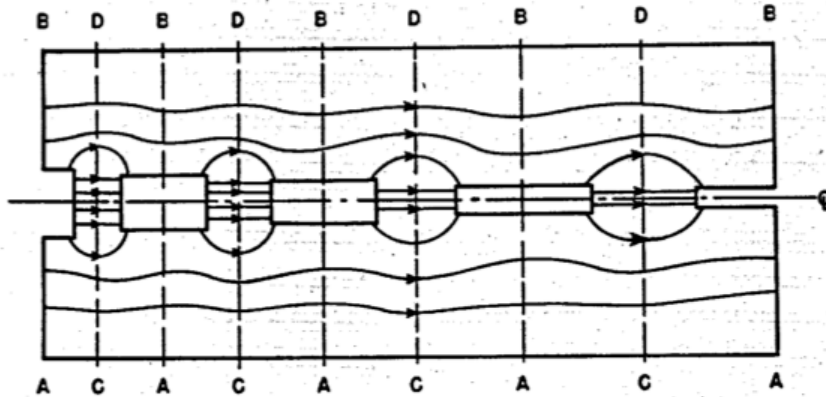


27 MHz Radio frequency



Alvarez structure – standing-wave linear accelerator

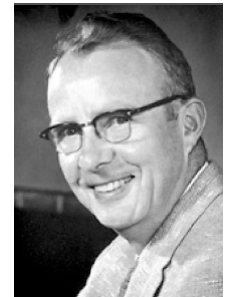
- ❖ The Wideroe linac is only efficient for low-energy heavy ions
- ❖ When using 10 MHz frequency, the length of the drift tubes would act more like antennas and becomes prohibitive for high-energy protons



- ❖ **Alvarez accelerator = resonant cavity**

Standing waves with E-field along direction of particle motion. While the electric fields point in the “wrong direction” the particles are shielded by the drift tubes.

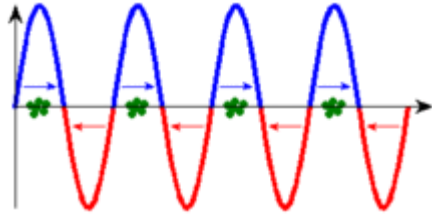
The accelerator consists of a long “tank” (radius determines frequency). Drift tubes are placed along the beam axis, so that the accelerating gaps satisfy synchronicity condition with drift tube length L given by $L = \beta\lambda_0$ where λ_0 is the free space wavelength at the operating frequency.



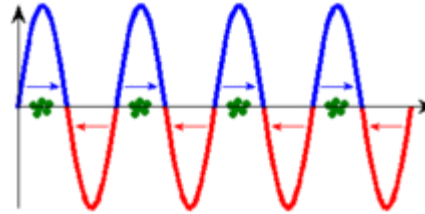
Luis W. Alvarez
(1911-1988)

Wideroe and Alvarez structure

Principal of an accelerated particle package



moving wave



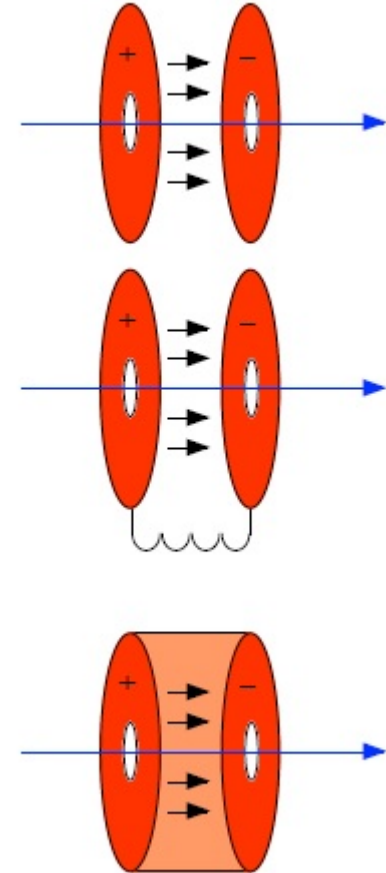
standing wave

RF Cavity

A parallel plate capacitor has an E-field inside, which can accelerate particles. Putting holes in the plates doesn't change the field much.

