Outline: Magnets

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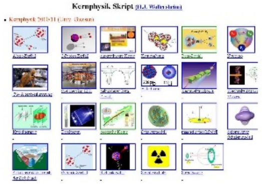
web-page: https://web-docs.gsi.de/~wolle/ and click on



Kernphysik Skript

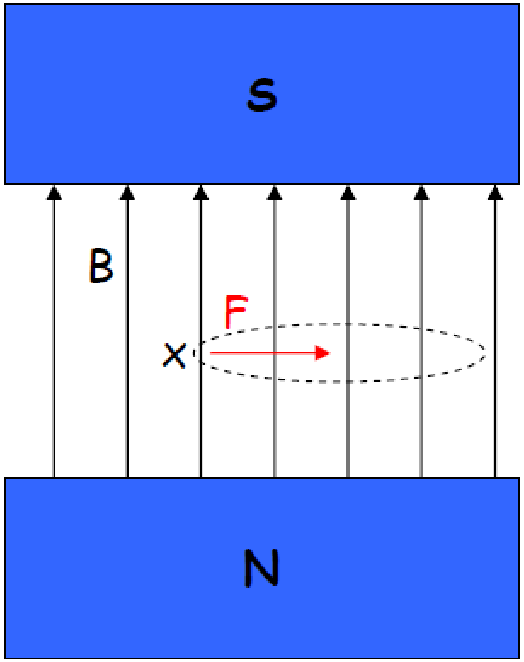
1. dipole magnets
2. quadrupole magnets
3. electromagnetic forces on charged particles
4. cyclotron frequency and K-value
5. cyclotron limits

Magnet examples

Quadrupole

Sextupole

Dipole magnets



A dipole magnet gives a constant B-field.

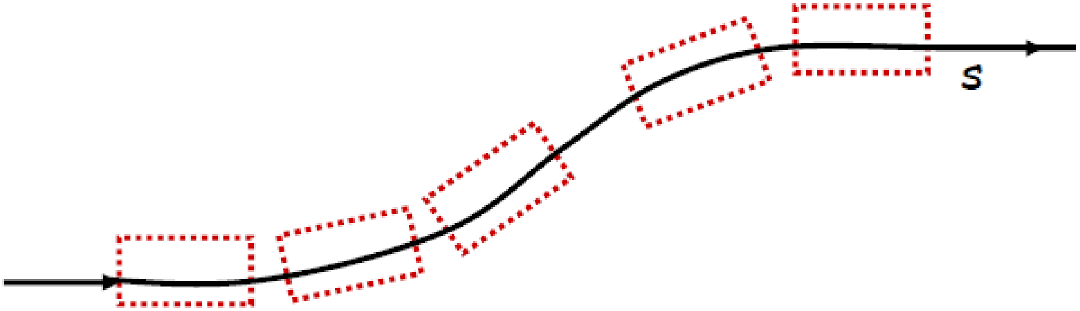
The field lines in a magnet run from North to South. The field shown at right is positive in the vertical direction.

Symbol convention:

× - traveling into the page
● - traveling out of page

In the field shown, for a positively charged particle traveling into the page, the force is to the right.

In an accelerator lattice, dipoles are used to bend the beam trajectory. The set of dipoles in a lattice defines the reference trajectory:



Field equations for a dipole

Let’s consider the dipole field force in more detail. Using the Lorentz Force equation, we can derive the following useful relations:

For a particle of mass ***m***, Energy ***E*** and momentum ***p*** in a uniform ***B***-field:

1) The bending radius of the motion of the particle in the dipole field is given by:

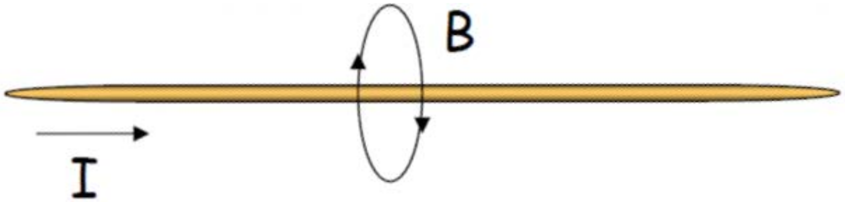
$$\boxed{\frac{1}{\rho} = \frac{eB}{pc}}$$

2) Re-arranging (1), we define the “magnetic rigidity” to be the required magnetic bending strength for given radius and energy:

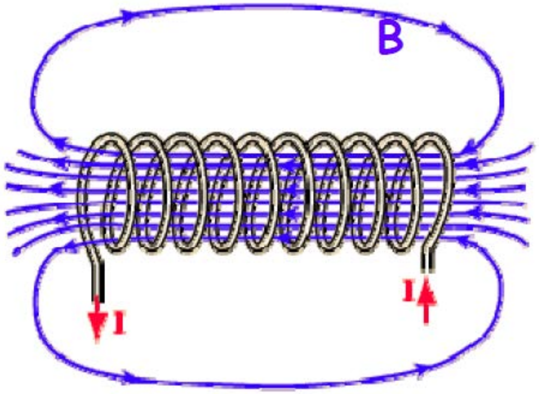
$$\boxed{\begin{gathered} B\rho = \frac{pc}{e} = \frac{\beta E}{e} \\ B\rho[T \cdot m] = \frac{10}{2.998} \cdot \beta E[GeV] \end{gathered}}$$

Generating a B-field from a current

Recall that a current in a wire generates a magnetic B-field which curls around the wire



Or, by winding many turns on a coil we can create a strong uniform magnetic field



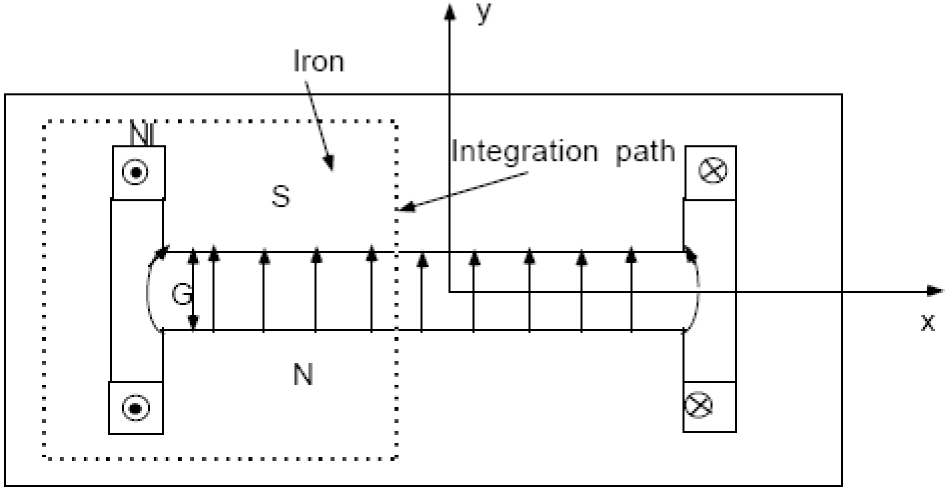
The field strength is given by one of Maxwell's equation:

$$\boxed{\Delta \times \frac{B}{\mu_r} = \frac{4\pi}{c} J}$$

$$\boxed{\mu_r = \frac{\mu_{material}}{\mu_0}}$$

The dipole current-to-field relationship

In an accelerator dipole magnet, we use current-carrying wires and metal cores of high μ to set up a strong dipole field:



Pure dipole: NI turns/pole

N turns of current ***I*** generate a small ***H* = *B/μ*** in the metal. Hence, the B-field across the gap, ***G***, is large.

Using Maxwell’s equation for B, we can derive the relationship between ***B*** in the gap, and ***I*** in the wires:

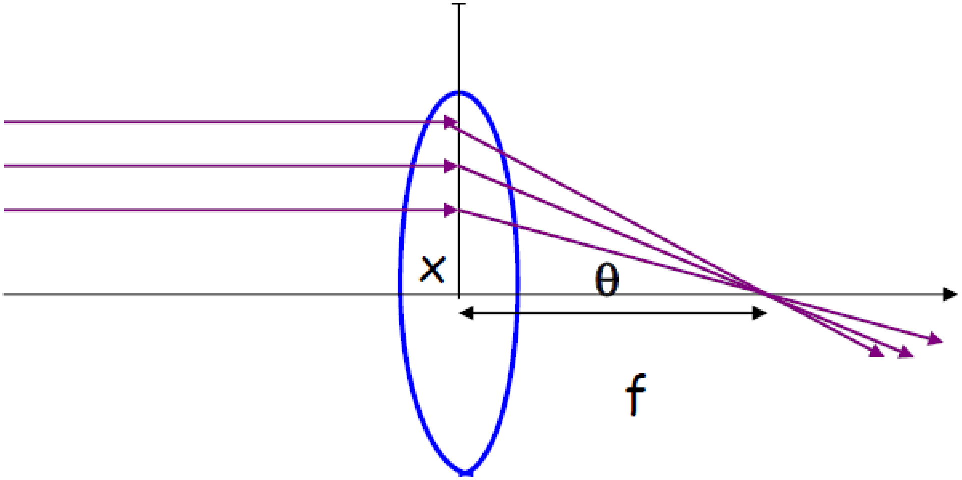
$$\boxed{I_{total} = 2 \cdot N \cdot I[A] = \frac{1}{0.4\pi} B_{\perp}[G] \cdot G[cm]}$$

Optical analogy for focusing

We have seen that a dipole produces a constant field that can be used to bend a beam.

Now we need something that can focus a beam. Without focusing, a beam will naturally diverge.

Consider the optical analogy of focusing a ray of light through a lens:



The rays come to a focus at the focal point, f. The focusing angle depends on the distance from center, x.

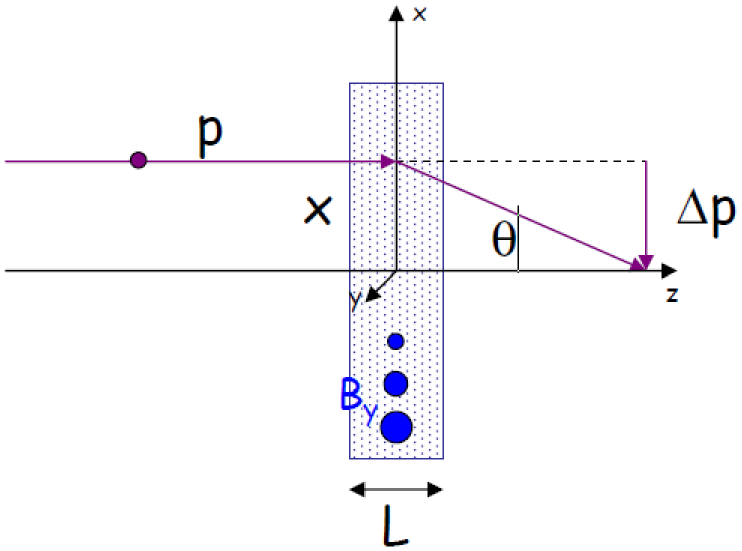
$$\tan\theta = \frac{x}{f}$$

$$\theta \approx \frac{x}{f} \qquad for\ small\ x$$

The farther off axis, the stronger the focusing effect!
The dependence is **linear** for small x.

Focusing particles with magnets

Now consider a **magnetic lens**. This lens imparts a transverse momentum kick, ***Δp***, to the particle beam with momentum ***p***.



For a field which increases linearly with x, the resulting kick, ***Δp***, will also increase linearly with ***x***.

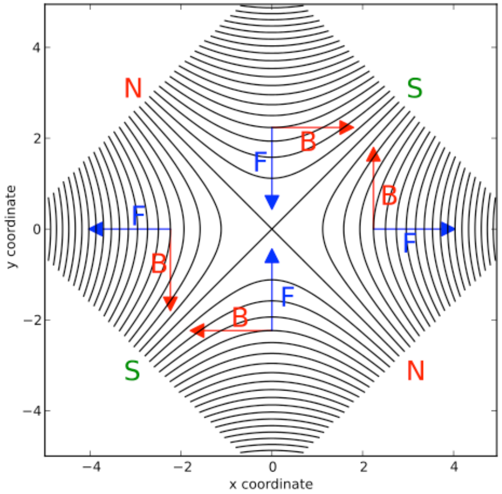
Beginning with the Lorentz force equation, we can solve for the focal length and focusing strength, ***k***:

$$\frac{1}{f} = \frac{e}{pc} \cdot gL = \frac{gL}{B\rho} \quad where \quad g = \frac{dB_y}{dx}$$

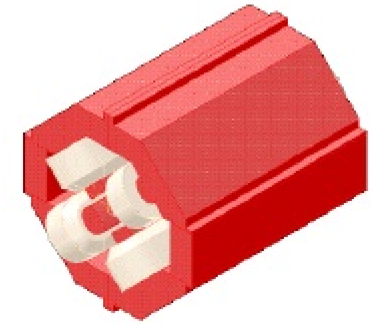
$$k[m^{-2}] = \frac{1}{fL} = \frac{e}{pc} \cdot g = \frac{0.299 \cdot g[T/m]}{\beta E[GeV]} = focusing\ strength$$

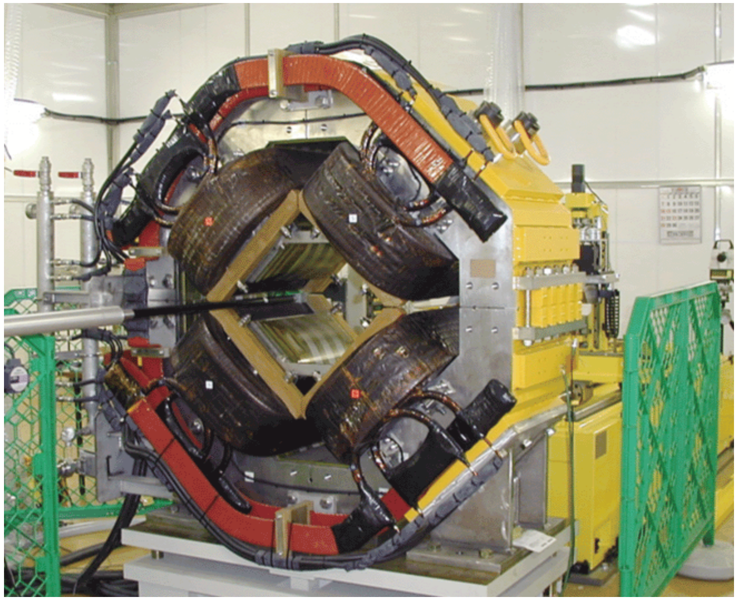
Magnet focusing

Quadrupole magnets are used to focus the beam



$B \sim x$



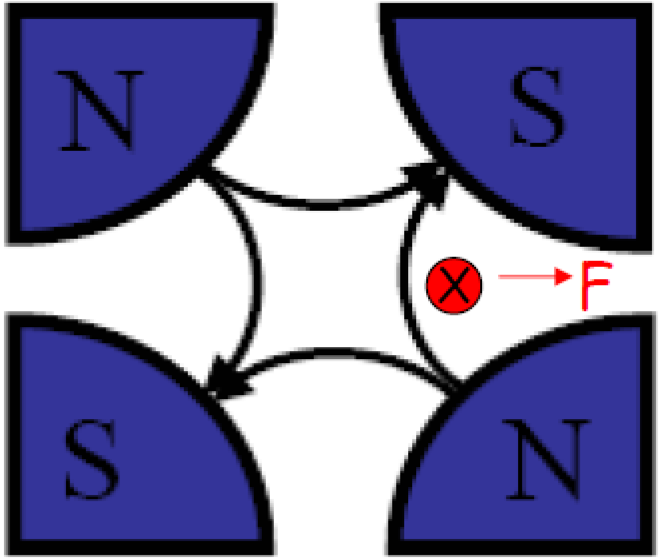


FQUADs (shown above) focus the beam horizontally.

