# Outline: Coulomb excitation

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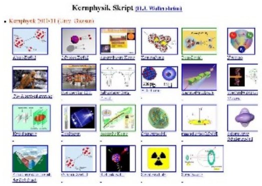
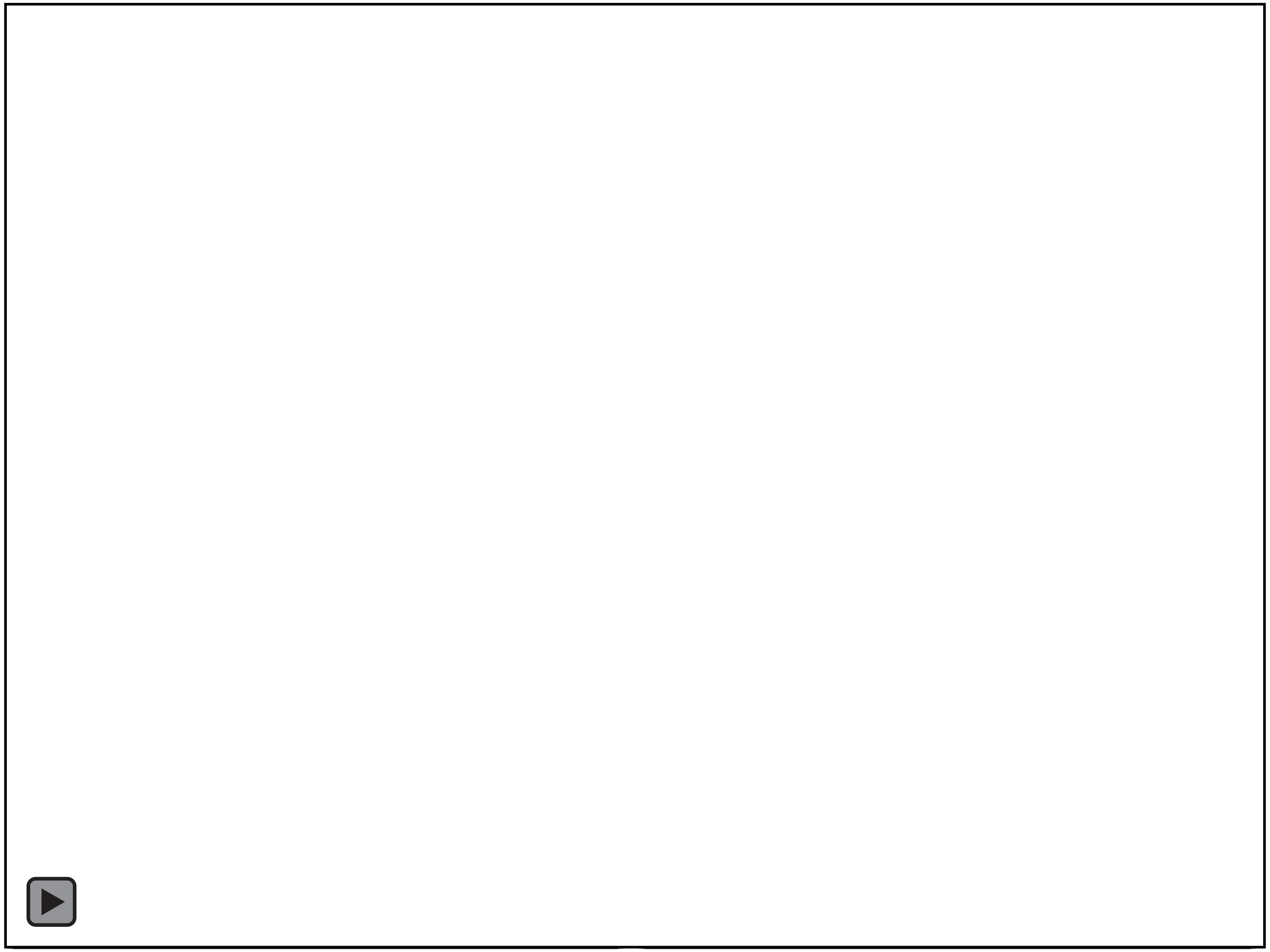
web-page: https://web-docs.gsi.de/~wolle/ and click on



1. particle detection
2. electric fields of multipoles
3. particle γ-ray coincidence measurement
4. Doppler shift correction
5. Conversion electrons
6. reorientation effect
7. octupole deformed nuclei

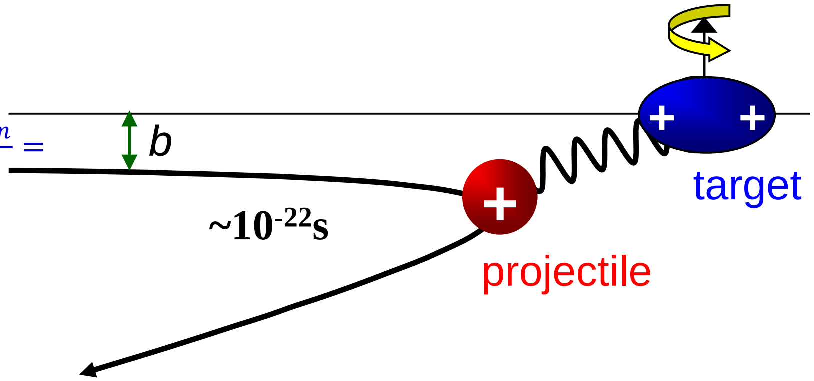
# Coulomb excitation



# Coulomb excitation particle detection

Nuclear excitation by electromagnetic field acting between nuclei.

$a \cdot cot \frac{\theta_{cm}}{2} = b$

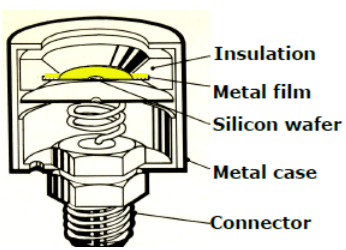



***observables:***

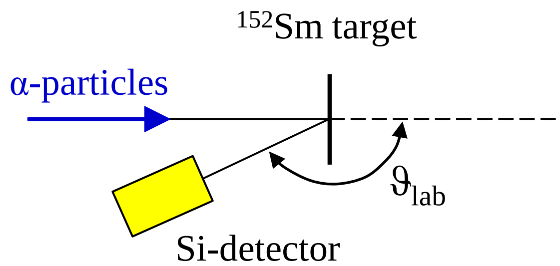
1. scattering angle $\vartheta_{lab} \Longrightarrow \theta_{cm}$

2. intensity $\frac{d\sigma_{Ruth}}{d\Omega_{cm}} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta_{cm}}{2}$

$$\frac{d\sigma_{inel}}{d\Omega_{cm}} = \left|\boldsymbol{a}_{i\rightarrow f}\right|^2 \cdot \frac{d\sigma_{Ruth}}{d\Omega_{cm}}$$

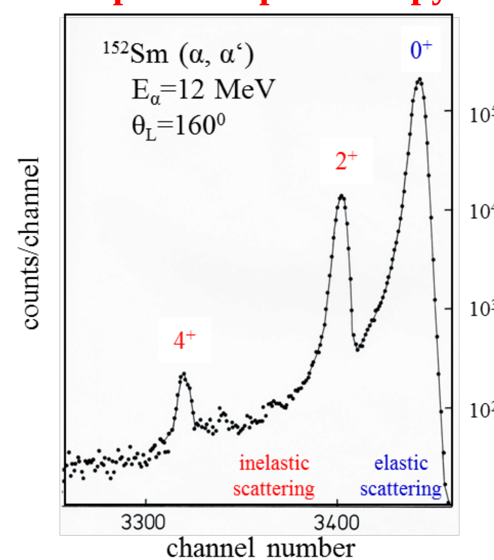



Possible:
depletion depth ~ 300µm
dead layer $d_d \leq 1\mu$
V ~ 0.5 V/µ
Over-bias reduces $d_d$



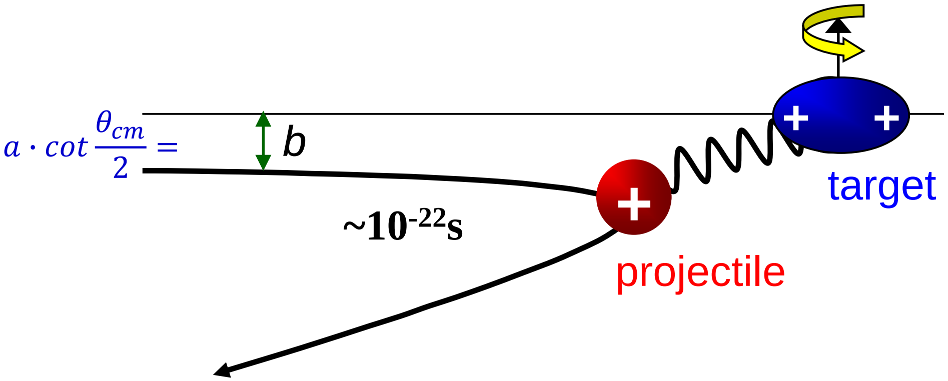

inelastic scattering: *kinetic energy is transferred into nuclear excitation energy*

**α-particle spectroscopy**




H.J. Wollersheim et al., Phys. Lett 48B (1974) 323

Nuclear excitation by electromagnetic field acting between nuclei.

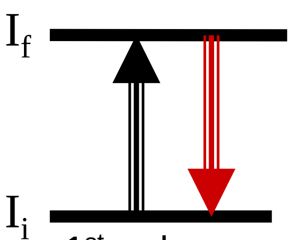



***observables:***

1. scattering angle $\vartheta_{lab} \Longrightarrow \theta_{cm}$
2. intensity $\frac{d\sigma_{Ruth}}{d\Omega_{cm}} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta_{cm}}{2}$

$$\frac{d\sigma_{inel}}{d\Omega_{cm}} = \left|a_{i\to f}\right|^2 \cdot \frac{d\sigma_{Ruth}}{d\Omega_{cm}}$$

*lifetime*

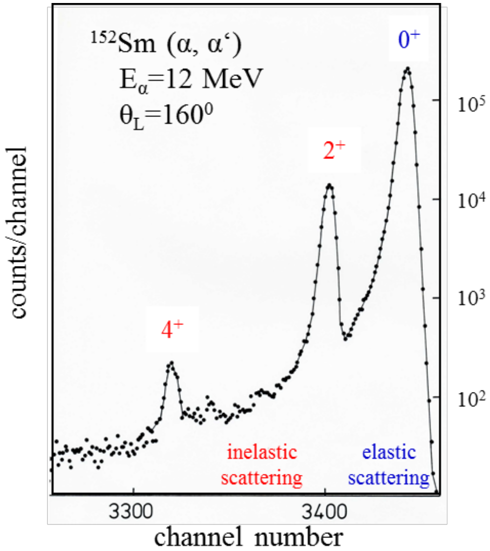



$\tau$ ~$10^{-12}$-$10^{-9}$s

$$\frac{1}{\tau} \cong 1.23 \cdot 10^{13} \cdot B(E2; I_f \to I_i) \cdot E_\gamma^5 \quad [s^{-1}]$$

$1^{st}$ order:

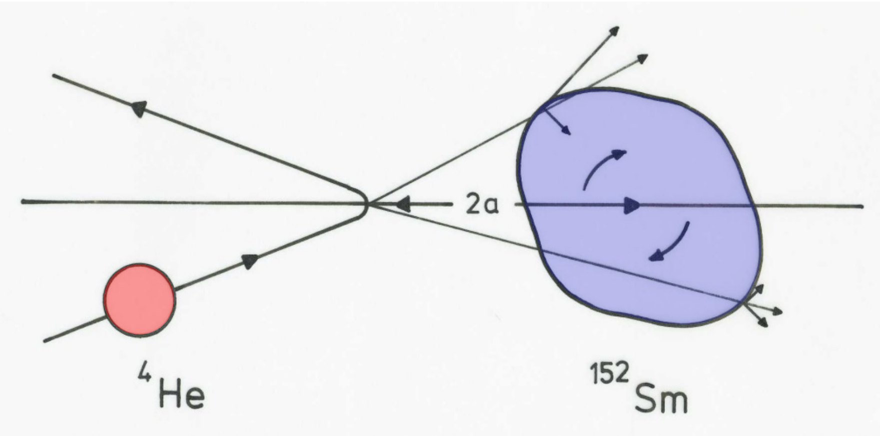
$$a_{i\to f}^{(1)} \propto \langle I_f \| \mathbf{M}(E2) \| I_i \rangle \qquad B(E2; I_i \to I_f) = \frac{1}{2I_i + 1} \langle I_f \| M(E2) \| I_i \rangle^2$$

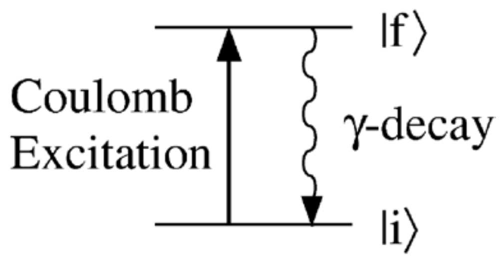


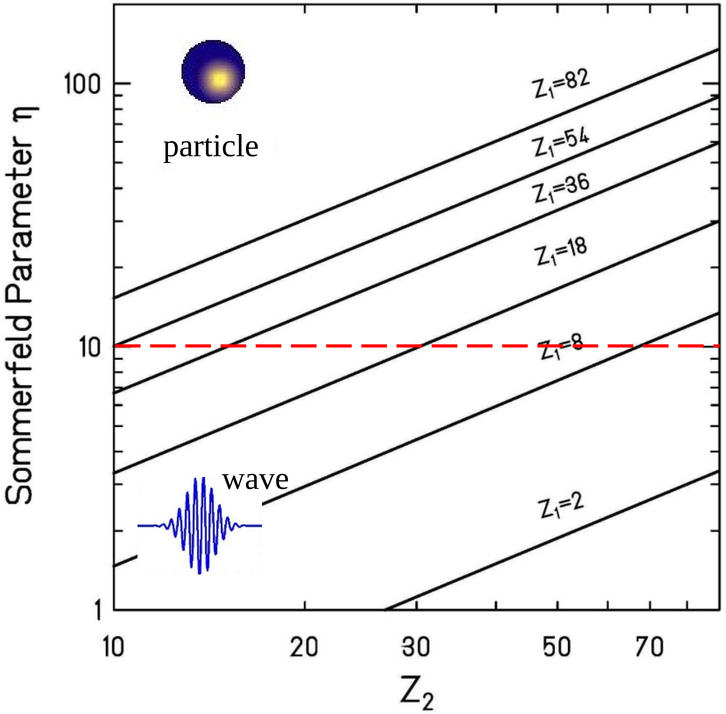

*The inelastic cross section* $d\sigma_{inel}/d\Omega_{cm}$
*is a direct measure of the E2 matrix elements*

H.J. Wollersheim et al., Phys. Lett 48B (1974) 323

# Coulomb excitation Sommerfeld parameter



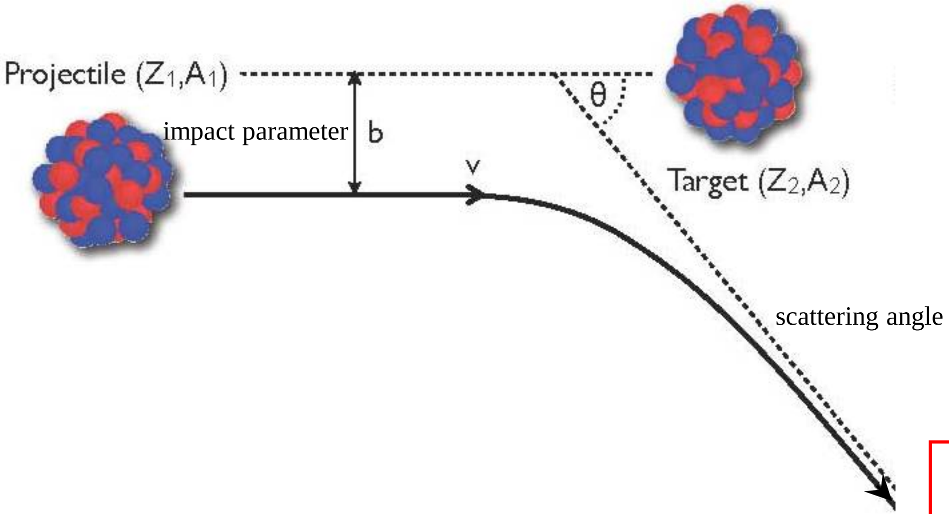

Sommerfeld parameter:

$$\eta = a \cdot k_\infty = \frac{Z_p \cdot Z_t \cdot e^2}{\hbar \cdot v_\infty} \gg 1$$

η >> 1 requirement for a (semi-) classical treatment of equations of motion (hyperbolic trajectories )

$^4$He (**Z=2**) projectiles behave like waves
quantum mechanical analysis is needed

# Classical Coulomb trajectories



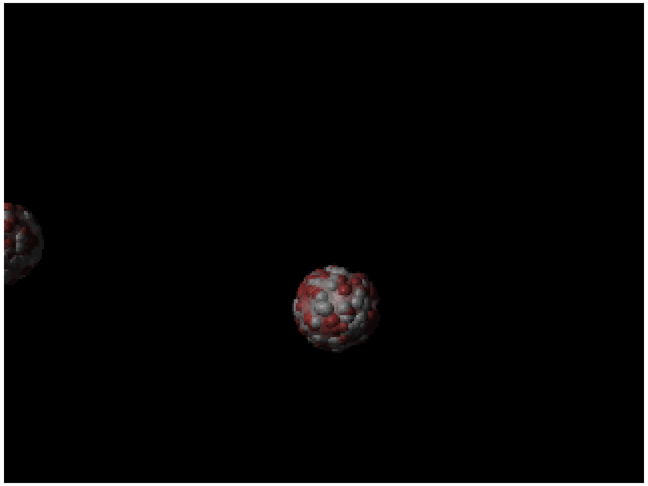

**Hyperbolic trajectory:**

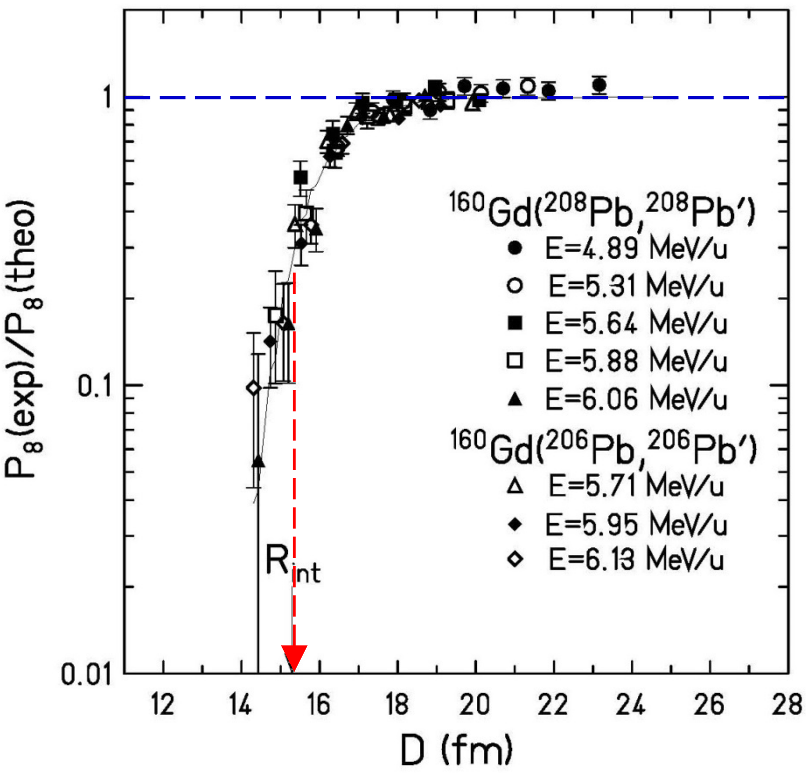
$$r = a \cdot [\varepsilon \cdot \cosh w + 1] \quad t = \frac{a}{v_\infty}[\epsilon \cdot sinh\omega + \omega]$$

$\varepsilon = \sin^{-1}(\theta_{cm}/2)$ eccentricity of orbit

- distance of closest approach: $D = a \cdot \left[sin^{-1}\frac{\theta_{cm}}{2} + 1\right]$
- impact parameter: $b = a \cdot cot\frac{\theta_{cm}}{2}$
- angular momentum : $\ell = k_\infty \cdot b = \eta \cdot cot\frac{\theta_{cm}}{2}$

# Safe bombarding energy pure electromagnetic interaction






Nuclear interaction radius:

$$R_{int} \cong C_p + C_t + 3fm$$

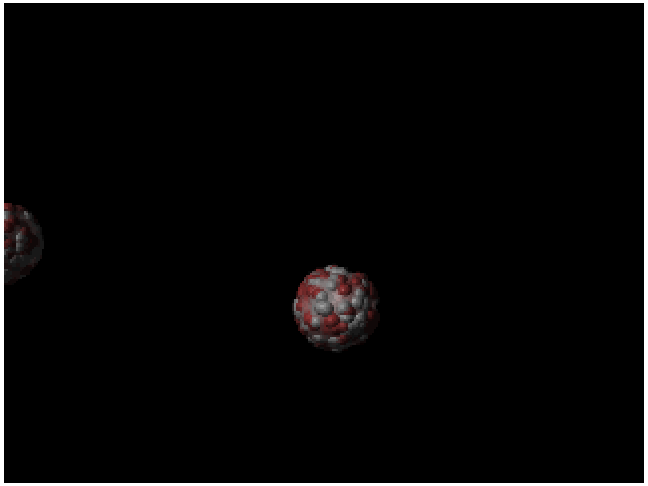
$C_P$, $C_T$ half-density radii

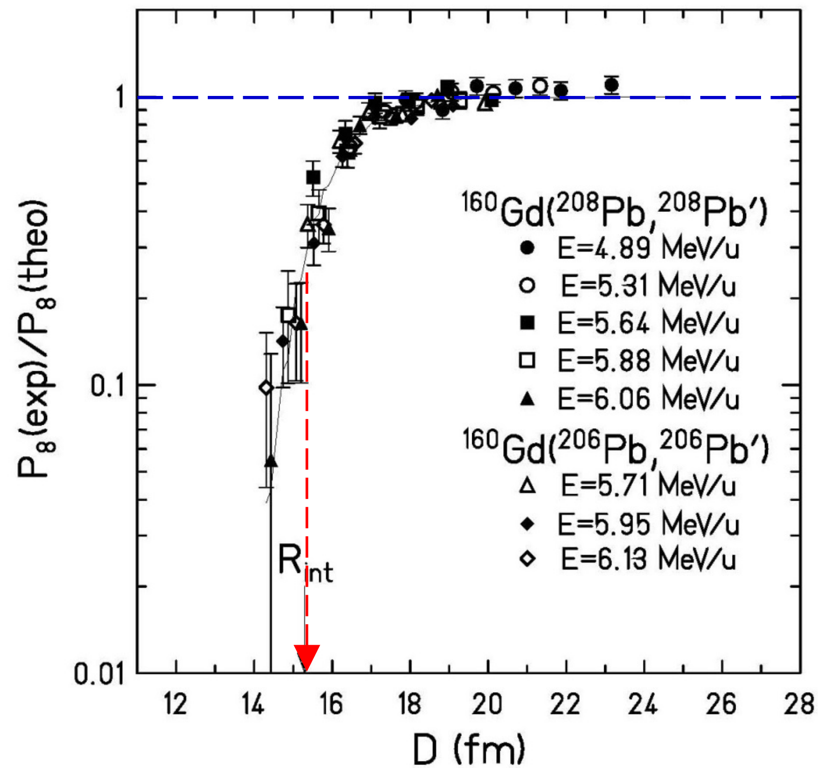
$$R_{int} = C_p + C_t + 4.49 - \frac{C_p + C_t}{6.35}$$

Pure Coulomb excitation requires a much larger distance between the nuclei

➔ ´safe bombarding energy´requirement

# Safe bombarding energy pure electromagnetic interaction






**Nuclear interaction radius:**

$$R_{int} \cong C_p + C_t + 3fm$$

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$$R_{int} = C_p + C_t + 4.49 - \frac{C_p + C_t}{6.35}$$

