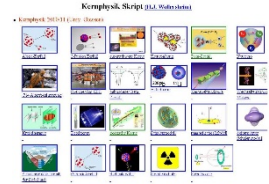


# Outline: Peripheral collisions

Lecturer: Hans-Jürgen Wollersheim

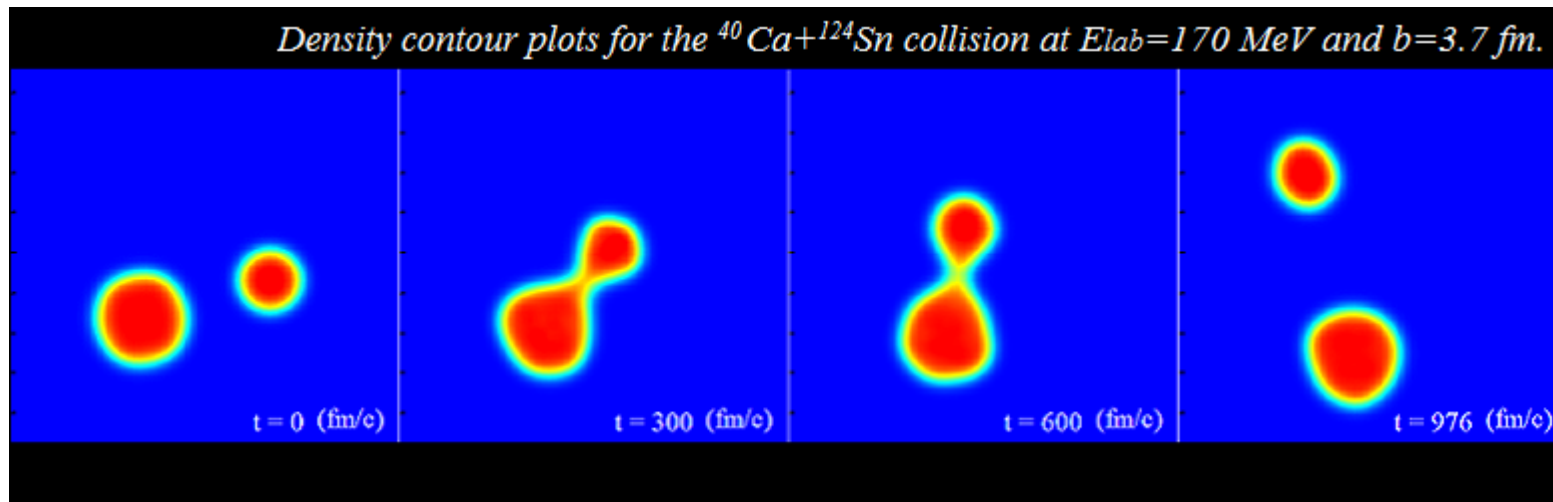
e-mail: [h.j.wollersheim@gsi.de](mailto:h.j.wollersheim@gsi.de)

web-page: <https://web-docs.gsi.de/~wolle/> and click on



1. reaction Q-value
2. sub-barrier transfer reactions
3. transfer probabilities for multi-nucleon transfer
4. PRISMA and HIRA spectrometers
5. transfer reactions with weakly bound nuclei

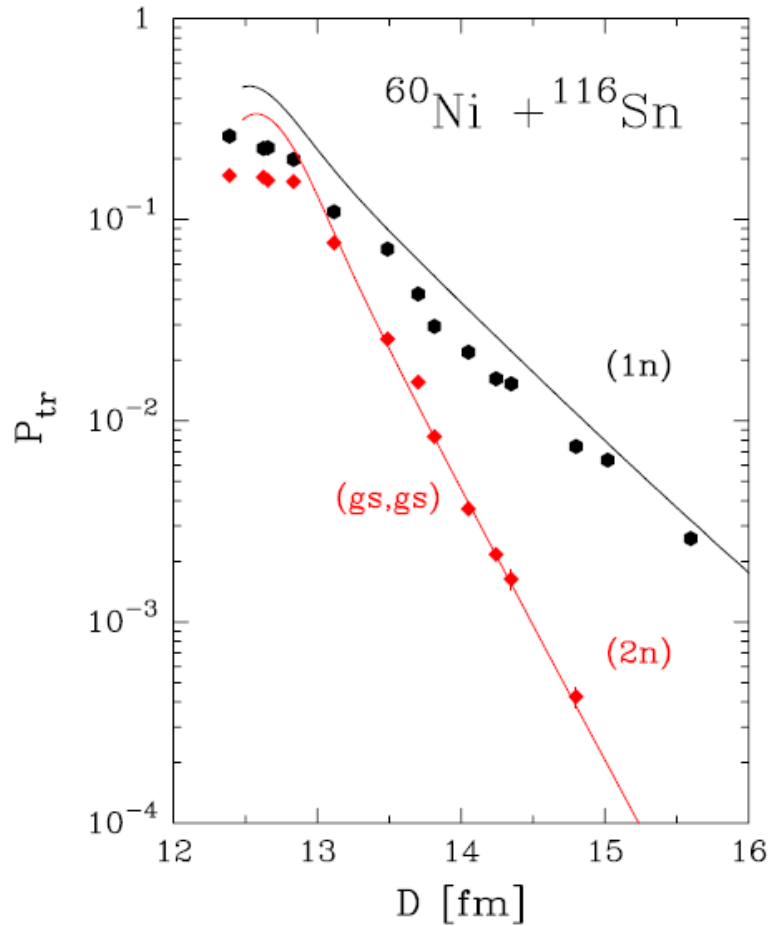
# Peripheral collisions



- ❖ probing single particle aspects and nucleon-nucleon correlations
- ❖ transition from quasi elastic to deep inelastic processes
- ❖ connection with other reaction channels (near and sub-barrier fusion)
- ❖ population of neutron-rich nuclei

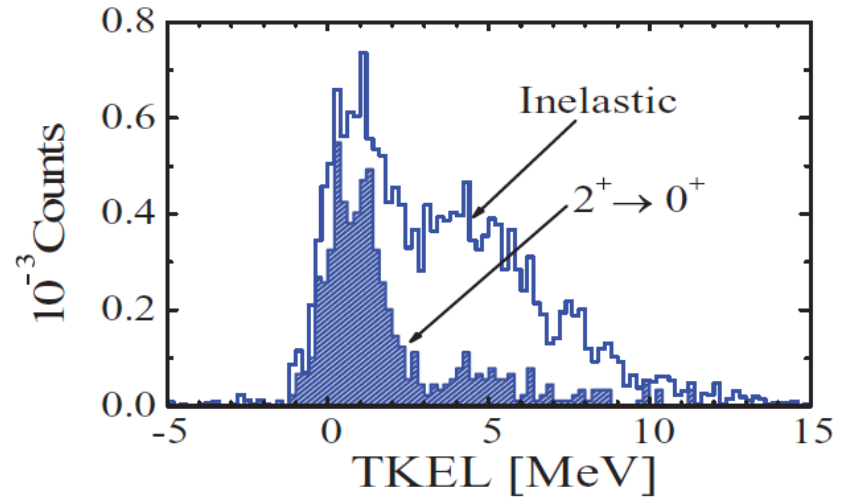
# Peripheral collisions

Sub-barrier transfer reactions  
study of nucleon-nucleon correlations



$C_p = 4.2$  fm,  $C_t = 5.5$  fm,  $R_{int} = 12.7$  fm

Multi-nucleon transfer  
study of secondary processes



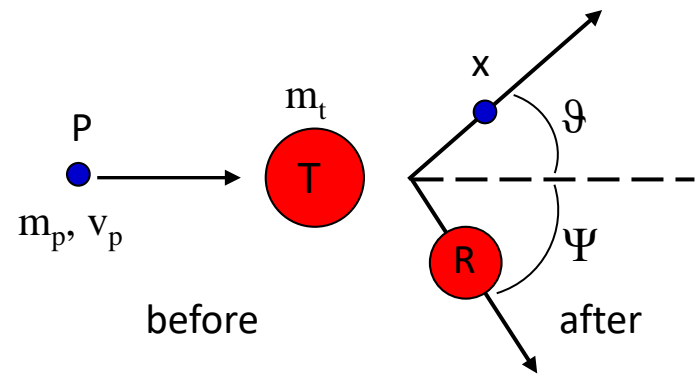
# Reaction Q-value

Consider the  $T(p,x)R$  reaction:

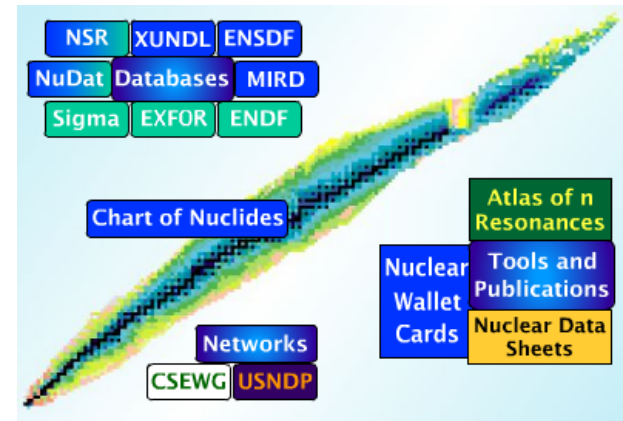
The *Q-value* of the reaction is defined as the difference in mass energies of the products and reactants, i.e.

$$Q_{gg} = [m_p + m_t - (m_x + m_R)] \cdot c^2$$

if  $Q$  is positive, the reaction is **exoergic** while if  $Q$  is negative, the reaction is **endoergic**.



<https://www.nndc.bnl.gov/qcalc/>



$$m_p c^2 + T_p + m_t c^2 = m_x c^2 + T_x + m_R c^2 + T_R$$

$$Q_{gg} = [m_p + m_t - m_x - m_R] c^2 = T_x + T_R - T_p$$

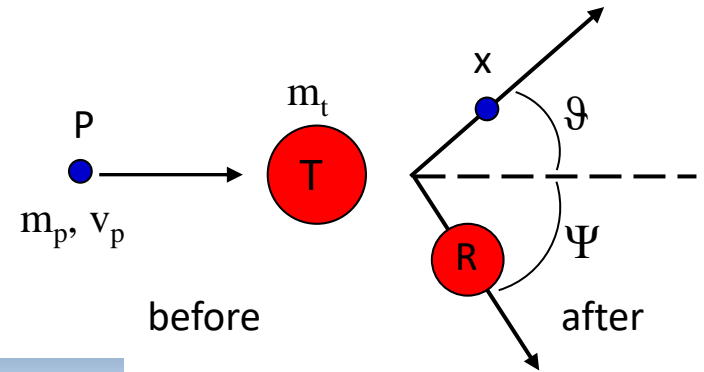
# Reaction Q-value

Consider the  $T(p,x)R$  reaction:

The  $Q$ -value of the reaction is defined as the difference in mass energies of the products and reactants, i.e.

$$Q_{gg} = [m_p + m_t - (m_x + m_R)] \cdot c^2$$

if  $Q$  is positive, the reaction is **exoergic** while is  $Q$  is negative, the reaction is **endoergic**.