DQUADs (as above, rotated 90^0) focus the beam vertically.

Quadrupole magnet

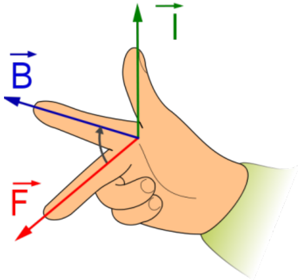
A **quadrupole magnet** imparts a force proportional to distance from the center. This magnet has 4 poles:



Consider a positive particle traveling into the page (into the magnet field).

According to the right hand rule, the force on a particle on the right side of the magnet is to the right, and the force on a similar particle on left side is to the left.

This magnet is horizontally defocusing. A distribution of particles in x would be defocused!



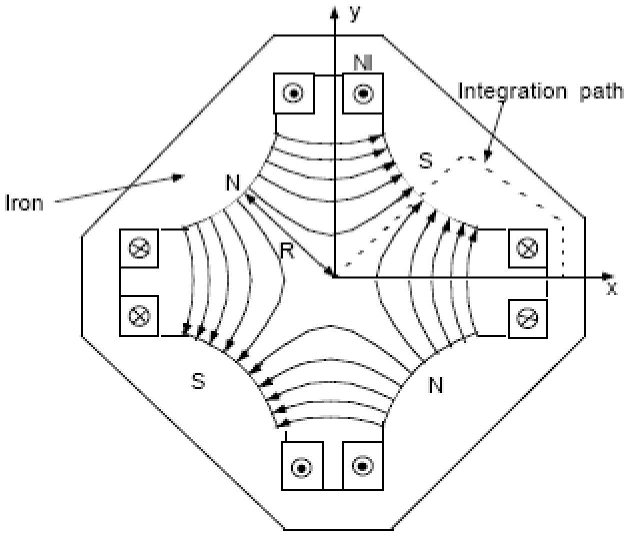
What about the vertical direction?

→ ***A quadrupole which defocuses in one plane focuses in the other.***

positive charge
right hand rule

Quadrupole current-to-field equation

As with a dipole, in an accelerator we use current-carrying wires wrapped around metal cores to create a quadrupole magnet:



Pure quadrupole, NI turns/pole

The field lines are denser near the edges of the magnet, meaning the field is stronger there. The strength of $\boldsymbol{B_y}$ is a function of $\boldsymbol{x}$, and visa-versa. The field at the center is zero!

Using Maxwell's equation for B, we can derive the relationship between ***B*** in the gap, and ***I*** in the wires:

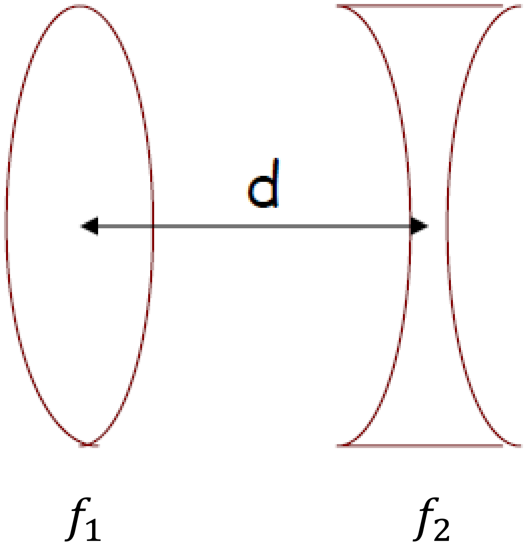
$$\dot{B} = \frac{dB_\varphi}{dr} = \frac{8\pi I}{cR^2}$$

$$\dot{B}[T/m] = \frac{2.52 \cdot I[A]}{R[mm]^2}$$

Focusing using arrays of quadrupoles

- Quadrupoles focus in one plane while defocusing in the other. So, how can this be used to provide net focusing in an accelerator?

Consider the optical analogy of two lenses, with focal lengths f_1 and f_2, separated by a distance d:



The combined f is:

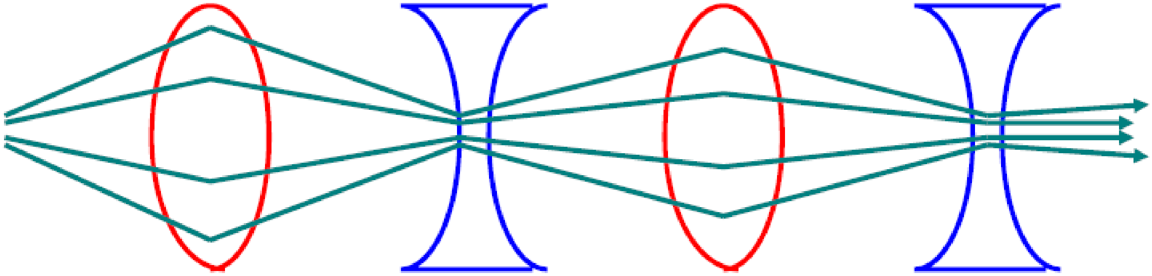
$$\frac{1}{f_{combined}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 \cdot f_2}$$

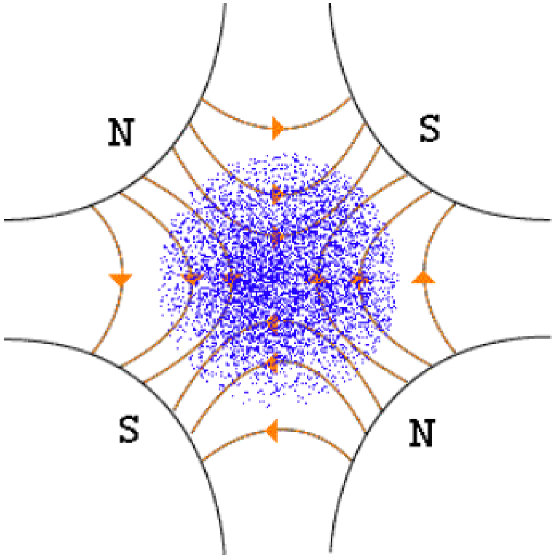
What if $f_1 = -f_2$?

The net effect is focusing, $\frac{1}{f_{combined}} = \frac{d}{f_1 \cdot f_2}$

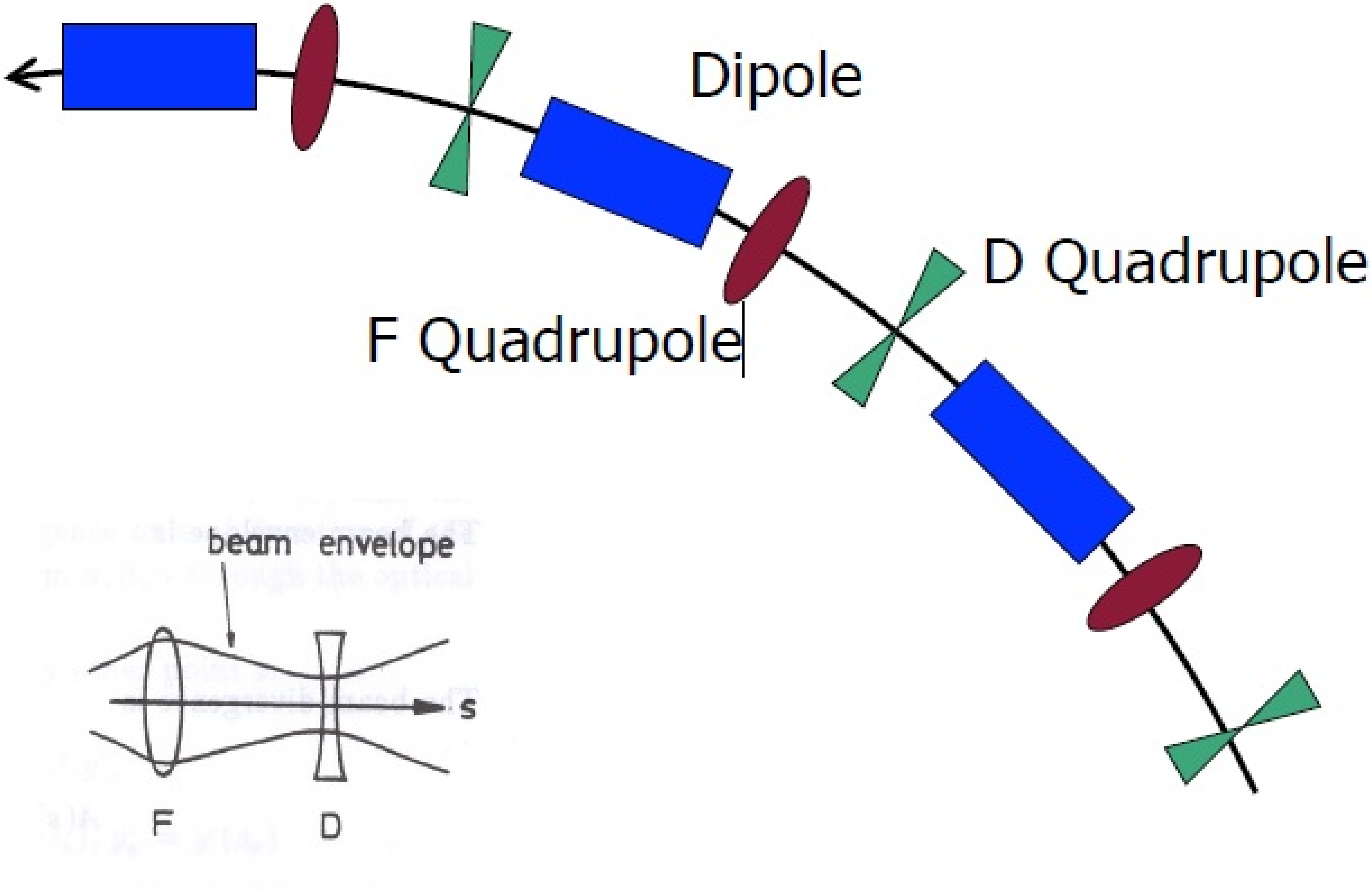
More on focusing particles …

The key is to alternate focusing and defocusing quadrupoles. This is called a FODO lattice (Focus-Drift-Defocus-Drift):





Strong focusing lattice



Dipole

D Quadrupole

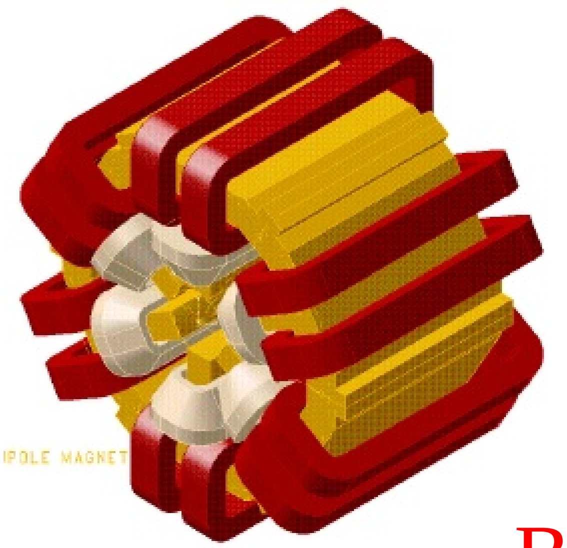
F Quadrupole

FODO Cell

Challenge: finite dispersion

Errors and nonlinearities – chromatic correction

Sextupoles – compensate off-energy particle tune shift



$$B \sim x^2$$

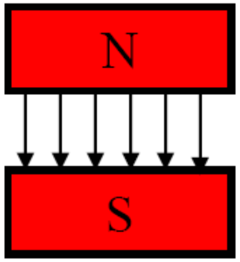
Located in finite dispersion region: focusing varies with energy offset

Other n-pole magnets

The general equation for B allows us to write the field for any n-pole magnet.