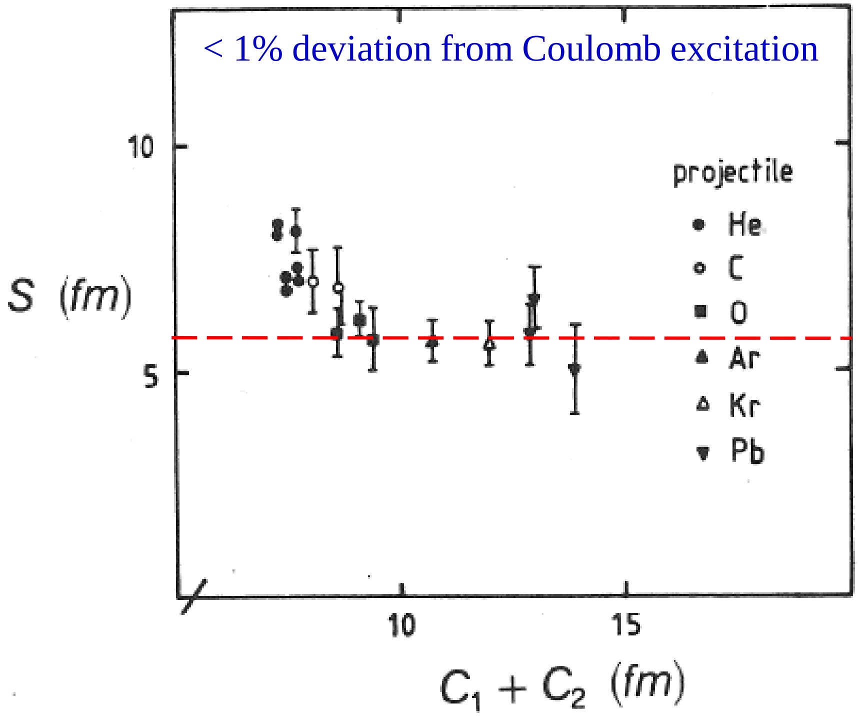
$\sigma_{total} \approx \sigma_{inel} + \sigma_{reaction}$ ıction

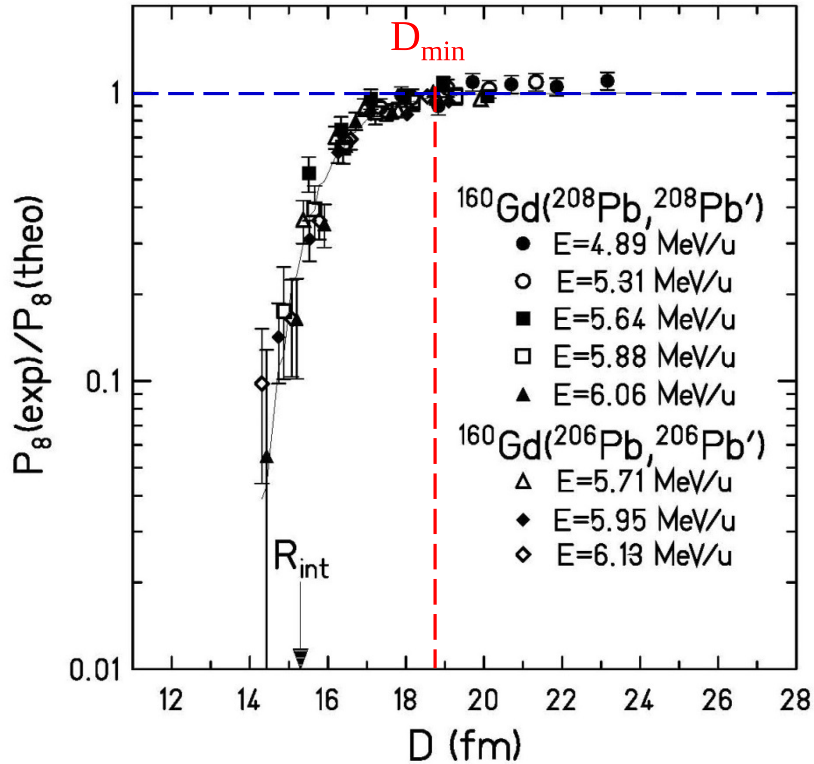
nuclear absorption: $$[1 - P_{abs}(D)] = exp\left\{-\frac{2}{\hbar}\int_{-\infty}^{+\infty} W[r(t)]dt\right\}$$

$$W[r(t)] = W_0 \cdot exp\left[-\frac{r(t) - C_1 - C_2}{a_I}\right]$$

$$[1 - P_{abs}(D)] = exp\left\{-\frac{2}{\hbar} \cdot W_0 \cdot exp\left[-\frac{D - C_1 - C_2}{a_I}\right] \cdot \frac{D}{v}\right\}$$

# Safe bombarding energy pure electromagnetic interaction




Rutherford scattering only if $D_{min}$ is large compared to nuclear radii + surfaces:

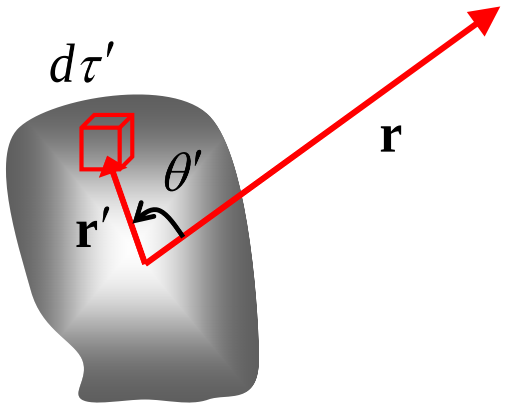
$$D_{min} > C_p + C_t + 5\ fm$$

$C_P$, $C_T$ half-density radii

➔ choose **adequate beam energy $E_{lab}$**
($D > D_{min}$ for all $\theta_{cm}$)

# Multipole Expansion of the electric field

# Electric fields of multipoles




In general the electric potential due to an arbitrary charge distribution is

$$U(\vec{r}) = \iiint \frac{\rho_p(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d\tau'$$

expansion

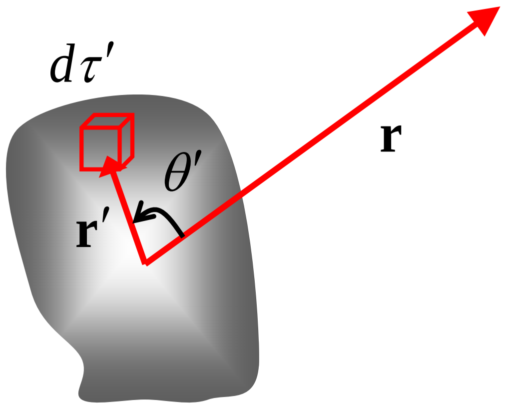
$$\frac{1}{|r - r'|} = \sum_{\ell=0}^{\infty} \frac{r'^{\ell}}{r^{\ell+1}} \frac{4\pi}{2\ell + 1} \sum_{m=-\ell}^{\ell} Y_{\ell m}(\vartheta, \varphi) Y^*_{\ell m}(\vartheta', \varphi')$$

special case: **electric monopole**

$\ell = m = 0 \qquad Y_{00}(\vartheta, \varphi) = Y_{00}(\vartheta', \varphi') = \frac{1}{\sqrt{4\pi}}$

$$\rightarrow \quad \frac{1}{|r - r'|} = \frac{1}{r}$$

# Electric fields of multipoles

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$$U(\vec{r}) = \iiint \frac{\rho_p(\vec{r}')}{r} d\tau'$$

homogenous charge distribution

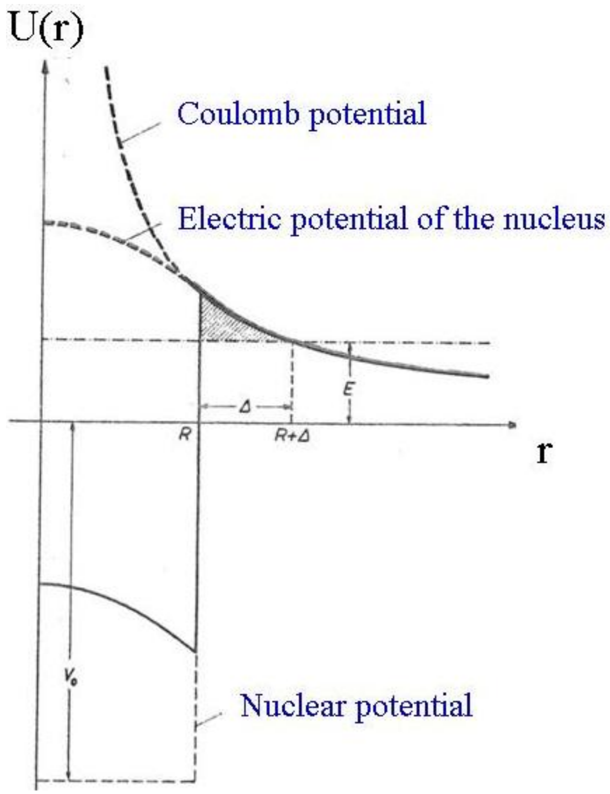
$$\rho_p\left(\overrightarrow{r'}\right) = \frac{3 \cdot Ze}{4\pi \cdot R_0^3}$$

special case: **electric monopole**

$$U(\vec{r}) = \frac{3 \cdot Ze}{4\pi \cdot R_0^3} \frac{1}{r} \iiint r'^2 dr' sin\vartheta' d\vartheta' d\varphi'$$

$$U(\vec{r}) = \frac{3 \cdot Ze}{4\pi \cdot R_0^3} \cdot \frac{1}{r} \cdot \frac{R_0^3}{3} \cdot 4\pi \;= \boldsymbol{\frac{Ze}{r}}$$

# Electric fields of multipoles

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In general the electric potential due to an arbitrary charge distribution is

$$U(\vec{r}) = \iiint \frac{\rho_p(\vec{r}\,')}{|\vec{r} - \vec{r}\,'|} d\tau'$$

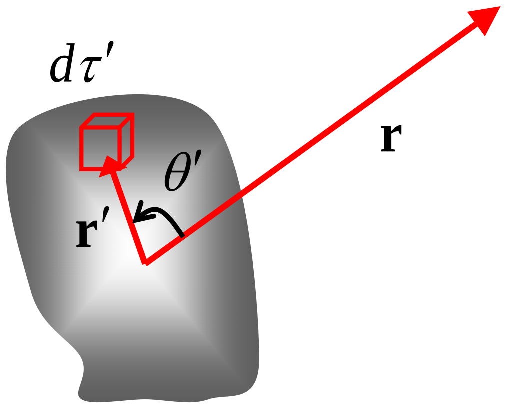
$$\frac{1}{|r - r'|} = \begin{Bmatrix} 1/r & for & r > r' \\ 1/r' & for & r < r' \end{Bmatrix}$$

$$U(\vec{r}) = 4\pi \cdot \frac{3 \cdot Ze}{4\pi \cdot R_0^3} \cdot \left[ \int_0^r \frac{1}{r} \cdot r'^2 dr' + \int_r^{R_0} \frac{1}{r'} \cdot r'^2 dr' \right]$$

$$U(\vec{r}) = \frac{3 \cdot Ze}{R_0^3} \cdot \left[ \frac{r^2}{3} + \frac{R_0^2}{2} - \frac{r^2}{2} \right]$$

$$U(\vec{r}) = \frac{Ze}{2R_0} \cdot \left[ 3 - \left( \frac{r}{R_0} \right)^2 \right]$$

# Electric fields of multipoles

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expansion

$$\frac{1}{|r - r'|} = \sum_{\ell=0}^{\infty} \frac{r'^{\ell}}{r^{\ell+1}} \frac{4\pi}{2\ell + 1} \sum_{m=-\ell}^{\ell} Y_{\ell m}(\vartheta, \varphi) Y^*_{\ell m}(\vartheta', \varphi')$$

multipole moments

$$M^*(\ell, m) = \iiint \rho_p(r') \cdot r'^{\ell} \cdot Y^*_{\ell m}(\vartheta', \varphi') d\tau'$$

special case: **electric quadrupole moment**

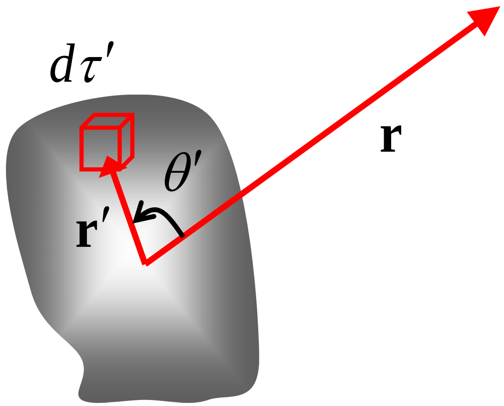
$$M^*(\ell = 2, m) = \frac{3 \cdot Ze \cdot R_0^2}{4\pi} \cdot \beta_2$$

reduced transition probability B(E2)-value:

$$B\left(E\ell = 2; I_i \to I_f\right) = \sum_{M_f m} \left\langle I_f M_f K_f | M(\ell = 2, m) | I_i M_i K_i \right\rangle^2$$

$$B\left(E\ell = 2; I_i \to I_f\right) = \frac{1}{2I_i + 1} \left|\left\langle I_f \| M(\ell = 2) \| I_i \right\rangle\right|^2$$

# Electric fields of multipoles deformation parameters




$$R(\vartheta',\varphi') = R_0 \cdot \{1 + \beta_2 Y_{20}(\vartheta',\varphi')\}$$

multipole moments

$$M^*(\ell,m) = \iiint \rho_p(r') \cdot r'^{\ell} \cdot Y^*_{\ell m}(\vartheta',\varphi') d\tau'$$

special case: electric quadrupole matrix element ($\ell = 2$, $m = 0$)

$$\rho_p\left(\vec{r'}\right) = \frac{3 \cdot Ze}{4\pi \cdot R_0^3}$$

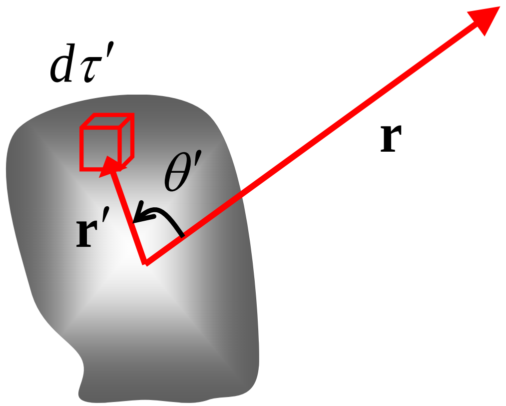
$$M^*(2,0) = \frac{3 \cdot Ze}{4\pi \cdot R_0^3} \iiint r'^2 \cdot Y^*_{20}(\vartheta',\varphi') \cdot r'^2 dr' sin\vartheta' d\vartheta' d\varphi'$$

$$M^*(2,0) = \frac{3 \cdot Ze}{4\pi \cdot R_0^3} \cdot \frac{R_0^5}{5} \iint \frac{1}{5}(1 + \beta_2 Y_{20})^5 \cdot Y^*_{20} \cdot sin\vartheta' d\vartheta' d\varphi'$$

$$M^*(2,0) \cong \frac{3 \cdot Ze \cdot R_0^2}{4\pi} \iint \frac{1}{5}(1 + 5 \cdot \beta_2 Y_{20}) \cdot Y^*_{20} \cdot d\Omega'$$

$$\boxed{M^*(2,0) \cong \frac{3 \cdot Ze \cdot R_0^2}{4\pi} \cdot \beta_2}$$

# Electric fields of multipoles Weisskopf estimate




multipole moments

$$\langle f\|M(E\ell)\|i\rangle = \iiint \psi_f M(E\ell)\,\psi_i d\tau$$

approximation:

if we take the radial parts of $\Psi_i$ and $\Psi_f$ to be constant for r<R (the nuclear radius) and to be =0 for r>R then the radial part of the transition probability is of the form:

$$\frac{\int_0^R r^\ell r^2 dr}{\int_0^R r^2 dr} = \frac{\frac{1}{\ell+3}R^{\ell+3}}{\frac{1}{3}R^3} = \frac{3}{\ell+3}R^\ell$$

For a transition from an excited state $I_i$ to the ground state $I_{gs}$ one finds in the electrical (Eℓ) case

$$B(E\ell; I_i \to I_{gs}) = \frac{1.2^{2\ell}}{4\pi}\cdot\left(\frac{3}{\ell+3}\right)^2 \cdot A^{2\ell/3} \quad e^2(fm)^{2\ell}$$

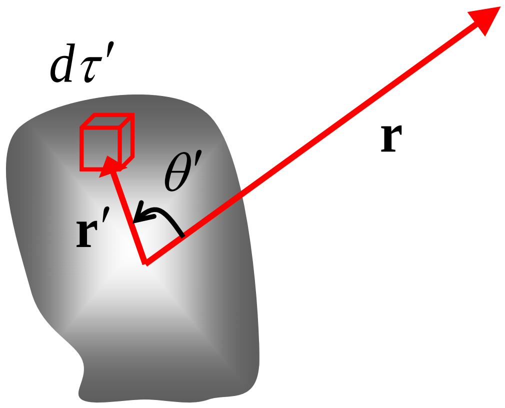
$$B(E1; I_i \to I_{gs}) = 6.446\cdot 10^{-4}\cdot A^{2/3} \quad e^2 b$$

$$B(E2; I_i \to I_{gs}) = 5.940\cdot 10^{-6}\cdot A^{4/3} \quad e^2 b^2$$

$$B(E3; I_i \to I_{gs}) = 5.940\cdot 10^{-8}\cdot A^{2} \quad e^2 b^3$$

$$B(E4; I_i \to I_{gs}) = 6.285\cdot 10^{-10}\cdot A^{8/3} \quad e^2 b^4$$

# Electric fields of multipoles

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expansion

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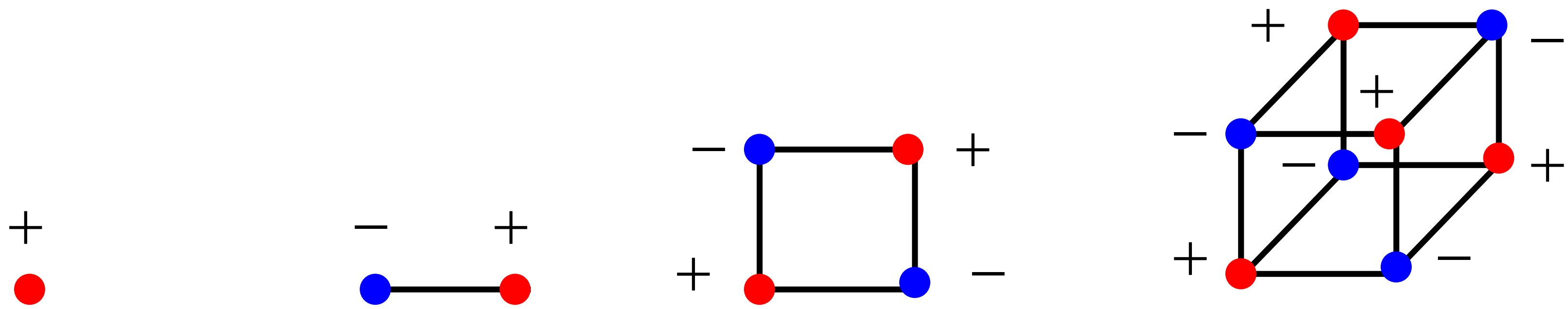
multipole moments

$$M^{*}(\ell,m) = \iiint \rho_p(r') \cdot r'^{\ell} \cdot Y_{\ell m}^{*}(\vartheta',\varphi') d\tau'$$

special case: **electric quadrupole potential**

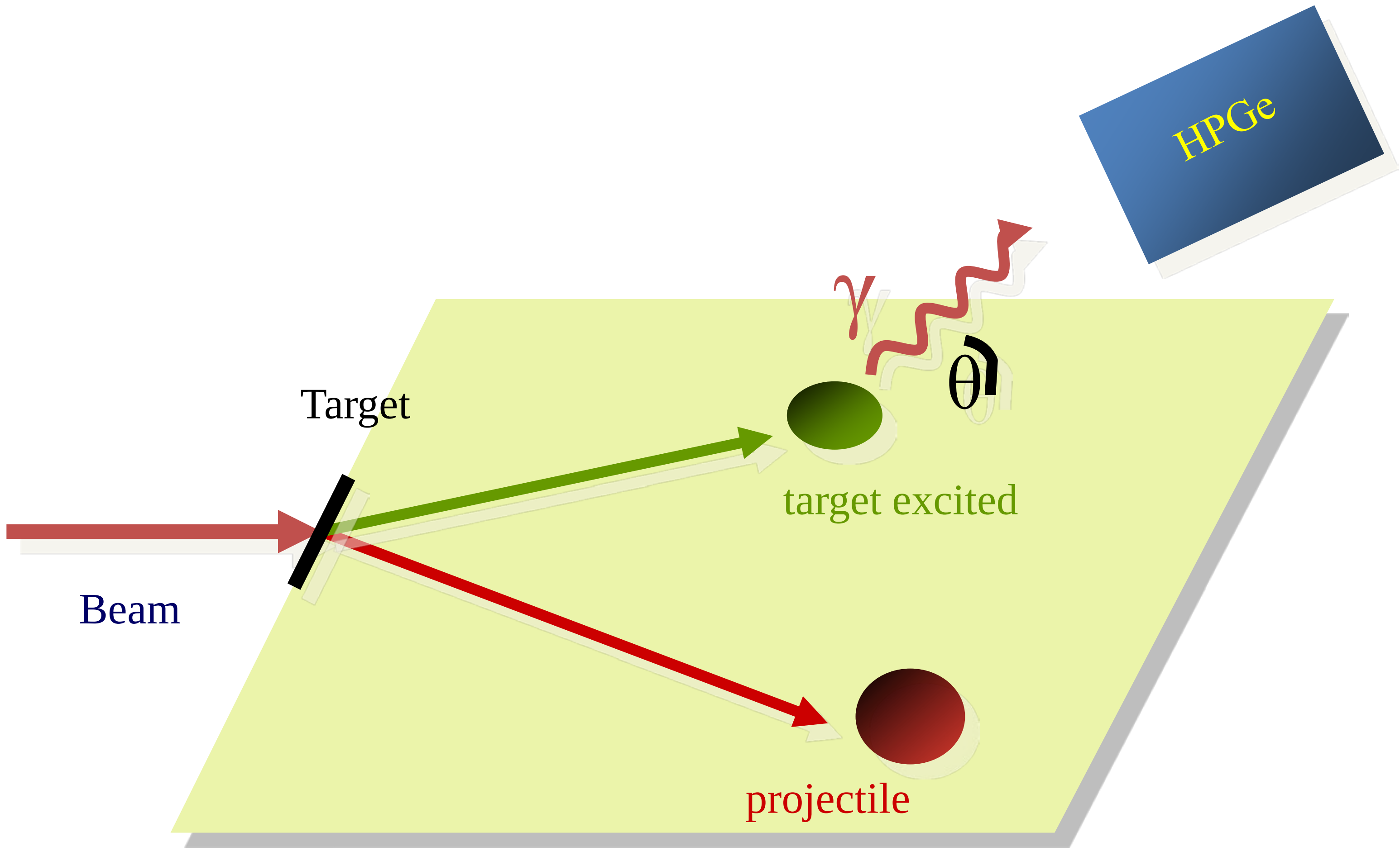
$$U(\vec{r}) = \sum_{m=-2}^{m=2} \frac{4\pi}{5} \cdot \frac{1}{r^3} \cdot Y_{\ell=2,m}(\vartheta,\varphi) \cdot M^{*}(\ell=2,m)$$

# Monopole, dipole, quadrupole, …

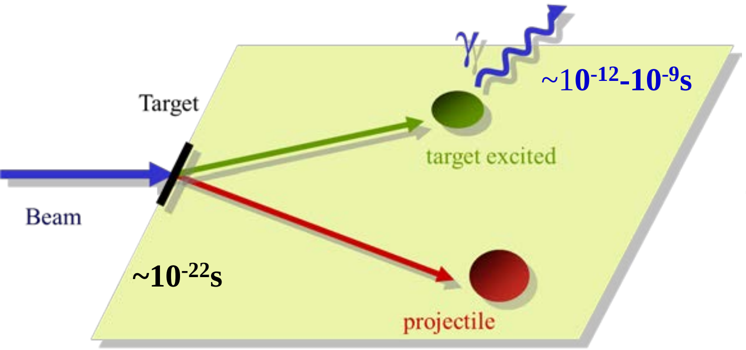



| Monopole | dipole | quadrupole | octopole |
|---|---|---|---|
| $U(r) \propto \frac{1}{r}$ | $U(r) \propto \frac{1}{r^2}$ | $U(r) \propto \frac{1}{r^3}$ | $U(r) \propto \frac{1}{r^4}$ |

# Particle γ-ray coincidence measurement



# Coulomb excitation particle – γ-ray coincidence measurement




$$\frac{d^2\sigma}{d\Omega_p^{lab}d\Omega_\gamma^{lab}} = \underbrace{\left|a_{i\to f}\right|^2}_{\equiv P_I} \frac{d\sigma_{Ruth}}{d\Omega_p^{cm}} \frac{d\Omega_p^{cm}}{d\Omega_p^{lab}} \cdot \frac{dW(\gamma_{f\to i})}{d\Omega_\gamma^{Rest}} \frac{d\Omega_\gamma^{Rest}}{d\Omega_\gamma^{lab}}$$

$\equiv P_I$ (excitation probability)