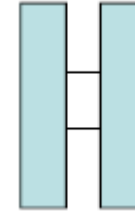
Connecting the plates by an inductor makes a resonant circuit with $\omega^2 = 1/L \cdot C$

Putting many inductors in parallel around the edges of the plates just lowers the inductance and raises the frequency. In fact, you can connect the plates with a metal cylinder!



Coupling of two cavities

- ❖ Suppose we couple two RF cavities together:
 - Each is an electrical oscillator with the same resonant frequency
 - A beampipe couples the two cavities

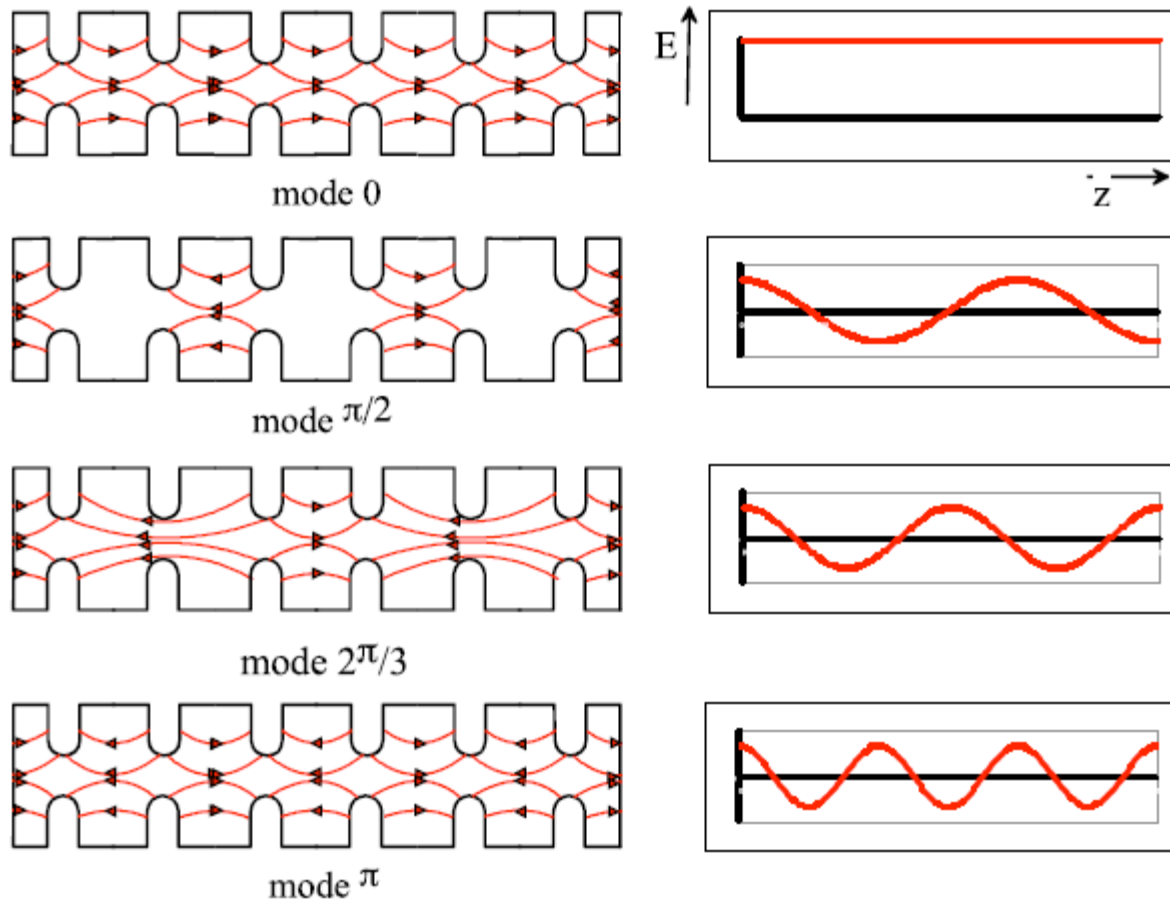


- ❖ Remember the case of mechanical coupling of two oscillators:
- ❖ Two mechanical modes are possible:
 - The “zero-mode”: $\phi_A - \phi_B = 0$, where each oscillates at natural frequency
 - The “pi-mode”: $\phi_A - \phi_B = \pi$, where each oscillates at a higher frequency



- ❖ Standing wave structures of coupled cavities are all driven so that the beam sees either the *zero* or π mode.