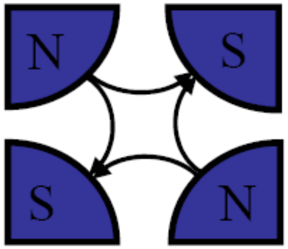
Examples of upright magnets:

n=1: Dipole



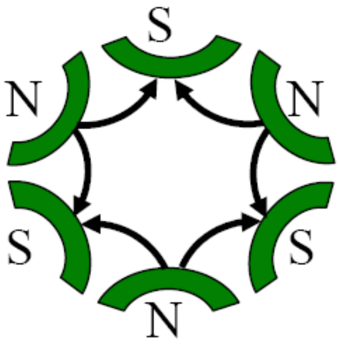
180^0 between poles

n=2: Quadrupole



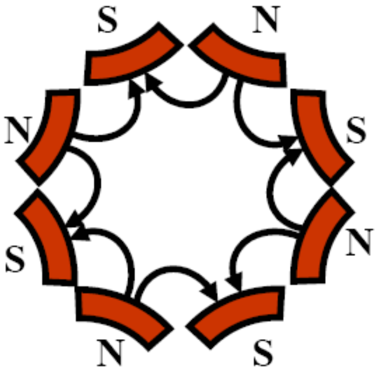
90^0 between poles

n=3: Sextupole



60^0 between poles

n=4: Octupole

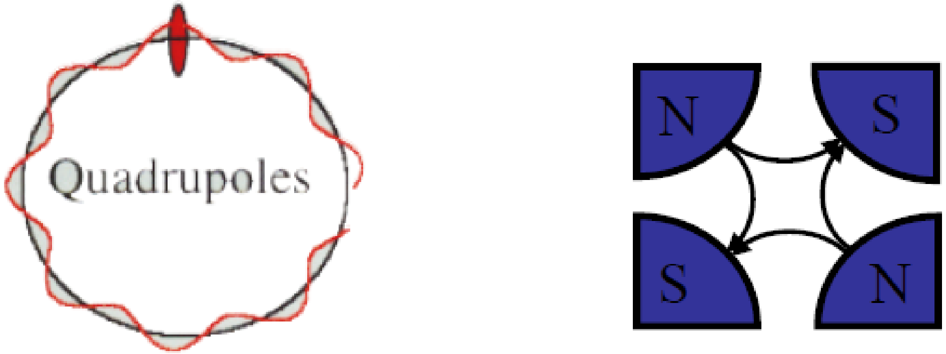


45^0 between poles

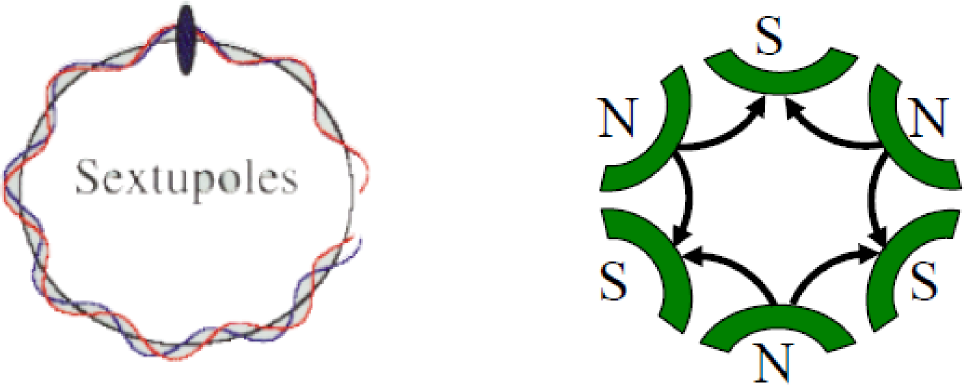
- In general, poles are 360°/2n apart.
- The skew version of the magnet is obtained by rotating the upright magnet by 180°/2n.

n-pole uses

Bending (following reference trajectory)



Focusing the beam

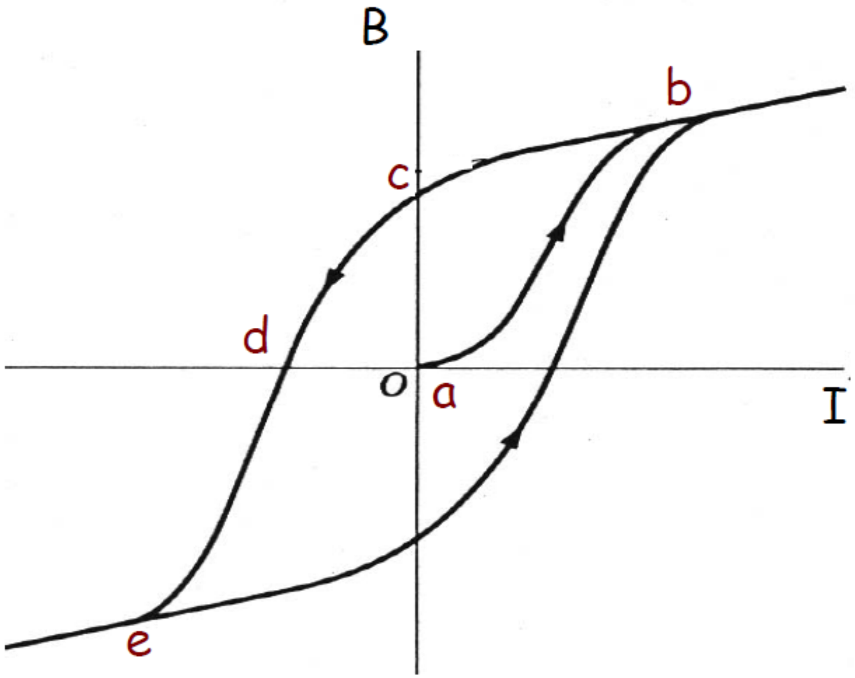


"Chromatic compensation"

Hysteresis and magnet cycling

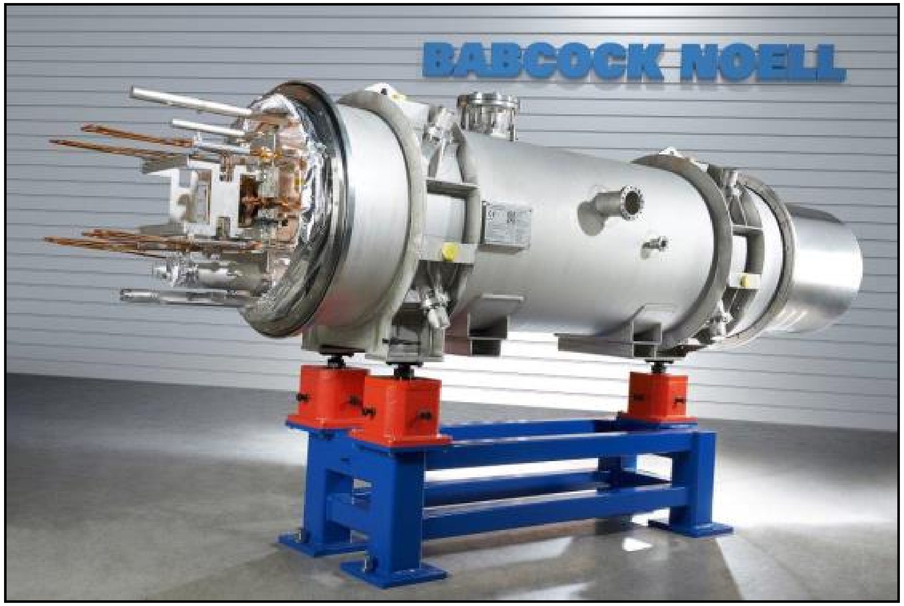
An external B-field, created by a current I, creates a B-field in iron by aligning tiny internal dipoles (electron spins) in the material.

However, if the current and external field are dropped to zero, the material remains partially magnetized. This gives rise to “hysteresis” and the need for magnet cycling.



a - start point
b - saturation
c - residual magnetization
d - B=0
e - saturation with -B

Superconducting magnets



Superconducting Accelerator Magnets

Who needs superconductivity anyway?

Abolish Ohm's Law

- no power consumption

  (although do need refrigeration power)
- high current density ⇒ compact windings, high gradients
- ampere turns are cheap, so we don't need iron

  (although often use it for shielding)

Consequences

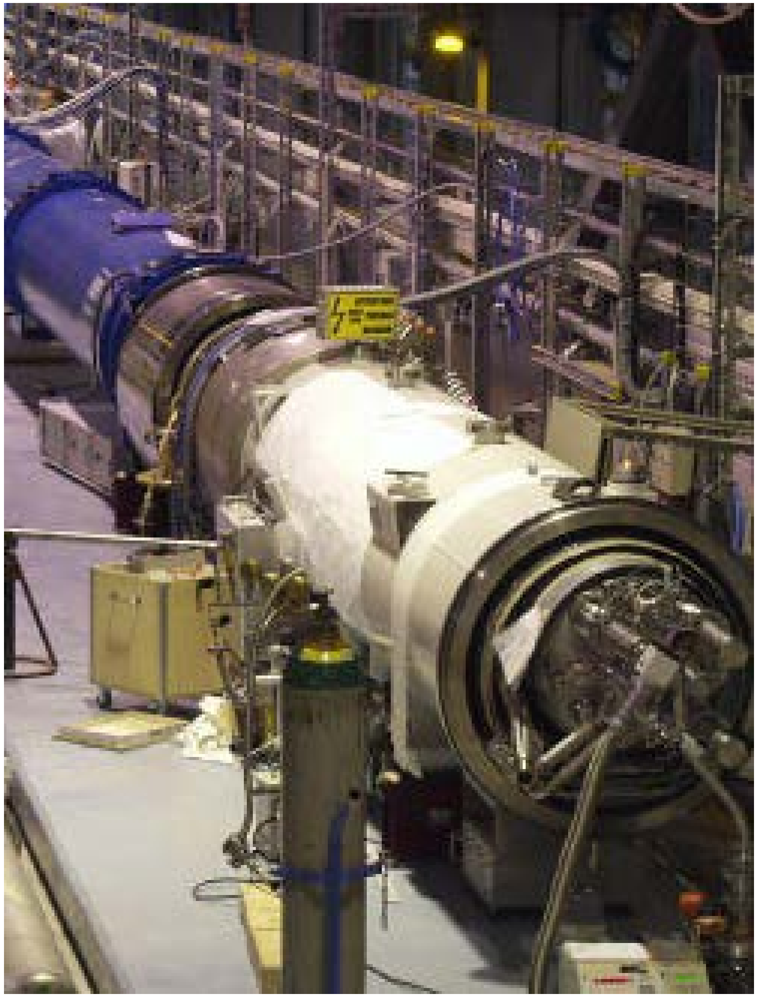
- low power bill
- higher magnetic fields mean reduced bend radius

  ⇒ smaller rings

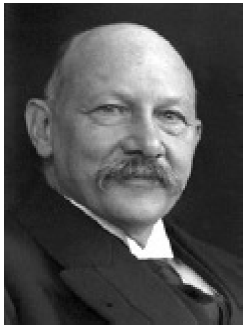
  ⇒ reduced capital cost

  ⇒ new technical possibilities (muon collider)
- higher quadrupole gradients

  ⇒ higher luminosity
- higher rf electric fields (continuous)



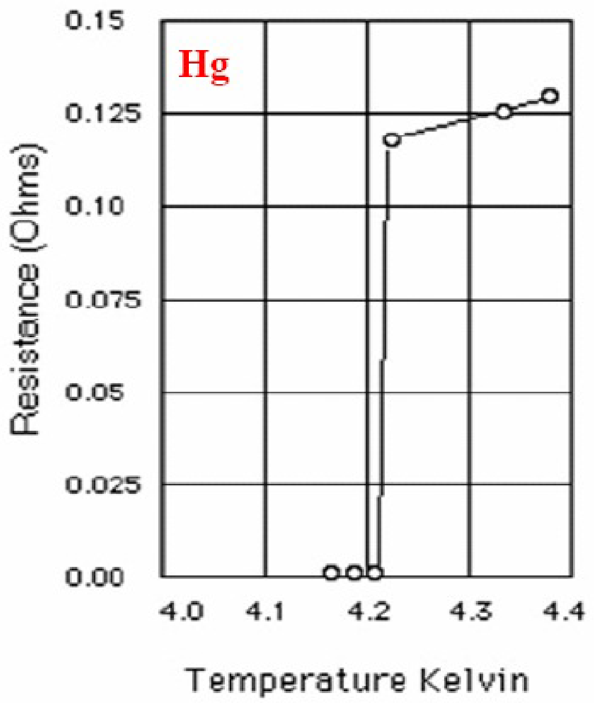
Discovery of superconductivity



Heike Kamerlingh Onnes

Gilles Holst

- Superconductivity is a phenomenon occurring in certain materials, when their electrical resistance vanishes below a characteristic critical temperature. It is accompanied by expulsion of the magnetic field from the material.



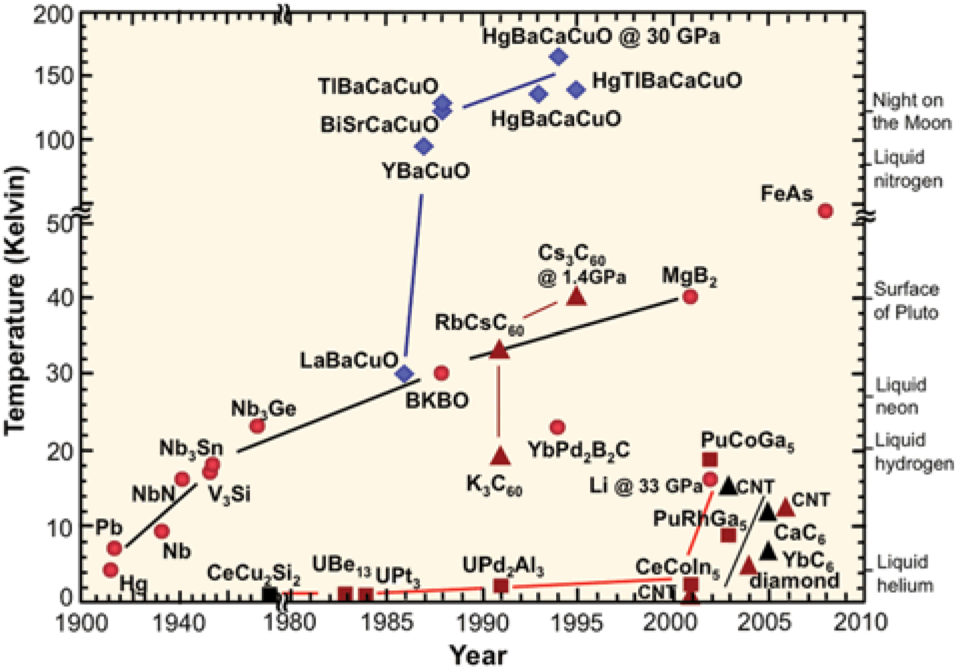
The superconducting transition is at 4.2K. Within 0.01K, the resistance jumps from unmeasurably small ($<10^{-6}$ Ω to 0.1 Ω).