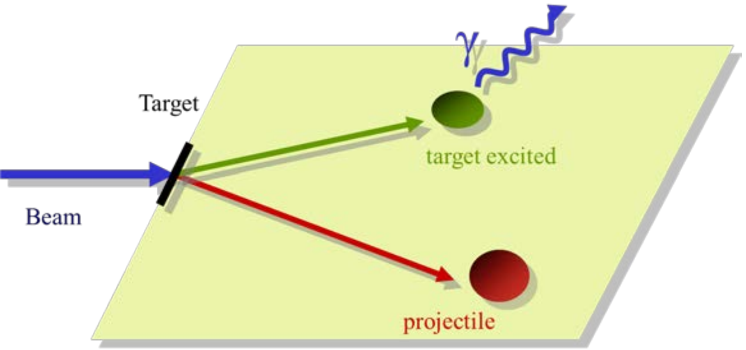
$$\frac{d\sigma_{Ruth}}{d\Omega_p^{cm}} = \frac{a^2}{4} \cdot sin^{-4}\frac{\theta_{cm}}{2}$$

$$\frac{d\Omega_p^{cm}}{d\Omega_p^{lab}} = 4 \cdot cos\vartheta_2$$

$$\frac{dW(\gamma_{f\to i})}{d\Omega_\gamma^{Rest}} = (4\pi)^{-1/2} \sum_{k=0,2,4} \sum_{-k\le\kappa\le k} A_{k\kappa} Q_k G_k F_k(I_M, I_N) Y_{k\kappa}(\theta_\gamma, \phi_\gamma)$$

$$\frac{d\Omega_\gamma^{Rest}}{d\Omega_\gamma^{lab}} = \left[\frac{E_\gamma}{E_{\gamma 0}}\right]^2 = \frac{[1-(v_i/c)^2]}{[1-v_i/c \cdot cos\vartheta_{\gamma i}]^2}$$

# Coulomb excitation particle – γ-ray coincidence measurement




$$\frac{d^2\sigma}{d\Omega_p^{lab}d\Omega_\gamma^{lab}} = \left|a_{i\to f}\right|^2 \frac{d\sigma_{Ruth}}{d\Omega_p^{cm}}\frac{d\Omega_p^{cm}}{d\Omega_p^{lab}} \cdot \frac{dW\left(\gamma_{f\to i}\right)}{d\Omega_\gamma^{Rest}}\frac{d\Omega_\gamma^{Rest}}{d\Omega_\gamma^{lab}}$$

$$\frac{d\sigma_{Ruth}}{d\Omega_p^{cm}} = \frac{a^2}{4}\cdot sin^{-4}\frac{\theta_{cm}}{2}$$

$$\frac{d\Omega_p^{cm}}{d\Omega_p^{lab}} = 4\cdot cos\vartheta_2$$

$$\frac{dW\left(\gamma_{f\to i}\right)}{d\Omega_\gamma^{Rest}} \cong a_0\cdot\left[1+\frac{a_2}{a_0}P_2\left(cos\vartheta_{\gamma 2}\right)+\frac{a_4}{a_0}P_4\left(cos\vartheta_{\gamma 2}\right)\right]$$

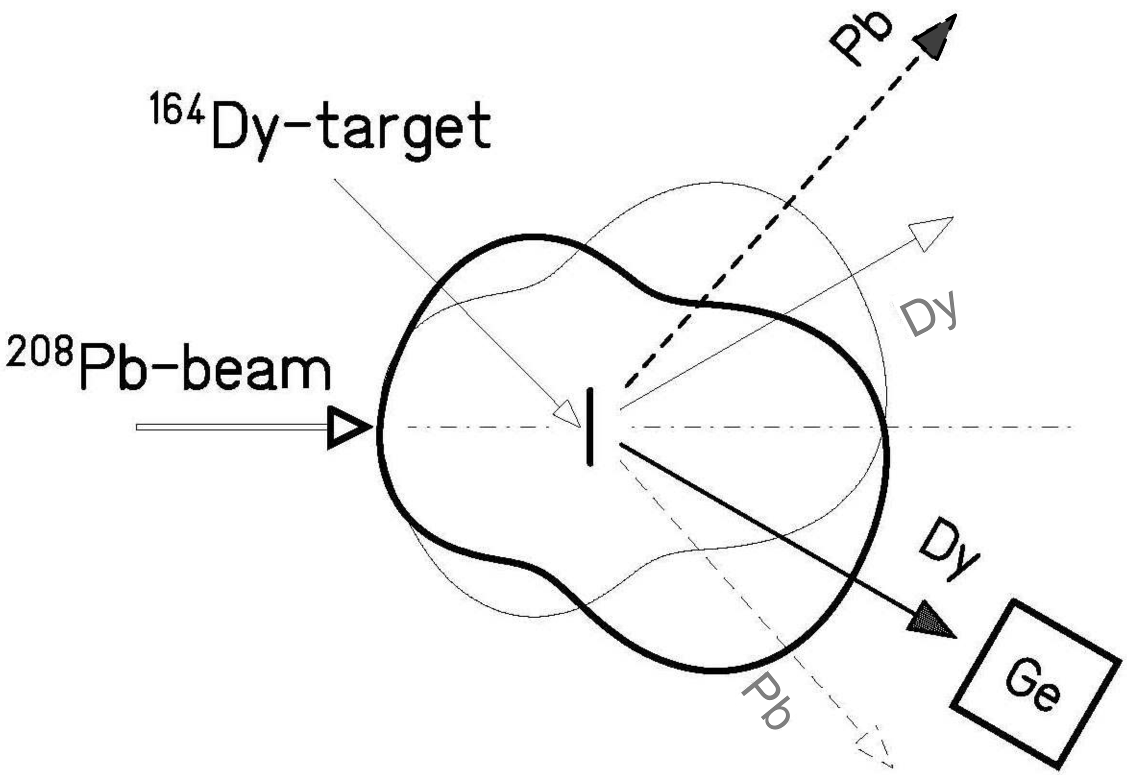
$$\frac{d\Omega_\gamma^{Rest}}{d\Omega_\gamma^{lab}} = \left[\frac{E_\gamma}{E_{\gamma 0}}\right]^2$$

$$a_0 = \frac{1}{1+\alpha_T(I\to I-2)}\frac{1}{4\pi}$$

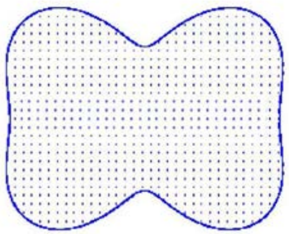
$$\frac{a_2}{a_0} = \frac{5}{7}\frac{I+1}{2I-1}$$

$$\frac{a_4}{a_0} = -\frac{3}{7}\frac{(I+1)\cdot(I+2)}{(2I-3)\cdot(2I-1)}$$

$$cos\vartheta_{\gamma 2} = cos\vartheta_\gamma\cdot cos\vartheta_2 + sin\vartheta_\gamma\cdot sin\vartheta_2\cdot cos\left(\varphi_\gamma-\varphi_2\right)$$



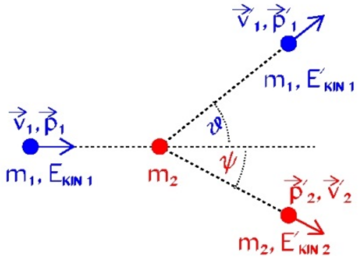
164Dy-target
208Pb-beam
Pb
Dy
Dy
Pb
Ge

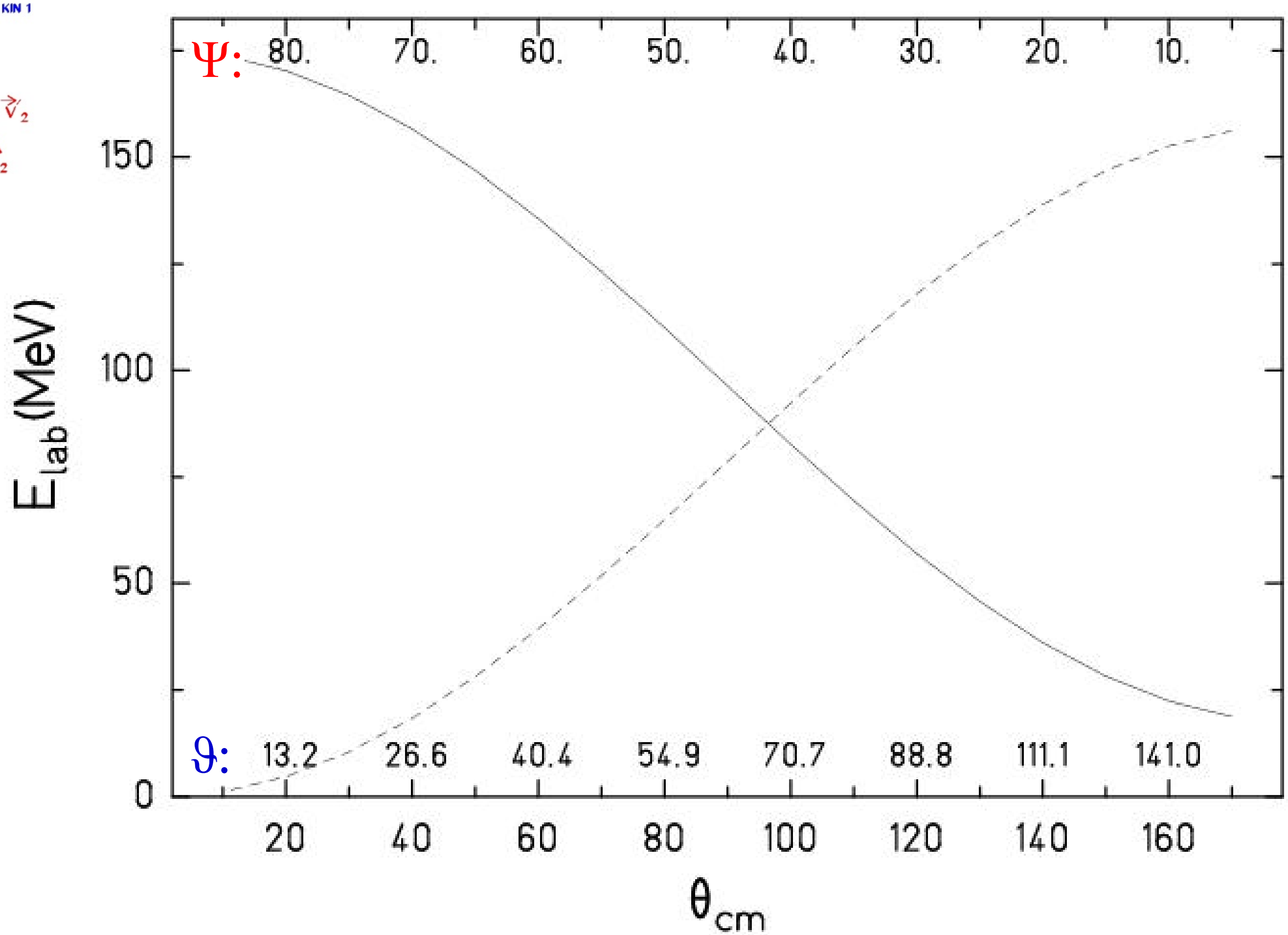
$4^+ \rightarrow 2^+$


$12^+ \rightarrow 11^-$

# Kinematics

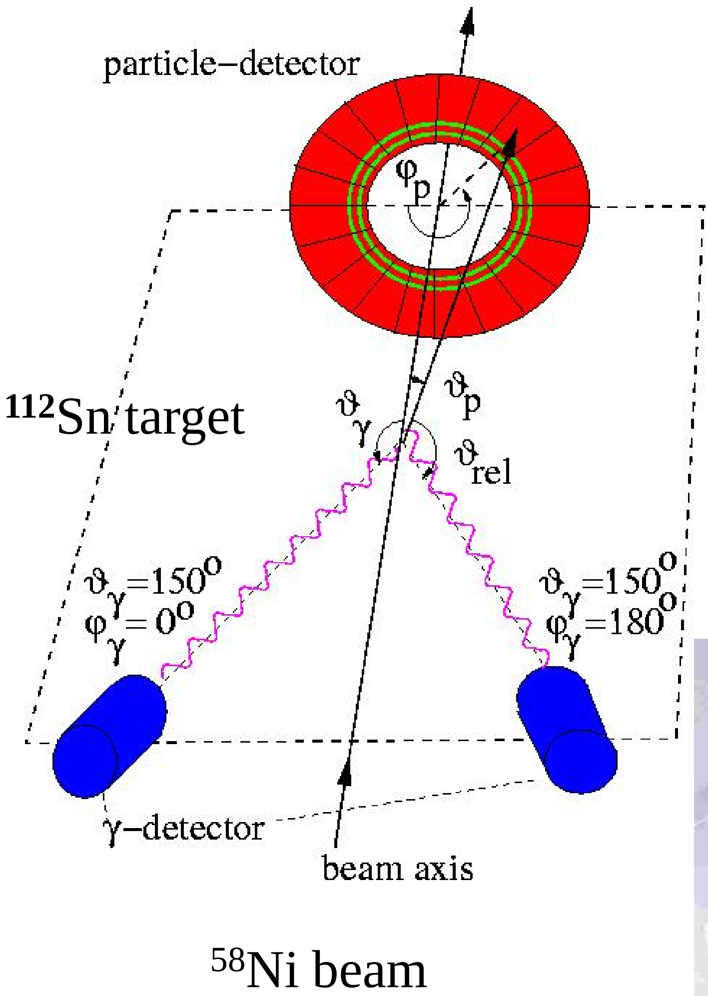



$^{58}Ni \rightarrow {}^{112}Sn$ @ 175 MeV




$= \frac{1}{2}(180^0 - \theta_{cm})$

# Coulomb excitation at IUAC



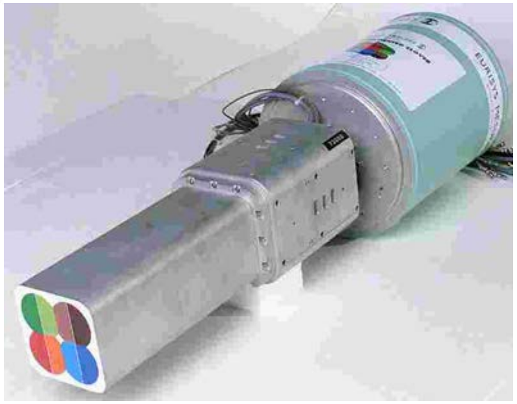

## $^{58}$Ni → $^{122}$Sn at 175 MeV

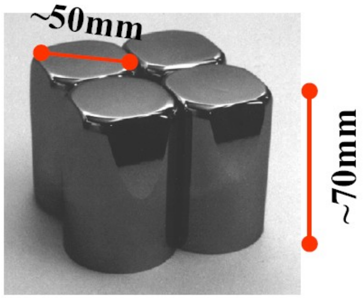
$E_{safe}$ = 202 MeV

beam intensity = 0.5 pnA
target thickness = 0.5 mg/cm$^2$
→ luminosity = $8 \cdot 10^{27}$ s$^{-1}$cm$^{-2}$

cross section ~ 70 mb
**event rate = 560 s$^{-1}$**

γ-efficiency = 0.005
**pγ-rate (Sn) = 3/s**

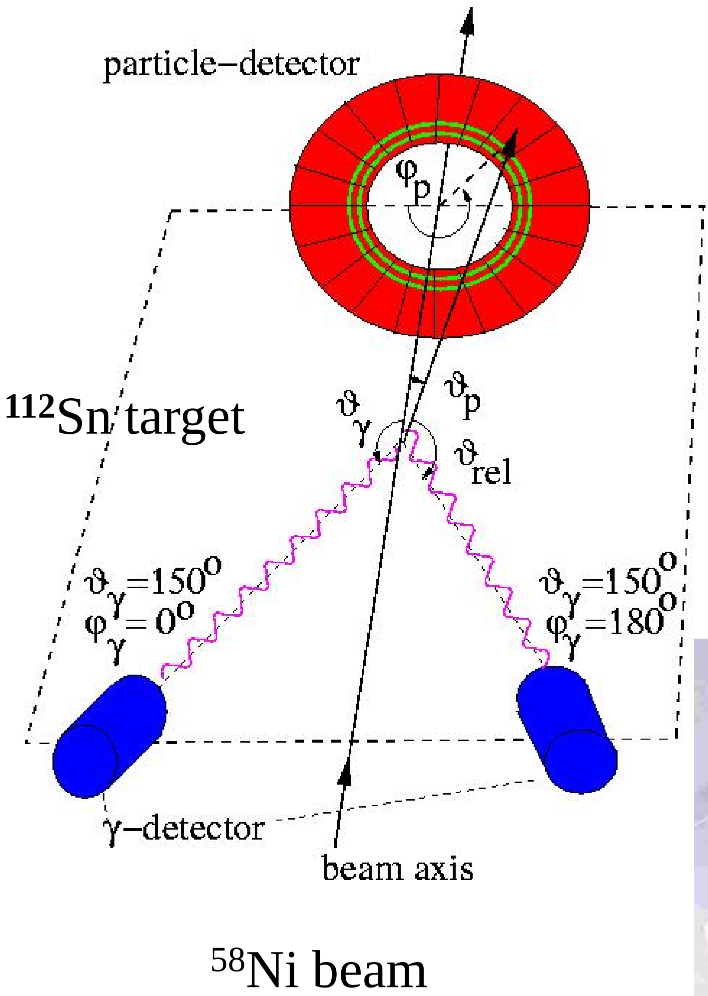




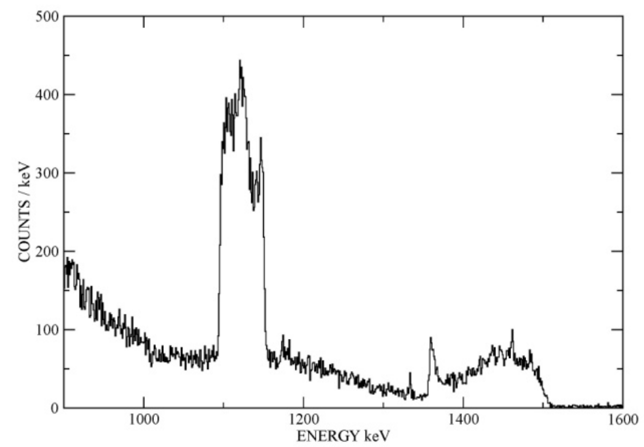

Clover Ge detector

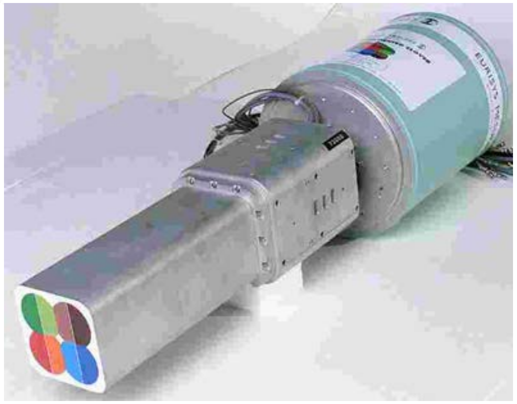
R. Kumar et al., Phys. Rev. C81, 024306 (2010)  M. Saxena et al., Phys. Rev. C90, 024316 (2014)

# Coulomb excitation at IUAC




**$^{58}Ni \rightarrow {}^{122}Sn$ at 175 MeV**



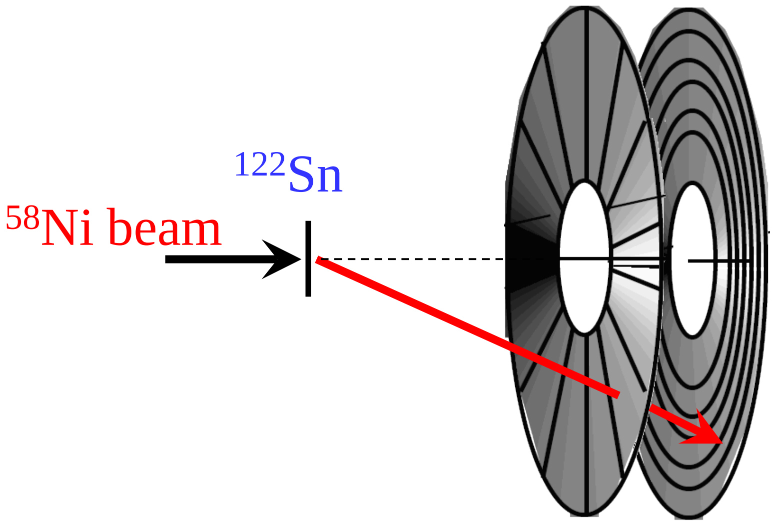



Clover Ge detector

R. Kumar et al., Phys. Rev. C81, 024306 (2010)

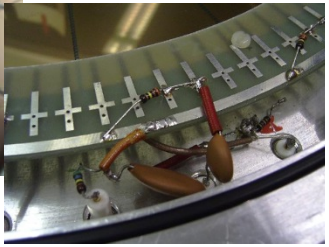
M. Saxena et al., Phys. Rev. C90, 024316 (2014)

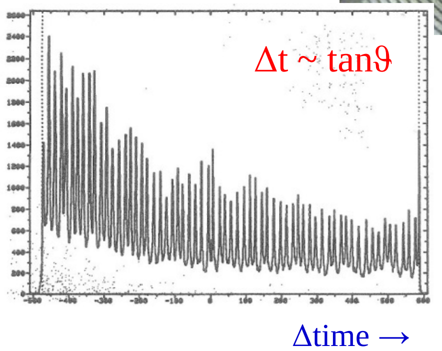
# Annular gas-filled parallel-plate avalanche counter (PPAC)

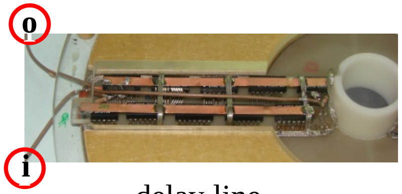



$V_0$ ~ 500 V
p = 5-10 Torr
gap ~ 3 mm (anode-cathode)

$\varphi_p$ $\approx \tan \vartheta_p$



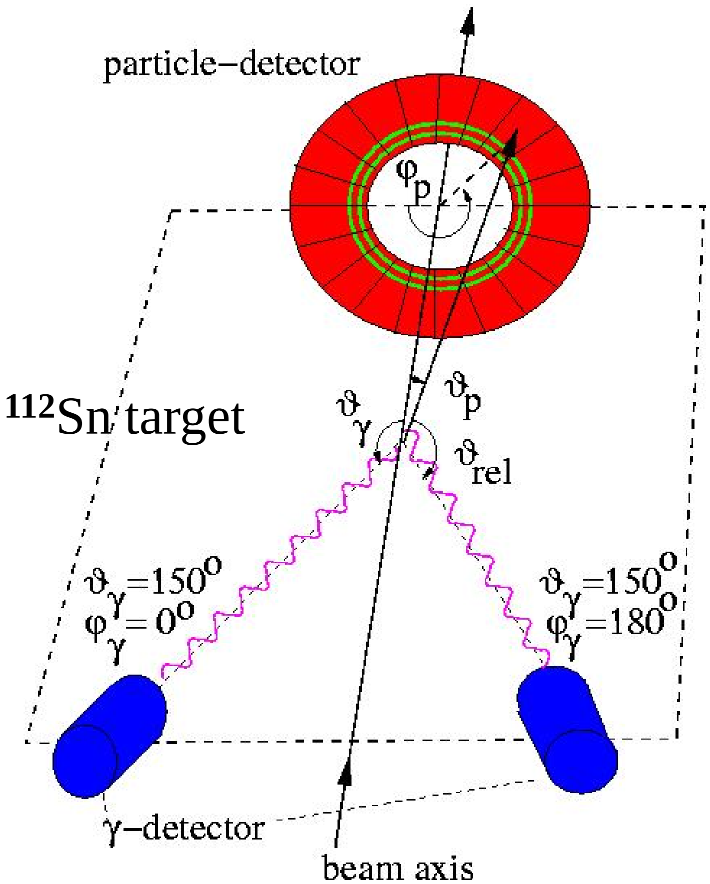



delay line

A. Jhingan detector laboratory at IUAC

# Doppler shift correction $^{58}$Ni + $^{122}$Sn at 175 MeV




$^{58}$Ni beam

delay line: inner – outer contact ≈ tanϑ

$$tan\vartheta = \frac{tan45^0 - tan15^0}{ch_2 - ch_1} \cdot (ch - ch_1) + tan15^0$$

