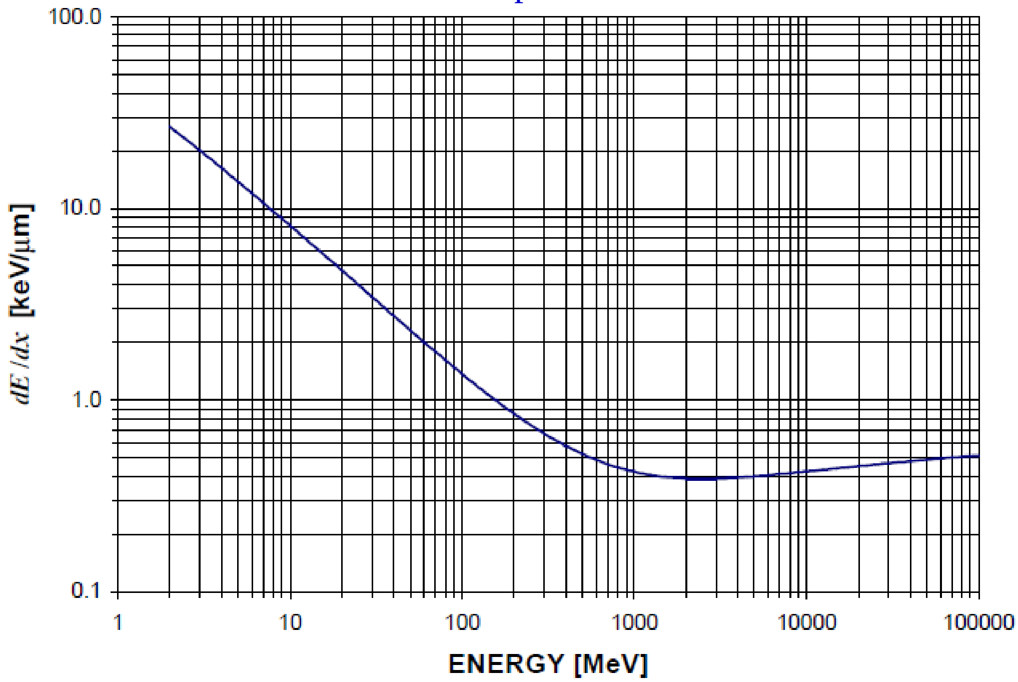
Bethe-Bloch formula describes the energy loss of heavy particles passing through matter

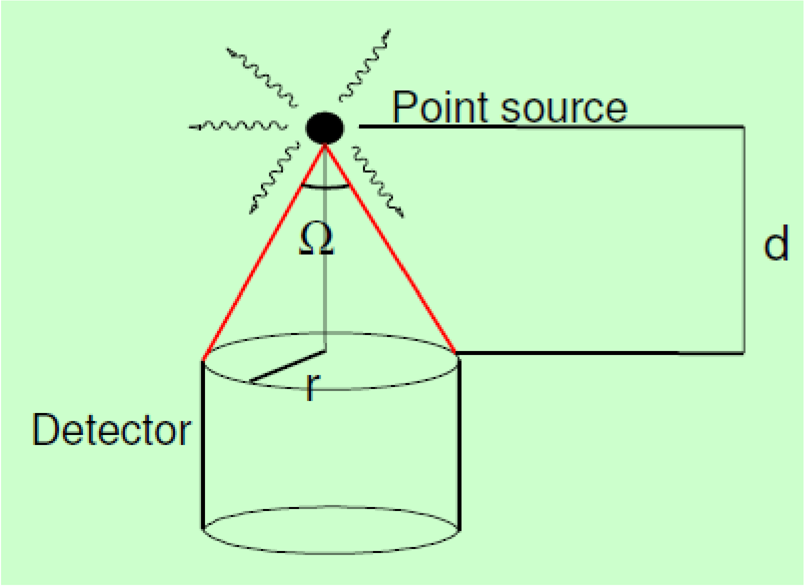
$$-\frac{dE}{dx} = \underbrace{4\cdot\pi\cdot r_e{}^2\cdot N_a\cdot m_e c^2}_{=\,0.3071\ \text{MeV g}^{-1}\text{cm}^2}\cdot\rho\cdot\frac{Z}{A}\cdot\frac{z^2}{\beta^2}\cdot\left[\frac{1}{2}ln\left(\frac{2\cdot m_e c^2\cdot\gamma^2\cdot\beta^2\cdot T_{max}}{I^2}\right)-\beta^2-\delta-2\cdot\frac{C}{Z}\right] \qquad \approx z^2\cdot\frac{Z}{A}\cdot f(\beta,I)$$

$$T_{max} = 2\cdot m_e c^2\cdot\beta^2\cdot\gamma^2$$

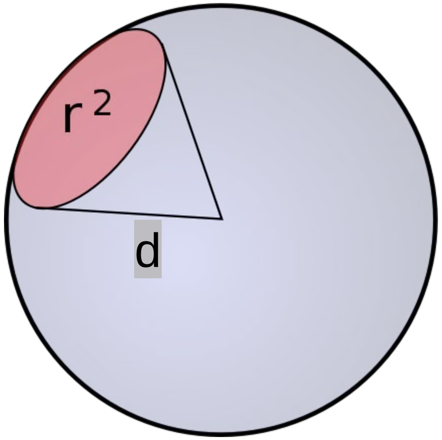
dE/dx vs. E of protons in silicon



Solid angle



$1\ \mathrm{Ci} = 3.7\cdot 10^{10}\ \mathrm{Bq}$



$$\frac{\Omega}{4\pi} \cong \frac{\pi r^2}{4\pi d^2} = \left(\frac{r}{2d}\right)^2$$