- Superconductivity was discovered by Dutch physicist Heike Kamerlingh Onnes and Gilles Holst on April 8, 1911 in Leiden.
  This discovery was made possible after Onnes was able to liquefy helium in 1908. He was awarded a Nobel prize in 1913 “*for his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium*”.

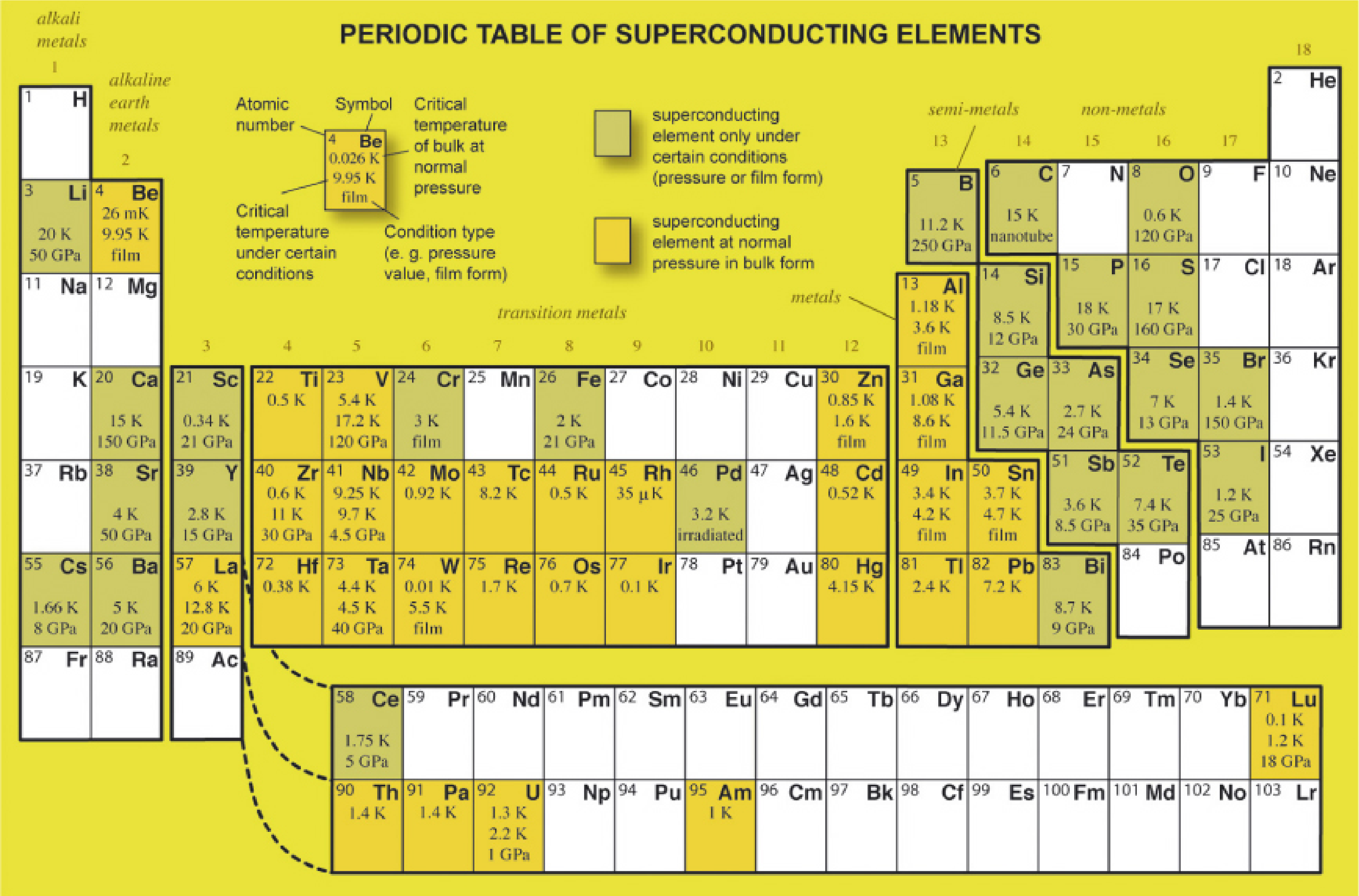
Types of superconductors

There are many ways to classify superconductors. The most common are:

- ❖ *Response to a magnetic field: **Type I*** SC has a single critical field, above which all superconductivity is lost; ***Type II*** SC has two critical fields, between which it allows partial penetration of the magnetic field.
- ❖ *By theory of operation:* A SC is ***conventional*** if it can be explained by the BCS theory or its derivatives, or ***unconventional***, otherwise.
- ❖ *By critical temperature:* A SC is generally considered ***high temperature (HTS)*** if it reaches a SC state when cooled using liquid nitrogen ($T_c > 77K$), or ***low temperature*** otherwise.
- ❖ *By material:* SC material classes include ***chemical elements*** (e.g. Hg or Pb), ***alloys*** (such as NbTi, Nb_3Sn or NbN), ***ceramics*** (YBCO and MgB2), or ***organic superconductors***



Superconducting elements



PERIODIC TABLE OF SUPERCONDUCTING ELEMENTS

alkali metals · *alkaline earth metals* · *transition metals* · *metals* · *semi-metals* · *non-metals*

Atomic number — Symbol — Critical temperature of bulk at normal pressure — Critical temperature under certain conditions — Condition type (e. g. pressure value, film form)

4 Be
0.026 K
9.95 K
film

▨ superconducting element only under certain conditions (pressure or film form)

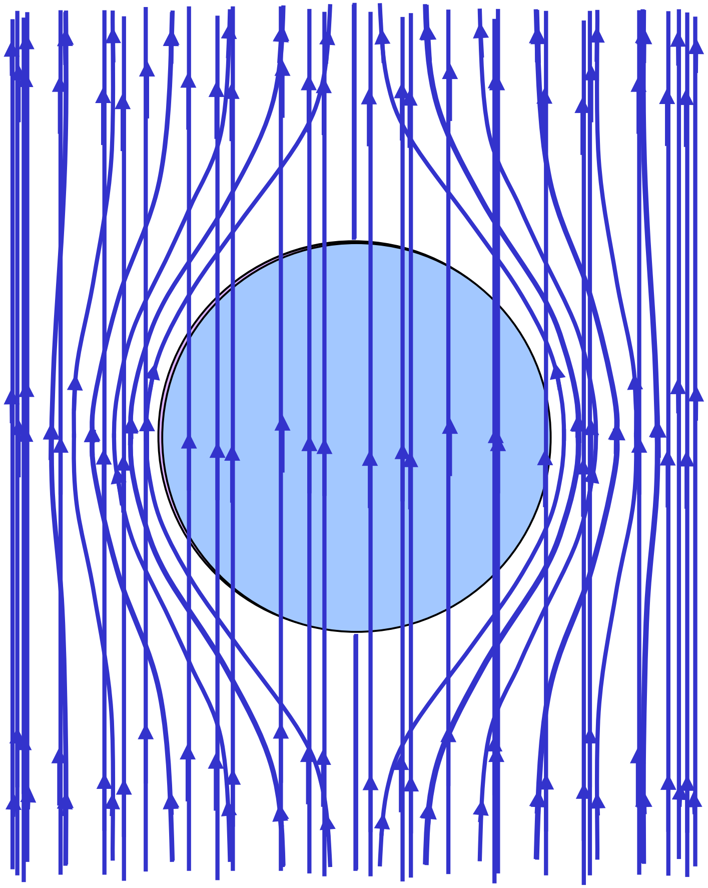
🟨 superconducting element at normal pressure in bulk form

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 H																	2 He
3 Li 20 K 50 GPa	4 Be 26 mK 9.95 K film											5 B 11.2 K 250 GPa	6 C 15 K nanotube	7 N	8 O 0.6 K 120 GPa	9 F	10 Ne
11 Na	12 Mg											13 Al 1.18 K 3.6 K film	14 Si 8.5 K 12 GPa	15 P 18 K 30 GPa	16 S 17 K 160 GPa	17 Cl	18 Ar
19 K	20 Ca 15 K 150 GPa	21 Sc 0.34 K 21 GPa	22 Ti 0.5 K	23 V 5.4 K 17.2 K 120 GPa	24 Cr 3 K film	25 Mn	26 Fe 2 K 21 GPa	27 Co	28 Ni	29 Cu	30 Zn 0.85 K 1.6 K film	31 Ga 1.08 K 8.6 K film	32 Ge 5.4 K 11.5 GPa	33 As 2.7 K 24 GPa	34 Se 7 K 13 GPa	35 Br 1.4 K 150 GPa	36 Kr
37 Rb	38 Sr 4 K 50 GPa	39 Y 2.8 K 15 GPa	40 Zr 0.6 K 11 K 30 GPa	41 Nb 9.25 K 9.7 K 4.5 GPa	42 Mo 0.92 K	43 Tc 8.2 K	44 Ru 0.5 K	45 Rh 35 µK	46 Pd 3.2 K irradiated	47 Ag	48 Cd 0.52 K	49 In 3.4 K 4.2 K film	50 Sn 3.7 K 4.7 K film	51 Sb 3.6 K 8.5 GPa	52 Te 7.4 K 35 GPa	53 I 1.2 K 25 GPa	54 Xe
55 Cs 1.66 K 8 GPa	56 Ba 5 K 20 GPa	57 La 6 K 12.8 K 20 GPa	72 Hf 0.38 K	73 Ta 4.4 K 4.5 K 40 GPa	74 W 0.01 K 5.5 K film	75 Re 1.7 K	76 Os 0.7 K	77 Ir 0.1 K	78 Pt	79 Au	80 Hg 4.15 K	81 Tl 2.4 K	82 Pb 7.2 K	83 Bi 8.7 K 9 GPa	84 Po	85 At	86 Rn
87 Fr	88 Ra	89 Ac															

58 Ce 1.75 K 5 GPa	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu 0.1 K 1.2 K 18 GPa
90 Th 1.4 K	91 Pa 1.4 K	92 U 1.3 K 2.2 K 1 GPa	93 Np	94 Pu	95 Am 1 K	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

Two kinds of superconductor: Type 1

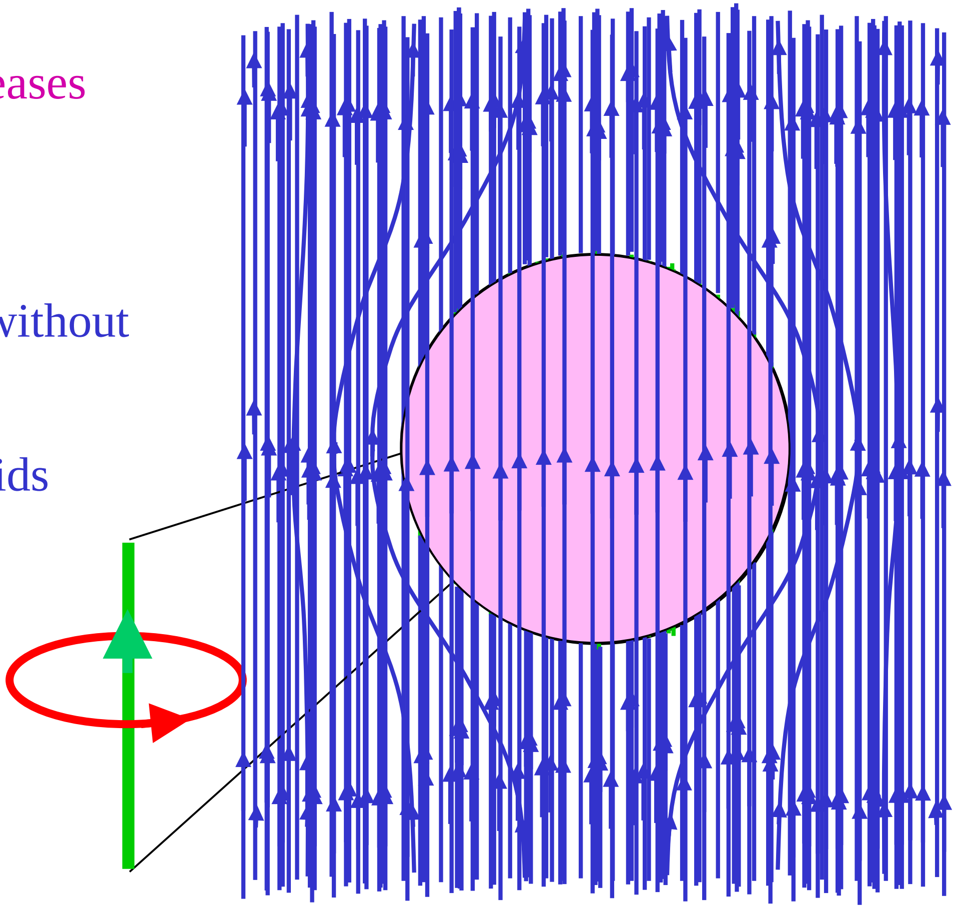
- the materials first discovered by Heike Kamerlingh Onnes in 1911 - soft metals like lead, tin mercury
- sphere of metal at room temperature
- apply magnetic field
- reduce the temperature - resistance decreases
- reduce the temperature some more
  - resistance decreases some more
- at the critical temperature θ_c the field is pushed out - the **Meissner effect** - superconductivity!
- increase the field - field is kept out
  - by Maxwell there must be surface currents
- increase the field some more - superconductivity is extinguished and the field jumps in
- thermodynamic critical field B_c is trade off between **reducing** energy via condensation to superconductivity and **increasing** energy by pushing out field ~ 0.1T



useless for magnets!

Two kinds of superconductor: Type 2

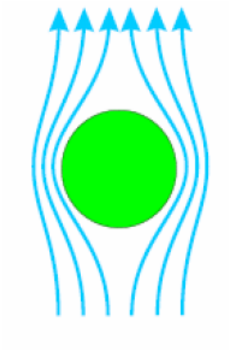
- apply magnetic field
- reduce the temperature - resistance decreases
- at the critical temperature θ_c the field is pushed out - surface currents again
- increase the field - field jumps back in without quenching superconductivity
- it does so in the form of quantized fluxoids
- lower critical field B_{c1}
- supercurrents encircle the resistive core of the fluxoid thereby screening field from the bulk material
- higher field ⇒ closer vortex spacing
- superconductivity is extinguished at the (much higher) upper critical field B_{c2}



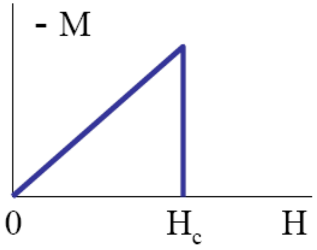
OK for magnets!