The screenshot shows a web browser window with the URL <http://nuclear.lu.se/database/masses/> and a tab titled "Q-value calculator". The interface includes a menu bar with "Datei", "Bearbeiten", "Ansicht", "Favoriten", and "Extras". Below the menu, there are buttons for "Reaction calculator", "?", and "About". The main content area displays the reaction  $^{116}\text{Sn} + ^{60}\text{Ni} \rightarrow ^{62}\text{Ni} + ^{114}\text{Sn}$ . Below the reaction, the binding energies (B.E.) for the nuclei are listed: B.E.(1) =  $988680.497 \pm 2.986$  keV, B.E.(2) =  $526841.574 \pm 1.420$  keV, B.E.(3) =  $545258.806 \pm 1.436$  keV, and B.E.(4) =  $971571.272 \pm 3.165$  keV. The calculated Q-value is **1308.007 keV**, and the uncertainty is 4.797 keV (ignoring correlations). The threshold is 0 keV. On the left side, there is a table for inputting reaction parameters:

	A	Symb.	Z
Projectile	116	Sn	50
Target	60	Ni	28
Ejectile	62	Ni	28
( Product	114	Sn	50

Buttons for "Check", "Calculate", and "Reset" are also visible.

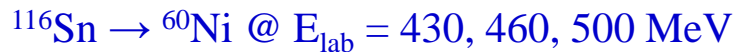
# Reaction Q-value neutron transfer

Consider the T(p,x)R reaction:

The *Q-value* of the reaction is defined as the difference in mass energies of the products and reactants, i.e.

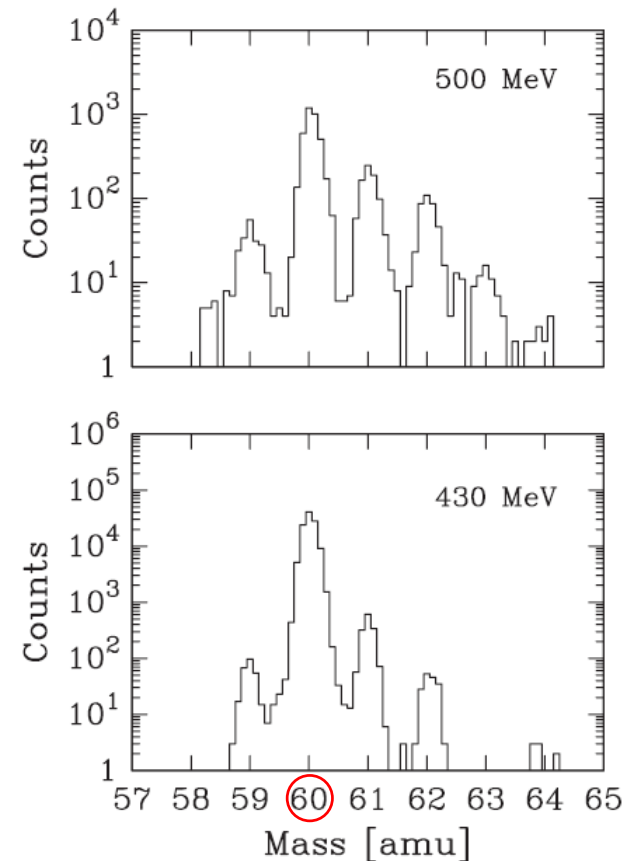
$$Q_{gg} = [m_p + m_t - (m_x + m_R)] \cdot c^2$$

if Q is positive, the reaction is **exoergic** while is Q is negative, the reaction is **endoergic**.



$$E_{\text{cm}} = 147, 156, 170 \text{ MeV}$$

$$V_C(R_{\text{int}}) = 159 \text{ MeV}$$



$(^{60}\text{Ni}, ^{58}\text{Ni})$ -2n	$(^{60}\text{Ni}, ^{59}\text{Ni})$ -1n	$(^{60}\text{Ni}, ^{60}\text{Ni})$ 0n	$(^{60}\text{Ni}, ^{61}\text{Ni})$ +1n	$(^{60}\text{Ni}, ^{62}\text{Ni})$ +2n	$(^{60}\text{Ni}, ^{63}\text{Ni})$ +3n	$(^{60}\text{Ni}, ^{64}\text{Ni})$ +4n
-4.12 MeV	-4.44 MeV	0 MeV	-1.74 MeV	+1.31 MeV	-2.15 MeV	-0.24 MeV

# Reaction Q-value proton transfer

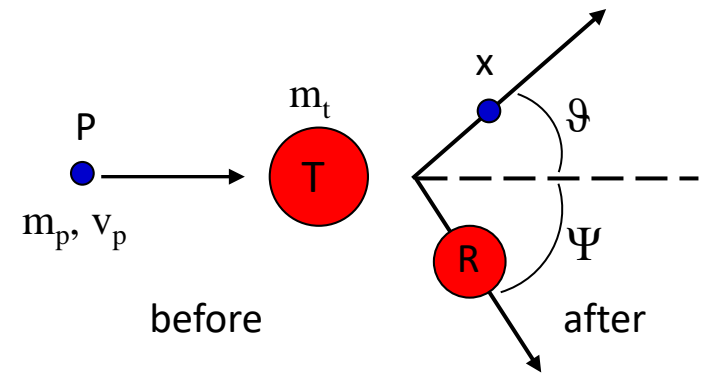
Consider the  $T(p,x)R$  reaction:

The *Q-value* of the reaction is defined as the difference in mass energies of the products and reactants, i.e.

$$Q_{gg} = [m_p + m_t - (m_x + m_R)] \cdot c^2$$

if  $Q$  is positive, the reaction is **exoergic** while is  $Q$  is negative, the reaction is **endoergic**.

The  $Q$ -value of the reaction will change for **proton transfer** due to the rearrangement of nuclear charge.



$$Q_{opt} = Q_{gg} - E^* = Q_{gg} - e^2 \left[ \frac{Z_p Z_t}{r_i} - \frac{(Z_p - z)(Z_t + z)}{r_f} \right]$$

$$Q_{opt} = Q_{gg} - \frac{Z_p Z_t e^2}{r_i} \cdot \left[ 1 - \frac{(Z_p - z)(Z_t + z) r_i}{Z_p Z_t r_f} \right]$$

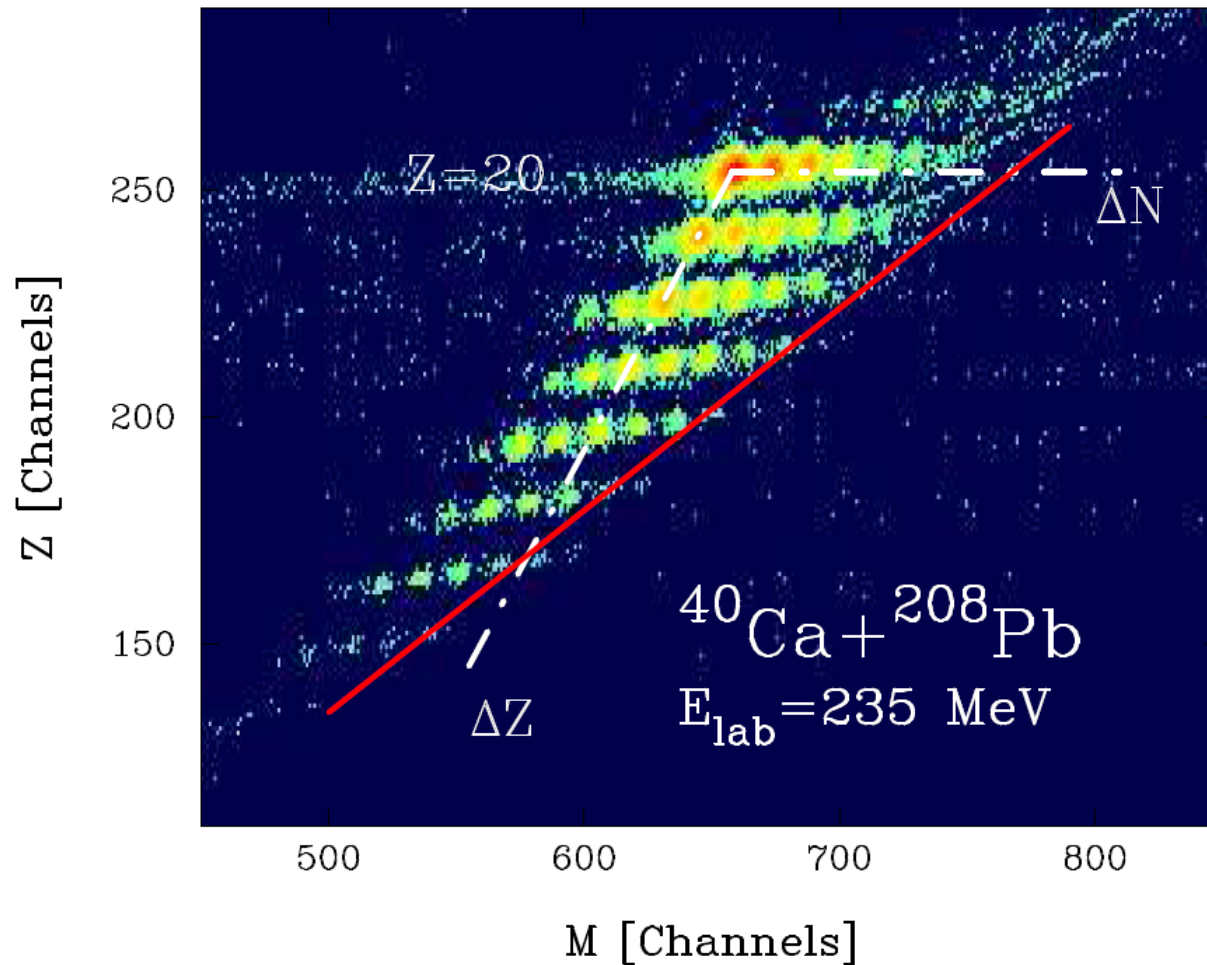
$$r_i = D = \frac{0.72 \cdot Z_1 Z_2}{E_{cm}} \left[ \sin^{-1} \frac{\theta_{cm}}{2} + 1 \right]$$

$$Q_{opt} = Q_{gg} - \frac{2E_{cm}}{\left[ \sin^{-1} \frac{\theta_{cm}}{2} + 1 \right]} \cdot \left[ 1 - \frac{(Z_p - z)(Z_t + z) r_i}{Z_p Z_t r_f} \right]$$

$$Q_{opt} \approx Q_{gg} - E_{cm} \cdot \left[ 1 - \frac{(Z_p - z)(Z_t + z)}{Z_p Z_t} \right]$$