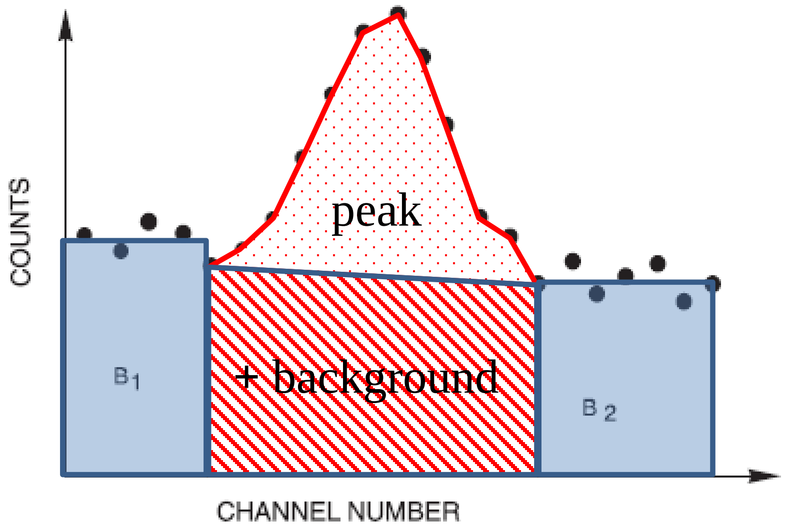
Ω = solid angle between source and detector (sr)

For a point source:

$$\frac{\Omega}{4\pi} = \frac{1}{2}\cdot\left(1 - \frac{d}{\sqrt{d^2 + r^2}}\right)$$

d (cm) r = 3cm	$\Omega/4\pi$ [%]	$\Omega/4\pi$ [%]
5	7.13	55
10	2.11	2.25
15	0.97	1

Statistical Error: Peak on top of Background



The area above the background represents the total counts between the vertical lines ***P*** minus the trapezoidal area ***B*** (red hatched). If the total counts are ***(P+B)*** and the end-points of the horizontal line are $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$ (width of B_1 + B_2 = width of B), then the net area is given by:

$$P = (P + B) - B$$

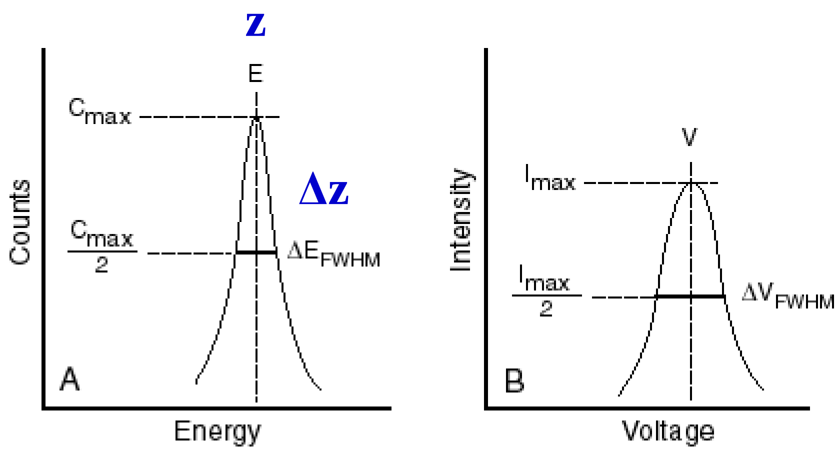
The ***standard deviation of ΔP*** is given by:

$$\Delta P = \sqrt{P + 2 \cdot B}$$

Quality of Measurements: Resolution

Resolution generally defined as 1 standard deviation (1σ) for a Gaussian distribution, or the FWHM $(\Delta z = 2.355 \cdot \sigma_z)$

If the measurement is dominated by Poissonian fluctuations:



$$\frac{\sigma_z}{\langle z \rangle} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$$

- lowest limit for the resolution apart from Fano factor correction

Fano factor F: fluctuations on N are reduced by correlation in the production of consecutive e-hole pairs. For Germanium detectors F ~ 0.1

$$\frac{\sigma_z}{\langle z \rangle} = \sqrt{\frac{F}{N}}$$

Luminosity

$$L = N_{projectiles} \cdot N_{target\ nuclei}$$

accelerator current: 1 nA

What is the number of projectiles?

1 particle / s ≡ $1.6 \cdot 10^{-19}$ C/s

$6.25 \cdot 10^{9}$ particles / s ≡ 1 nA

^{28}Si target thickness: 1 mg/cm^2

How many target nuclei?

28 g/cm^2 Silicon ≡ $6.02 \cdot 10^{23}$ atoms/cm^2

1 mg/cm^2 Silicon ≡ $2.15 \cdot 10^{19}$ atoms/cm^2

Luminosity = $6.25 \cdot 10^{9} \cdot 2.15 \cdot 10^{19} = 1.34 \cdot 10^{29}$ [s^{-1} cm^{-2}]

event rate [s^{-1}] = luminosity [s^{-1} cm^{-2}] · cross section [cm^2]

= $1.34 \cdot 10^{29}$ [s^{-1} cm^{2}] · cross section [~ mb = 10^{-27} cm^2] ≈ 10^2 [s^{-1}]