Meissner effect (1933)

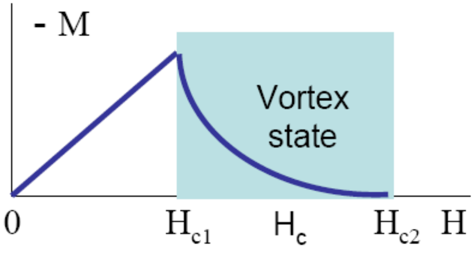
- Superconductivity is a quantum mechanical phenomenon. It is characterized by the Meissner effect (Meissner and Ochsenfeld, 1933), the complete ejection of magnetic field lines from the interior of the superconductor as it transitions into the superconducting state.
- The occurrence of the Meissner effect indicates that superconductivity cannot be understood simply as the idealization of perfect conductivity in classical physics.



Superconductor in Meissner state = ideal diamagnetic



Complete Meissner effect in type-I superconductors



High-field partial Meissner effect in type-II superconductors

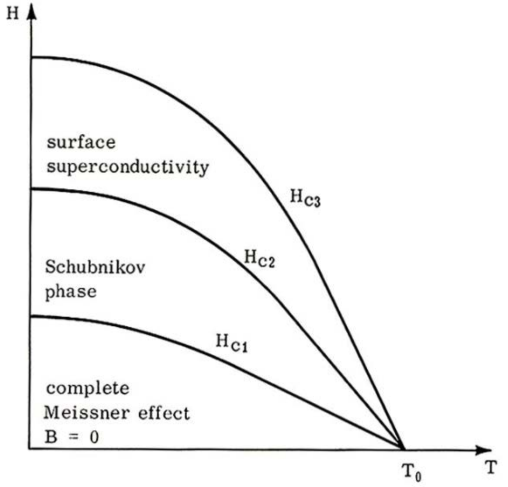


Figure 3-1
Phase diagram for a long cylinder of a Type II superconductor.

- Type I: Meissner state $B = H + M = 0 \; for \; H < H_c$; normal state at $H > H_c$
- Type II: Meissner state $B = H + M = 0 \; for \; H < H_{c1}$; partial flux penetration for $H_{c1} < H < H_{c2}$; normal state for $H > H_{c2}$

Microscopic theory of superconductivity

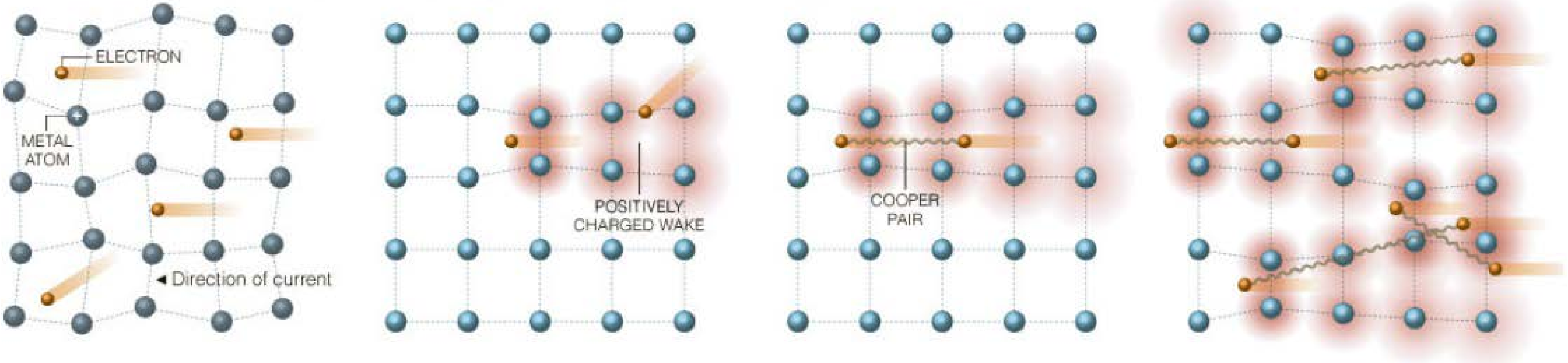
Bardeen-Cooper-Schrieffer (BCS) theory (1957)
Nobel prize in 1972

The New York Times

January 7, 2008

Low-Temperature Superconductivity

December was the 50th anniversary of the theory of superconductivity, the flow of electricity without resistance that can occur in some metals and ceramics.



ELECTRICAL RESISTANCE
Electrons carrying an electrical current through a metal wire typically encounter resistance, which is caused by collisions and scattering as the particles move through the vibrating lattice of metal atoms.

CRITICAL TEMPERATURE
As the metal is cooled to low temperatures, the lattice vibration slows. A moving electron attracts nearby metal atoms, which create a positively charged wake behind the electron. This wake can attract another nearby electron.

COOPER PAIRS
The two electrons form a weak bond, called a Cooper pair, which encounters less resistance than two electrons moving separately. When more Cooper pairs form, they behave in the same way.

SUPERCONDUCTIVITY
If a pair is scattered by an impurity, it will quickly get back in step with other pairs. This allows the electrons to flow undisturbed through the lattice of metal atoms. With no resistance, the current may persist for years.

Sources: Oak Ridge National Laboratory; Philip W. Phillips

JONATHAN CORUM/THE NEW YORK TIMES

BCS Theory

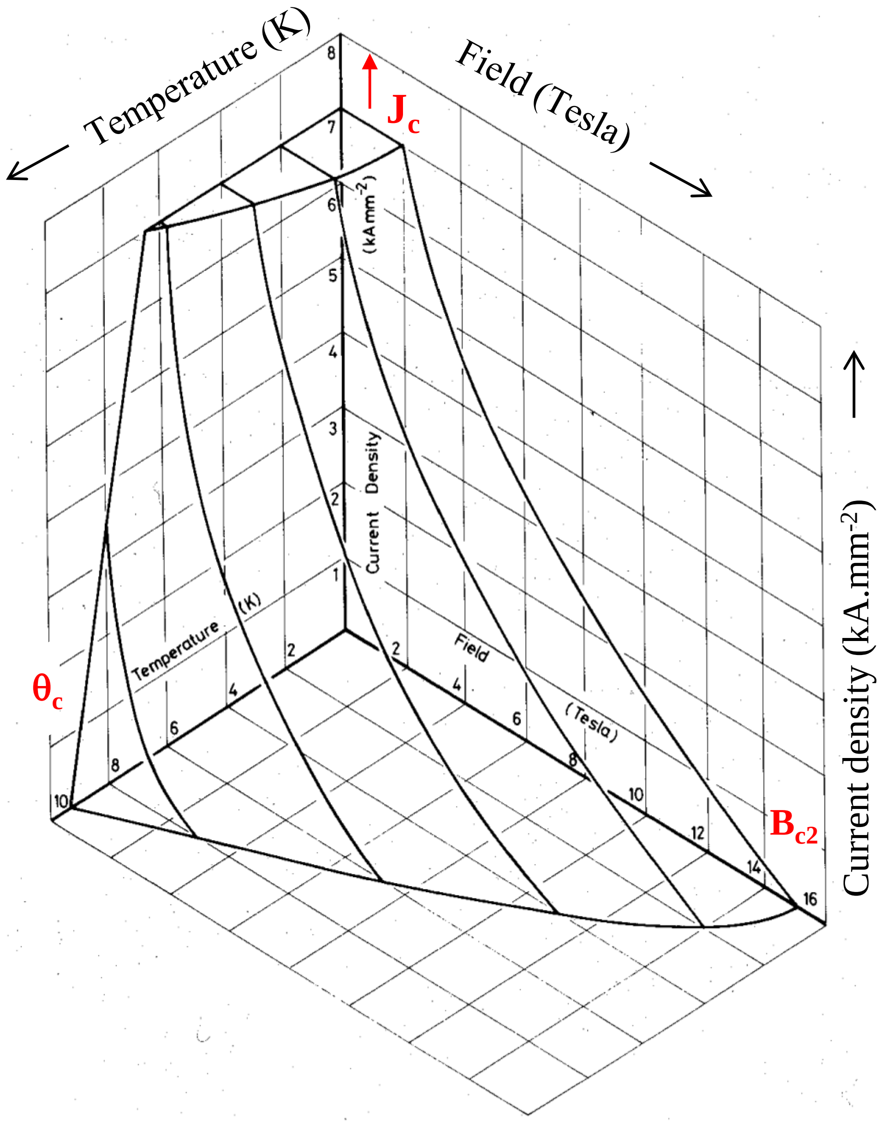
What is the phase coherence?

Incoherent (normal) crowd:
each electron for itself

Phase-coherent (superconducting) condensate
of electrons

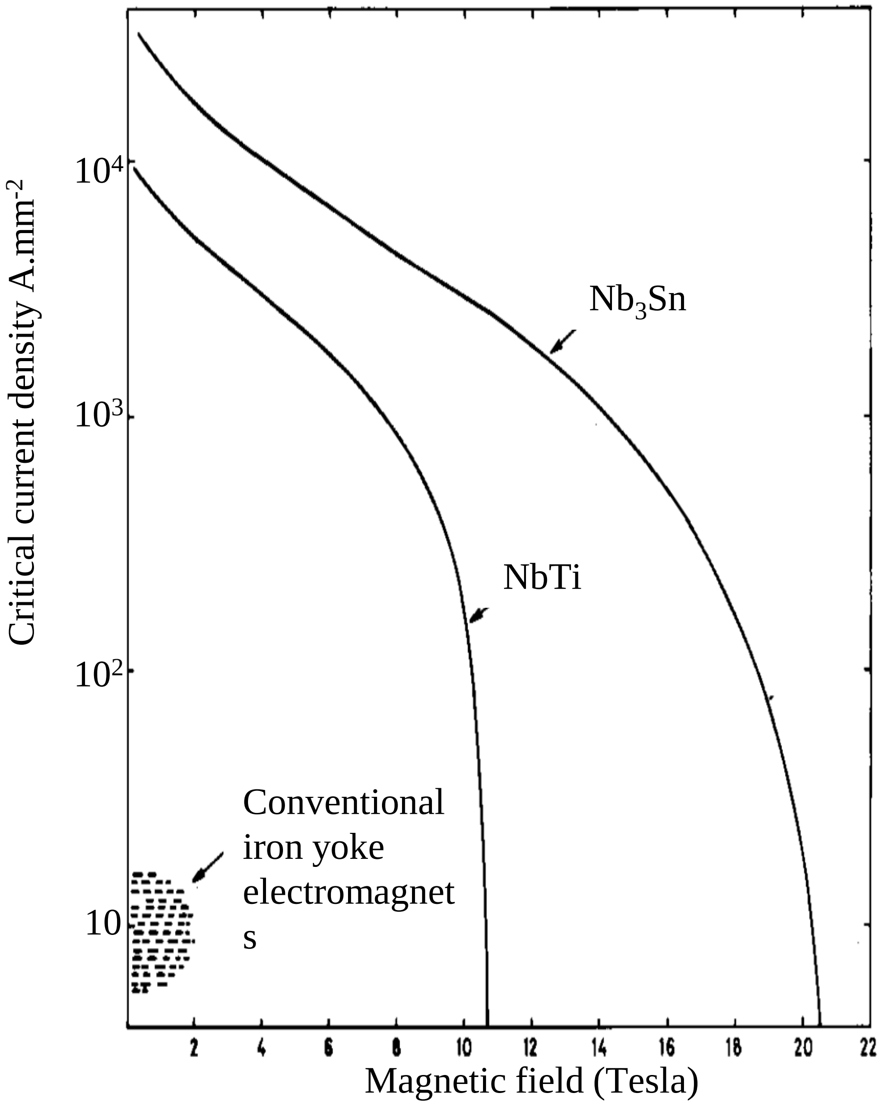
- Attraction between electrons with antiparallel momenta k and spins due to exchange of lattice vibration quanta (phonons)
- Instability of the normal Fermi surface due to bound states of electron (Cooper) pairs.
- Bose condensation of overlapping Cooper pairs in a coherent superconducting state.
- Scattering on electrons does not cause the electric resistance because it would break the Cooper pair.
- ***The strong overlap of many Cooper pairs results in the macroscopic phase coherence.***

The critical surface for niobium titanium



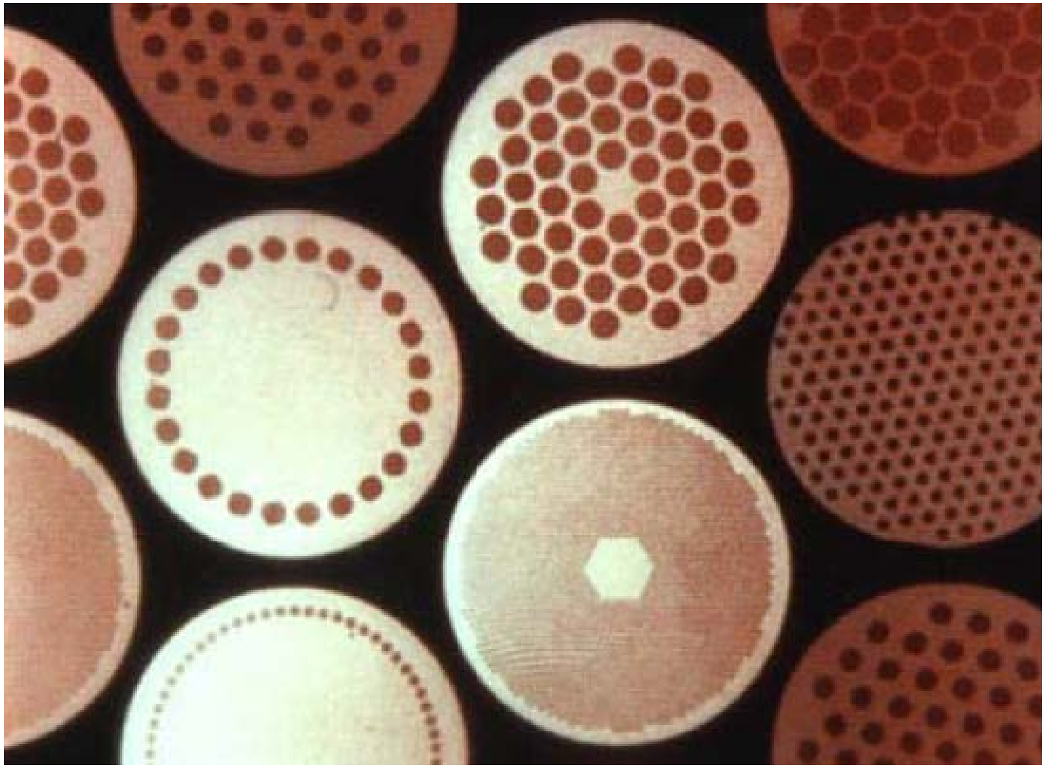
- Niobium titanium **NbTi** is the standard ‘work horse’ of the superconducting magnet business
- it is a ductile alloy
- picture shows the **critical surface**, which is the boundary between superconductivity and normal resistivity in 3 dimensional space
- superconductivity prevails everywhere below the surface, resistance everywhere above it
- we define an upper critical field B_{c2} (at zero temperature and current) and critical temperature θ_c (at zero field and current) which are characteristic of the alloy composition
- critical current density $J_c(B,\theta)$ depends on processing