# Reaction Q-value

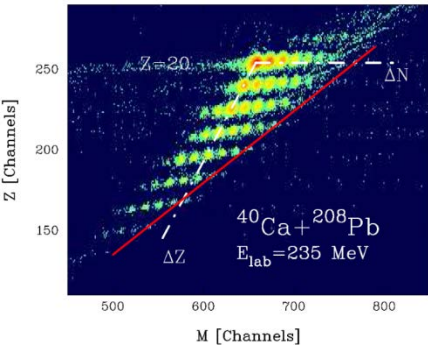
The population in the  $(N,Z)$  plane is governed by  $Q_{\text{opt}}$



$$E_{\text{cm}} = 197 \text{ MeV} \quad V_C(R_{\text{int}}) = 178 \text{ MeV}$$



# Reaction Q-value



The population in the  $(N,Z)$  plane is governed by  $Q_{\text{opt}}$

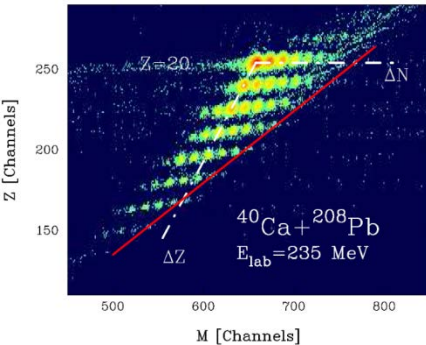
	$E^*$
$^{22}\text{Ti}$	-14.4
$^{21}\text{Sc}$	-7.3
$^{20}\text{Ca}$	0
$^{19}\text{K}$	+7.6
$^{18}\text{Ar}$	+15.4
$^{17}\text{Cl}$	+23.4
$^{16}\text{S}$	+31.7
$^{15}\text{P}$	+40.3
$^{14}\text{Si}$	+49.0

$$E_{\text{cm}} \cdot [1 - V_{\text{C}}(f)/V_{\text{C}}(i)] \text{ (MeV)}$$

$$Q_{\text{gg}} \text{ (MeV)}$$

$$[V_{\text{C}}(i) - V_{\text{C}}(f)] \text{ (MeV)}$$

# Reaction Q-value



The population in the  $(N, Z)$  plane is governed by  $Q_{\text{opt}}$

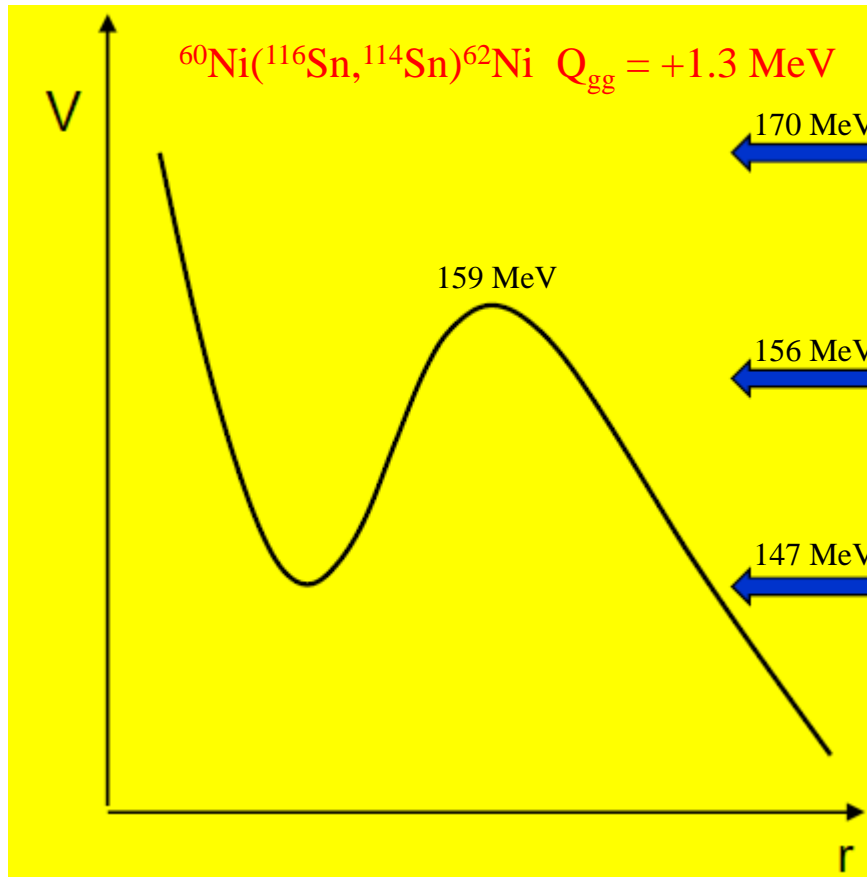
	$E^*$	-2n	-1n	<b>0n</b>	1n	2n	3n	4n	5n	6n	7n	8n
$^{22}\text{Ti}$	-14.4	-47.7	-37.4	-23.0	-17.2	-6.3	-3.9	+3.5	+4.9	+10.5	+10.8	+15.4
$^{21}\text{Sc}$	-7.3	0.0	-25.9	-13.2	-8.3	-2.4	+0.1	+4.9	+6.1	+10.1	+10.3	+13.8
<b><math>^{20}\text{Ca}</math></b>	<b>0</b>	-20.4	-12.0	<b>0</b>	+1.3	+6.2	+6.4	+11.1	+10.3	+14.0	+12.8	+15.9
$^{19}\text{K}$	+7.6	-13.9	-6.7	+2.1	+2.6	+6.1	+5.9	+8.7	+7.8	+9.6	+7.9	+9.0
$^{18}\text{Ar}$	+15.4	-3.2	-0.1	+7.5	+6.6	+9.8	+7.8	+10.4	+7.5	+8.9	+5.5	+6.4
$^{17}\text{Cl}$	+23.4	-1.1	+1.6	+7.2	+5.9	+7.0	+4.6	+5.4	+2.6	+2.3	-2.7	-3.1
$^{16}\text{S}$	+31.7	+4.8	+5.3	+10.4	+7.0	+8.1	+3.9	+4.6	-0.5	-1.1	-7.0	-8.1
$^{15}\text{P}$	+40.3	+4.1	+3.8	+7.0	+2.6	+2.2	-2.9	-4.0	-9.2	-12.2	-18.5	-21.4
$^{14}\text{Si}$	+49.0	+6.6	+4.1	+6.2	+0.6	-0.6	-7.2	-9.4	-16.5	-19.4	-27.4	-30.4

$$E_{\text{cm}} \cdot [1 - V_{\text{C}}(\text{f})/V_{\text{C}}(\text{i})] \text{ (MeV)}$$

$$Q_{\text{gg}} - [V_{\text{C}}(\text{i}) - V_{\text{C}}(\text{f})] \text{ (MeV)}$$

# Sub-barrier transfer reactions

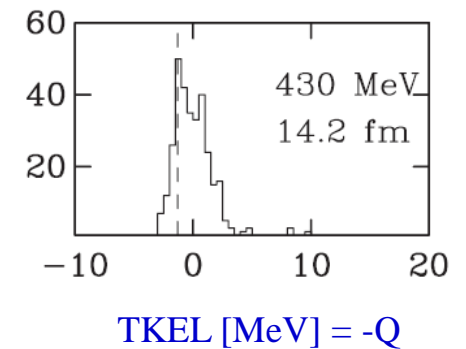
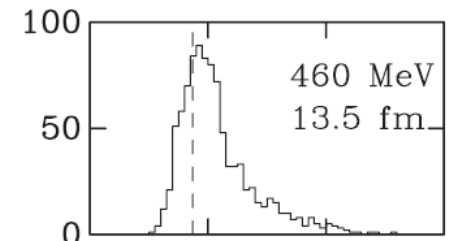
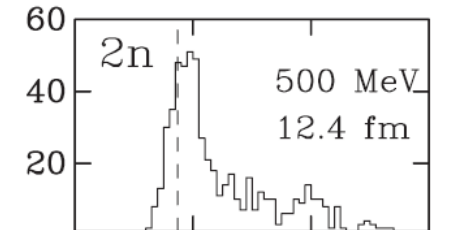
A smooth transition between quasi-elastic and deep inelastic processes



$E > E_B$

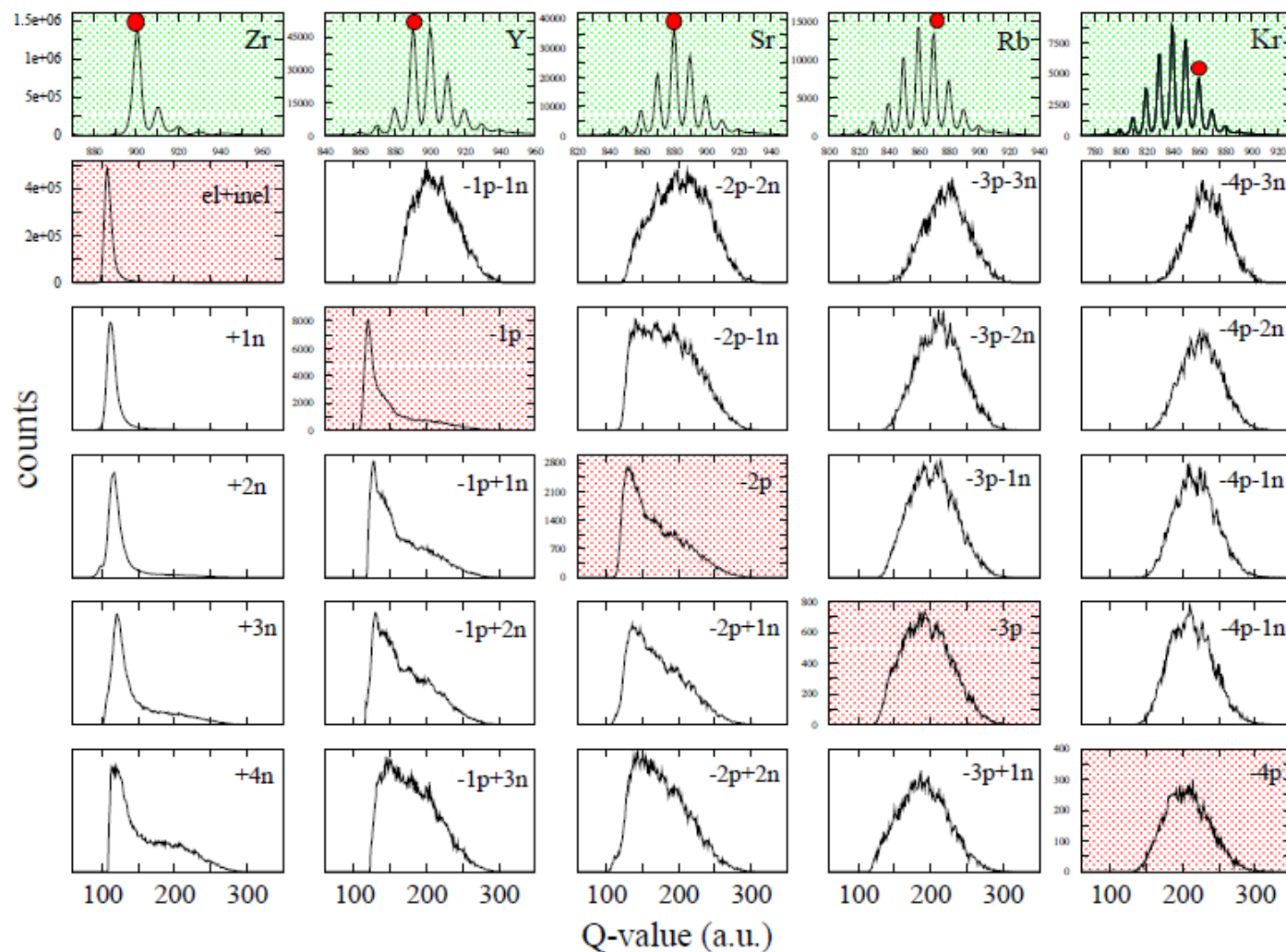
$E \sim E_B$

$E < E_B$



Below the barrier Q-values gets very narrow and without deep inelastic components