φ-segmentation : 36$^0$, 72$^0$, 108$^0$, etc

$^{58}$Ni **projectile measured with PPAC** ($^{122}$Sn **target excitation**)

index 1 ≡ projectile ($^{58}$Ni) index 2 ≡ target nucleus ($^{122}$Sn)

$$v_{cm} = 0.04634 \cdot (1 + A_2/A_1)^{-1}\sqrt{E_{lab}/A_1} \quad ( = 0.02594)$$

$$\theta_{cm} = \vartheta_1 + arcsin\left(\frac{A_1}{A_2} sin\vartheta_1\right)$$

$$\vartheta_2 = 0.5 \cdot (180^0 - \theta_{cm})$$

$$v_2 = 2 \cdot v_{cm} \cdot cos\vartheta_2$$

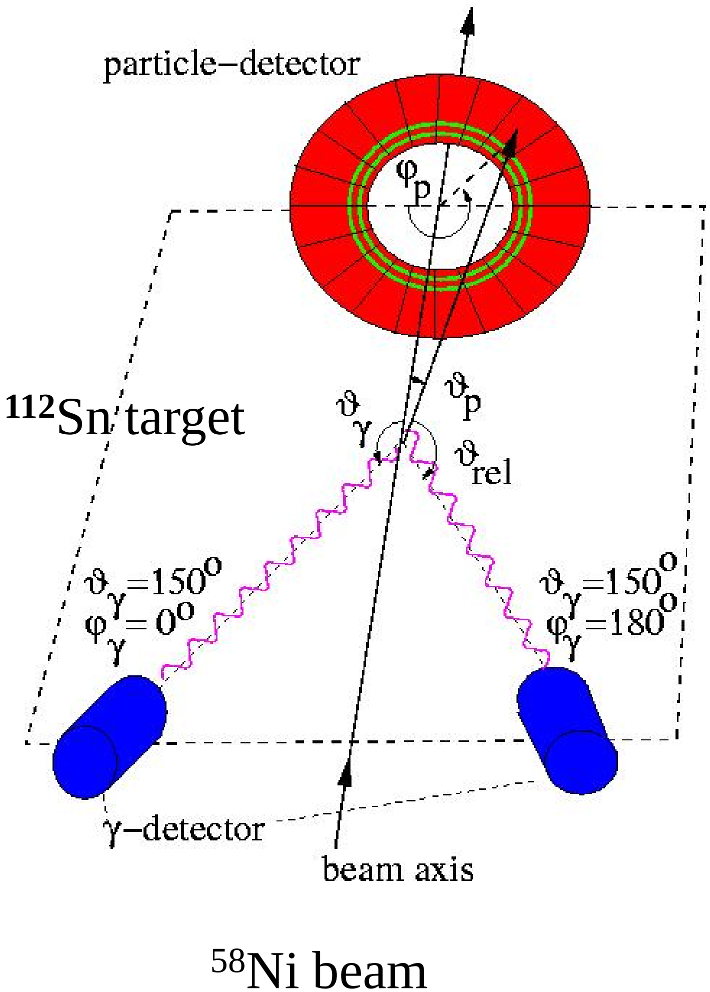
$$cos\vartheta_{\gamma 2} = cos\vartheta_\gamma \cdot cos\vartheta_2 - sin\vartheta_\gamma \cdot sin\vartheta_2 \cdot cos(\varphi_\gamma - \varphi_2)$$

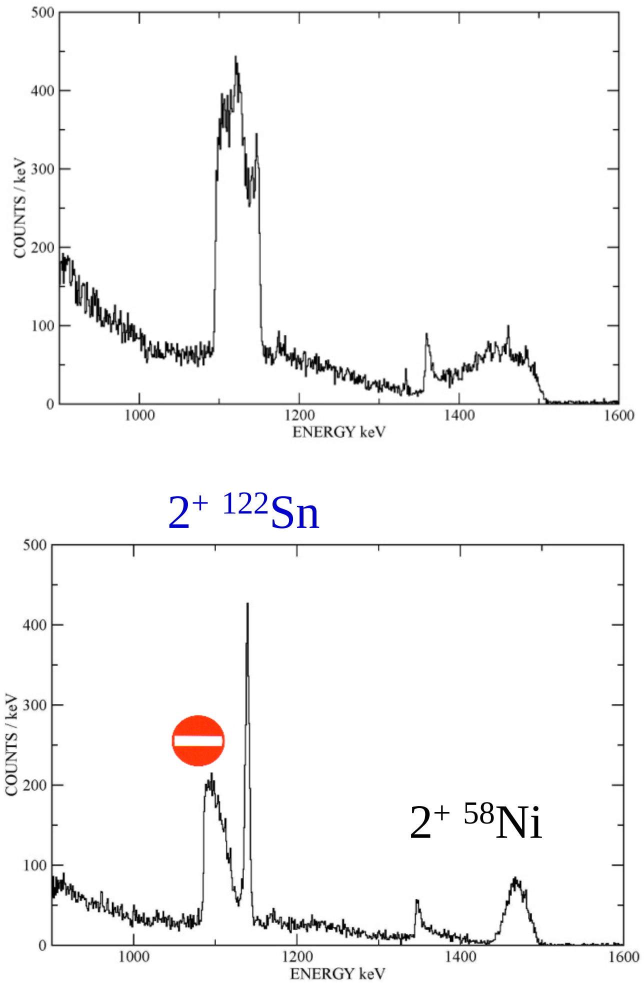
$$cos(\varphi_\gamma - \varphi_1) = cos\varphi_\gamma \cdot cos\varphi_1 + sin\varphi_\gamma \cdot sin\varphi_1$$

$$\frac{E_{\gamma 0}}{E_\gamma} = \frac{1 - v_2 \cdot cos\vartheta_{\gamma 2}}{\sqrt{1 - v_2^2}}$$

D. Schwalm et al. Nucl.Phys. A192 (1972), 449

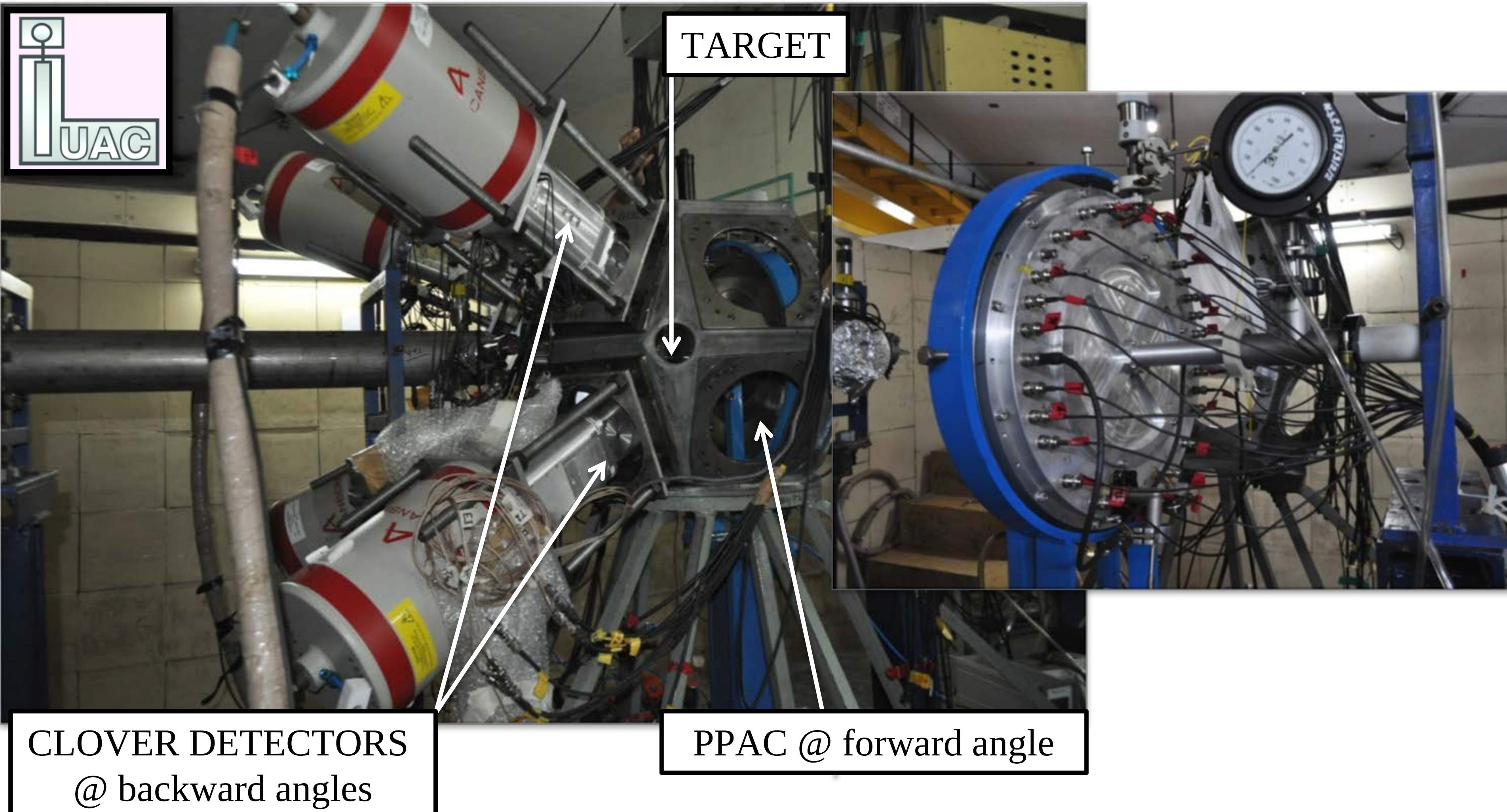
# Doppler shift correction $^{58}$Ni + $^{122}$Sn at 175 MeV



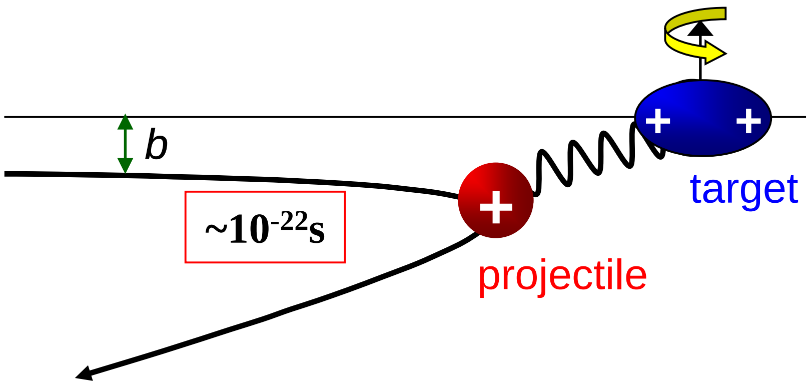



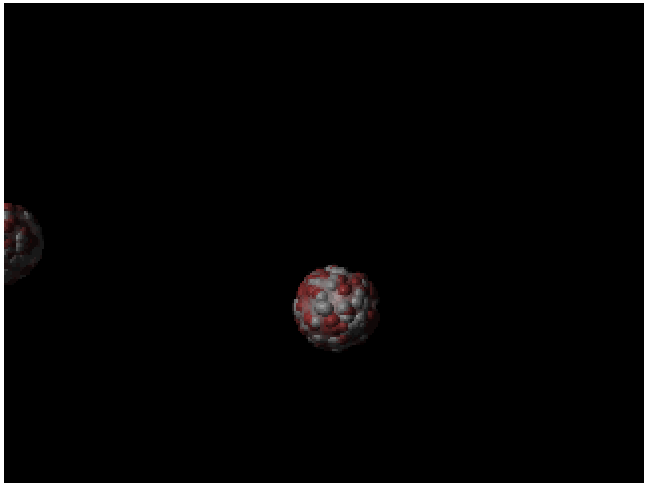

R. Kumar, A. Jhingan @ IUAC

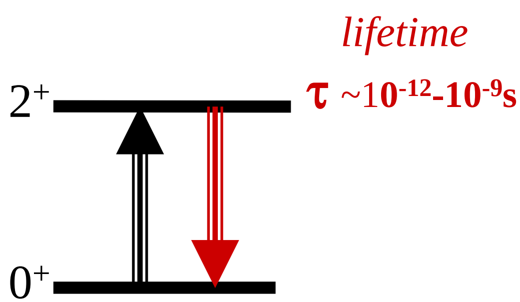
# Experimental set-up



# Particle-gamma coincidence spectroscopy






1st order:

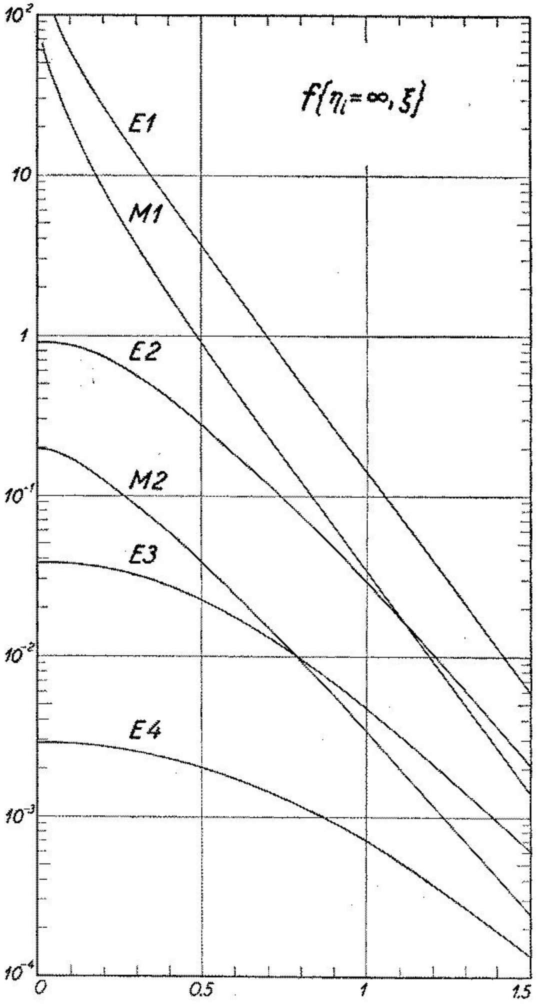
$$a^{(1)}_{i\to f} \propto \langle I_f \| \mathbf{M}(E2) \| I_i \rangle$$

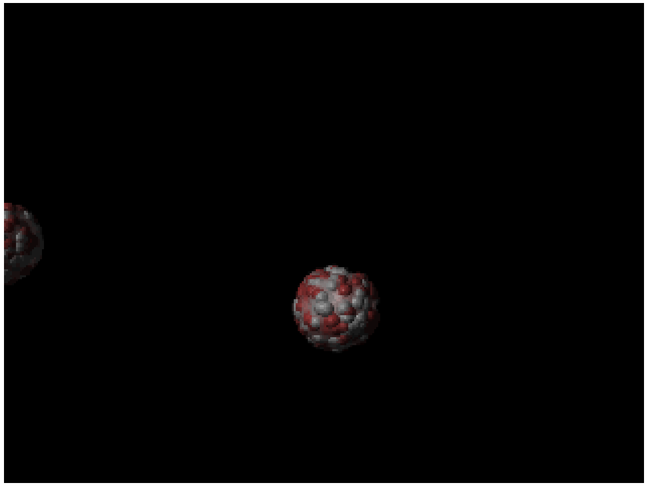
Coulomb excitation:

$$\boldsymbol{\sigma_{E2}}(\mathbf{2}^+)[b] = 4.819 \cdot (1 + A_1/A_2)^{-2} \cdot \frac{A_1}{Z_2^2} \cdot (E_{MeV} - \Delta E'_{MeV}) \cdot \boldsymbol{B}(\boldsymbol{E2}, \mathbf{0}^+ \to \mathbf{2}^+)_{e^2b^2} \cdot f_{E2}(\xi)$$

$$\xi = \frac{Z_1 \cdot Z_2 \cdot A_1^{1/2} \cdot \Delta E'_{MeV}}{12.65 \cdot (E_{MeV} - 0.5 \cdot \Delta E'_{MeV})^{3/2}} \cdot \left(1 + \frac{5}{32}\left(\frac{\Delta E'}{E}\right)^2 + \cdots\right)$$

# Particle-gamma coincidence spectroscopy



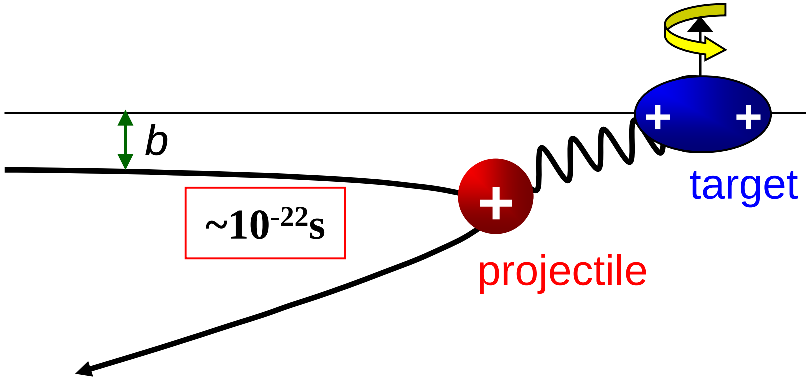
Coulomb excitation:

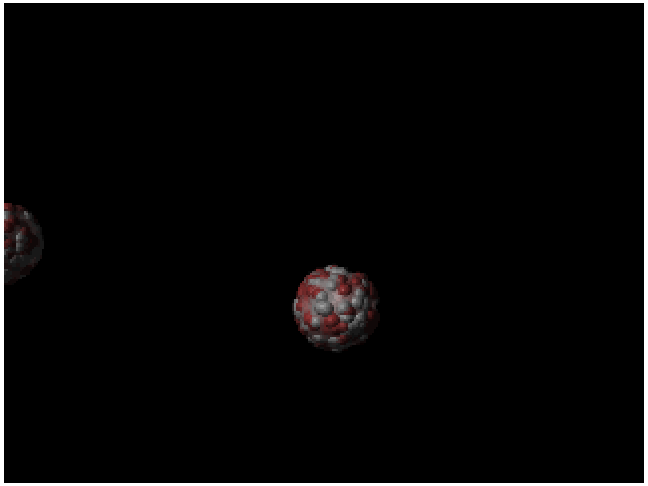
$$\boldsymbol{\sigma_{E2}}(\mathbf{2^+})[b] = 4.819 \cdot (1 + A_1/A_2)^{-2} \cdot \frac{A_1}{Z_2^2} \cdot (E_{MeV} - \Delta E'_{MeV}) \cdot \boldsymbol{B(E2, 0^+ \rightarrow 2^+)}_{e^2b^2} \cdot f_{E2}(\xi)$$

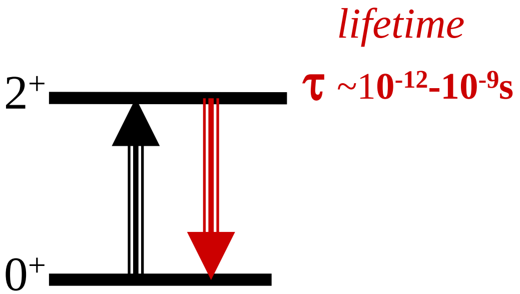
$$\xi = \frac{Z_1 \cdot Z_2 \cdot A_1^{1/2} \cdot \Delta E'_{MeV}}{12.65 \cdot (E_{MeV} - 0.5 \cdot \Delta E'_{MeV})^{3/2}} \cdot \left(1 + \frac{5}{32}\left(\frac{\Delta E'}{E}\right)^2 + \cdots\right)$$

K. Alder et al., RMP 28 (56) 432

# Particle-gamma coincidence spectroscopy






1st order:

$$a_{i\to f}^{(1)} \propto \langle I_f \| \mathbf{M}(E2) \| I_i \rangle$$

Coulomb excitation:

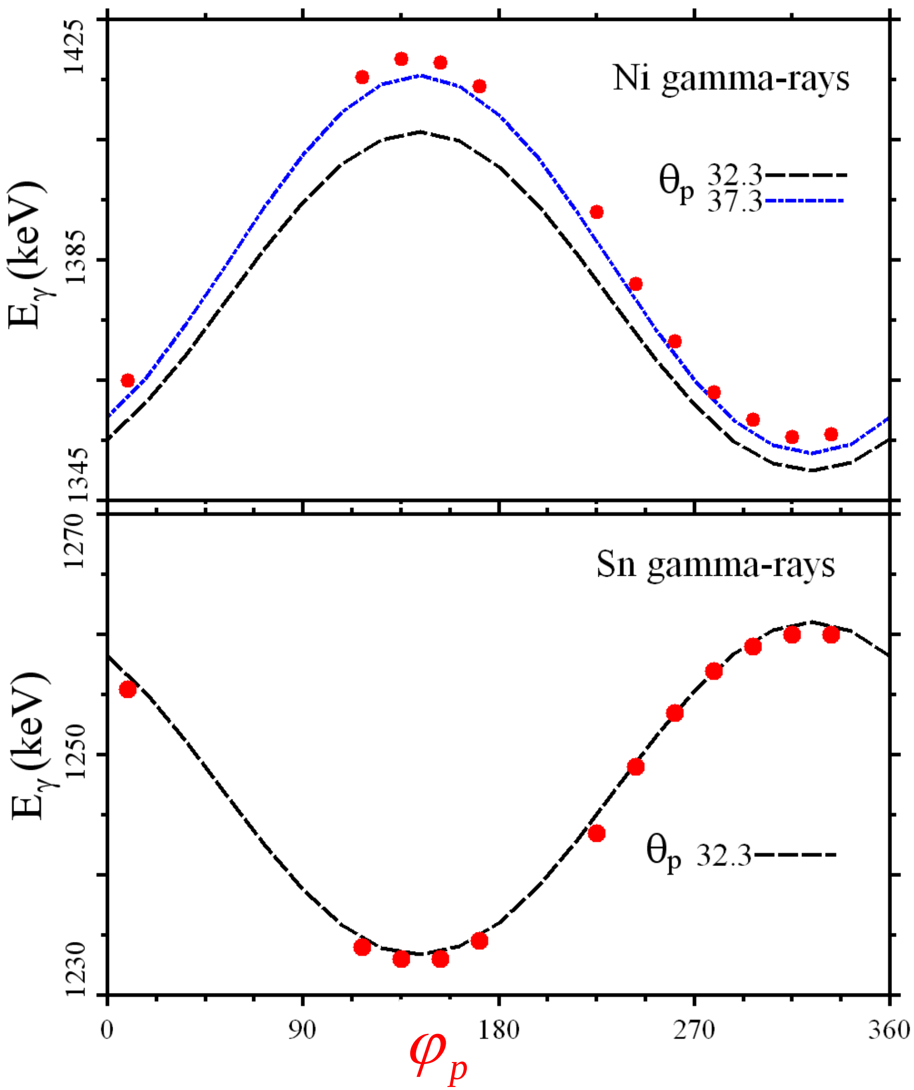
$$\boldsymbol{\sigma_{E2}}(\mathbf{2^+})[b] = 4.819 \cdot (1 + A_1/A_2)^{-2} \cdot \frac{A_1}{Z_2^2} \cdot (E_{MeV} - \Delta E'_{MeV}) \cdot \boldsymbol{B}(\mathbf{E2}, \mathbf{0^+} \to \mathbf{2^+})_{e^2b^2} \cdot f_{E2}(\xi)$$

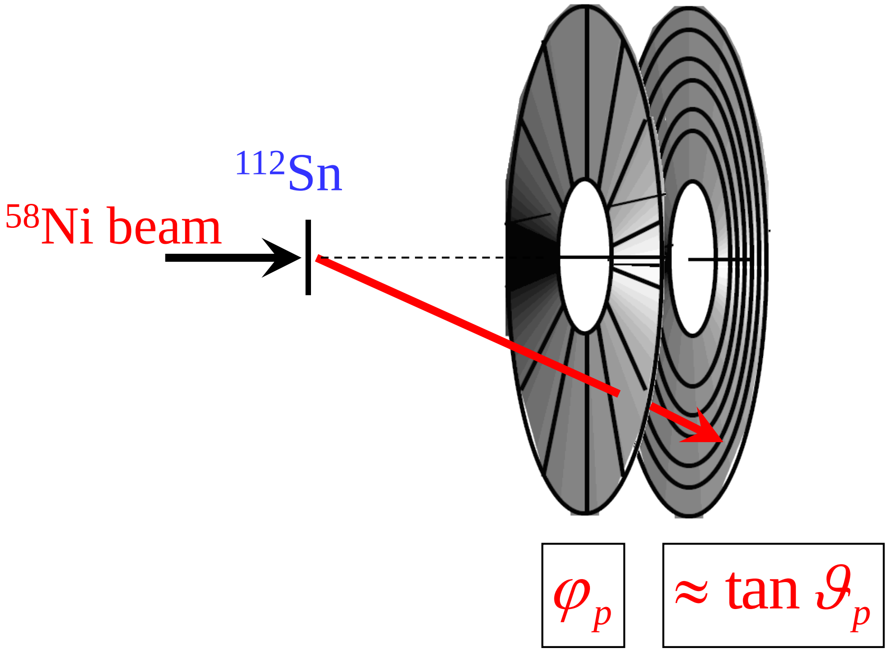
Particle-γ coincidence spectroscopy: $Y_\gamma(I_i \rightarrow I_f) \rightarrow \sigma_{E2}(I_i)$