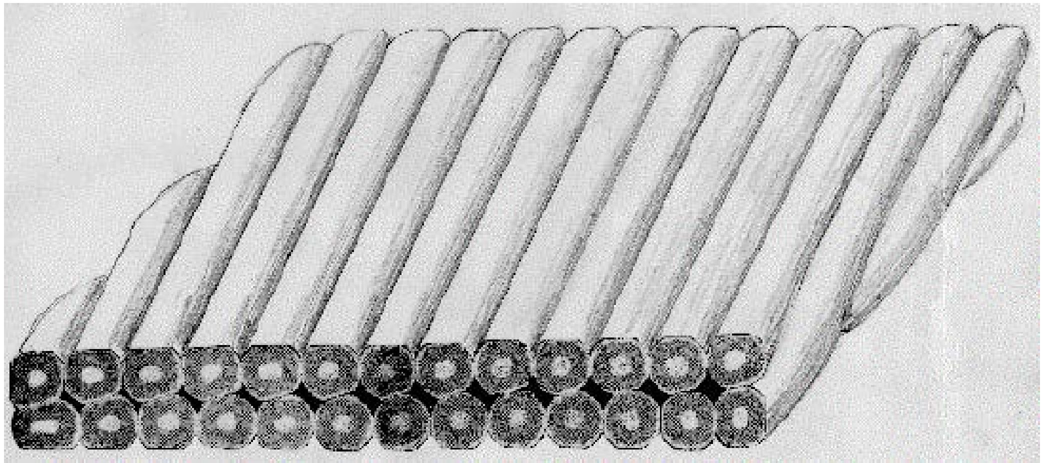
The critical line at 4.2K



- because magnets usually work in boiling liquid helium, the critical surface is often represented by a curve of current versus field at 4.2K
- niobium tin Nb_3Sn has a much higher performance in terms of critical current field and temperature than NbTi
- but it is brittle intermetallic compound with poor mechanical properties
- note that both the field and current density of both superconductors are way above the capability of conventional electromagnets

Practical superconductors for magnets





- superconducting materials are always used in combination with a good normal conductor such as copper

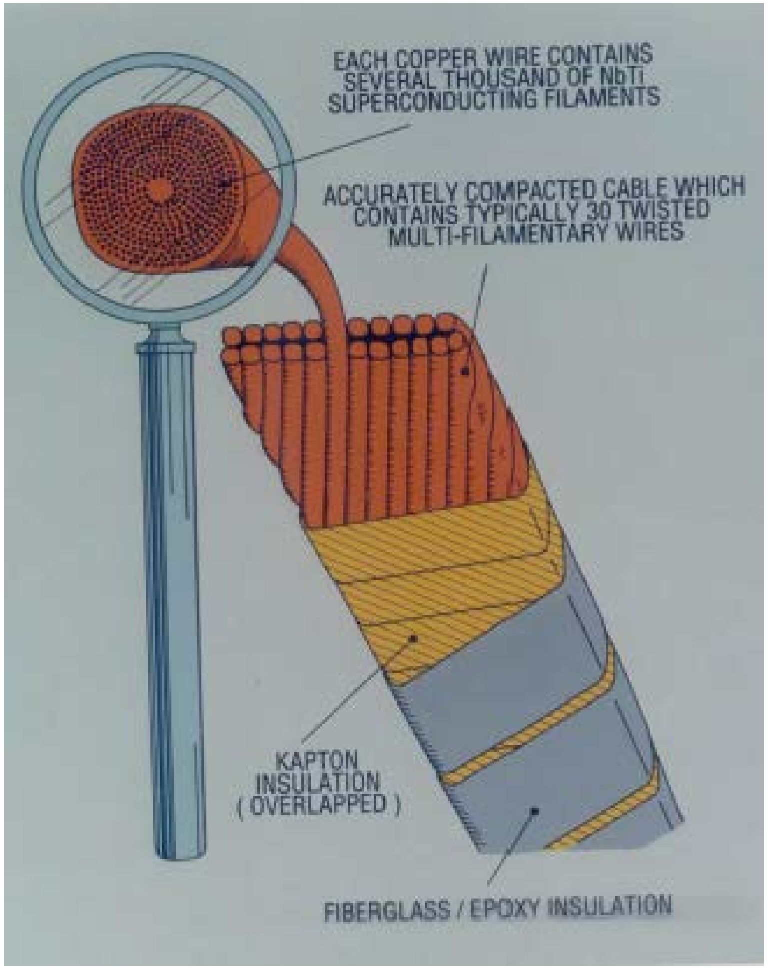
- to ensure intimate mixing between the two, the superconductor is made in form of fine filaments embedded in a matrix of copper

- typical dimensions are:
  - wire diameter: 0.3 – 1.0 mm
  - filament diameter: 10 – 60 μm

- for electromagnetic reasons, the composite wires are twisted so that the filaments look like a rope

- for accelerators, many wires are combined in a cable

A typical superconducting cable



Filament in an actual cable

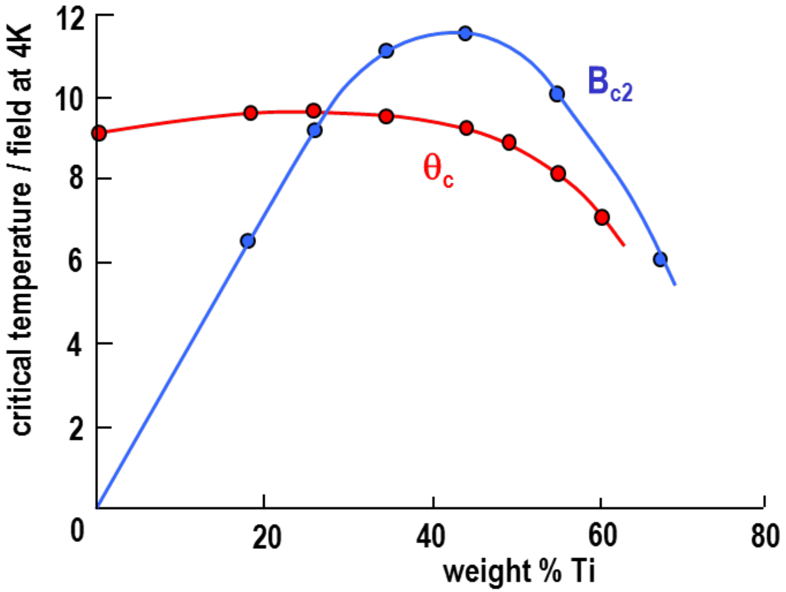
(Filament size in SSC/RHIC magnets: 6 micron

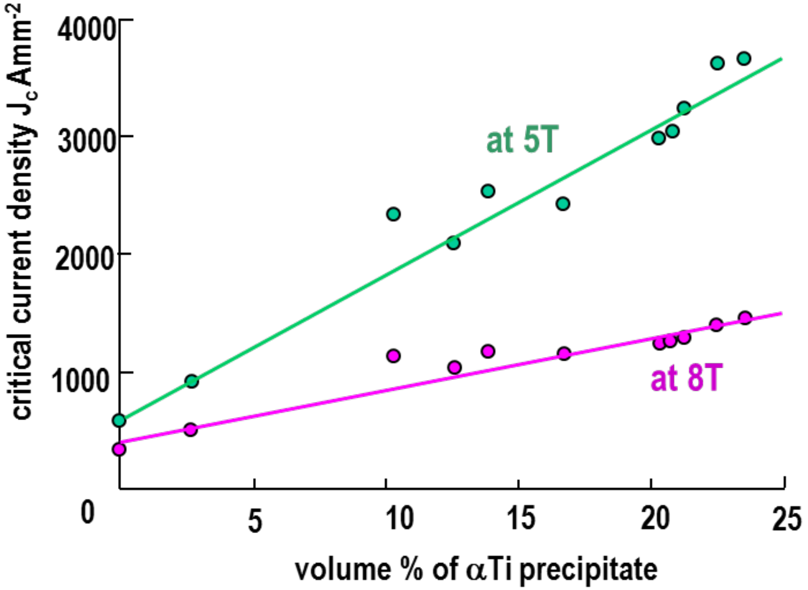
Critical properties

- **Critical temperature θ_c**: choose the right material to have a large energy gap or 'depairing energy'
  property of the material

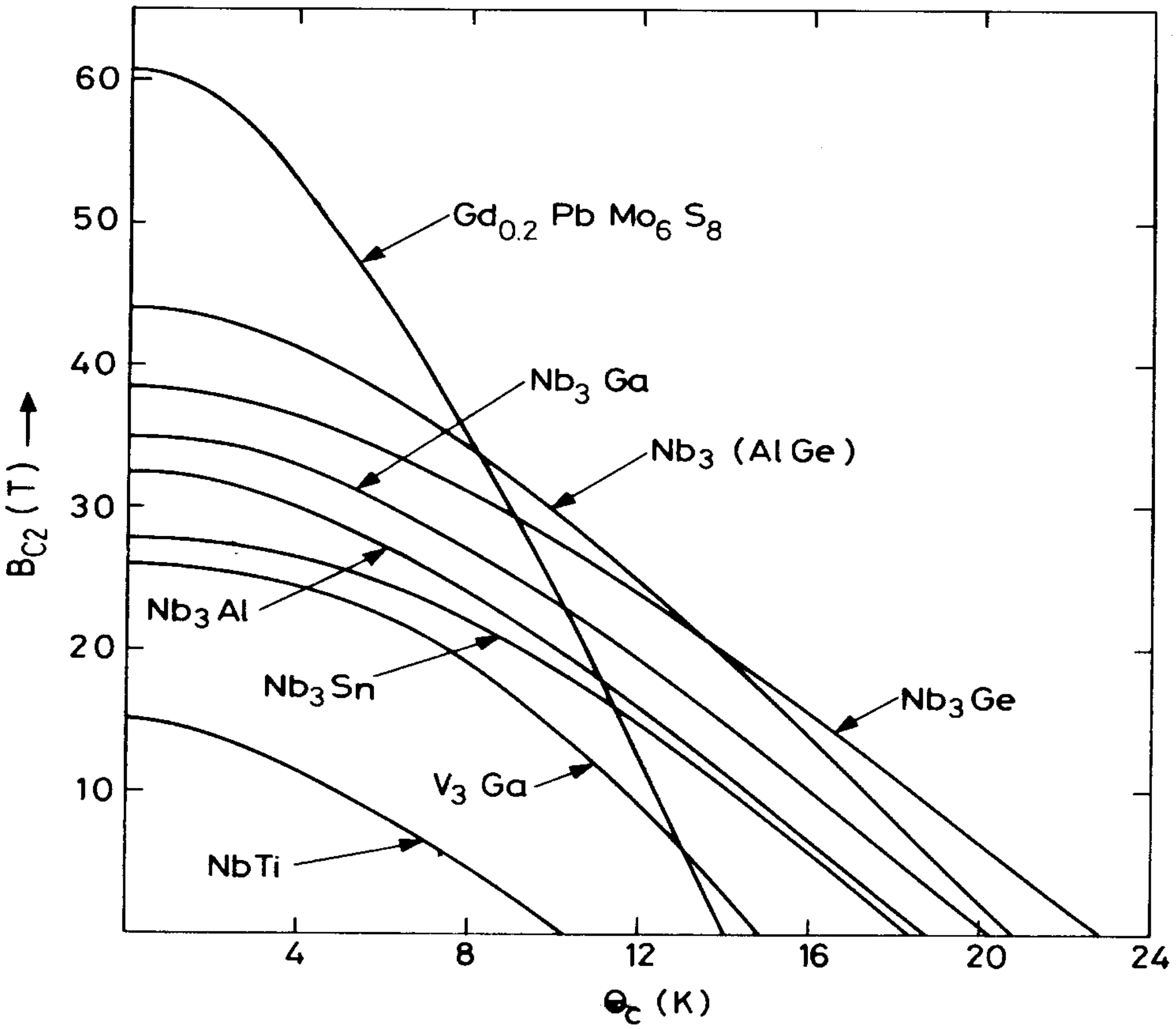
- **Upper Critical field B_{c2}:** choose a Type 2 superconductor with a high critical temperature and a high normal state resistivity
  property of the material

- **Critical current density J_c**: mess up the microstructure by cold working and precipitation heat treatments
  hard work by the producer