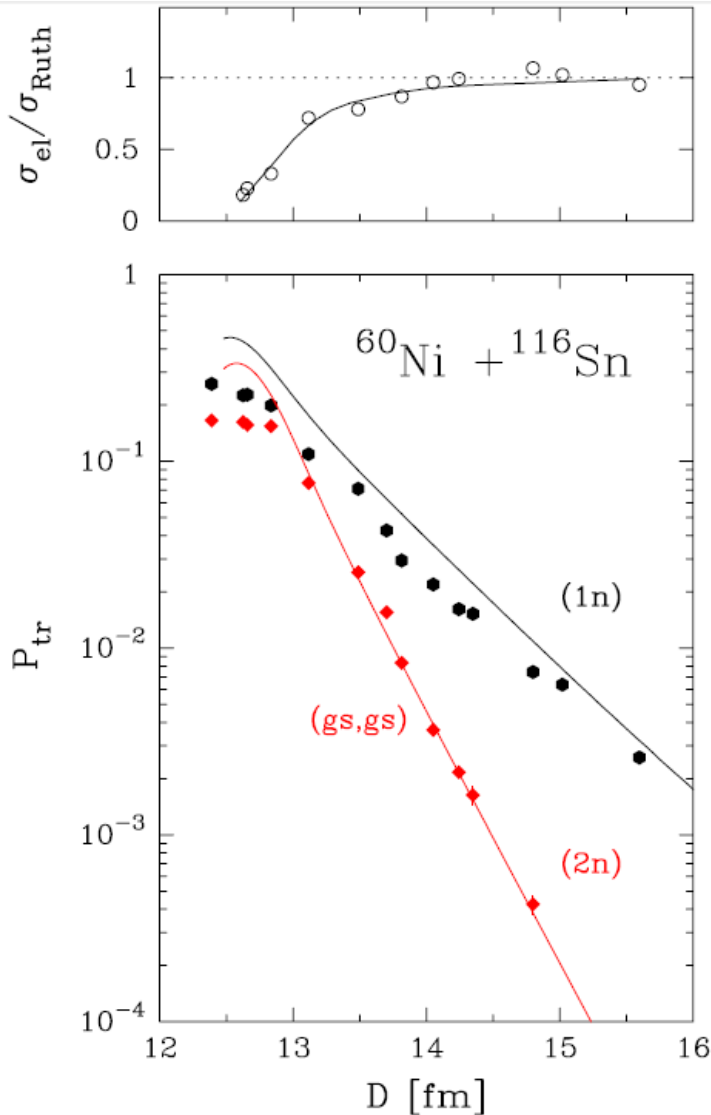
# From quasi-elastic to deep-inelastic regime $^{90}\text{Zr} + ^{208}\text{Pb}$ at $E=560$ MeV (PRISMA)



$$E_{\text{cm}}/V_{\text{C}}(R_{\text{int}}) = 1.19$$

# Sub-barrier transfer reactions

$^{60}\text{Ni}(^{116}\text{Sn}, ^{114}\text{Sn})^{62}\text{Ni}$   $Q_{gg} = +1.3 \text{ MeV}$



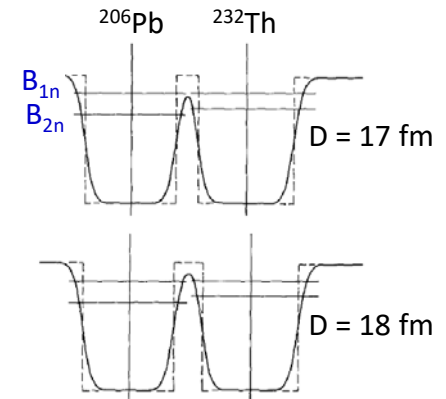
slopes of  $P_{tr}$  versus  $D$  are expected from the binding energy

$$\frac{P_{tr}}{\sin(\theta_{cm}/2)} \propto \exp(-2\alpha \cdot D) \quad \alpha = \sqrt{\frac{2\mu B}{\hbar^2}}$$

$B \rightarrow$  binding energy

$$\alpha_{xn} [fm^{-1}] = 0.21874 \sqrt{x \cdot B_{MeV}}$$

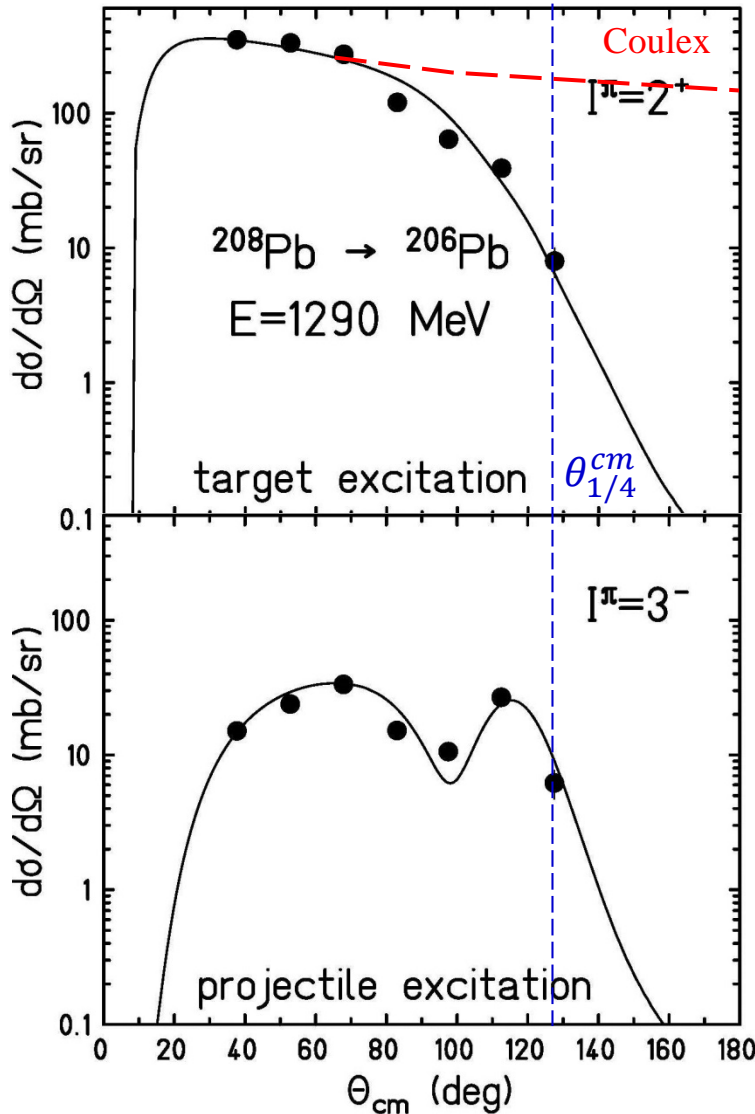
one probes tunneling effects between interacting nuclei, which enter into contact through the tail of their density distributions



$$D = \frac{Z_1 Z_2 e^2}{2E_{cm}} \cdot (1 + \sin^{-1}(\theta_{cm}/2))$$

# Inelastic scattering close to the Coulomb barrier

## electromagnetic and nuclear excitation



$208\text{Pb} + 206\text{Pb}$  at  $E_{cm} = 641.7 \text{ MeV}$

$C_p = 6.81 \text{ fm}$ ,  $C_t = 6.79 \text{ fm}$ ,  $R_{int} = 15.95 \text{ fm}$ ,  $V_C(R_{int}) = 607.0 \text{ MeV}$

$\theta_{1/4}^{cm} = 127.6^\circ$

$$\frac{d\sigma_{inel}}{d\Omega_{cm}} = \{1 - P_{abs}(D, \theta_{cm})\} \cdot \frac{d\sigma_{coul}}{d\Omega_{cm}}$$

$$\sigma_{reac} = P_{abs}(D, \theta_{cm}) \cdot \sigma_{Ruth}$$

$$[1 - P_{abs}(D)] = \exp\left\{-\frac{2}{\hbar} \int_{-\infty}^{+\infty} W[r(t)] dt\right\}$$

$$W[r(t)] = W_0 \cdot \exp\left[-\frac{r(t) - C_1 - C_2}{a_I}\right]$$

$$[1 - P_{abs}(D)] = \exp\left\{-\frac{2}{\hbar} \cdot W_0 \cdot \exp\left[-\frac{D - C_1 - C_2}{a_I}\right] \cdot \frac{D}{v}\right\}$$

# Transfer studies at energies below the Coulomb barrier

- ✓ only a few reaction channels are open  
one reduces uncertainties with nuclear potentials
- ✓ Q-value distributions get much narrower  
one can probe nucleon correlations close to the ground state

but

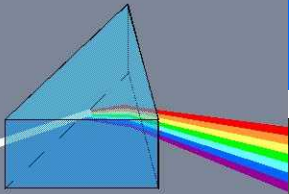
1. angular distributions are backward peaked  
projectile-like particles have low kinetic energy
2. a complete identification of final reaction products in A,Z and Q-values becomes difficult
3. cross sections get very small (need for high efficiency)

solutions:

- use Recoil Mass Separator
- use Magnetic Spectrometers with inverse kinematics



# Prisma spectrometer



**PRISMA: a large acceptance  
magnetic spectrometer**

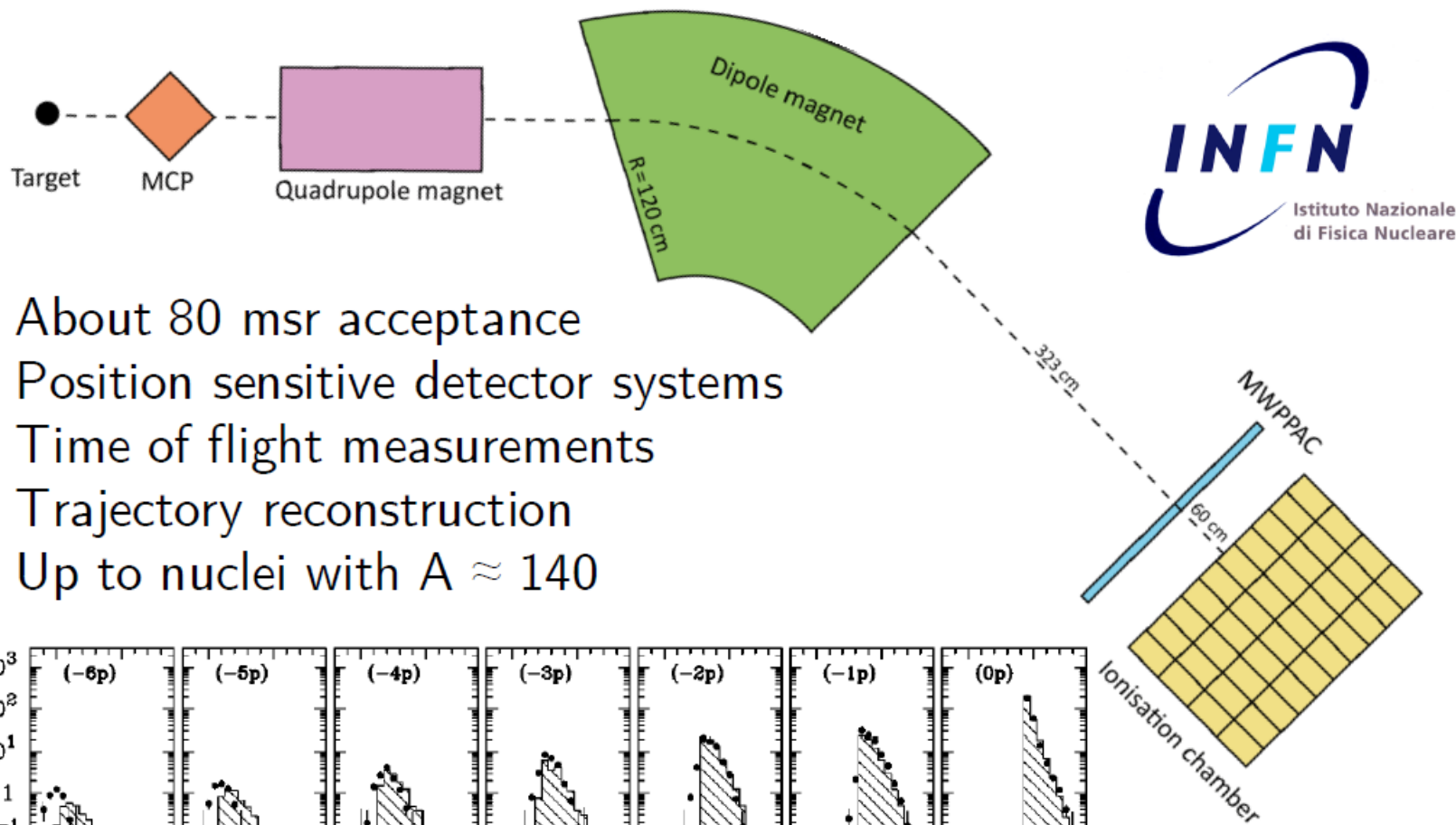
$\Omega \approx 80$  msr;  $B\rho_{\max} = 1.2$  Tm