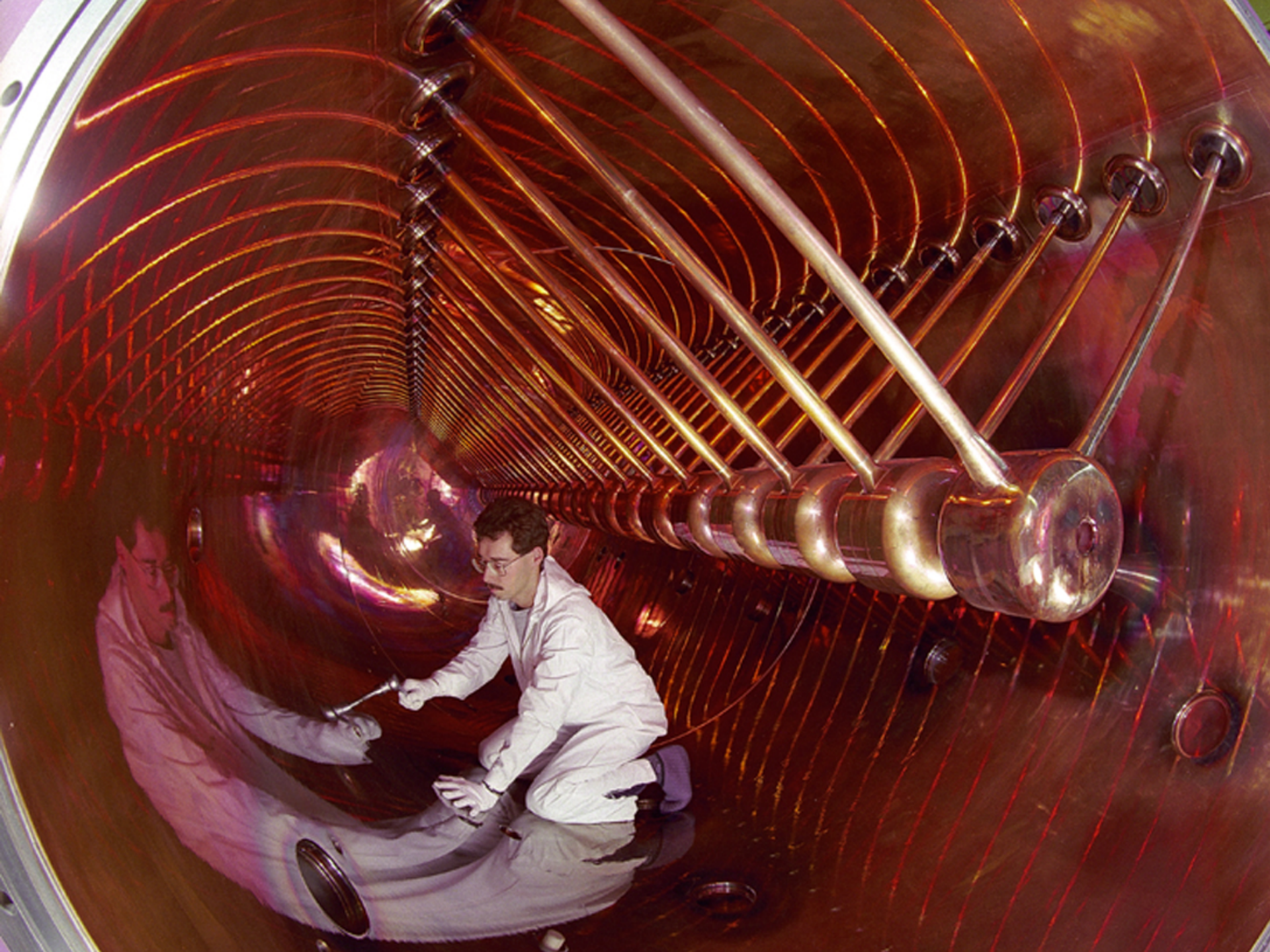
Standing wave cavities



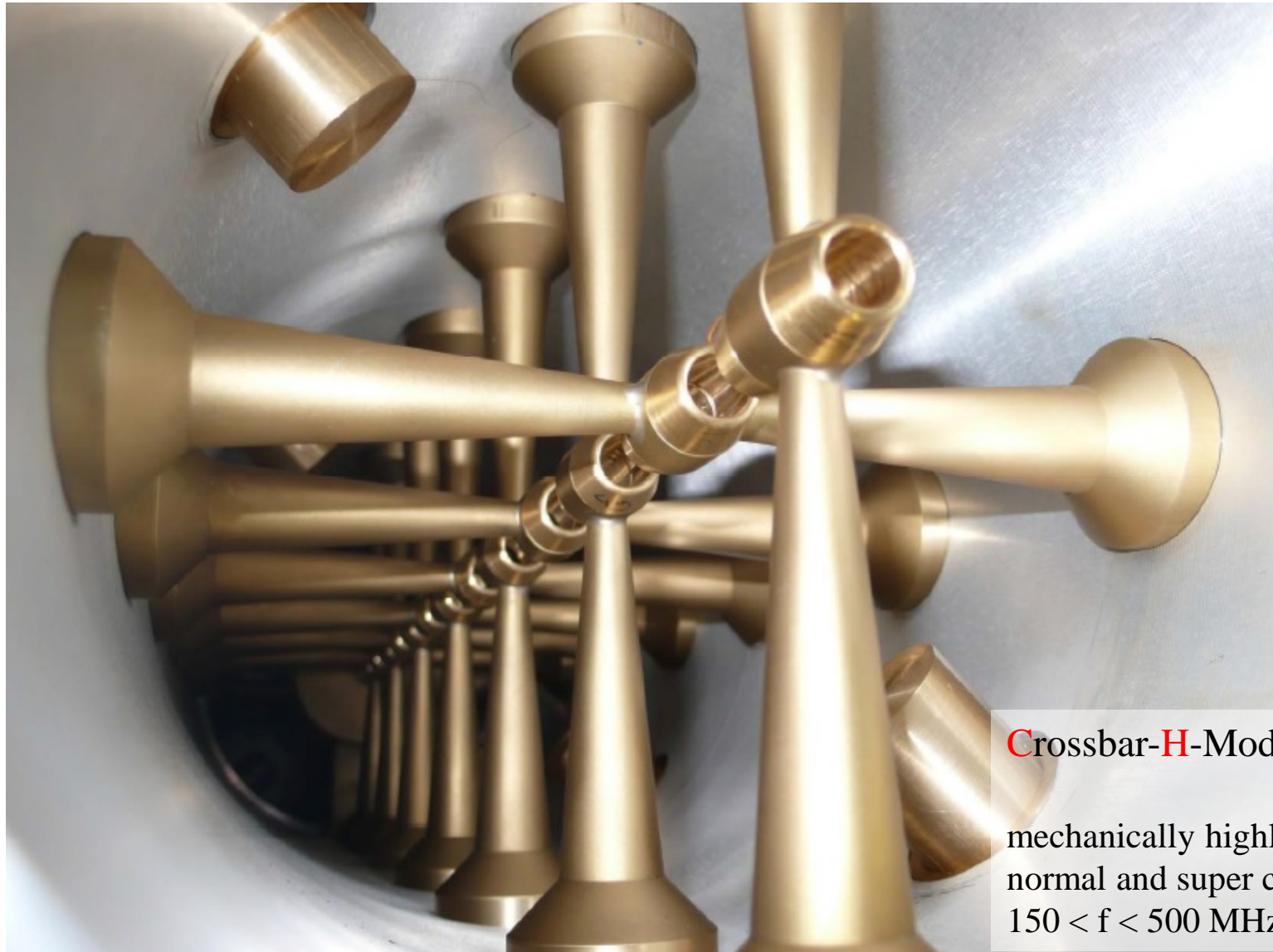
The mode names correspond to the phase difference from one cell to the next