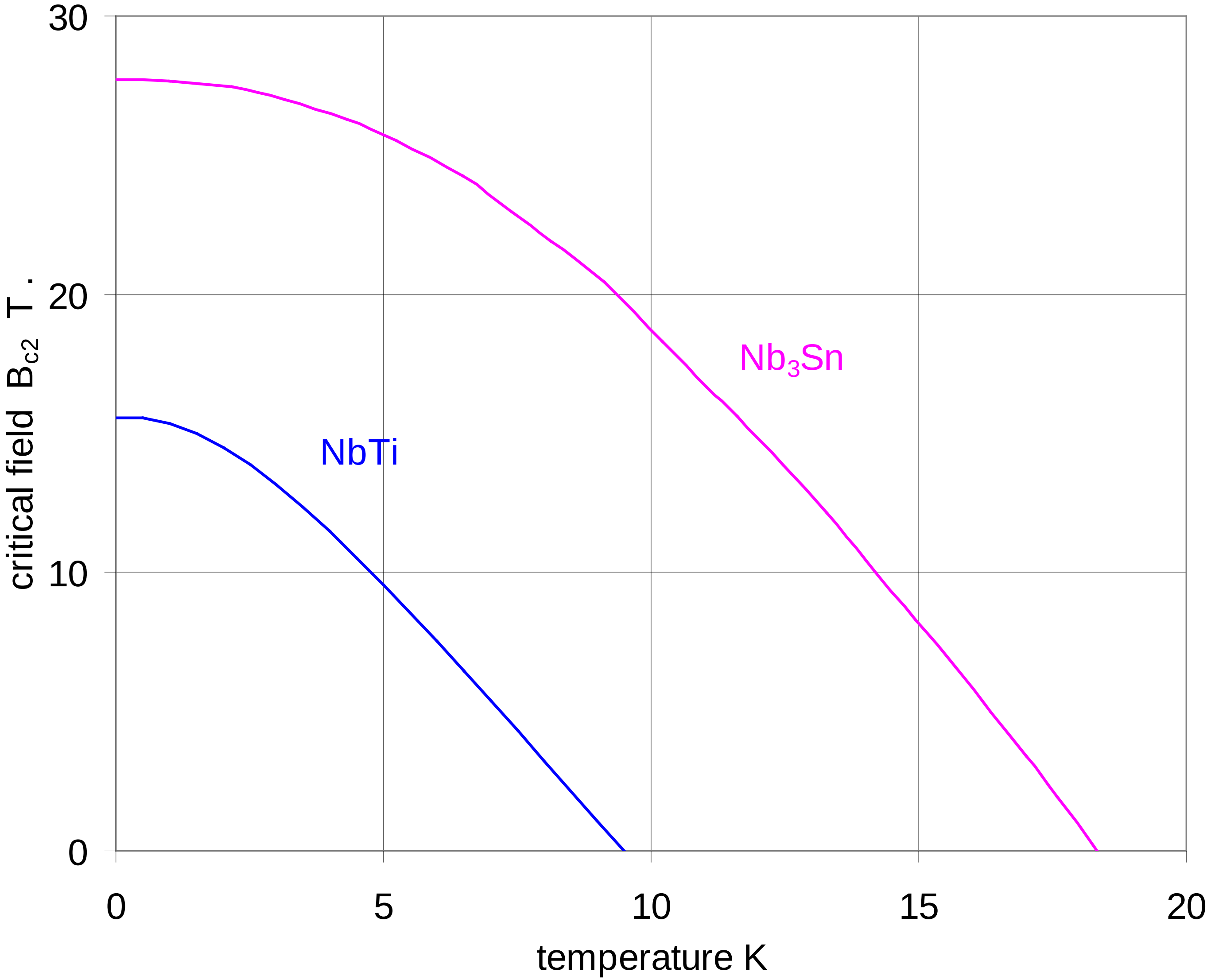
Critical field & temperature of metallic superconductors



Note: of all the metallic superconductors, only NbTi is ductile.

All the rest are brittle intermetallic compounds

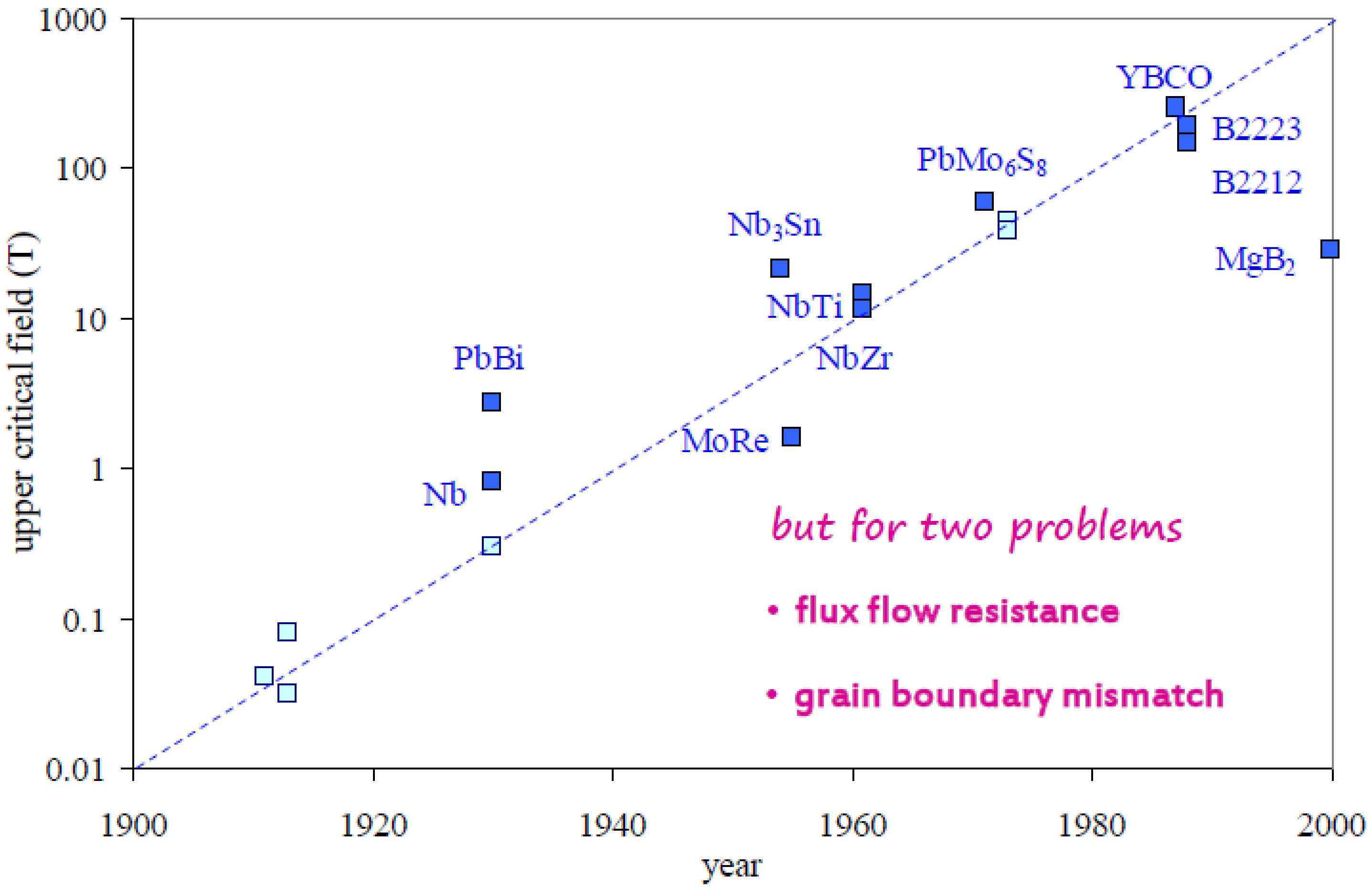
Critical field & temperature of metallic superconductors



To date, all superconducting accelerators have used NbTi.

Of the intermetallics, only Nb_3Sn has found significant use in magnets

Wonderful materials for magnets

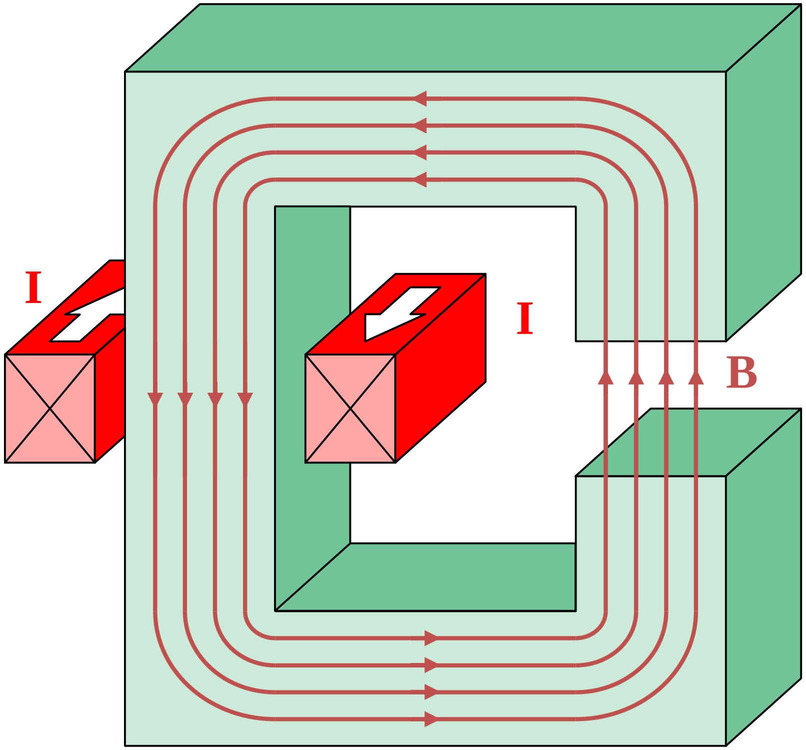


but for two problems

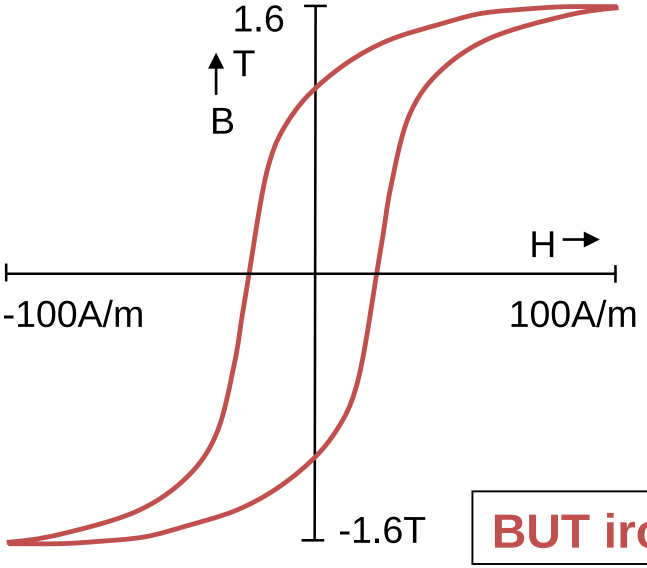
- **flux flow resistance**
- **grain boundary mismatch**

Magnetic fields and ways to create them: (1) Iron

- Conventional electromagnets
- iron yoke reduces magnetic reluctance
    - ⇒ reduces ampere turns required
    - ⇒ reduces power consumption
- iron guides and shapes the field



Iron electromagnet
– for accelerator, HEP experiment
transformer, motor, generator, etc.

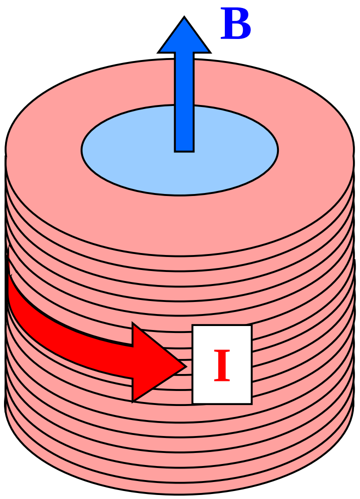


BUT iron saturates at ~ 2T

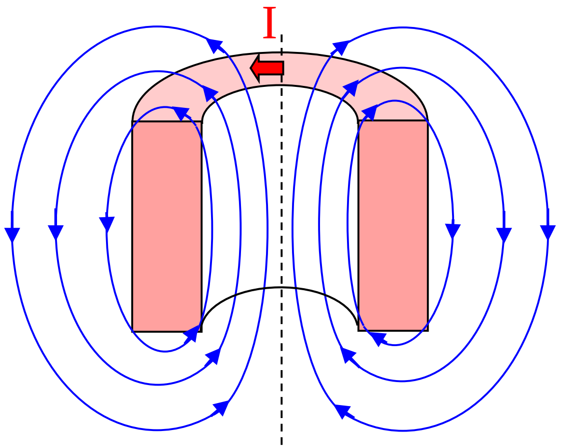
Magnetic fields and ways to create them: (2) Solenoids

- no iron – field shape is set solely by the winding
- cylindrical winding
- azimuthal current flow

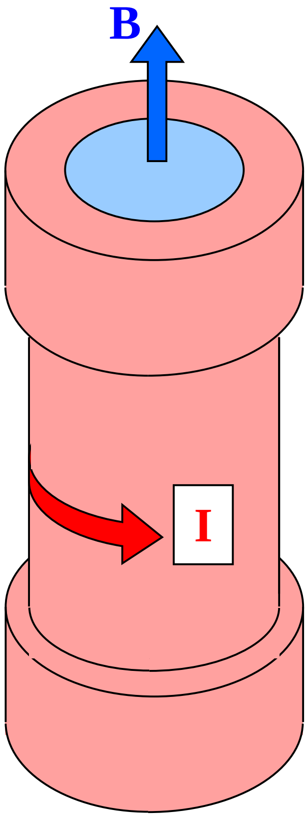
  - eg wire wound on bobbin
- axial field



- field lines curve outwards at the ends
- this curvature produces non uniformity of field
- very long solenoids have less curvature and more uniform field