$\Delta A/A \sim 1/200$

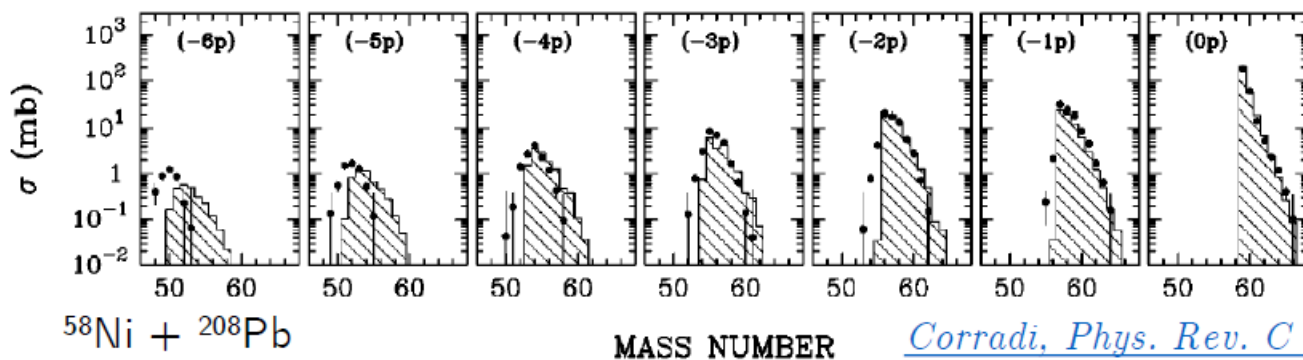
Energy acceptance  $\sim \pm 20\%$



# Prisma spectrometer



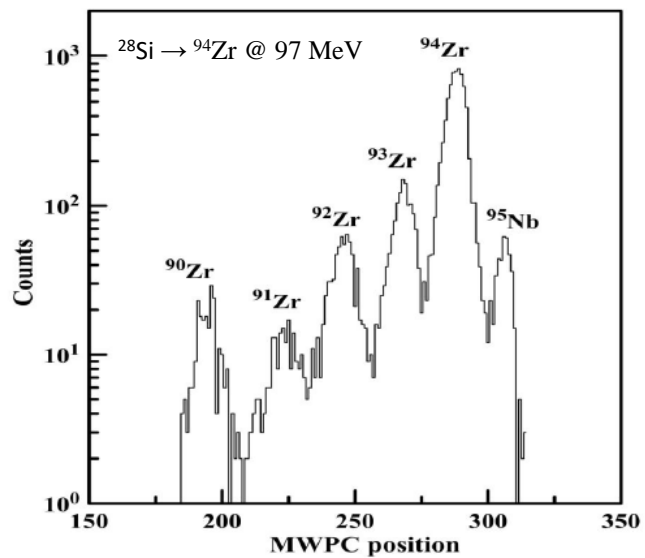
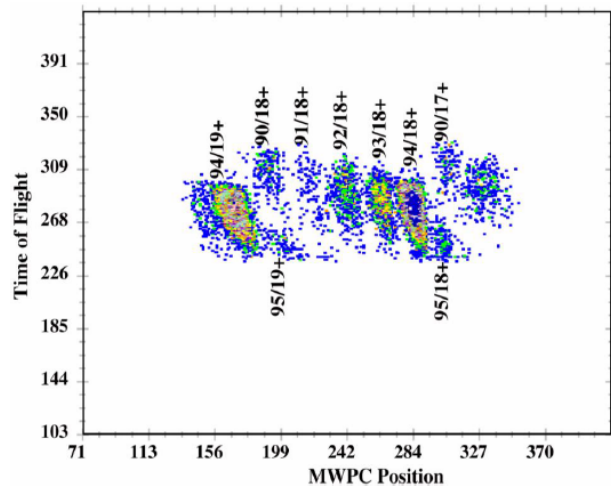
- About 80 msr acceptance
- Position sensitive detector systems
- Time of flight measurements
- Trajectory reconstruction
- Up to nuclei with  $A \approx 140$



[Corradi, Phys. Rev. C 66, 024606 \(2002\)](#)

MCP = micro channel plate

# Heavy Ion Reaction Analyzer (HIRA)

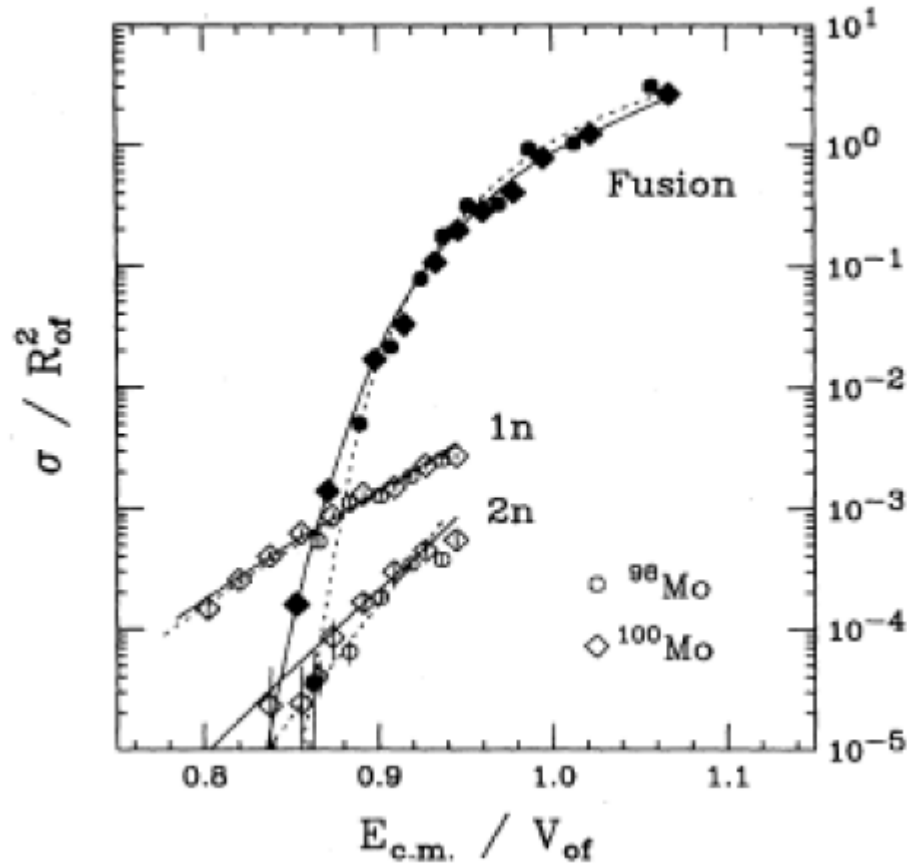


$$^{28}\text{Si} \rightarrow ^{90,94}\text{Zr} @ E_{\text{lab}} = 83.3, 86.4, 89.5, 92.5, 95.5 \text{ MeV}$$

$$^{28}\text{Si} \rightarrow ^{90}\text{Zr} @ E_{\text{cm}} = 63.5, 65.9, 68.3, 70.6, 72.8 \text{ MeV} \quad V_{\text{C}} = 71.5 \text{ MeV}$$

$$^{28}\text{Si} \rightarrow ^{94}\text{Zr} @ E_{\text{cm}} = 64.2, 66.6, 69.0, 71.3, 73.6 \text{ MeV} \quad V_{\text{C}} = 71.1 \text{ MeV}$$

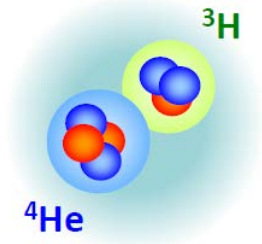
# Why should we measure sub-barrier transfer?



one probes transfer and fusion  
in an overlapping region of  
energies and angular momenta

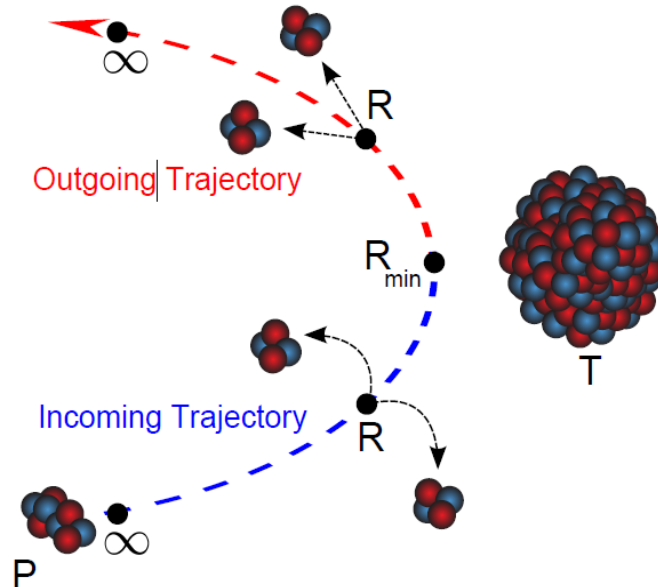
# Transfer reactions with weakly bound nuclei ${}^7\text{Li} + {}^{209}\text{Bi}$

${}^7\text{Li}$



breakup threshold energy:

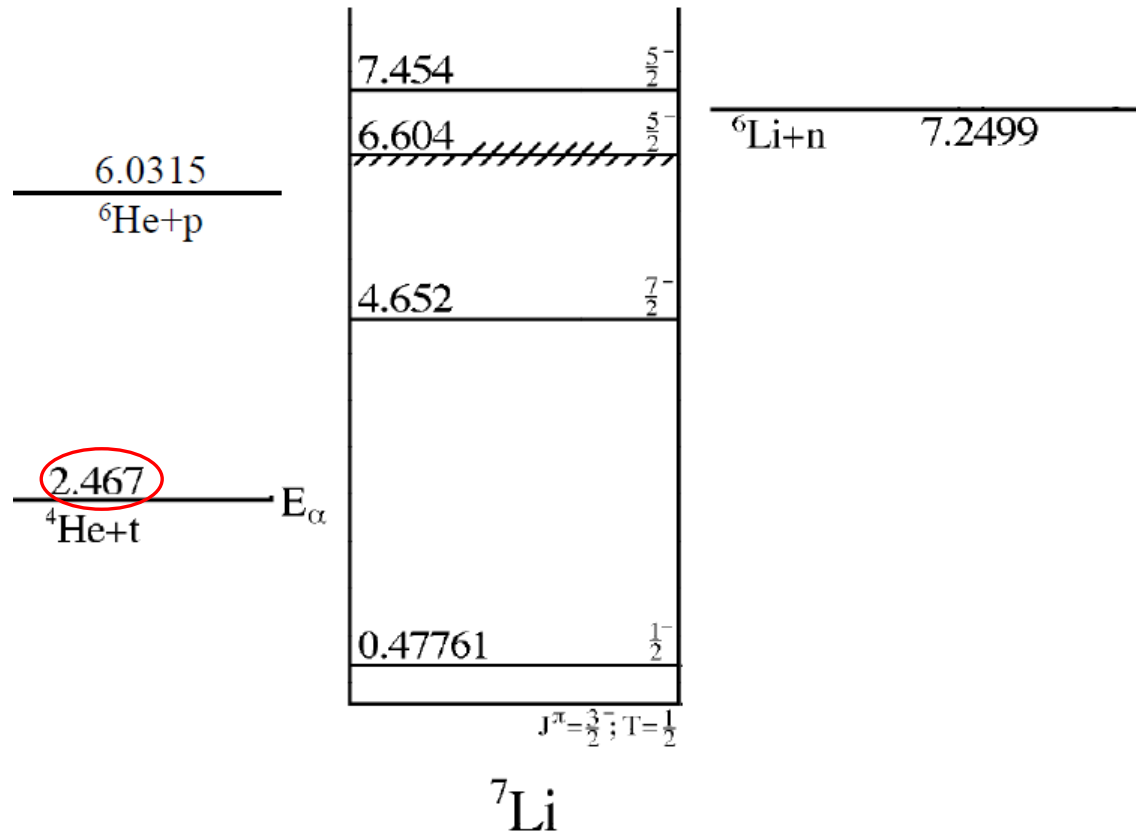
$$Q_{\text{breakup}} = -2.467 \text{ MeV}$$