UNILAC Alvarez Accelerator





CH-cavity for FAIR p-Linac

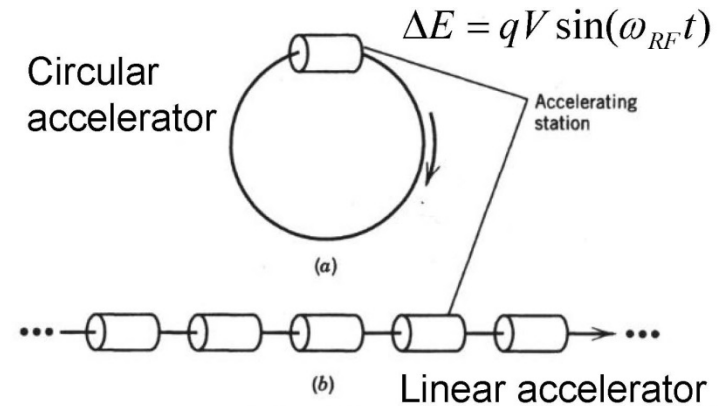


Crossbar-H-Mode-Cavity

mechanically highly stable
normal and super conducting
 $150 < f < 500$ MHz

Synchrotron

As linacs are dominated by cavities, circular machines are dominated by magnets



- Both the accelerating field frequency and the magnetic field strength change synchronously with time to match energy and keep revolution radius constant.
- Magnetic field produced by several bending magnets increases with momentum. For high energy:

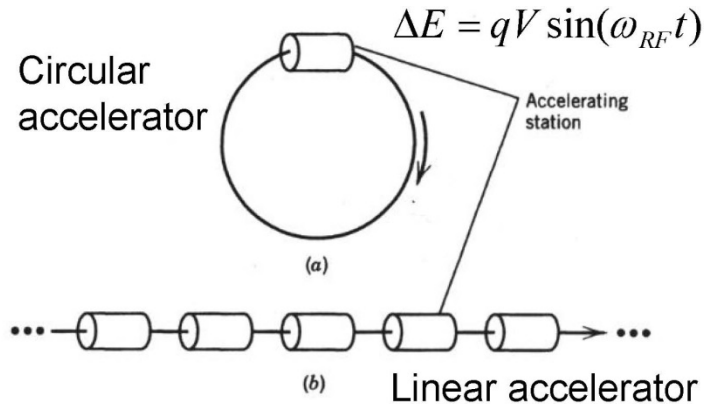
$$E_{proton}[GeV] \approx 0.3 \cdot B\rho[T \cdot m]$$

- Practical limitations for magnetic field \rightarrow high energies only at large radius.

example: 100 GeV protons

- **Fe-magnet** $B \sim 1.5 \text{ T} \rightarrow R = 222 \text{ m}$
- **superconductive magnet** $B \sim 5 \text{ T} \rightarrow R = 67 \text{ m}$