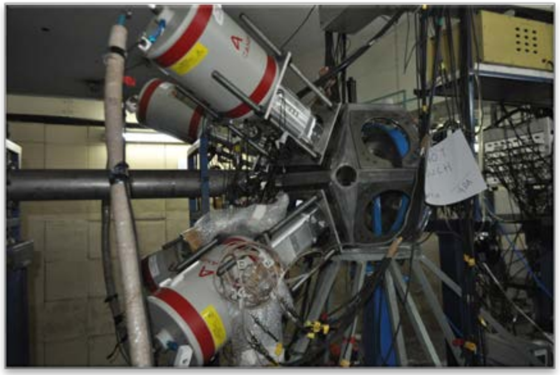
(indirect measurement)

- need to take into account branching ratios, particle-γ angular correlations, electron conversion

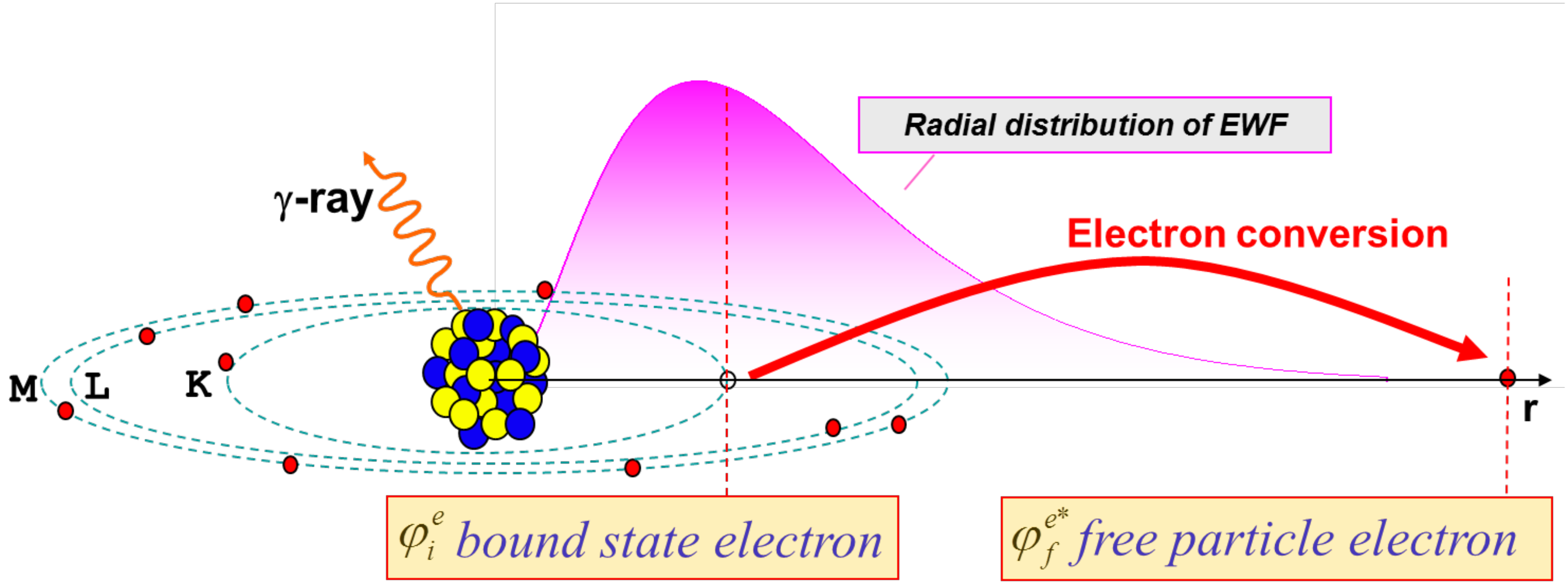
# Particle-gamma angular correlation



# Conversion electrons




**Energetics of CE-decay (i=K, L, M,….)**

$$E_i = E_f + E_{ce,i} + E_{BE,i}$$

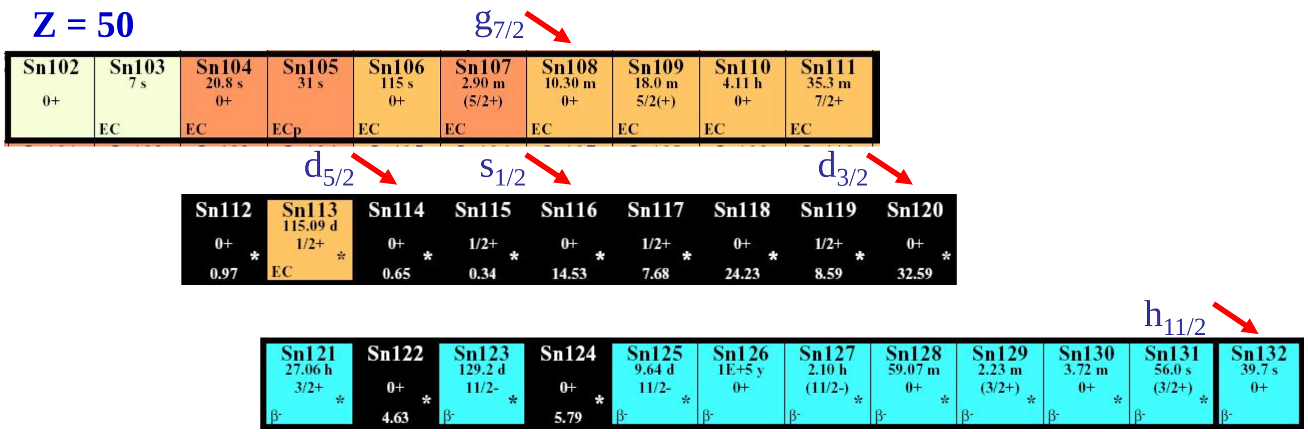
**γ- and CE-decays are independent; transition probability (λ ~ Intensity)**

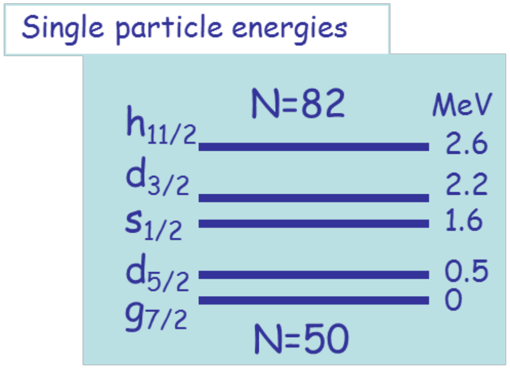
$$\lambda_T = \lambda_\gamma + \lambda_{CE} = \lambda_\gamma + \lambda_K + \lambda_L + \lambda_M \ldots\ldots$$

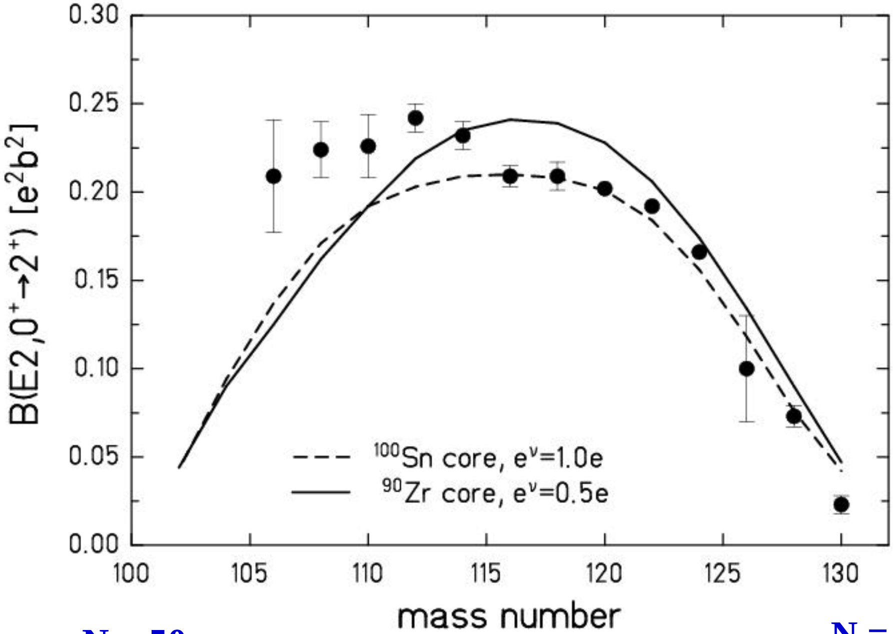
**Conversion coefficient**

$$a_i = \lambda_{CE,i} / \lambda_\gamma$$

# Coulomb excitation results of semi-magic Sn isotopes




N = 50 N = 82

**single-particle transition:**
(Weisskopf estimate)

$$B(E2; I_i \rightarrow I_{gs}) = 5.94\ 10^{-6} \cdot A^{4/3} \quad [e^2b^2]$$

$$B(E2; 2^+ \rightarrow 0^+) = 14\ [spu] \quad \text{for } ^{114}\text{Sn}$$

$$B(E2; 0^+ \rightarrow 2^+) = 0.232\ [e^2b^2] \quad \text{for } ^{114}\text{Sn}$$

$$B(E2; I_i \rightarrow I_f) = \frac{2I_f + 1}{2I_i + 1} \cdot B(E2; I_f \rightarrow I_i)$$

# Coulomb excitation results of semi-magic Sn isotopes

**Z = 50**

$g_{7/2}$ → (Sn108)

| Sn102 | Sn103 | Sn104 | Sn105 | Sn106 | Sn107 | Sn108 | Sn109 | Sn110 | Sn111 |
|---|---|---|---|---|---|---|---|---|---|
| | 7 s | 20.8 s | 31 s | 115 s | 2.90 m | 10.30 m | 18.0 m | 4.11 h | 35.3 m |
| 0+ | | 0+ | | 0+ | (5/2+) | 0+ | 5/2(+) | 0+ | 7/2+ |
| | EC | EC | ECp | EC | EC | EC | EC | EC | EC |

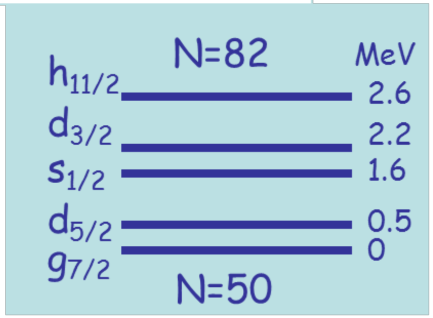
$d_{5/2}$ → (Sn114), $s_{1/2}$ → (Sn116), $d_{3/2}$ → (Sn120)

| Sn112 | Sn113 | Sn114 | Sn115 | Sn116 | Sn117 | Sn118 | Sn119 | Sn120 |
|---|---|---|---|---|---|---|---|---|
| | 115.09 d | | | | | | | |
| 0+ * | 1/2+ * | 0+ * | 1/2+ * | 0+ * | 1/2+ * | 0+ * | 1/2+ * | 0+ * |
| 0.97 | EC | 0.65 | 0.34 | 14.53 | 7.68 | 24.23 | 8.59 | 32.59 |

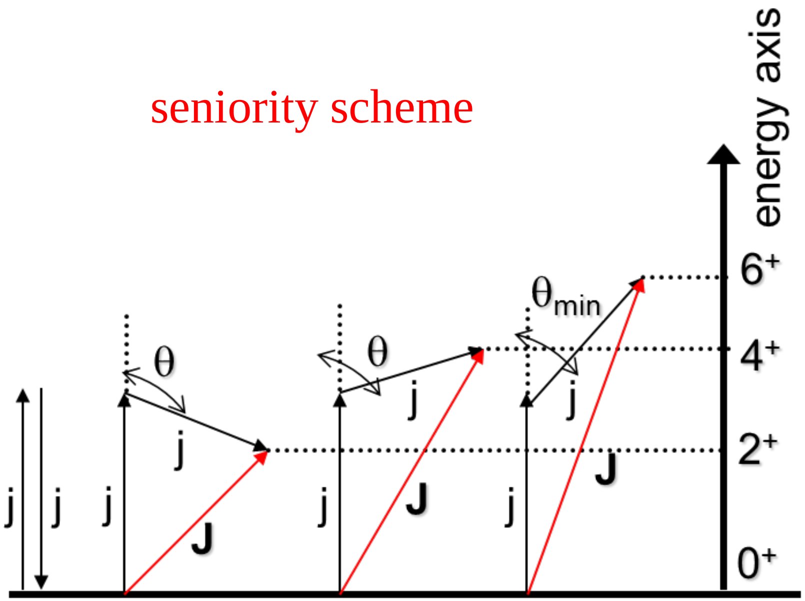
$h_{11/2}$ → (Sn132)

| Sn121 | Sn122 | Sn123 | Sn124 | Sn125 | Sn126 | Sn127 | Sn128 | Sn129 | Sn130 | Sn131 | Sn132 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 27.06 h | | 129.2 d | | 9.64 d | 1E+5 y | 2.10 h | 59.07 m | 2.23 m | 3.72 m | 56.0 s | 39.7 s |
| 3/2+ * | 0+ * | 11/2- * | 0+ * | 11/2- * | 0+ | (11/2-) * | 0+ * | (3/2+) * | 0+ * | (3/2+) * | 0+ |
| β- | 4.63 | β- | 5.79 | β- | β- | β- | β- | β- | β- | β- | β- |

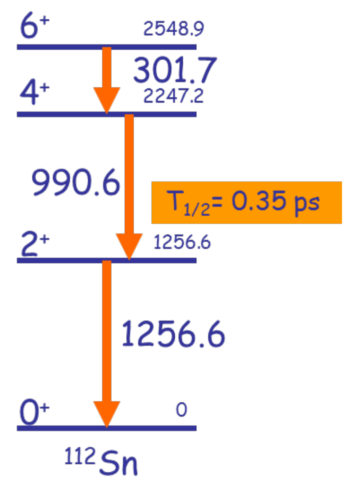
Single particle energies

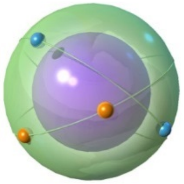



seniority scheme

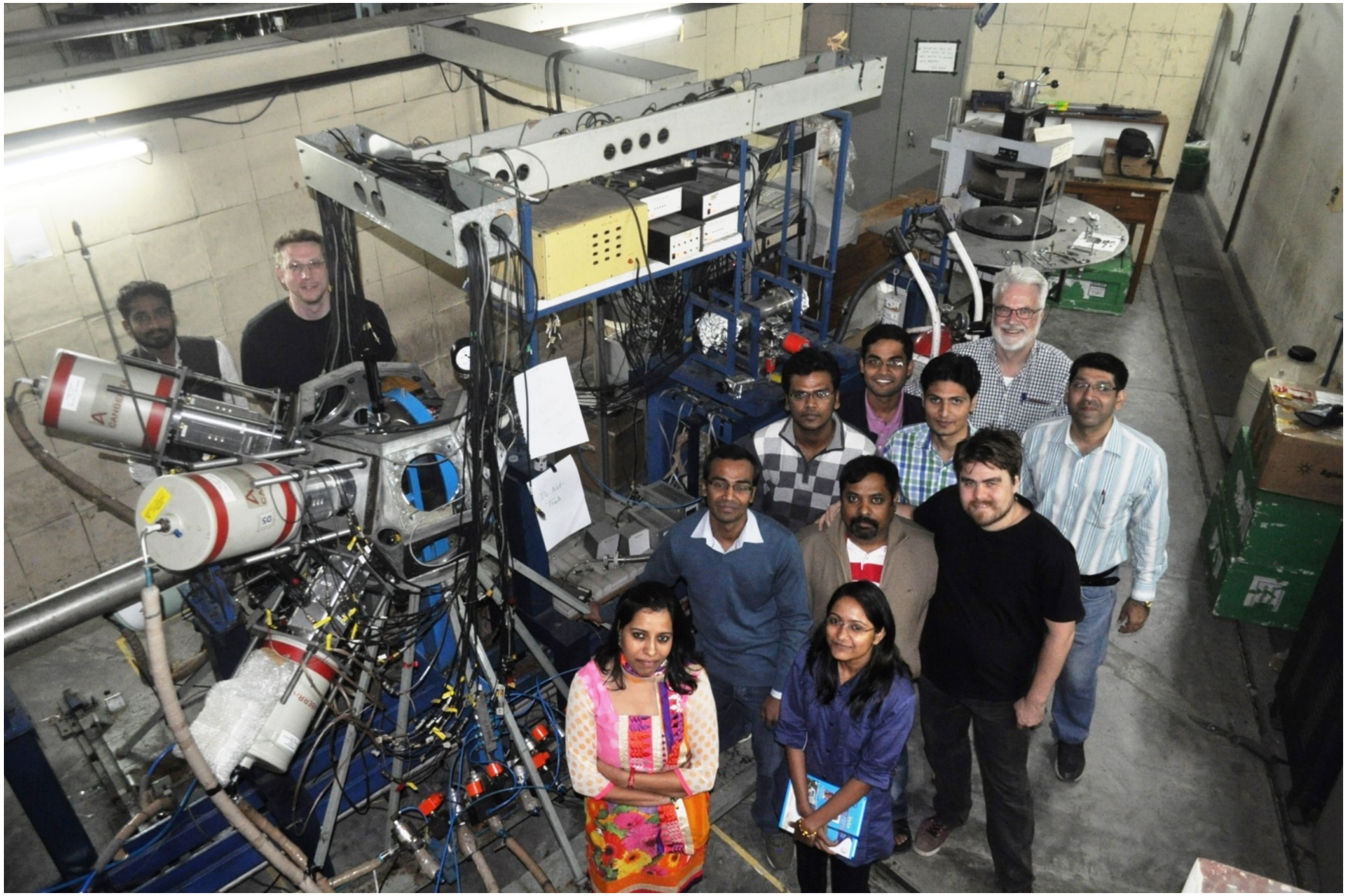



exp. level scheme



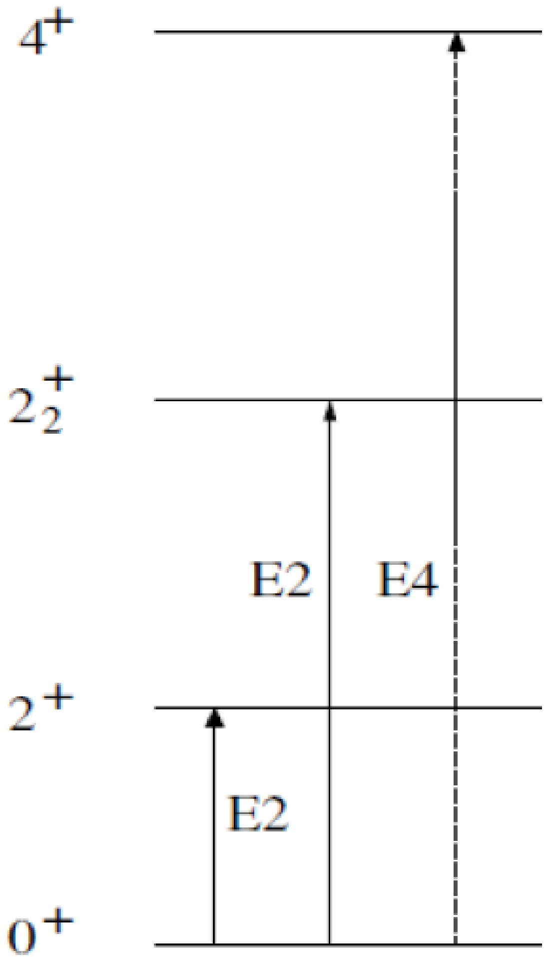



# Coulomb excitation team



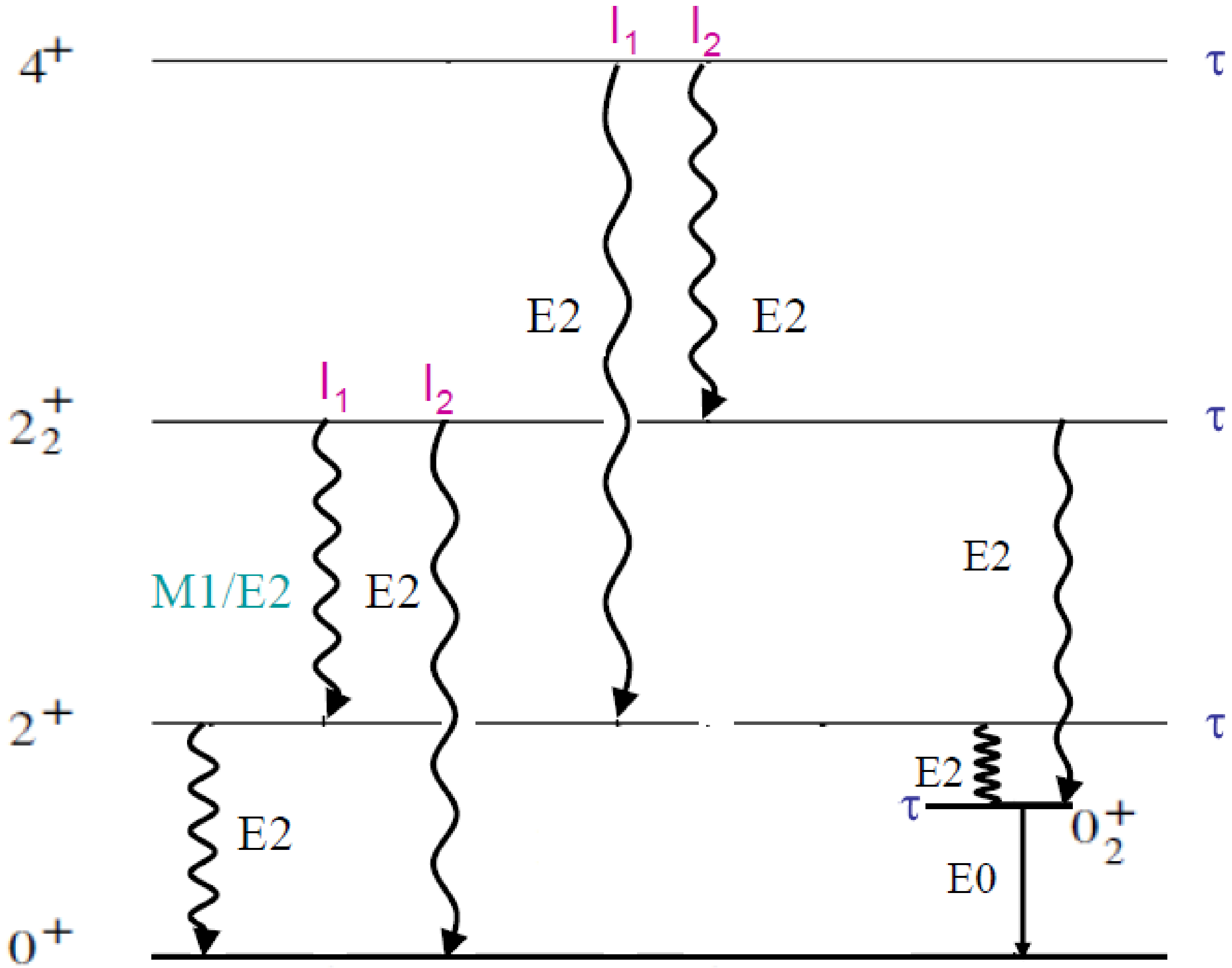
Vivek Mishra, Pieter Doornenbal, Mansi Saxena, Chhavi Joshi, Sunil Prajapati, Rakesh Kumar, Paer-Anders Soederstroem, Mohit Kumar, Sunil Dutt, Akhil Jhingan, Aakashrup Banerjee, Hans Jürgen Wollersheim.

# Multiple (multi-step) Coulomb excitation

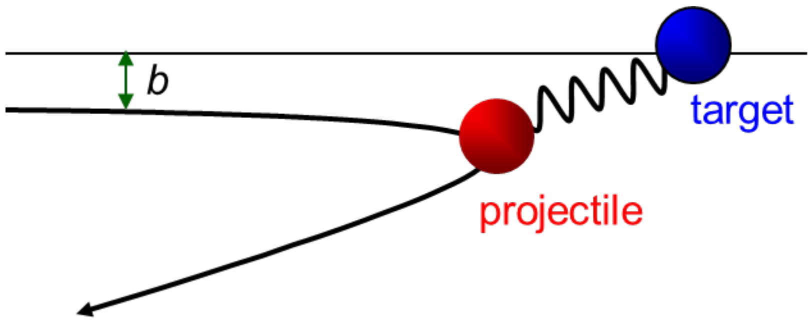



1st order

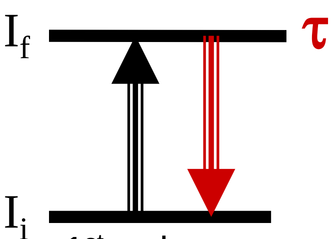
# γ-ray decay after multiple Coulomb excitation



# The reorientation effect

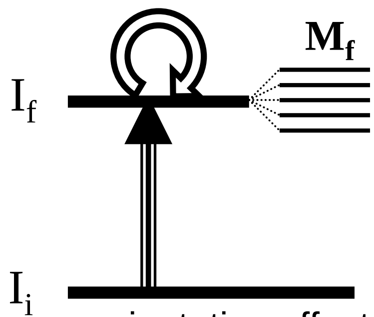



The excitation cross section is a direct measure of the $E\lambda$ matrix elements.