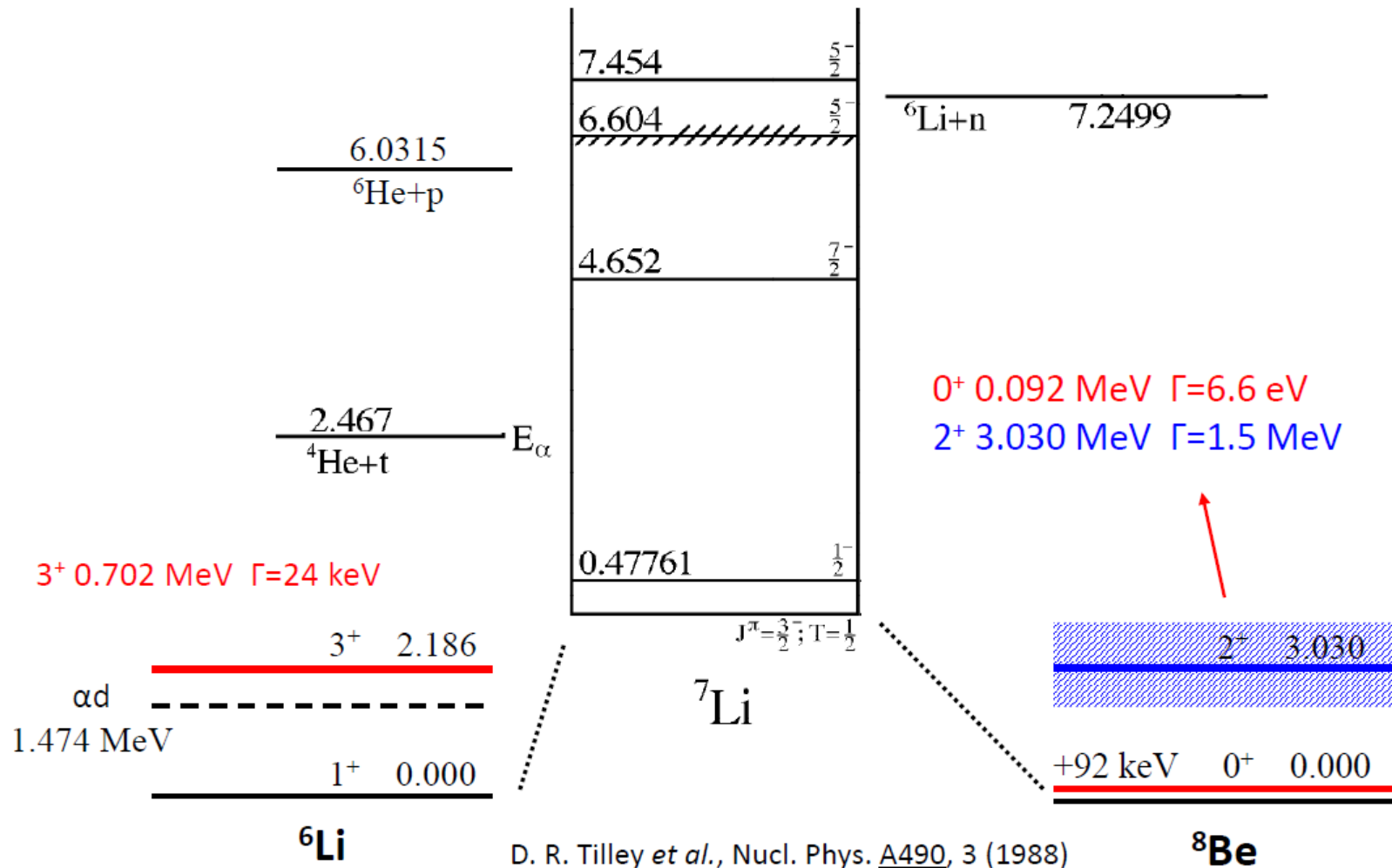
$({}^7\text{Li}, {}^5\text{Li})$ -2n	$({}^7\text{Li}, {}^6\text{Li})$ -1n	$({}^7\text{Li}, {}^8\text{Li})$ +1n	$({}^7\text{Li}, {}^9\text{Li})$ +2n	$({}^7\text{Li}, {}^6\text{He})$ -1p	$({}^7\text{Li}, {}^8\text{Be})$ +1p
-3.18 MeV	-2.65 MeV	-5.43 MeV	-8.25 MeV	-4.99 MeV	<b>+13.46 MeV</b>

${}^5\text{Li} \rightarrow {}^4\text{He} + {}^1\text{H}$	${}^6\text{Li} \rightarrow {}^4\text{He} + {}^2\text{H}$				${}^8\text{Be} \rightarrow {}^4\text{He} + {}^4\text{He}$
+1.965 MeV	-1.474 MeV				+0.092 MeV

# Structure and thresholds

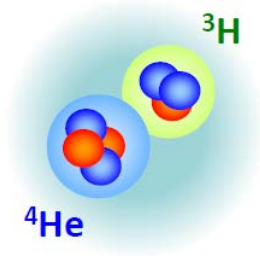


# Structure and thresholds

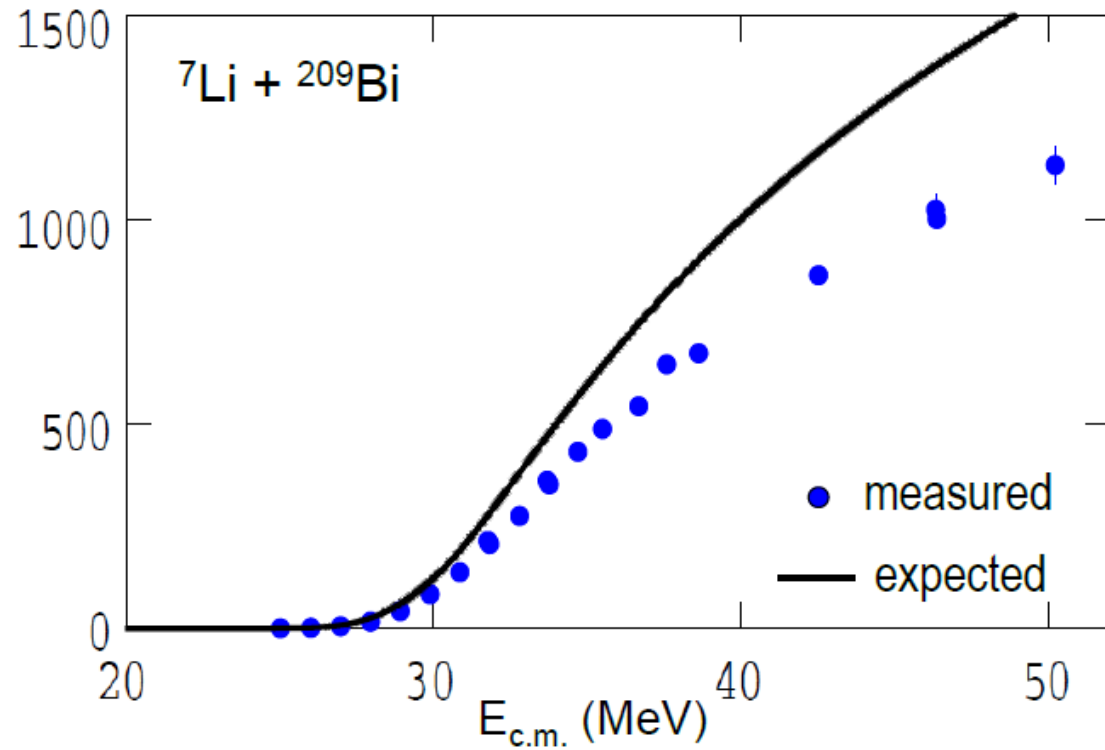


# What causes the reduction in fusion?

${}^7\text{Li}$

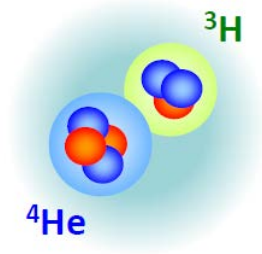


breakup threshold energy:  
 $Q_{\text{breakup}} = -2.467 \text{ MeV}$



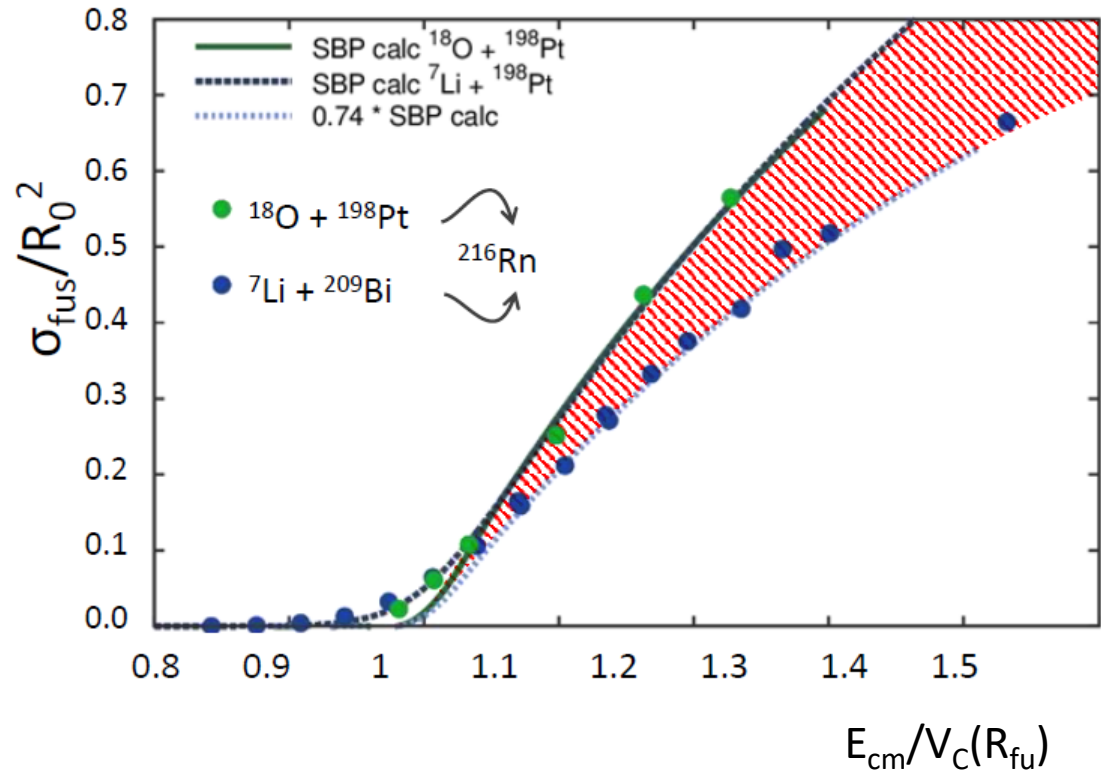
# What causes the reduction in fusion?