Synchrotron



Mark Oliphant
(1901-2000)

- The bending field changes with particle beam energy to maintain a constant radius:

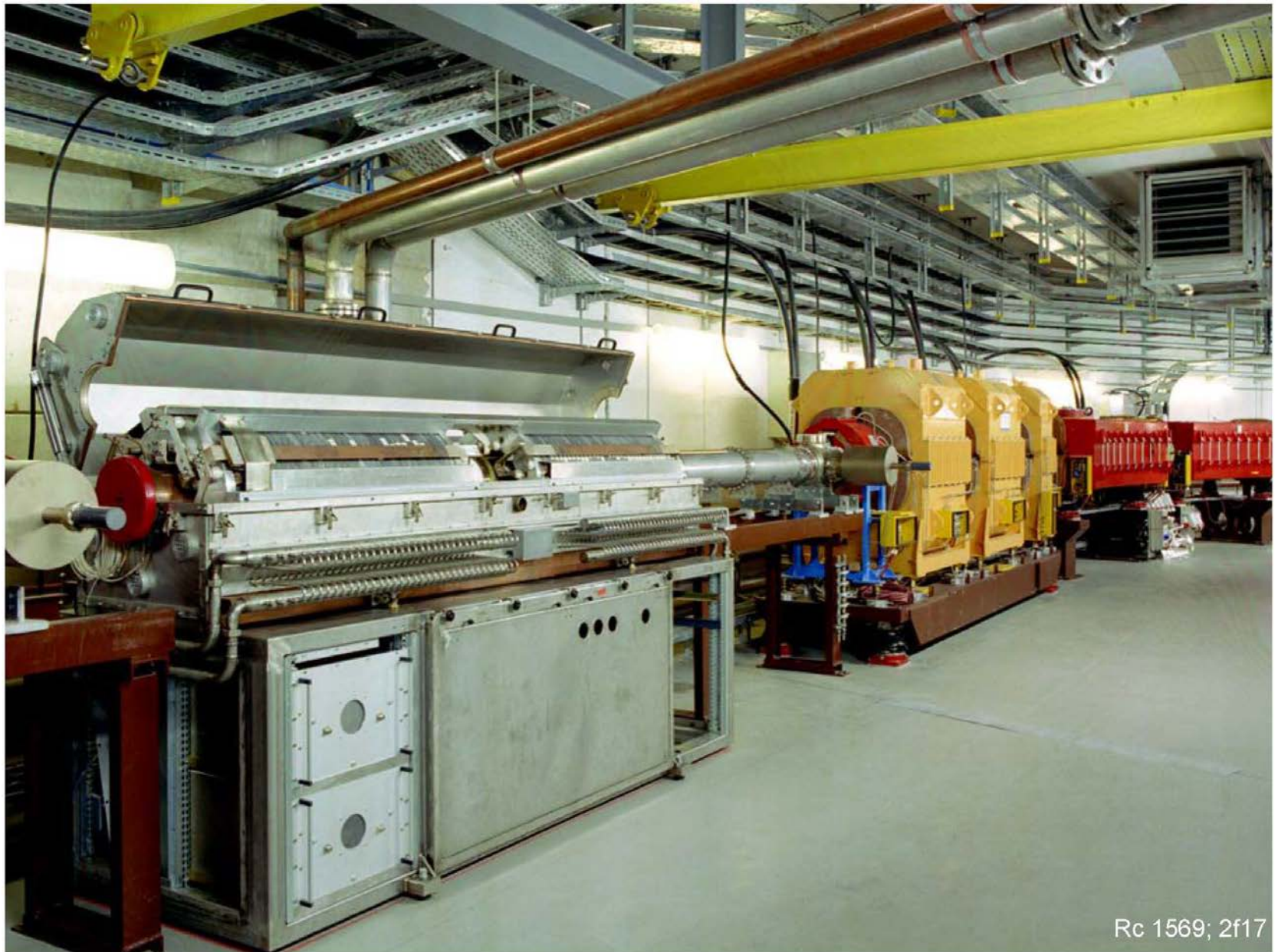
$$\frac{1}{\rho[m]} = 0.3 \frac{B[T]}{\beta E[GeV]} = 0.3 \frac{B[T]}{cp[GeV]}$$