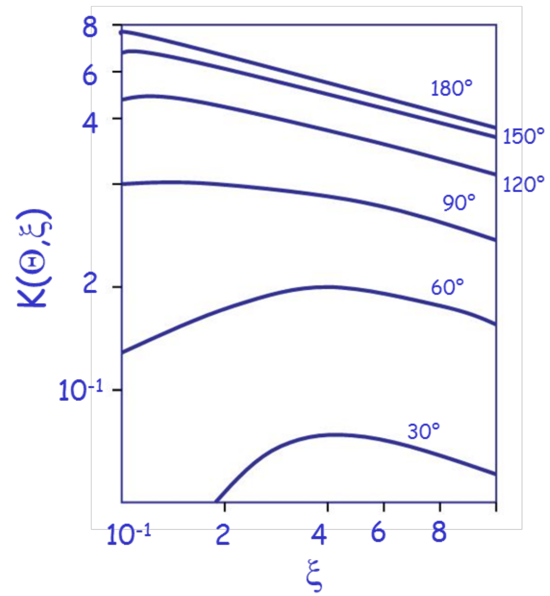

$1^{st}$ order:

$$a^{(1)}_{i\to f} \propto \langle I_f \| \mathbf{M}(E2) \| I_i \rangle$$




reorientation effect:

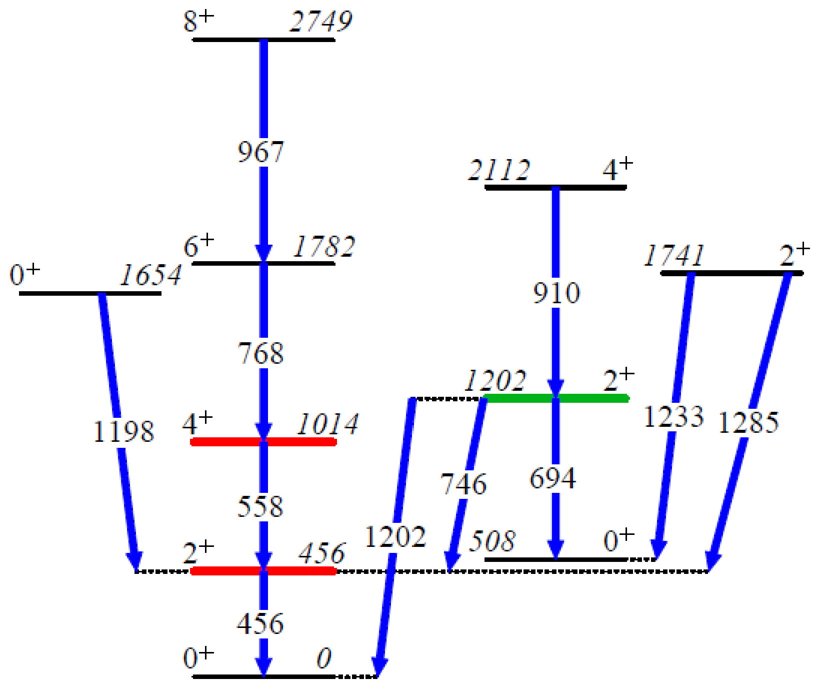
$$a^{(2)}_{i\to f} \propto \langle I_f \| \mathbf{M}(E2) \| I_f \rangle \langle I_f \| \mathbf{M}(E2) \| I_i \rangle$$

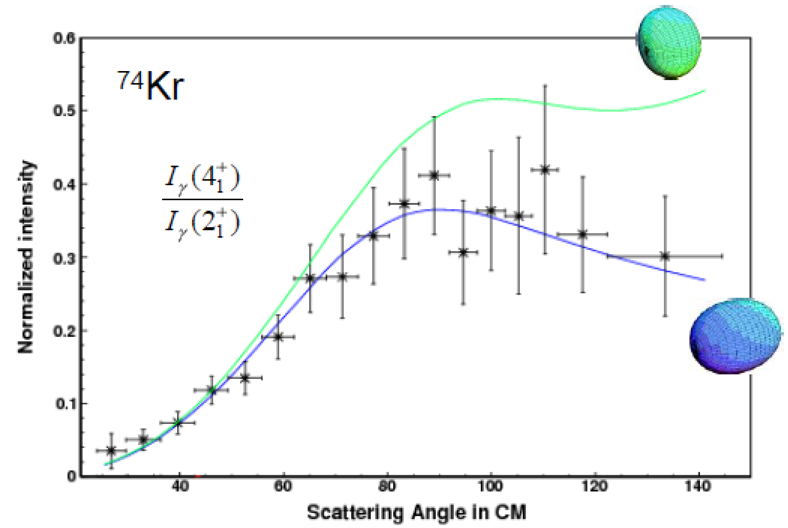


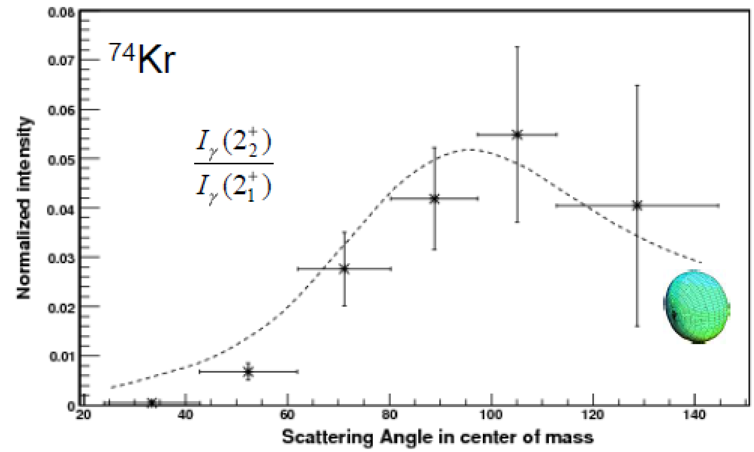

$$P^{(2)}_{0\to 2}(\theta,\xi) = P^{(1)}_{0\to 2}(\theta,\xi) \cdot \left[ 1 + \sqrt{\frac{7}{2\pi}\frac{5}{4}} \cdot \frac{A_p}{Z_p} \cdot \frac{\Delta E}{1 + A_p/A_t} \cdot Q_2 \cdot K(\theta,\xi) \right]$$

$$Q(2^+) = -\sqrt{\frac{2\pi}{7}\frac{4}{5}} \cdot \langle 2 \| M(E2) \| 2 \rangle$$

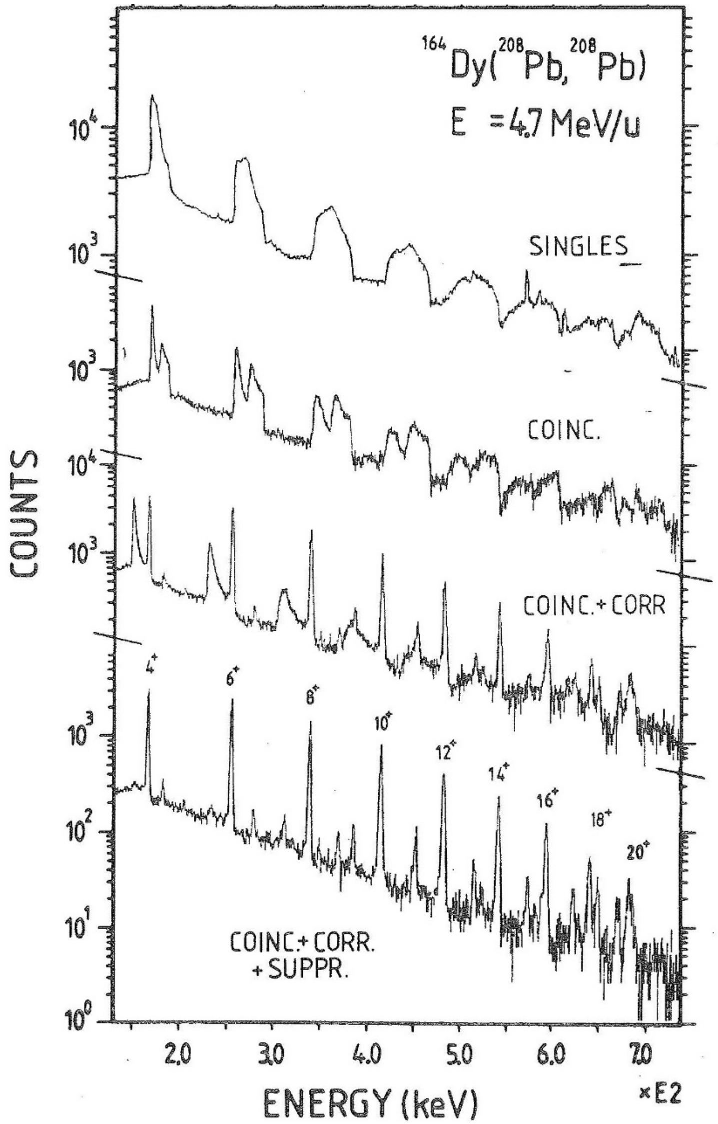
# Shape coexistence in $^{74}Kr$

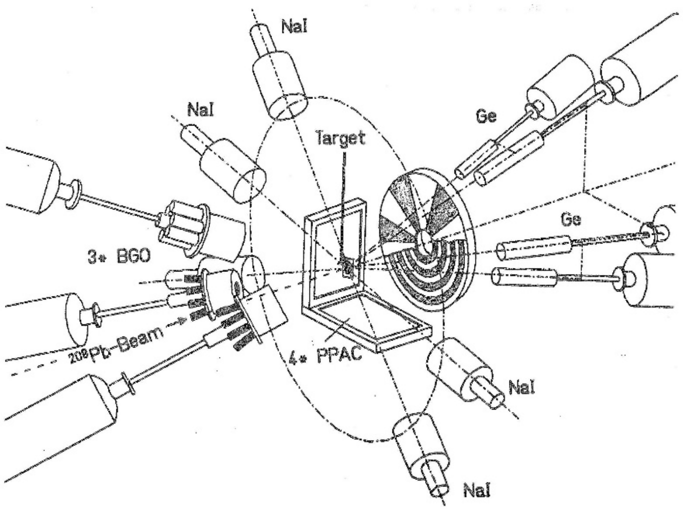


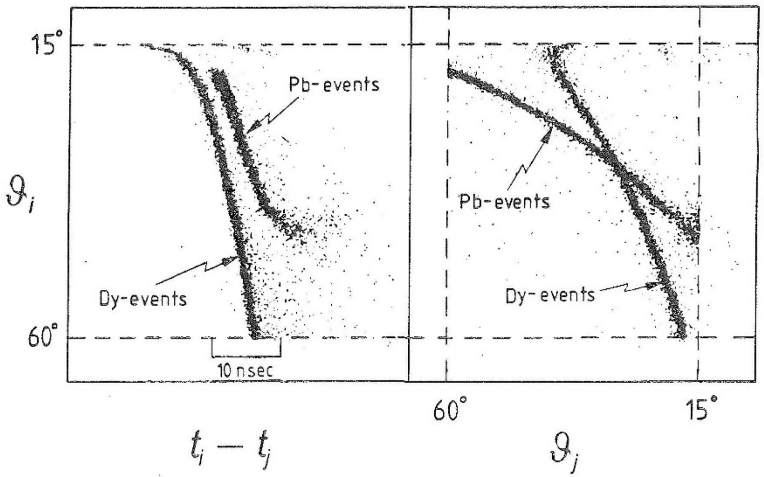



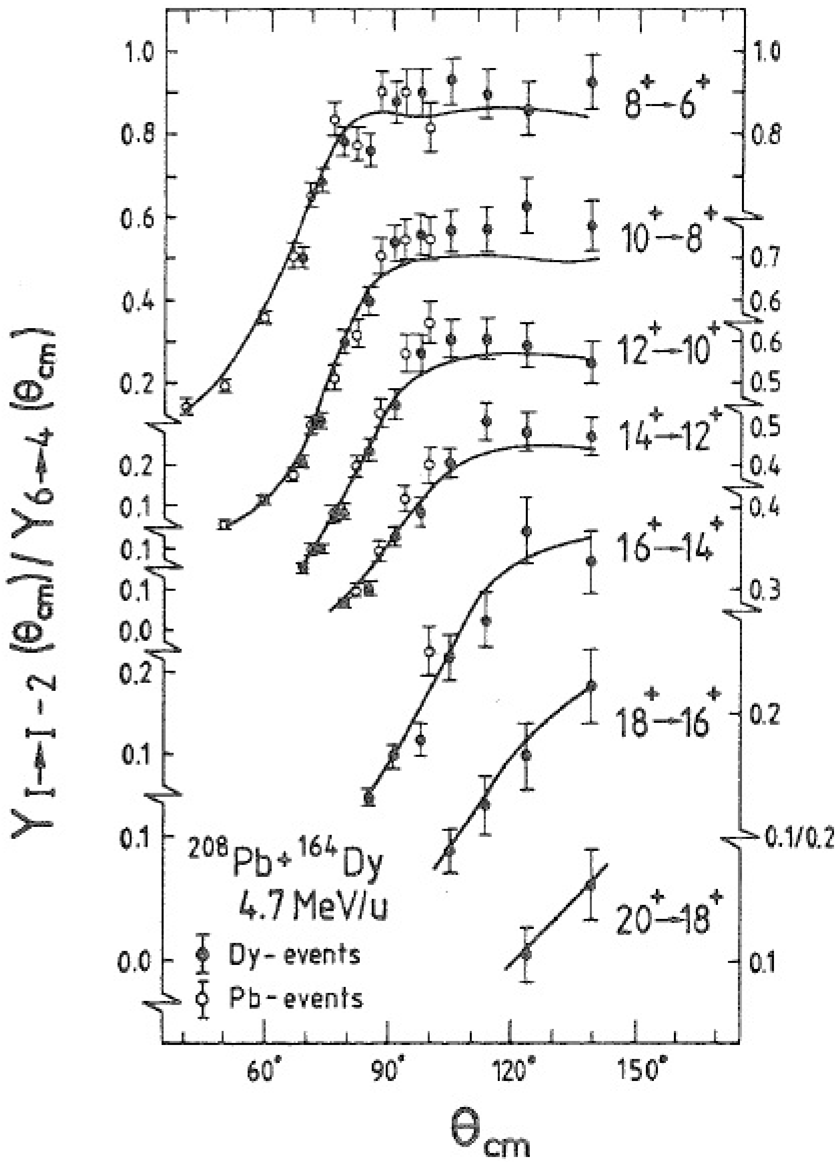
# Doppler shift correction $^{208}$Pb + $^{164}$Dy at 978 MeV

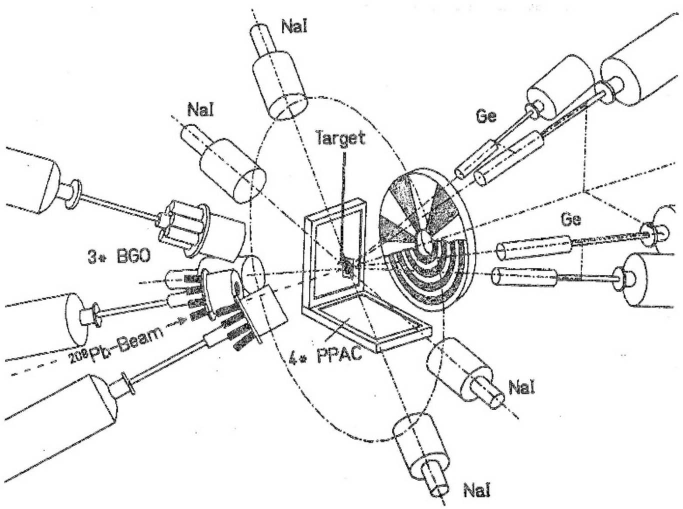


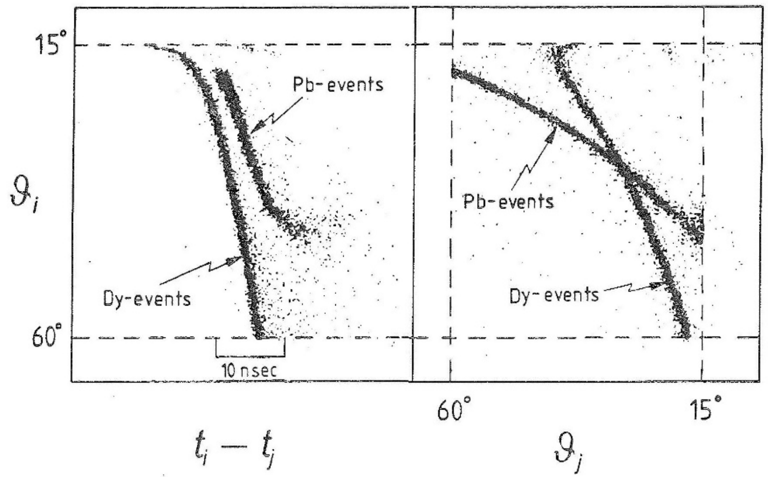



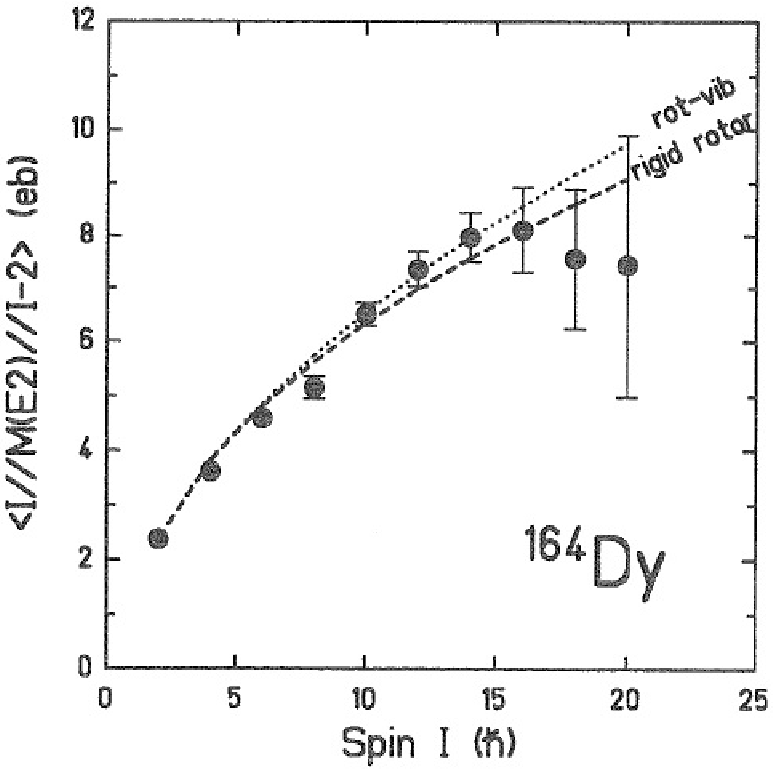
# Doppler shift correction $^{208}Pb + ^{164}Dy$ at 978 MeV

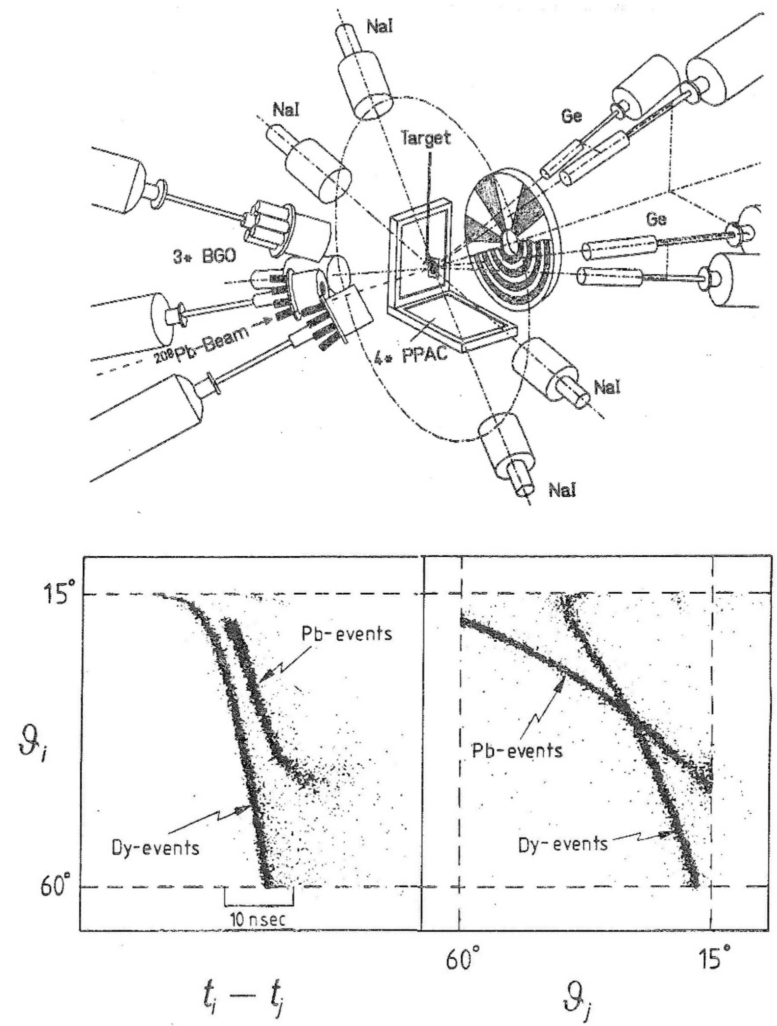


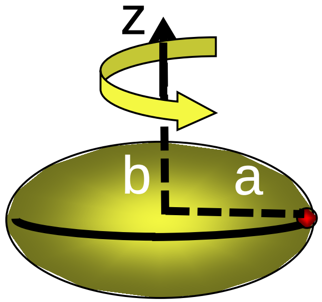
# Doppler shift correction $^{208}Pb + ^{164}Dy$ at 978 MeV




B(E2)-values in good agreement with the rigid rotor model

# Deformed nuclei collective rotation and nucleon pairing




$$R(\theta,\phi) = R_0 \cdot [1 + \beta \cdot Y_{20}(\theta,\phi)] \qquad \beta = \frac{4}{3}\sqrt{\frac{\pi}{5}}\frac{\Delta R}{\bar{R}} \qquad \Delta R = a - b \qquad \bar{R} = \frac{a+b}{2}$$

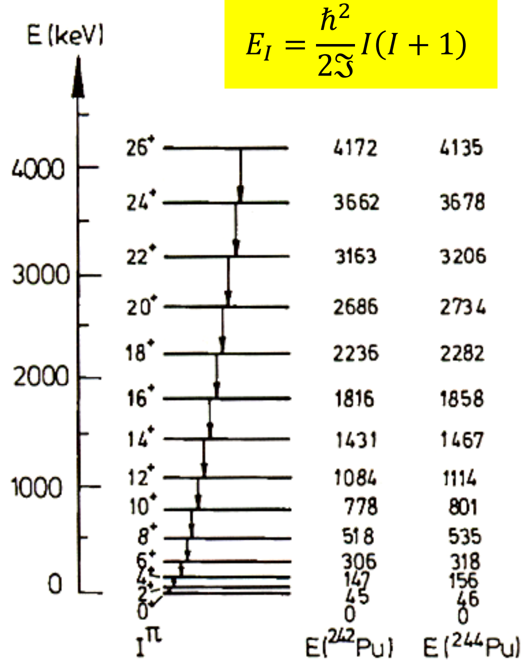
$$E_\gamma = E_I - E_{I-2} = \frac{\hbar^2}{2\Im}(4I-2)$$