${}^7\text{Li}$



breakup threshold energy:

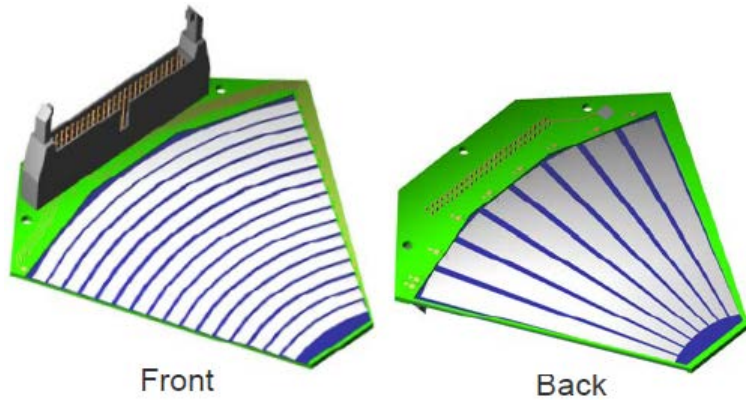
$$Q_{\text{breakup}} = -2.467 \text{ MeV}$$



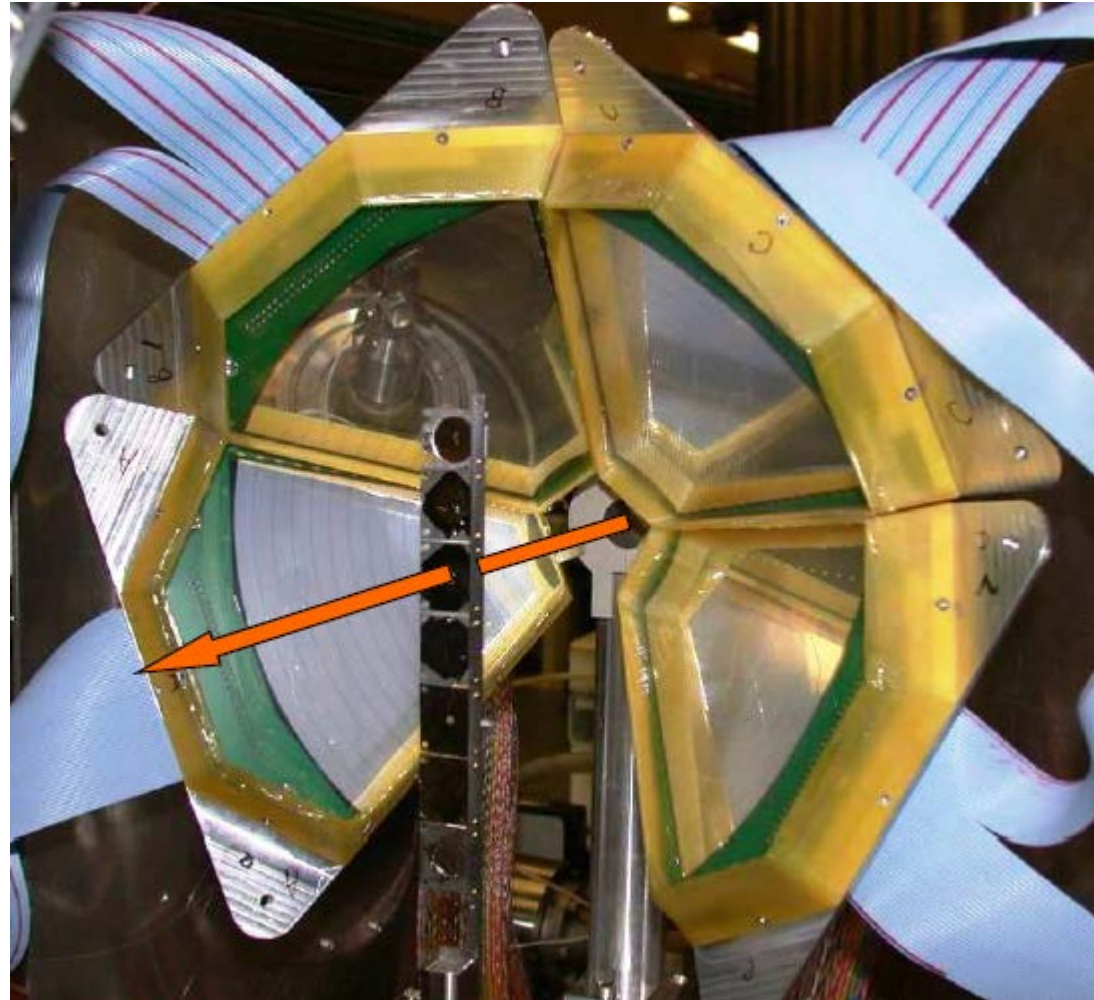
Fusion of weakly bound  ${}^7\text{Li} + {}^{209}\text{Bi}$  suppressed relative to single-barrier calculation in contrast to  ${}^{18}\text{O} + {}^{198}\text{Pt}$



# Experimental set-up at ANU



- $60^\circ$  wedge detectors  
Micron semiconductor Ltd
- Large angular coverage ( $0.83 \pi$  sr)
- Detectors with high pixellation  
(512 pixels)



# Reconstruction of Q-value non-relativistic implementation

## 1. energy conservation:

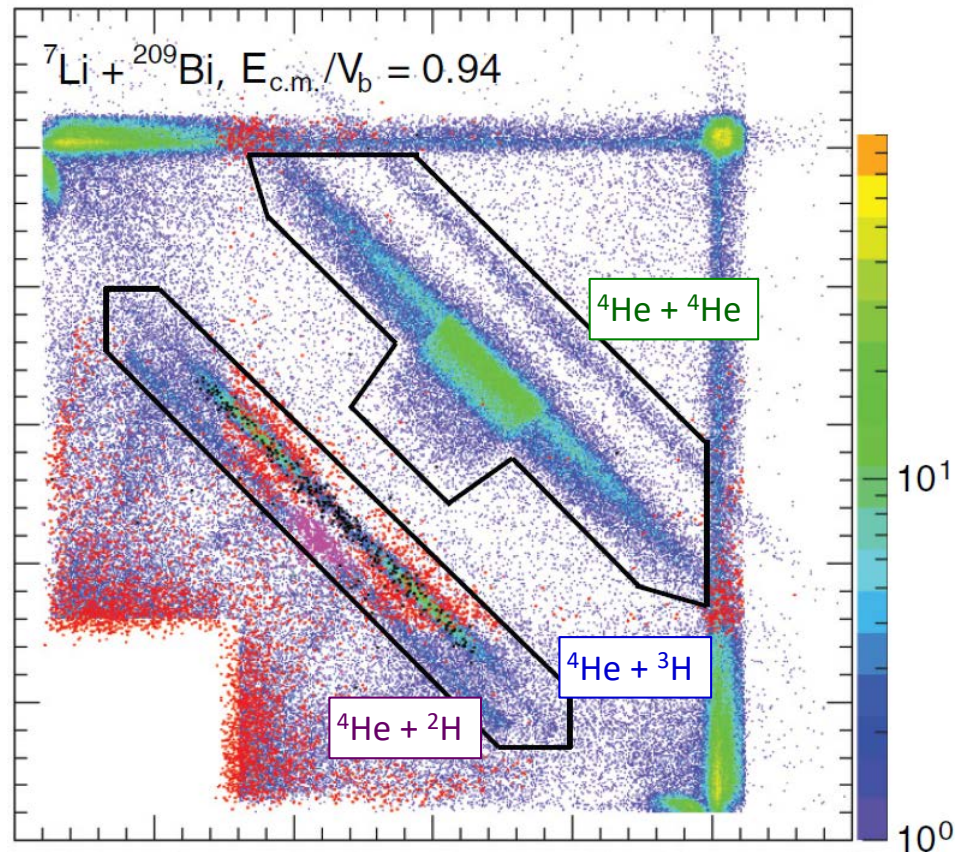
$$Q = (E_1 + E_2 + E_{recoil}) - E_{beam}$$

measured      from momentum conservation      known

## 2. momentum conservation (3-body breakup)

$$\vec{P}_{beam} = \vec{P}_1 + \vec{P}_2 + \vec{P}_{recoil}$$

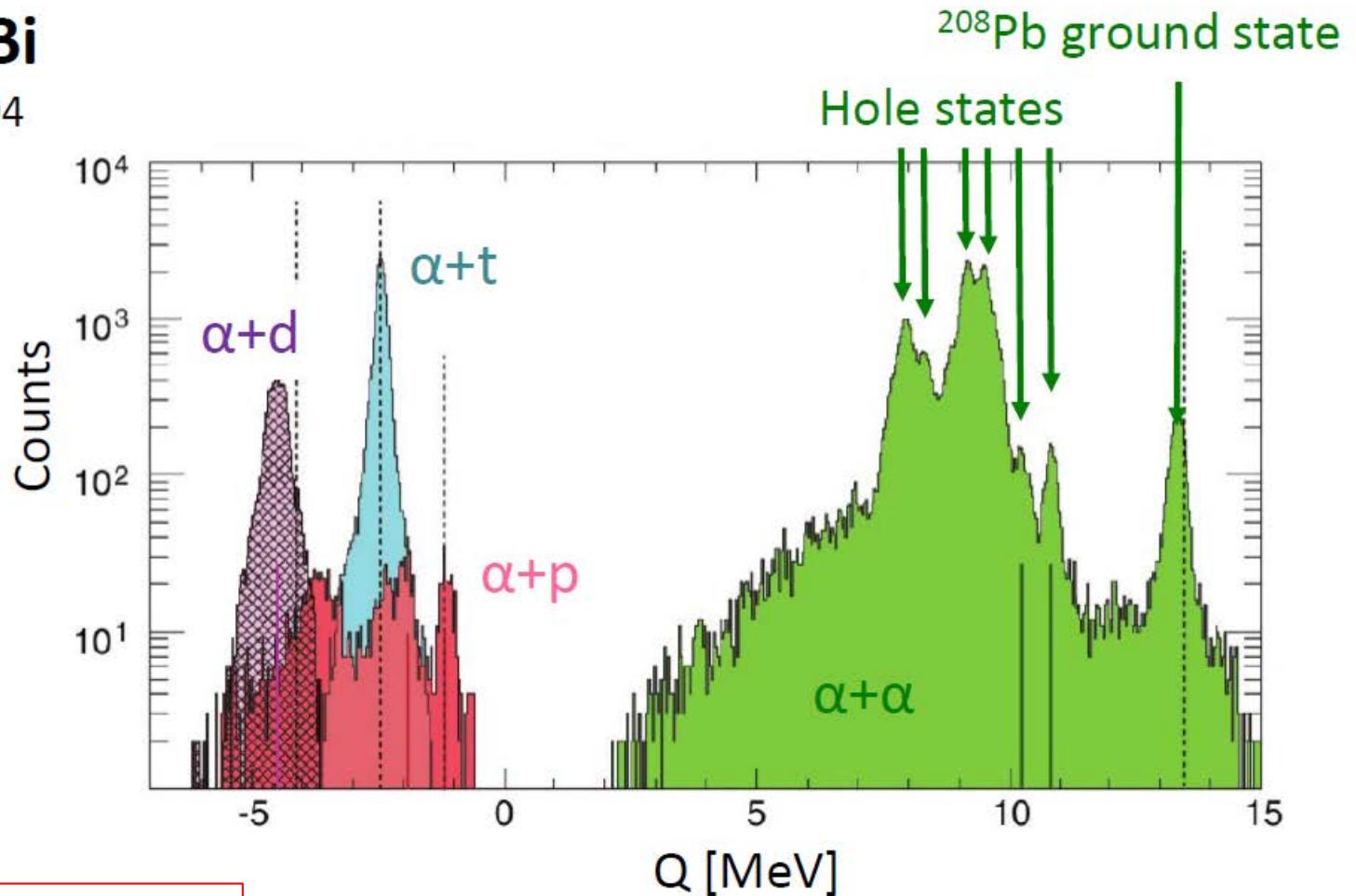
$$E_{recoil} = \frac{|\vec{P}_{recoil}|^2}{2 \cdot m_{recoil}}$$