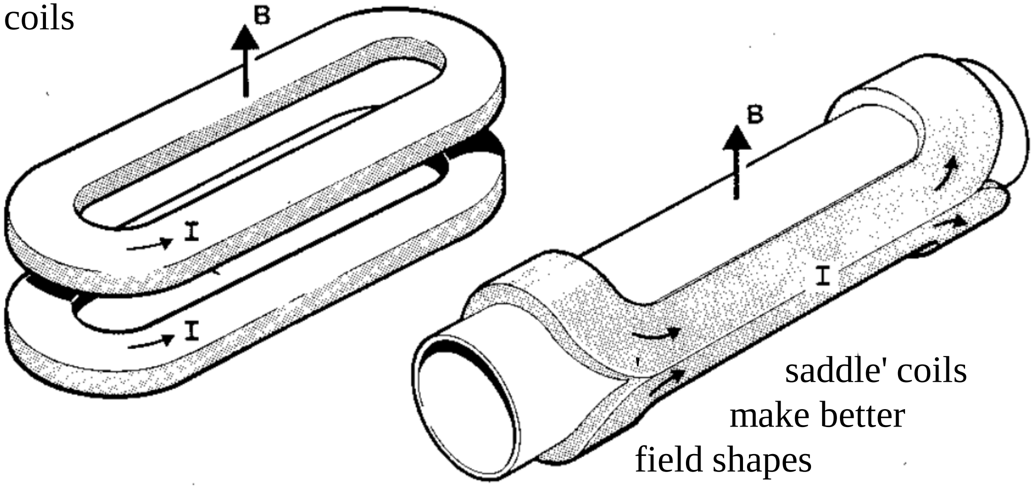
- can also reduce field curvature by making the winding thicker at the ends
- this makes the field more uniform



- more complicated winding shapes can be used to make very uniform fields

Magnetic fields and ways to create them: (3) transverse uniform fields

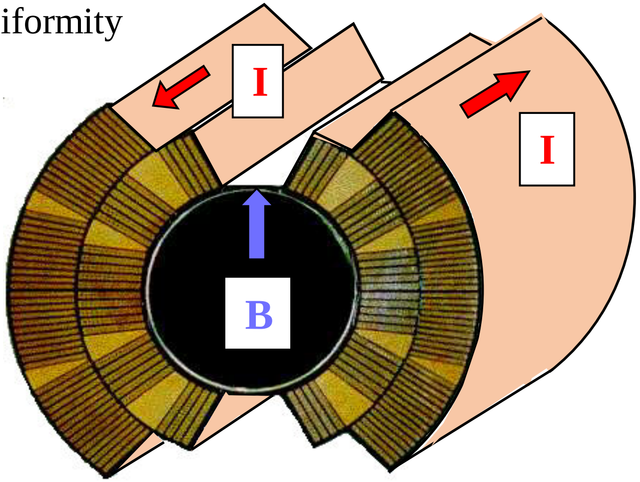
simplest winding uses racetrack coils

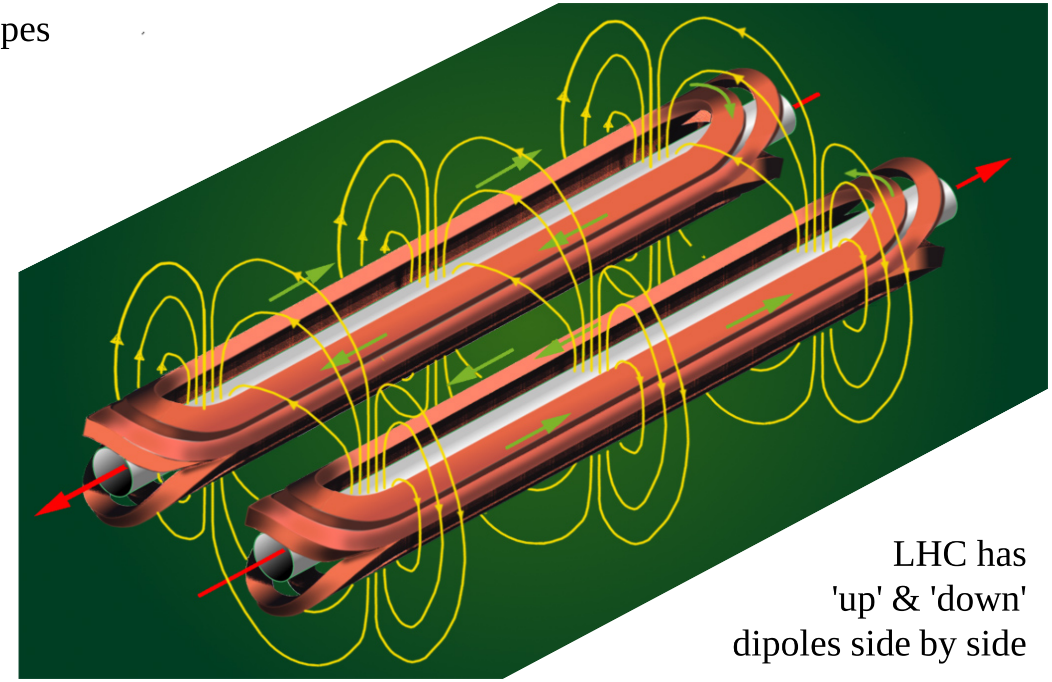


saddle' coils make better field shapes

- some iron - but field shape is set mainly by the winding
- used when the long dimension is transverse to the field, eg. accelerator magnets
- known as ***dipole*** magnets (because the iron version has 2 poles)

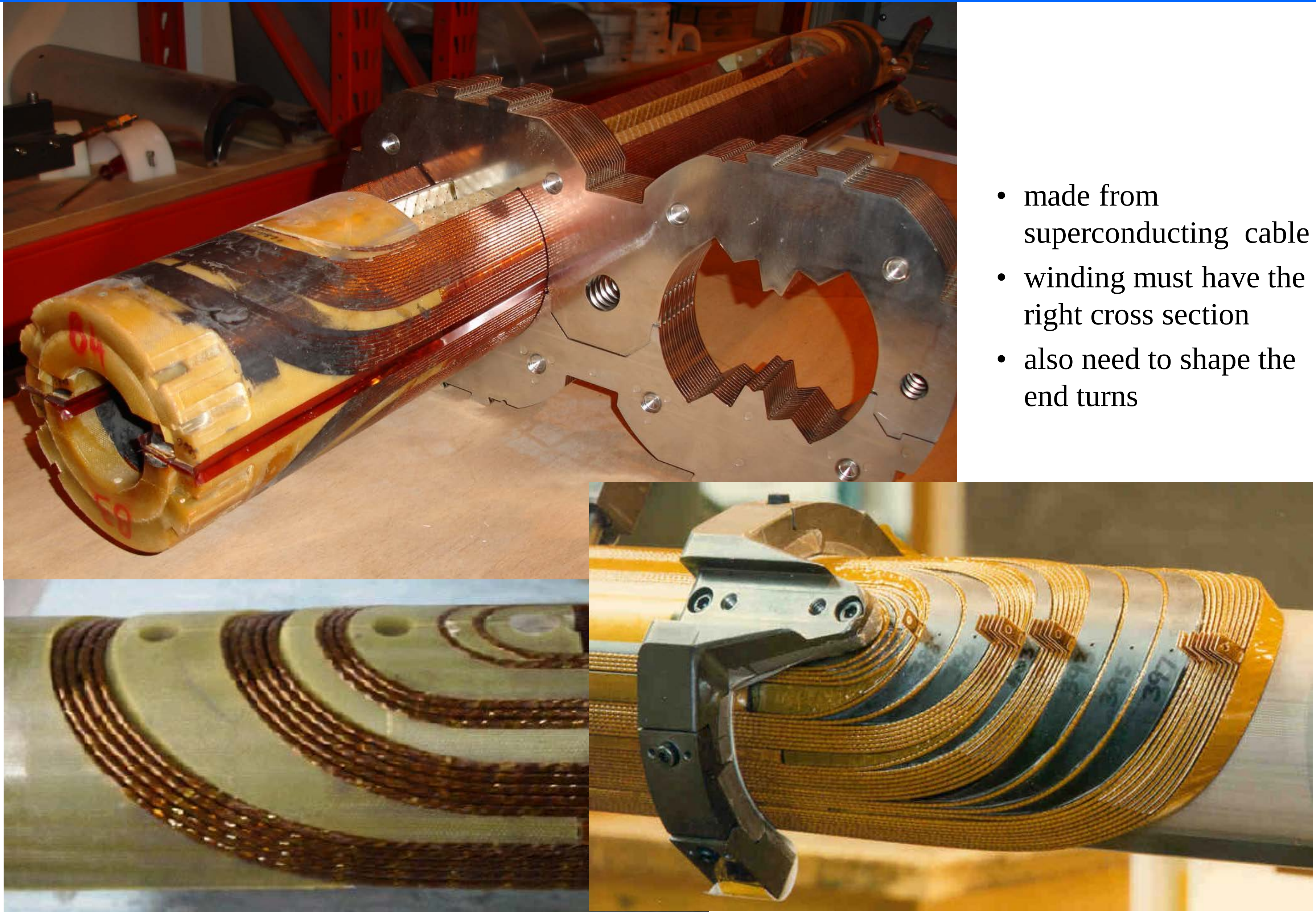
special winding cross sections for good uniformity





LHC has 'up' & 'down' dipoles side by side

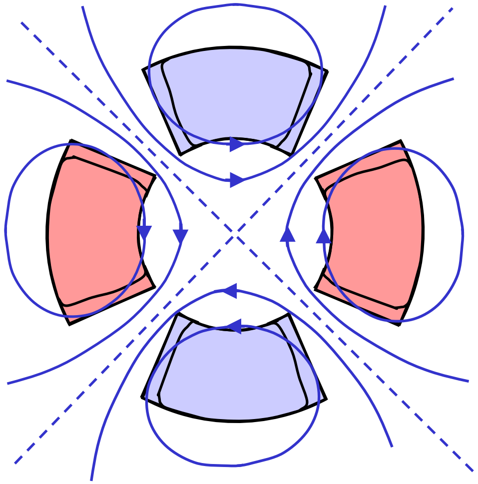
Dipole magnets

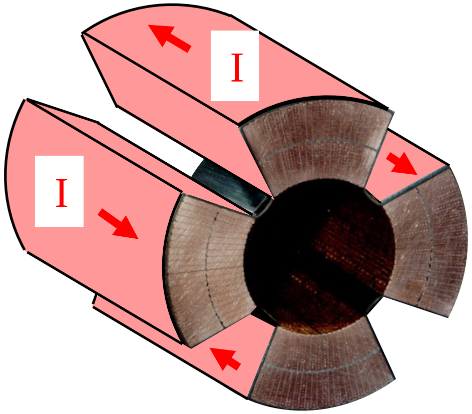


- made from superconducting cable
- winding must have the right cross section
- also need to shape the end turns

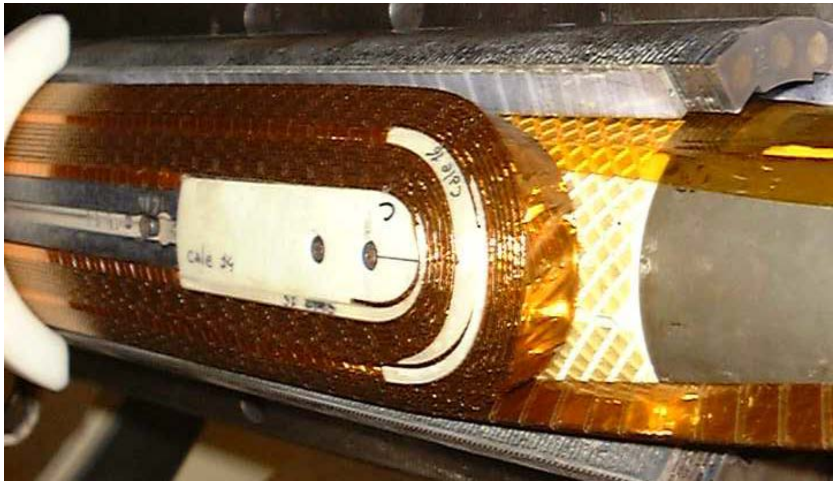
Fields and ways to create them: (4) transverse gradient fields

- gradient fields produce focussing
- quadrupole windings



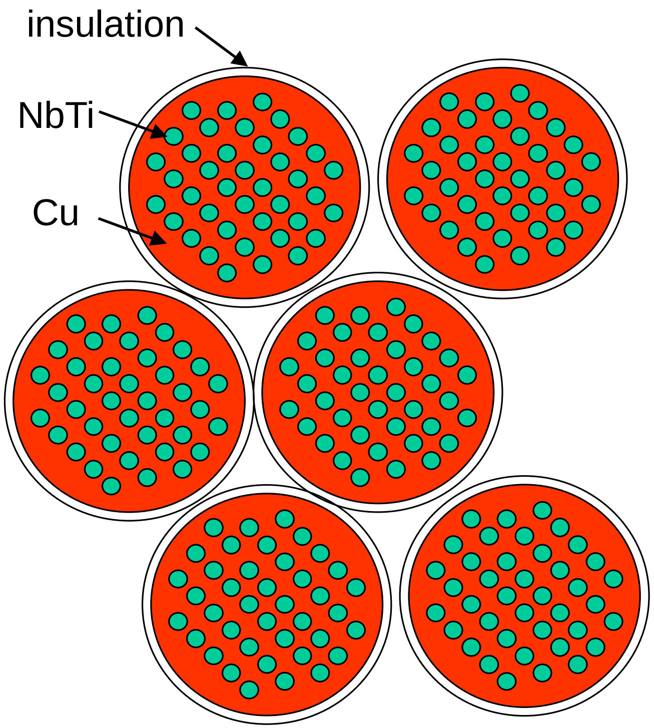


$B_x = ky$ $B_y = kx$



Engineering current density

In designing a magnet, what really matters is the overall 'engineering' current density $\boldsymbol{J_{eng}}$



$$J_{eng} = \frac{current}{unit\ \ cell\ \ area} = J_{supercon} \times \lambda_{metal} \times \lambda_{winding}$$

fill factor in the wire $\lambda_{metal} = \dfrac{1}{(1+mat)}$

where *mat* = matrix : superconductor ratio

typically:

for NbTi *mat* = 1.5 to 3.0 ie λ_{metal} = 0.4 to 0.25

for $\mathrm{Nb_3Sn}$ *mat* ~ 3.0 ie λ_{metal} ~ 0.25

for B2212 *mat* = 3.0 to 4.0 ie λ_{metal} = 0.25 to 0.2

$\lambda_{\mathrm{winding}}$ takes account of space occupied by insulation, cooling channels, mechanical reinforcement etc and is typically 0.7 to 0.8

So typically $\boldsymbol{J_{eng}}$ is only 15% to 30% of $\boldsymbol{J_{supercon}}$