- So ***B* ramps in proportion to the momentum**. The revolution frequency also changes with momentum.
- The synchronicity condition, including now the relativistic term, is

$$\omega = \frac{qB}{m\gamma}$$

- For an **electron synchrotron**, the injected beam is already relativistic, so only the magnetic field changes with beam energy.
- For a **proton synchrotron**, the injected beam is not yet relativistic, so the RF accelerating frequency and the magnetic field both ramp with energy.

SIS - SchwerIonenSynchrotron



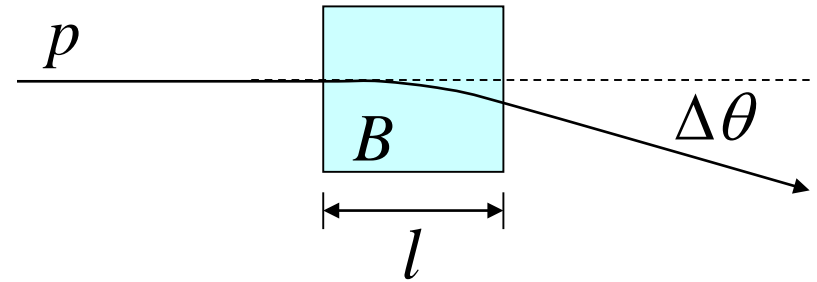
Rc 1569; 2f17

Thin lens approximation and magnetic “kick”

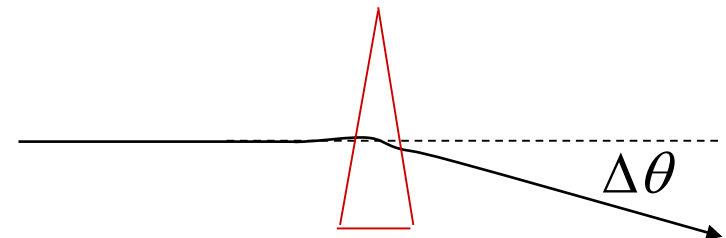
- If the path length through a transverse magnetic field is short compared to the bend radius of the particle, then we can think of the particle receiving a transverse “kick”, which is proportional to the integrated field