# Q-value spectrum (target states)

${}^7\text{Li} + {}^{209}\text{Bi}$

$E_{\text{CM}}/V_{\text{B}} = 0.94$



${}^7\text{Li} + {}^{209}\text{Bi} \rightarrow {}^8\text{Be} + {}^{208}\text{Pb}$	$Q_{\text{gg}} = 13.457 \text{ MeV}$
$\rightarrow {}^5\text{Li} + {}^{211}\text{Bi}$	$Q_{\text{gg}} = -3.175 \text{ MeV}$
$\rightarrow {}^6\text{Li} + {}^{210}\text{Bi}$	$Q_{\text{gg}} = -2.645 \text{ MeV}$

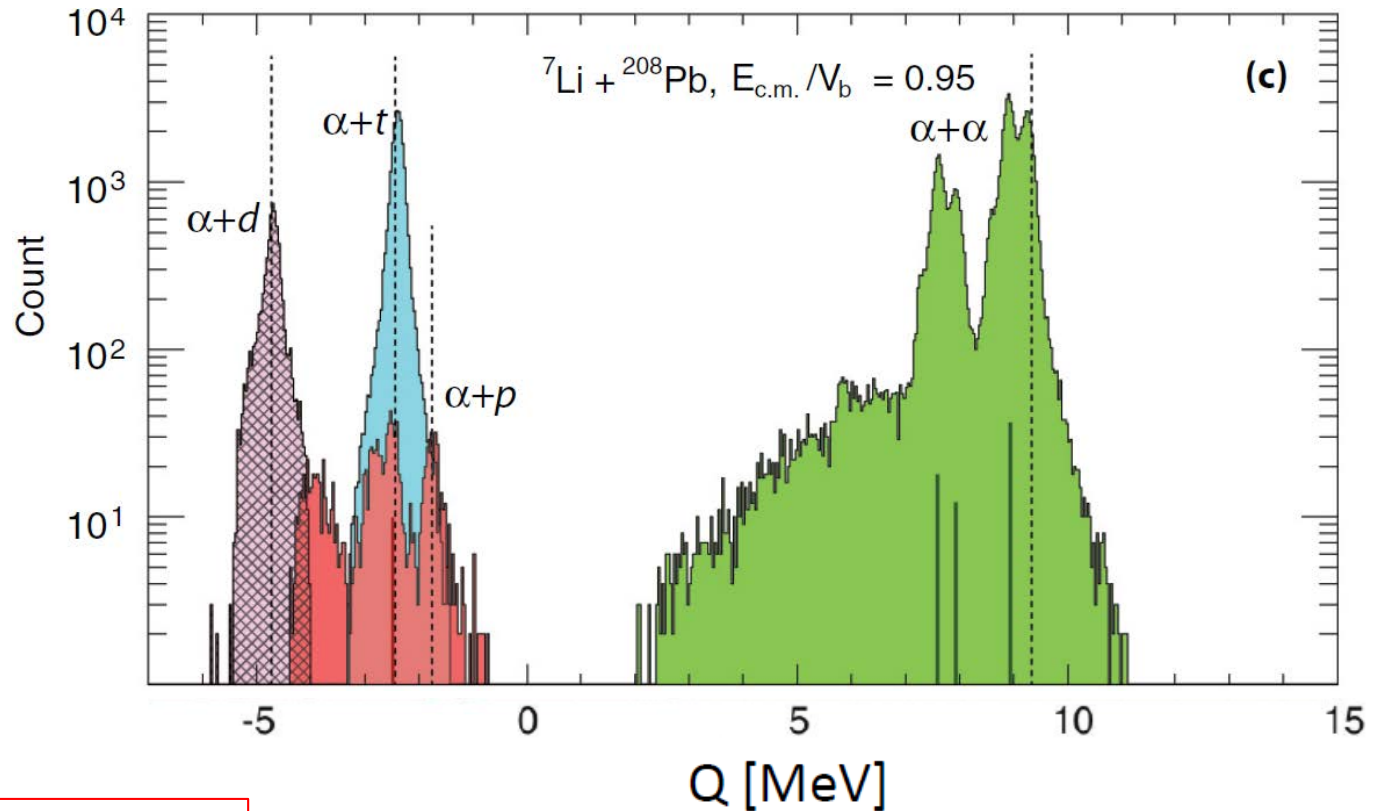
${}^8\text{Be} \rightarrow {}^4\text{He} + {}^4\text{He}$	$Q_{\text{gg}} = +0.092 \text{ MeV}$
${}^5\text{Li} \rightarrow {}^4\text{He} + {}^1\text{H}$	$Q_{\text{gg}} = +1.965 \text{ MeV}$
${}^6\text{Li} \rightarrow {}^4\text{He} + {}^2\text{H}$	$Q_{\text{gg}} = -1.474 \text{ MeV}$



# Q-value spectrum (target states)

**${}^7\text{Li} + {}^{208}\text{Pb}$**

$E_{\text{CM}}/V_B = 0.95$



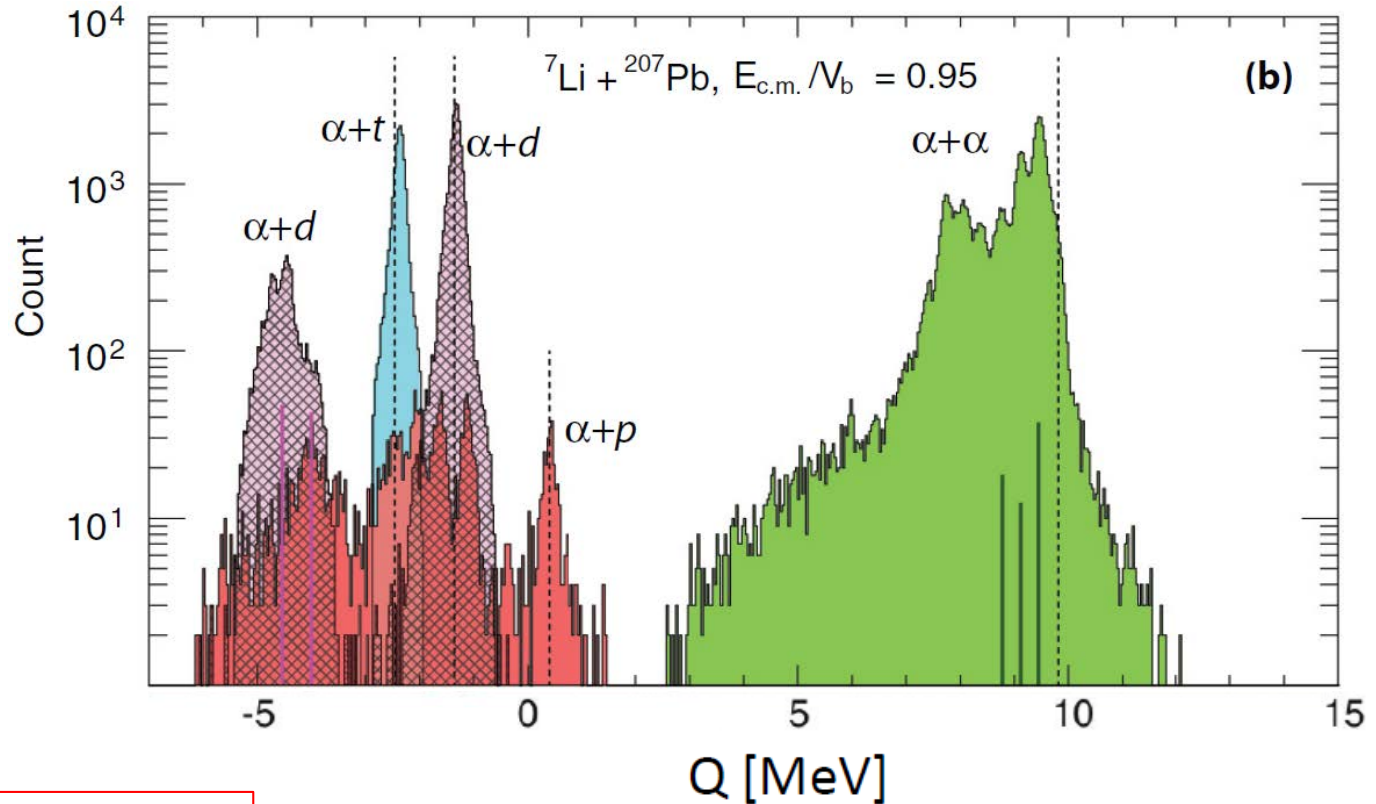
${}^7\text{Li} + {}^{208}\text{Pb} \rightarrow {}^8\text{Be} + {}^{207}\text{Tl}$	$Q_{\text{gg}} = 9.246 \text{ MeV}$
$\rightarrow {}^5\text{Li} + {}^{210}\text{Pb}$	$Q_{\text{gg}} = -3.792 \text{ MeV}$
$\rightarrow {}^6\text{Li} + {}^{209}\text{Pb}$	$Q_{\text{gg}} = -3.313 \text{ MeV}$

${}^8\text{Be} \rightarrow {}^4\text{He} + {}^4\text{He}$	$Q_{\text{gg}} = +0.092 \text{ MeV}$
${}^5\text{Li} \rightarrow {}^4\text{He} + {}^1\text{H}$	$Q_{\text{gg}} = +1.965 \text{ MeV}$
${}^6\text{Li} \rightarrow {}^4\text{He} + {}^2\text{H}$	$Q_{\text{gg}} = -1.474 \text{ MeV}$

# Q-value spectrum (target states)

**${}^7\text{Li} + {}^{207}\text{Pb}$**

$E_{\text{CM}}/V_B = 0.95$



${}^7\text{Li} + {}^{207}\text{Pb} \rightarrow {}^8\text{Be} + {}^{206}\text{Tl}$	$Q_{\text{gg}} = 9.766 \text{ MeV}$	${}^8\text{Be} \rightarrow {}^4\text{He} + {}^4\text{He}$	$Q_{\text{gg}} = +0.092 \text{ MeV}$
$\rightarrow {}^5\text{Li} + {}^{209}\text{Pb}$	$Q_{\text{gg}} = -1.610 \text{ MeV}$	${}^5\text{Li} \rightarrow {}^4\text{He} + {}^1\text{H}$	$Q_{\text{gg}} = +1.965 \text{ MeV}$
$\rightarrow {}^6\text{Li} + {}^{208}\text{Pb}$	$Q_{\text{gg}} = 0.118 \text{ MeV}$	${}^6\text{Li} \rightarrow {}^4\text{He} + {}^2\text{H}$	$Q_{\text{gg}} = -1.474 \text{ MeV}$