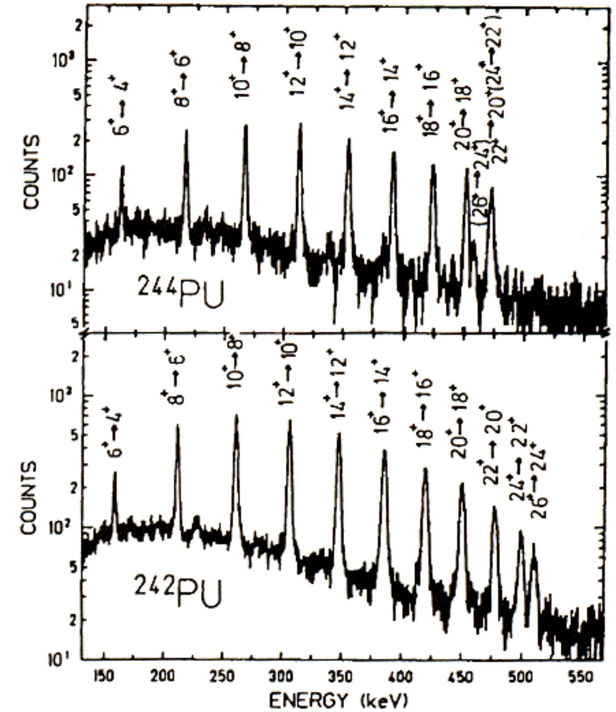
$$B(E2; I \to I-2) = \frac{15}{32\pi}\frac{I(I+1)}{(2I-1)(2I+1)} \cdot Q_2$$

$$E_I = \frac{\hbar^2}{2\Im} I(I+1)$$

$$Q_2 = \frac{3ZR_0^2}{\sqrt{5\pi}} \cdot \beta$$

$$\Im = \frac{2}{5} A \cdot M \cdot R_0^2 \cdot \beta^2$$

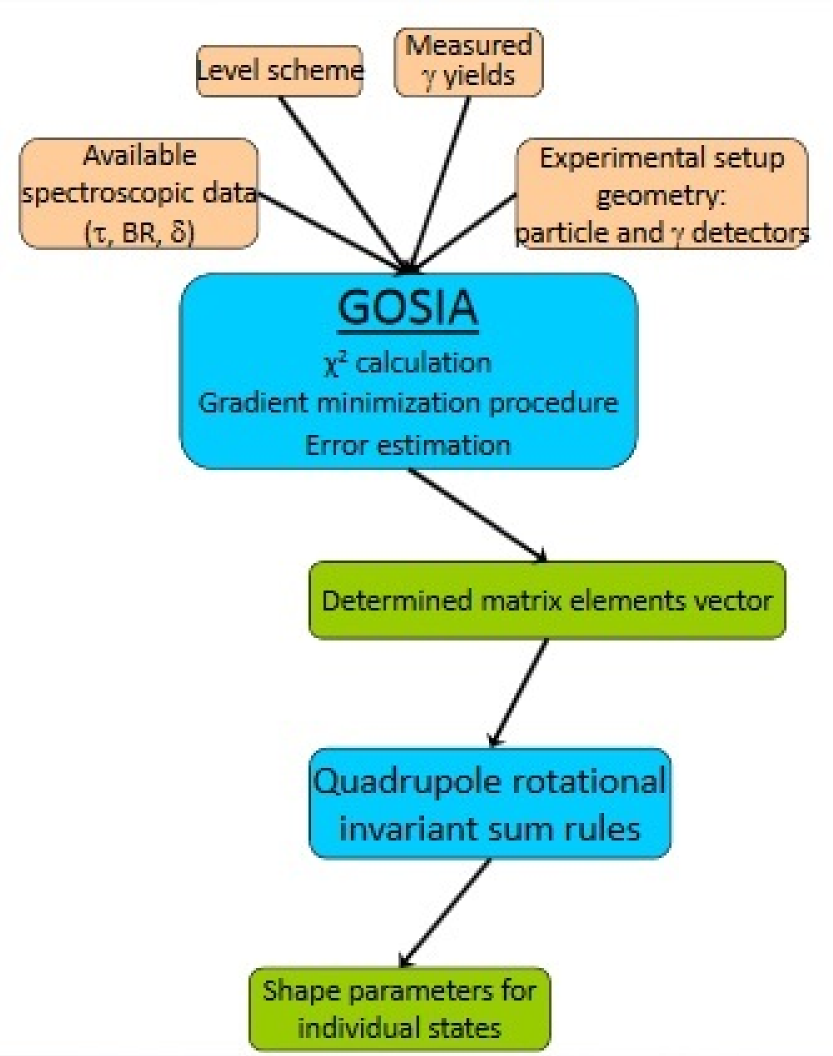


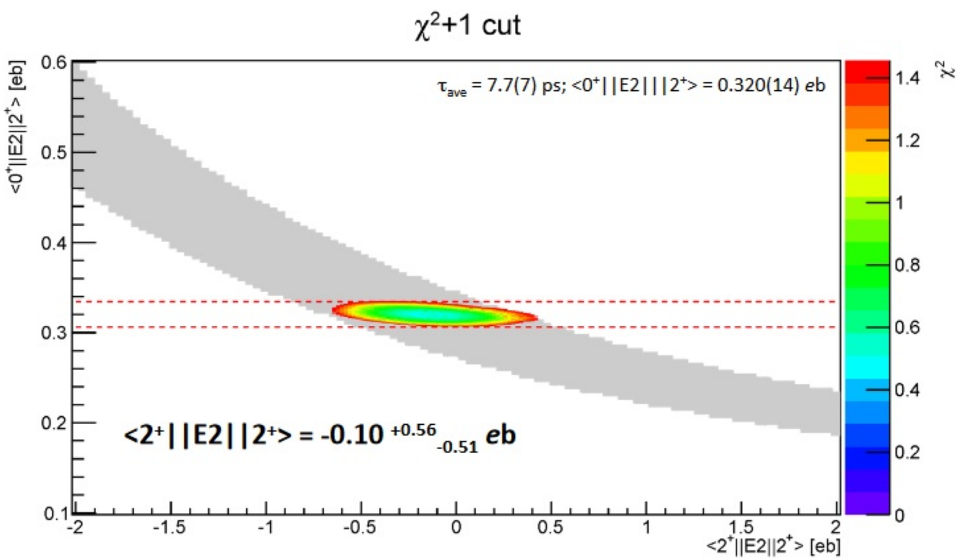

❖ analysis with GOSIA code

W. Spreng et al., Phys. Rev. Lett. 51 (1983), 1522

# GOSIA code

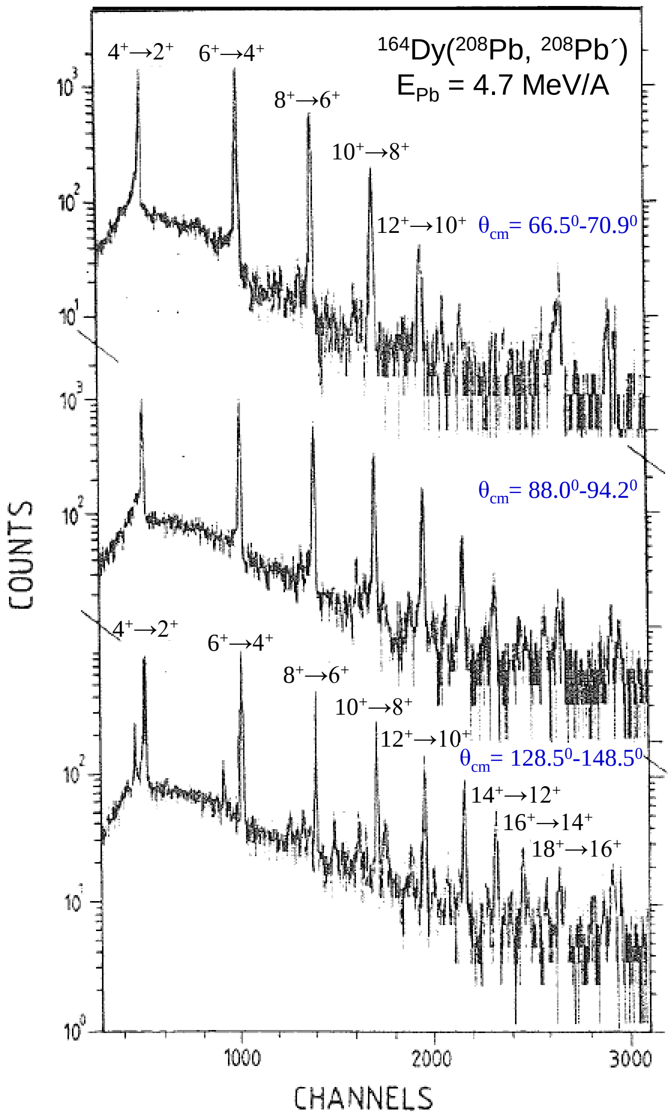


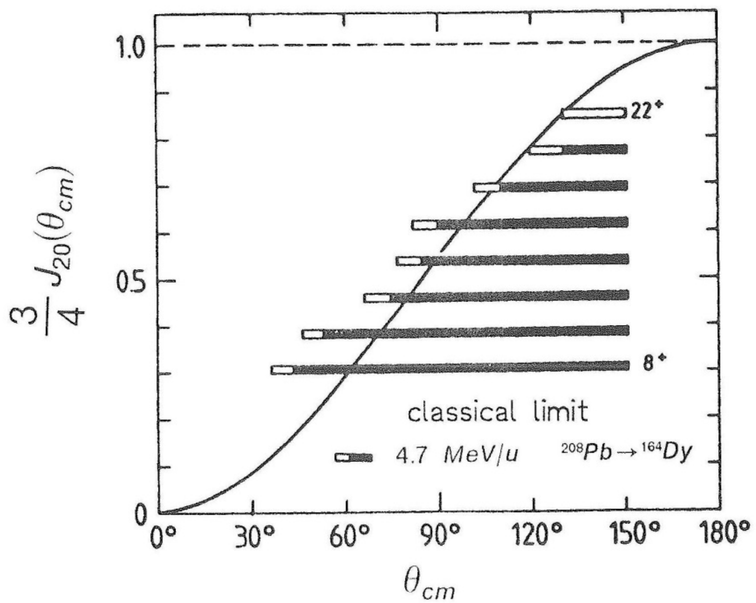
Level scheme
Measured γ yields
Available spectroscopic data (τ, BR, δ)
Experimental setup geometry: particle and γ detectors
GOSIA
χ² calculation
Gradient minimization procedure
Error estimation
Determined matrix elements vector
Quadrupole rotational invariant sum rules
Shape parameters for individual states

χ²+1 cut
τave = 7.7(7) ps; <0+||E2||2+> = 0.320(14) eb
<2+||E2||2+> = -0.10 +0.56 -0.51 eb
<0+||E2||2+> [eb]
<2+||E2||2+> [eb]
χ²

# Coulomb excitation angular momentum transfer



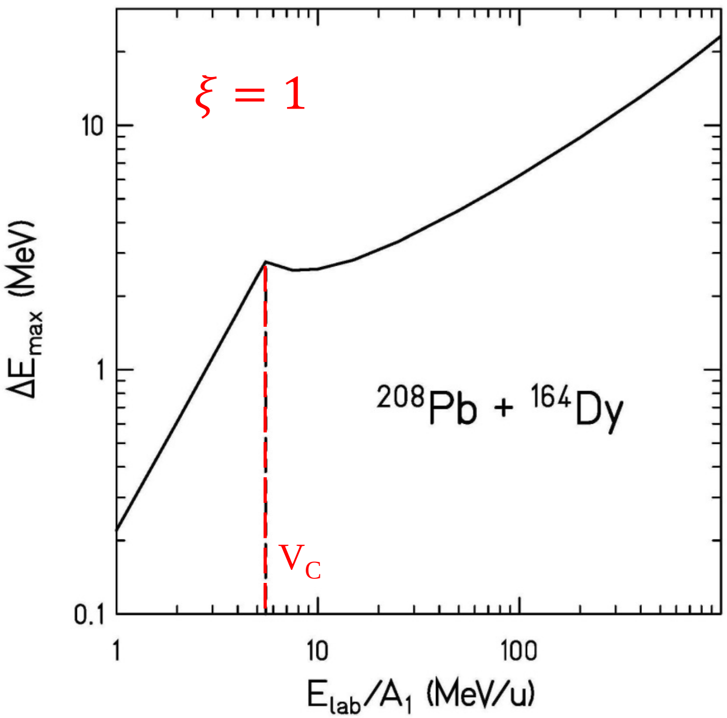




$$\Delta L_{max} = \frac{3}{2} \cdot J_{20}(\theta_{cm}) \cdot q$$

$$q = \frac{Z_p \cdot e^2 \cdot Q_0}{4 \cdot \hbar \cdot v \cdot a^2}$$

$$J_{20}(\theta_{cm}) = sin^2\frac{\theta_{cm}}{2} + tan^2\frac{\theta_{cm}}{2}\left[1 - \frac{\pi - \theta_{cm}}{2} tan\frac{\theta_{cm}}{2}\right]$$

$$J_{20}(\theta_{cm}) \cong \frac{2}{3}(1 - cos\theta_{cm})$$




ξ measures suddenness of interaction

$$\xi(\theta_{cm}) = \frac{\Delta E_{exc}}{\hbar \cdot c} \cdot \frac{D - a}{\gamma \cdot \beta}$$

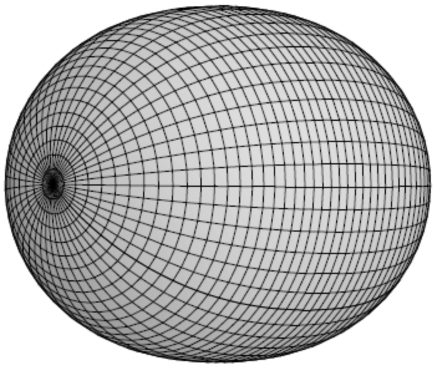
"adiabatic limit" for (single step) excitation **ξ = 1**

**maximum energy transfer:** $\Delta E_{exc} = \hbar \cdot c \cdot \dfrac{\beta \cdot \gamma}{D - a}$

# Shape parameterization

$$R(\theta,\phi) = R_0 \cdot \left[ 1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu} \cdot Y_{\lambda\mu}(\theta,\phi) \right]$$

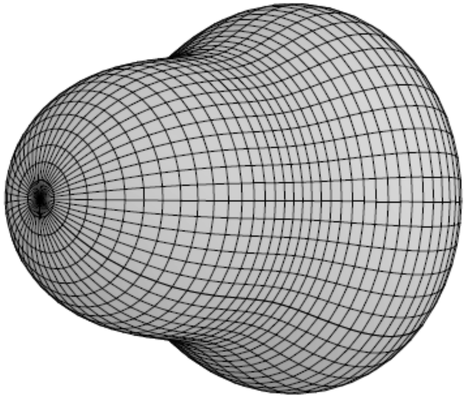
axially symmetric quadrupole



**λ=2**

$\alpha_{20} \neq 0$, $\alpha_{2\pm1} = \alpha_{2\pm2} = 0$

axially symmetric octupole

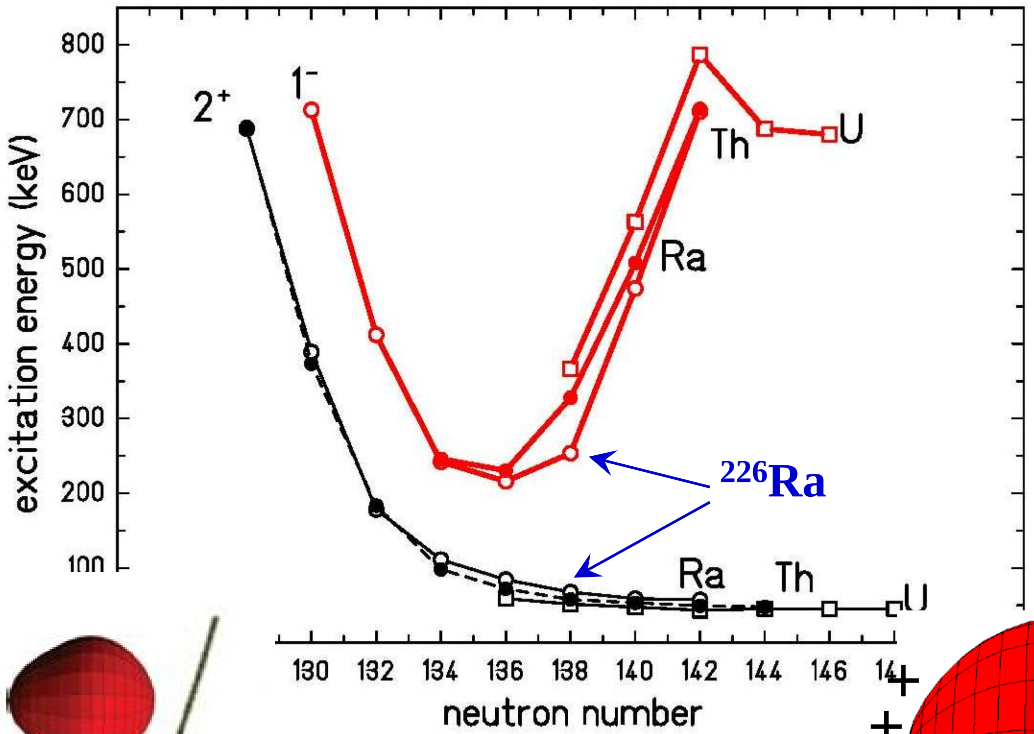


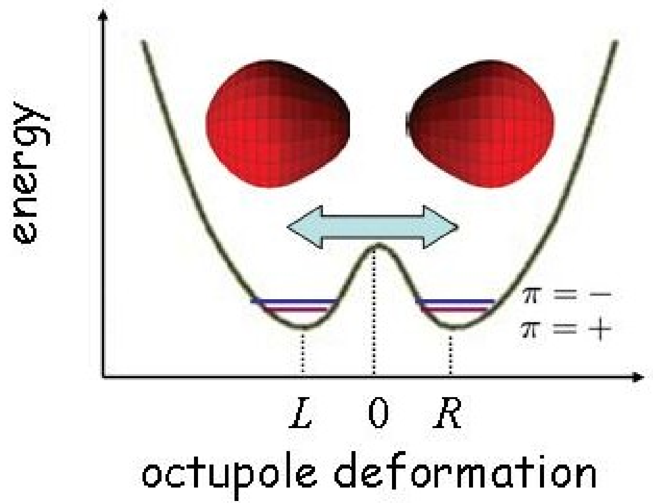
**λ=3**

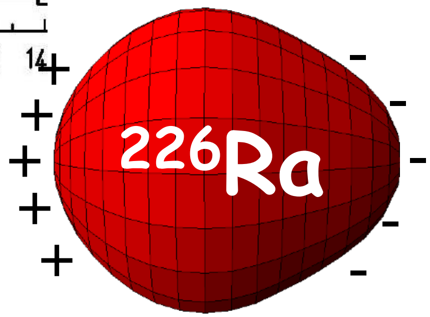
$\alpha_{30} \neq 0$, $\alpha_{3\pm1,2,3} = 0$

$\alpha_{20} \neq 0$, $\alpha_{2\pm1,2} = 0$

# Octupole collectivity

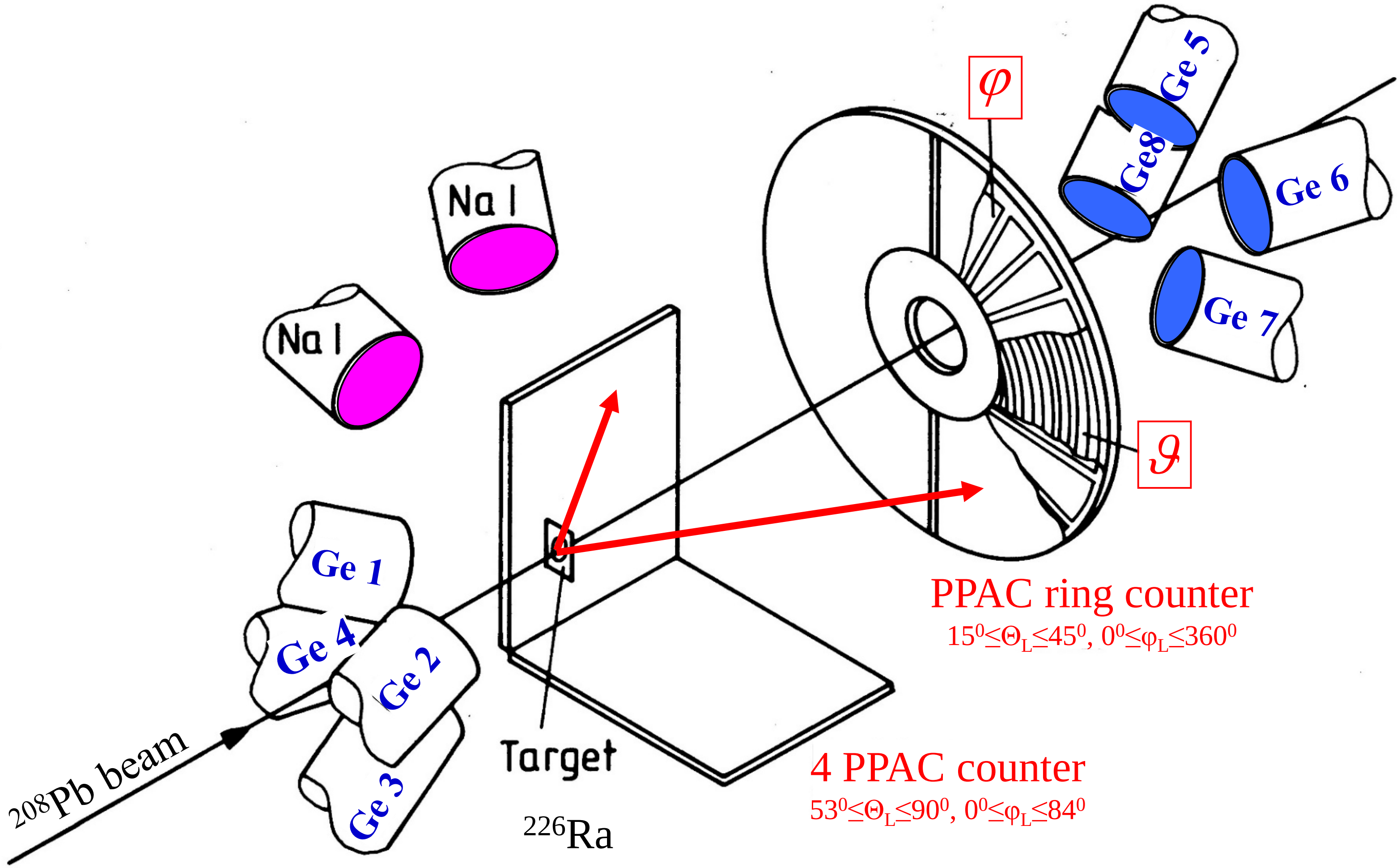


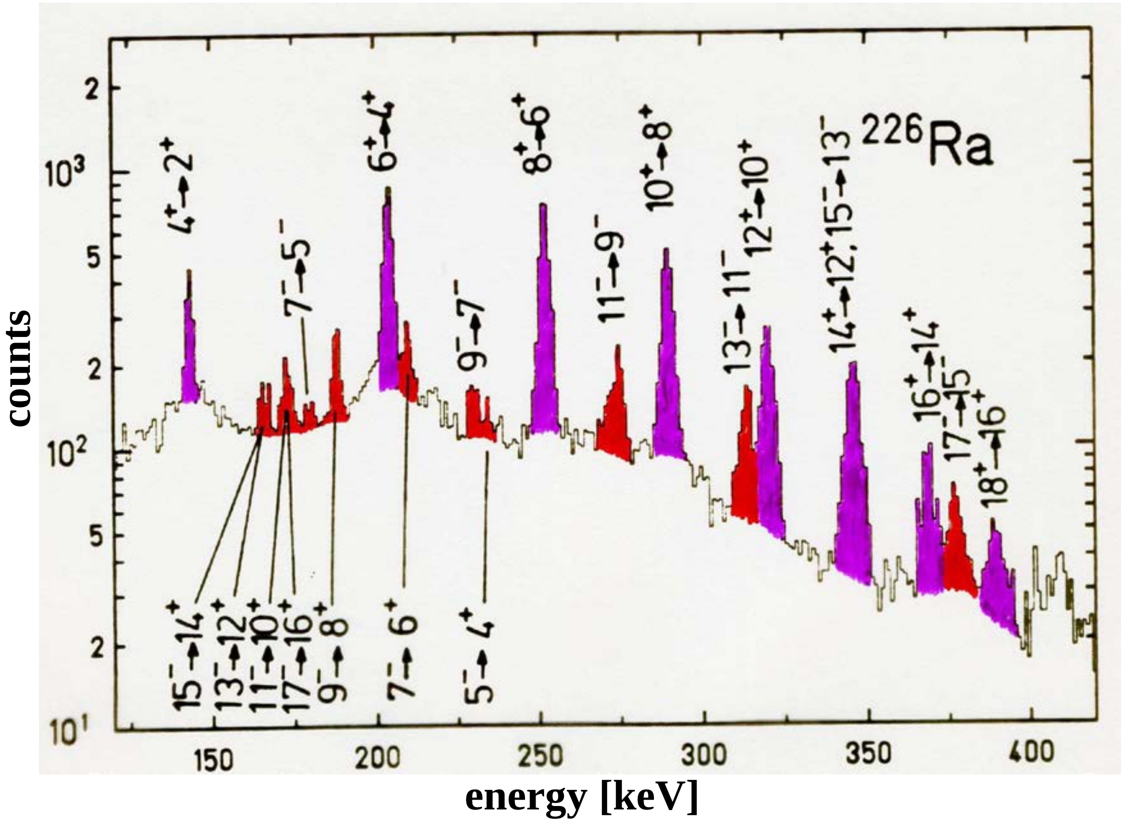

In an **octupole** deformed nucleus the center of mass and center of charge tend to separate, creating a non-zero **electric dipole moment**.