$$p_{\perp} \approx qvBt = qvB(l/v) = qBl$$

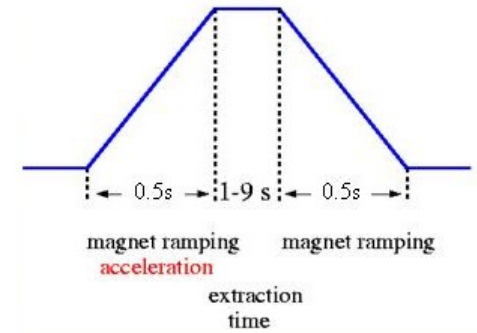
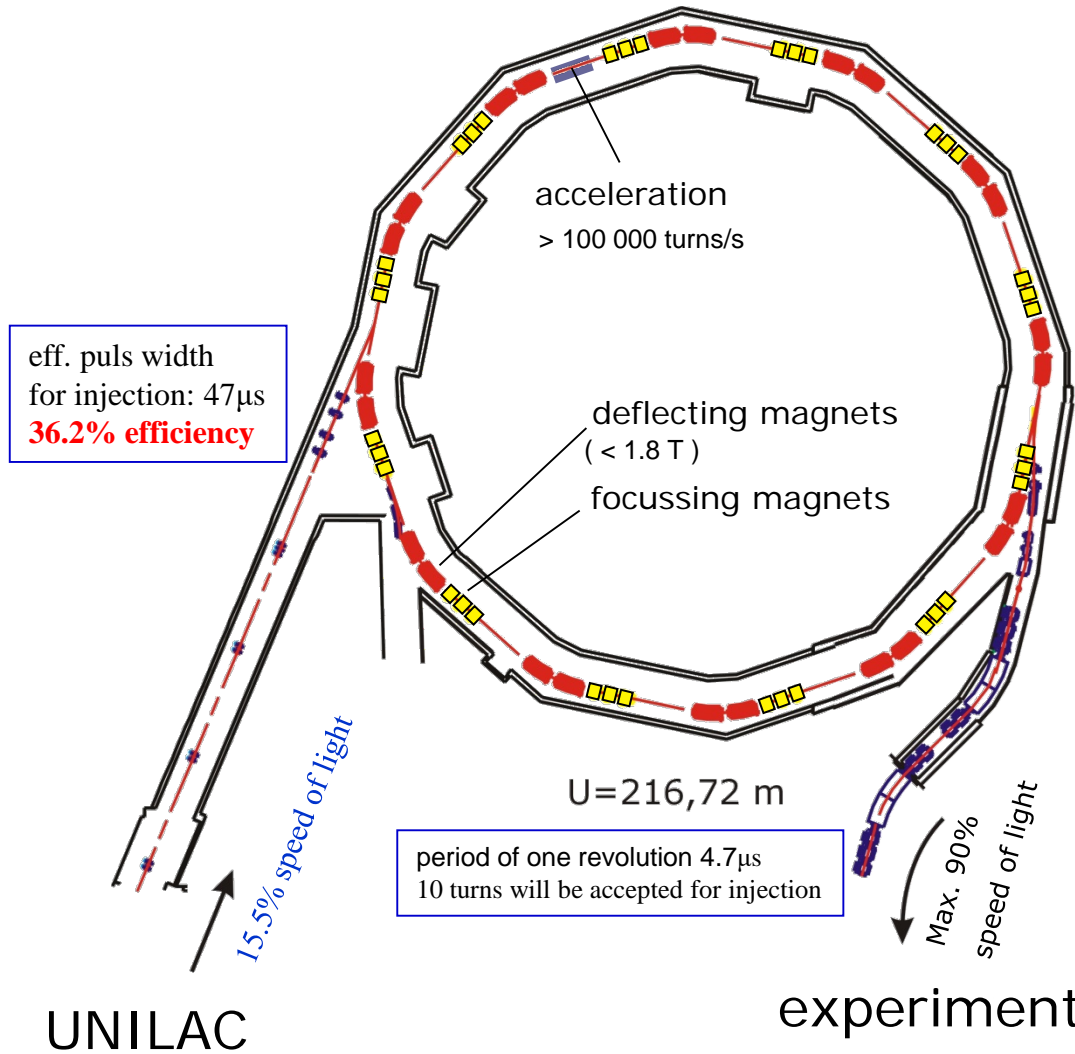
and it will be bent through small angle



$$\Delta\theta \approx \frac{p_{\perp}}{p} = \frac{Bl}{(B\rho)}$$



SIS - SchwerIonenSynchrotron



eff. puls width
for injection: 47 μs
36.2% efficiency

Ion	Number of injections	Intensity [spill ⁻¹] at FRS	Ion source	Date
⁵⁸ Ni	1	6*10 ⁹	MEVVA	3.2006
¹⁰⁷ Ag	1	3*10 ⁹	MEVVA	2.2006
¹²⁴ Xe	1	5*10 ⁹	MUCIS	3.2008
¹³⁶ Xe	4	5*10 ⁹	MEVVA	7.2006
²⁰⁸ Pb	30	1.3*10 ⁹	PIG	3.2006
²³⁸ U	1	2.0*10 ⁹	PIG	9.2009

$$\text{intensity}[\text{s}^{-1}] = 0.5 * \text{intensity}[\text{spill}^{-1}]$$