# Experimental set-up




$^{226}RaBr_2$ (400 μg/cm²) on C-backing (50 μg/cm²) and covered by Be (40 μg/cm²)

# γ-ray spectrum of $^{226}Ra$



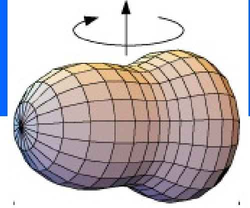

$^{208}$**Pb** → $^{226}$**Ra**

$E_{lab}$ = 4.7 AMeV

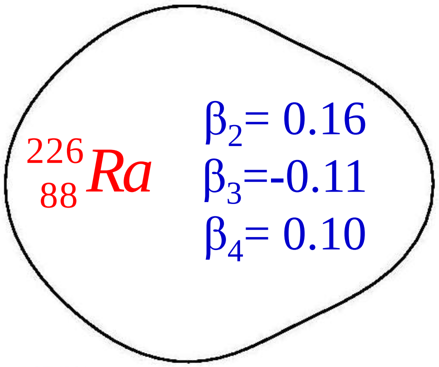
$15^0 \leq \theta_{lab} \leq 45^0$

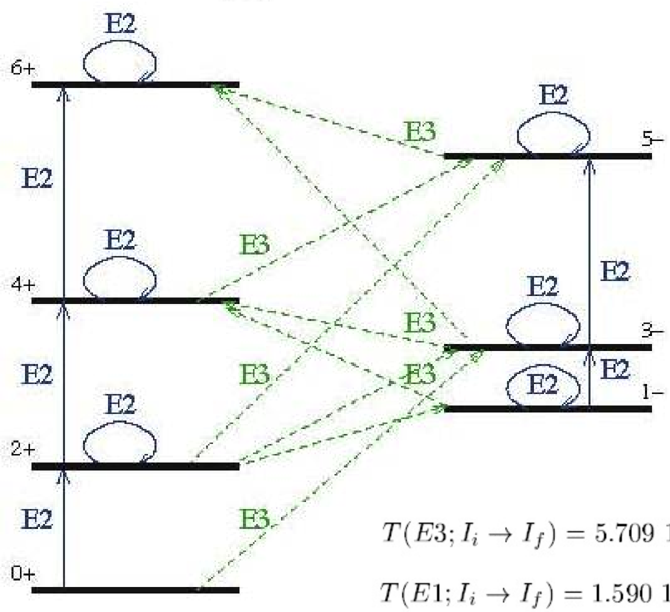
$0^0 \leq \varphi_{lab} \leq 360^0$

# Signature of an octupole deformed nucleus



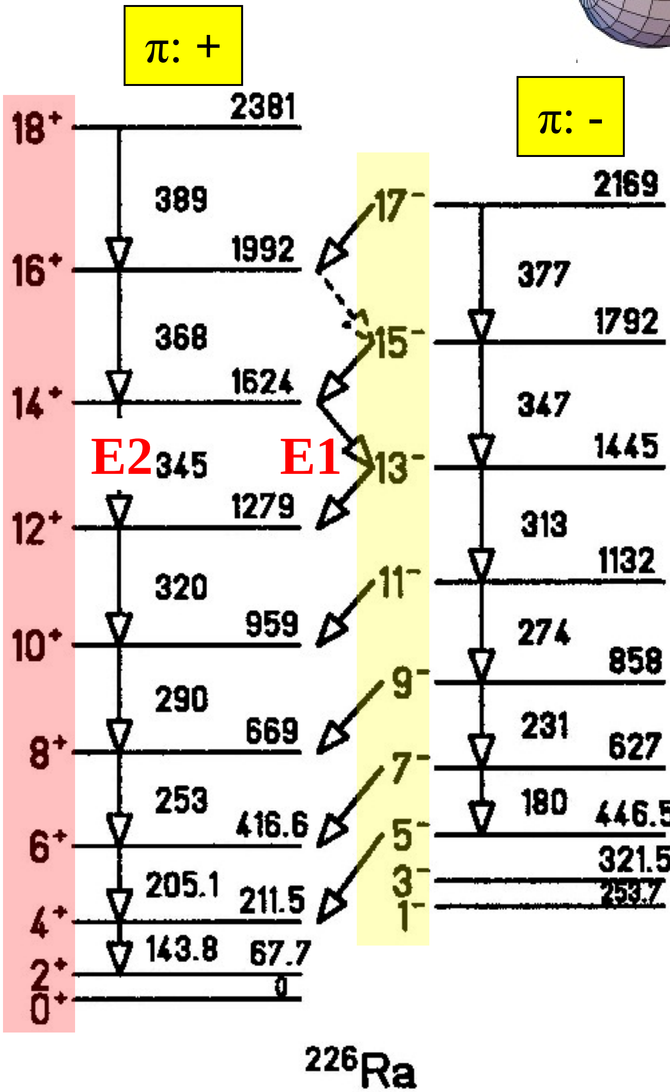
$$R(\theta) = R_0 \cdot \left[1 + \beta_2 \cdot Y_{20}(\theta) + \beta_3 \cdot Y_{30}(\theta) + \beta_4 \cdot Y_{40}(\theta)\right]$$



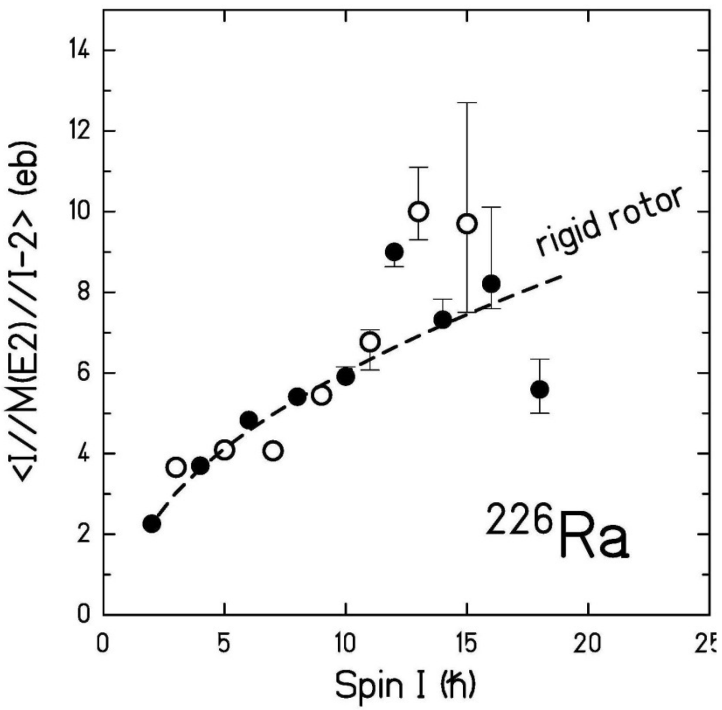




$$T(E3; I_i \to I_f) = 5.709\ 10^8\ E_\gamma^7\ B(E3; I_i \to I_f)$$

$$T(E1; I_i \to I_f) = 1.590\ 10^{17}\ E_\gamma^3\ B(E1; I_i \to I_f)$$



# Electric transition quadrupole moments in $^{226}$Ra




○ negative parity states
● positive parity states

**rigid rotor model:**

$$\langle I-2 \| M(E2) \| I \rangle = \sqrt{\frac{15}{32\cdot\pi}} \cdot \sqrt{\frac{I\cdot(I-1)}{2I-1}} \cdot Q_2 \cdot e$$

**liquid drop:**

$$Q_2 = \frac{3\cdot Z\cdot R_0^2}{\sqrt{5\cdot\pi}} \cdot (\beta_2 + 0.360\beta_2^2 + 0.336\beta_3^2 + 0.328\beta_4^2 + 0.967\beta_2\beta_4)\ [fm^2]$$

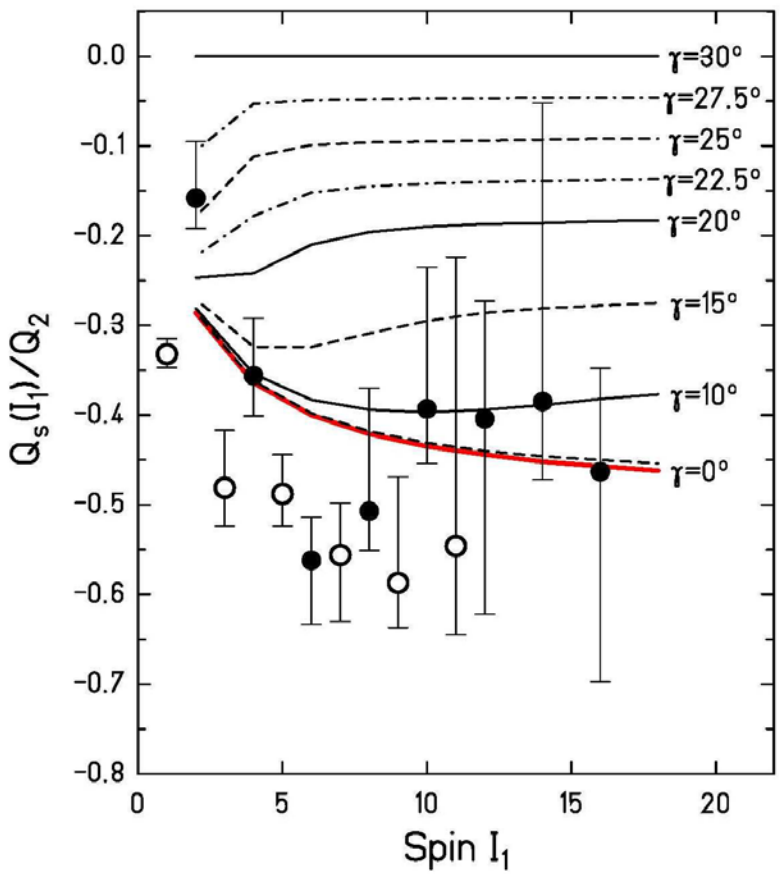
$Q_2(\mathrm{exp}) = 750\ \mathrm{fm}^2$

$Q_2(\mathrm{theo}) = 680\ \mathrm{fm}^2$

$\beta_2 = 0.21$

*H.J. Wollersheim et al.; Nucl. Phys. A556 (1993) 261* *W. Nazarewicz et al.; Nucl. Phys. A467 (1987) 437*

# Static quadrupole moments in $^{226}$Ra




○ negative parity states
● positive parity states

**rigid rotor model:**

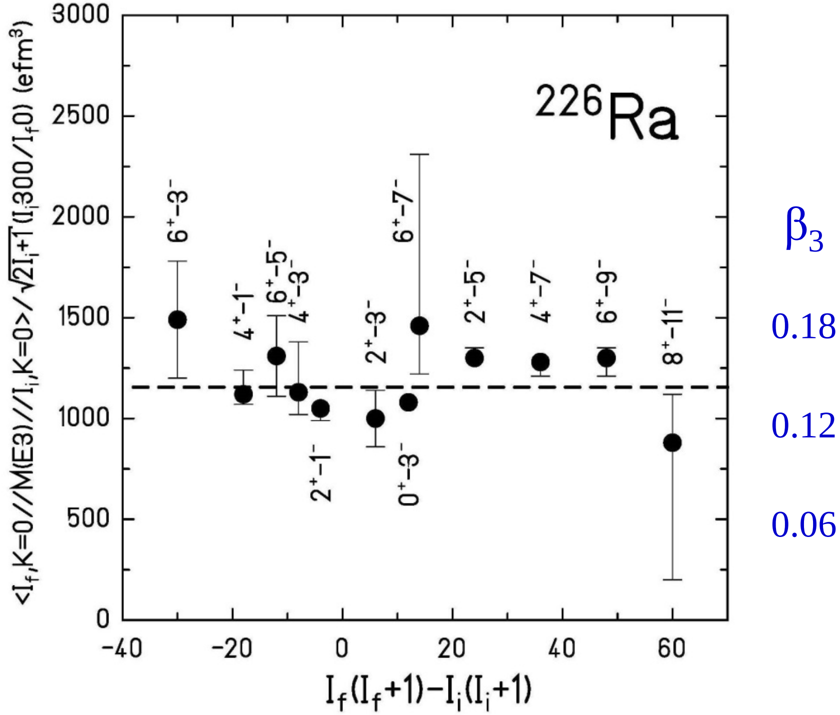
$$\frac{Q_S(I)}{Q_0} = \sqrt{\frac{I\cdot(2I-1)}{(I+1)\cdot(2I+1)\cdot(2I+3)}} \cdot \frac{\langle I\|M(E2)\|I\rangle}{\langle 2_1\|M(E2)\|0_1\rangle}$$

**rigid triaxial rotor model:**

$$\frac{Q_s(2_1)}{Q_0} = -\frac{6\cdot\cos(3\gamma)}{7\cdot\sqrt{9-8\cdot\sin^2(3\gamma)}}$$

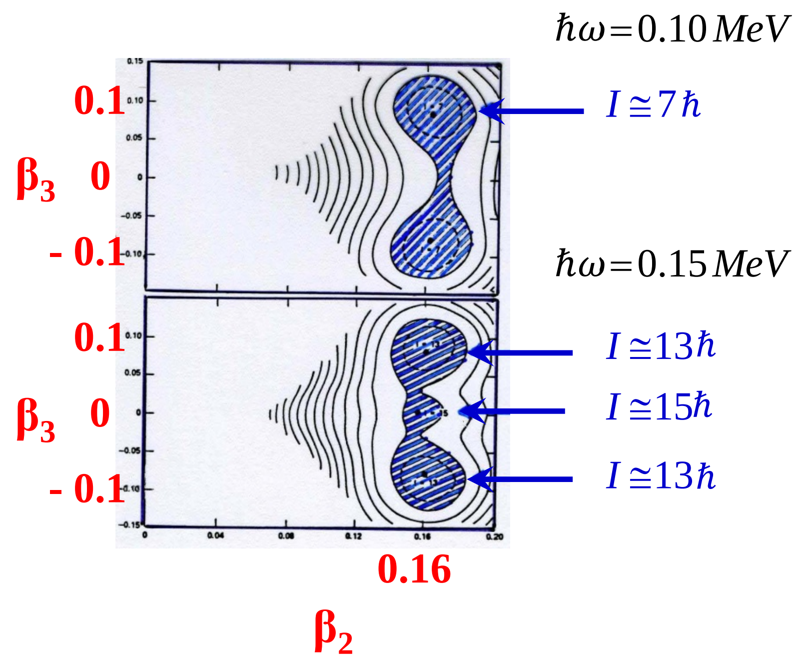
*H.J. Wollersheim et al.; Nucl. Phys. A556 (1993) 261* *Davydov and Filippov, Nucl. Phys. 8, 237 (1958)*

# Electric transition octupole moments in $^{226}$Ra




**liquid drop:**

$$Q_3 = \frac{3 \cdot Z \cdot R_0^3}{\sqrt{7 \cdot \pi}} \cdot \left(\beta_3 + 0.841 \beta_2 \beta_3 + 0.769 \beta_3 \beta_4\right) \left[fm^3\right]$$

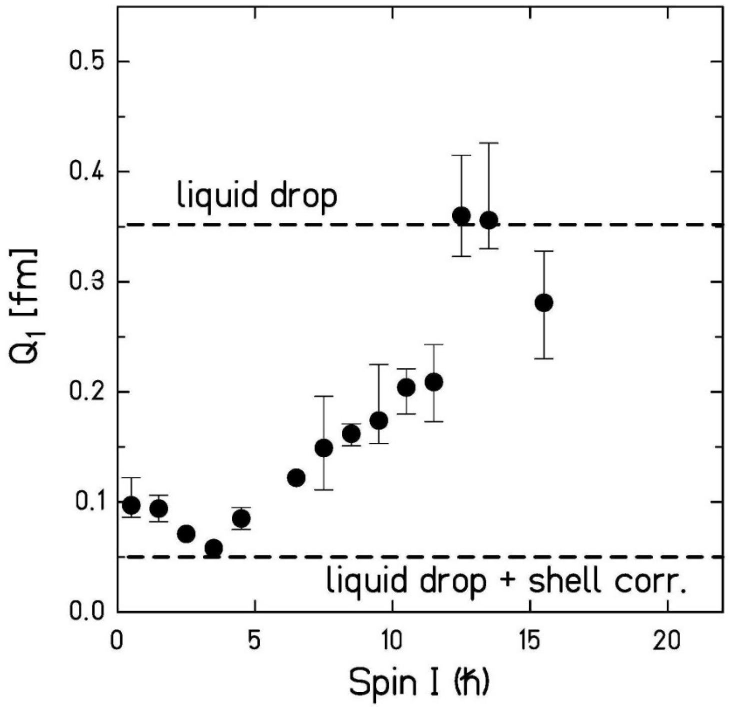


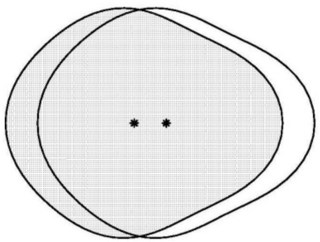

$$\langle I-3 \| M(E3) \| I \rangle = -\sqrt{\frac{35}{32\pi}} \cdot \sqrt{\frac{I \cdot (I-1) \cdot (I-2)}{(2I-3) \cdot (2I+3)}} \cdot Q_3 \cdot e$$

$$\langle I-1 \| M(E3) \| I \rangle = \sqrt{\frac{21}{32\pi}} \cdot \sqrt{\frac{(I-1) \cdot I \cdot (I+1)}{(2I-3) \cdot (2I+3)}} \cdot Q_3 \cdot e$$

*H.J. Wollersheim et al.; Nucl. Phys. A556 (1993) 261* *W. Nazarewicz et al.; Nucl. Phys. A467 (1987) 437*

# Intrinsic electric dipole moments in $^{226}$Ra



**liquid-drop contribution:**

$$Q_1^{LD} = C_{LD} \cdot A \cdot Z \cdot \left(\beta_2\beta_3 + 1.458 \cdot \beta_3\beta_4\right)$$

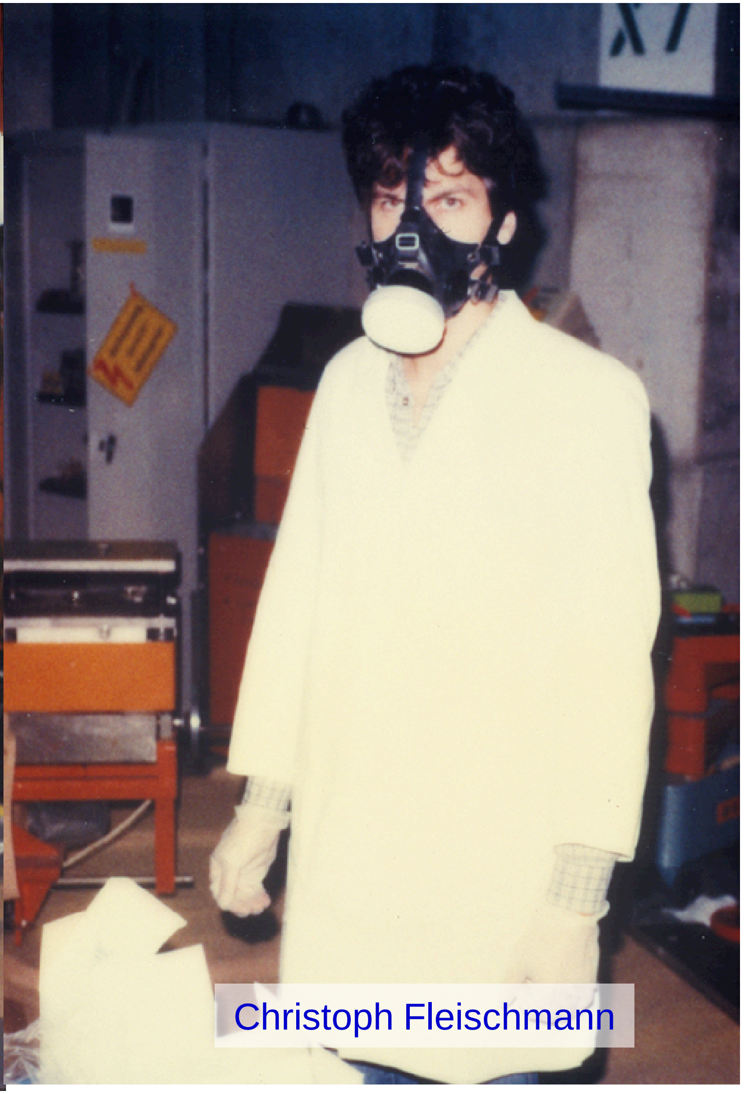
$$with\ \ C_{LD} = 5.2 \cdot 10^{-4}\ \ [fm]$$

**rigid rotor model:**

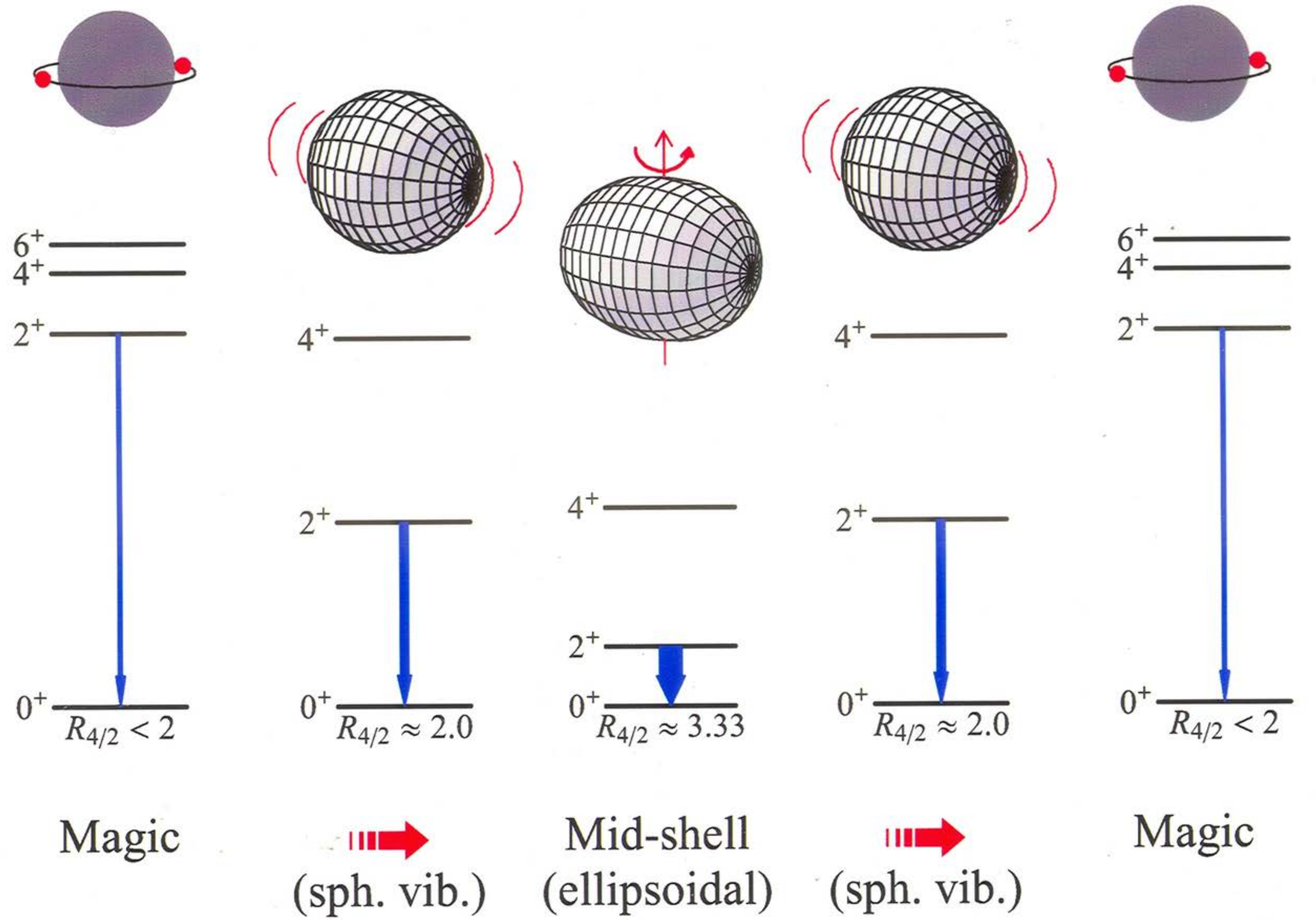
$$\left\langle I-1 \left\| M(E1) \right\| I \right\rangle = -\sqrt{\frac{3}{4\pi}} \cdot \sqrt{I} \cdot Q_1 \cdot e$$

*H.J. Wollersheim et al.; Nucl. Phys. A556 (1993) 261* *G. Leander et al.; Nucl. Phys. A453 (1986) 58*

# Coulomb excitation of $^{226}$Ra


*H.J. Wollersheim et al.; Nucl. Phys. A556 (1993) 261*



6+
4+
2+
0+
$R_{4/2} < 2$
4+
2+
0+
$R_{4/2} \approx 2.0$
4+
2+
0+
$R_{4/2} \approx 3.33$
4+
2+
0+
$R_{4/2} \approx 2.0$
6+
4+
2+
0+
$R_{4/2} < 2$
Magic
(sph. vib.)
Mid-shell
(ellipsoidal)
(sph. vib.)
Magic