Outline: Symmetries

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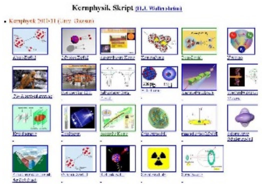


1. definition and consequences
2. isospin symmetry (mirror nuclei: ^{54}Ni, ^{54}Fe)
3. seniority-pairing: ^{98}Cd, ^{130}Cd
4. rotational nuclei SU(3): ^{254}No, ^{152}Dy
5. octupole deformation: ^{226}Ra

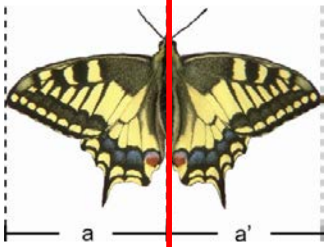
Symmetry: definition

by Hermann Weyl, Richard P. Feynman:

object, natural law **transformation**

„… a thing is symmetrical, if you can do something to it and after you have done it, it looks the same as before …“

invariance



why symmetries ?

Symmetry: ordering principle
predictions
connection to unobserved quantities
conservation laws
structure of interactions
Noether-theorem (1918)

Emmy Amalie Noether
(1882-1935)

The symmetry properties of a physical system are intimately related to conservation laws!

Example

1. Free particle

- Lagrange function $L(q,\dot{q},t)=\frac{m}{2}\cdot\dot{q}^2$
- Transformed Lagrange function $T: q \rightarrow Q = q + s \;\; und \;\; \dot{Q} = \dot{q}$
- $L(q,\dot{q},t)=L(Q,\dot{Q},t)=\frac{m}{2}\cdot\dot{Q}^2$ i.e. invariant or symmetrical concerning transformation T
- according to the theorem of Noether exists a conservation quantity. *Which?*
- one find it in the Euler – Lagrange equation $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}}-\frac{\partial L}{\partial q}=0$
- $\frac{\partial L}{\partial q}=0 \rightarrow \frac{d}{dt}\frac{\partial L}{\partial \dot{q}}=0 \quad \rightarrow \frac{\partial L}{\partial \dot{q}}=m\cdot\dot{q}$
- $m\cdot\dot{q}=const. \rightarrow$ momentum conservation

From the translation invariance results the momentum conservation

Example

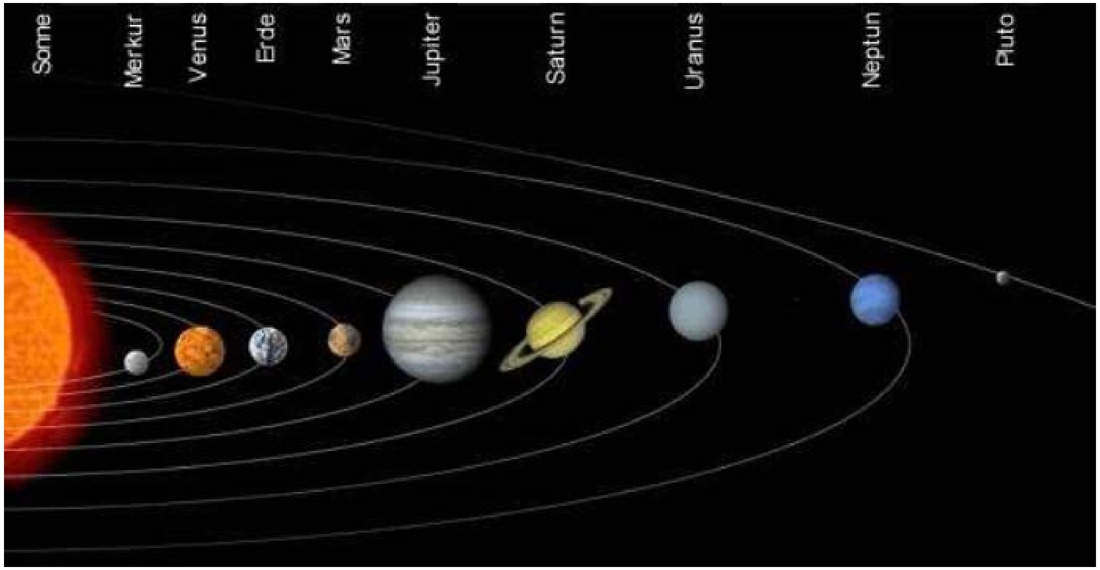
2. Kepler problem

- mass m in a central field $V = -\frac{\alpha}{r}$
- Lagrange function in spherical coordinates r, ϑ, φ is $L = \frac{m}{2} \cdot \left(\dot{r}^2 + r^2 \dot{\vartheta}^2 + r^2 \sin^2 \vartheta \cdot \dot{\varphi}^2 \right) + \frac{\alpha}{r}$
- $\frac{\partial L}{\partial \varphi} = 0$ i.e. the Lagrange function is independent of φ
- Lagrange function is invariant with respect to rotations of the angle φ
- according to the theorem of Noether exists a conservation quantity. *Which?*
- one find it in the Euler – Lagrange equation
- $\frac{d}{dt} \frac{\partial L}{\partial \dot{\varphi}} - \frac{\partial L}{\partial \varphi} = 0$
- $\frac{d}{dt} \frac{\partial L}{\partial \dot{\varphi}} = 0 \rightarrow \frac{\partial L}{\partial \dot{\varphi}} = m \cdot r^2 \cdot \sin^2 \vartheta \cdot \dot{\varphi} = const. = L_z$
- as the z-component of the angular momentum is an independent quantity $\vec{L} = m \cdot \vec{r} \times \dot{\vec{r}} = const.$

In a central field exists conservation of angular momentum

Conservation quantities in space-time symmetries

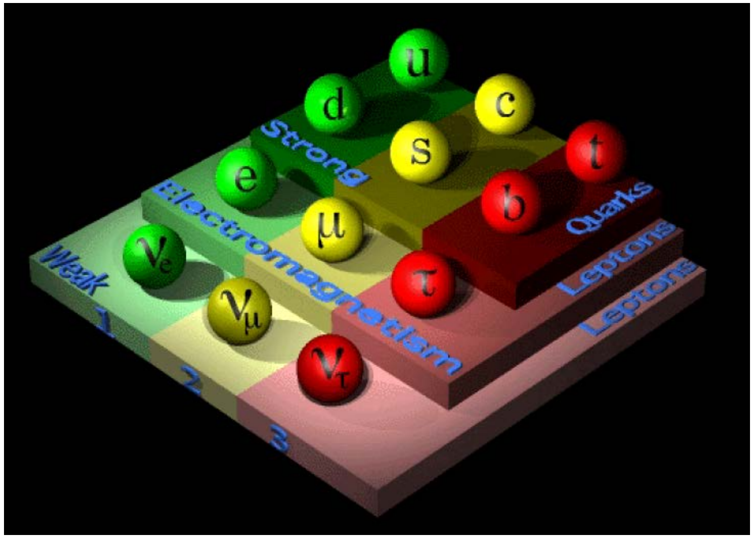
- Homogeneity of space
  momentum conservation
- Isotropy of space
  angular momentum conservation
- Homogeneity of time
  energy conservation



solar system

energy - and angular momentum conservation

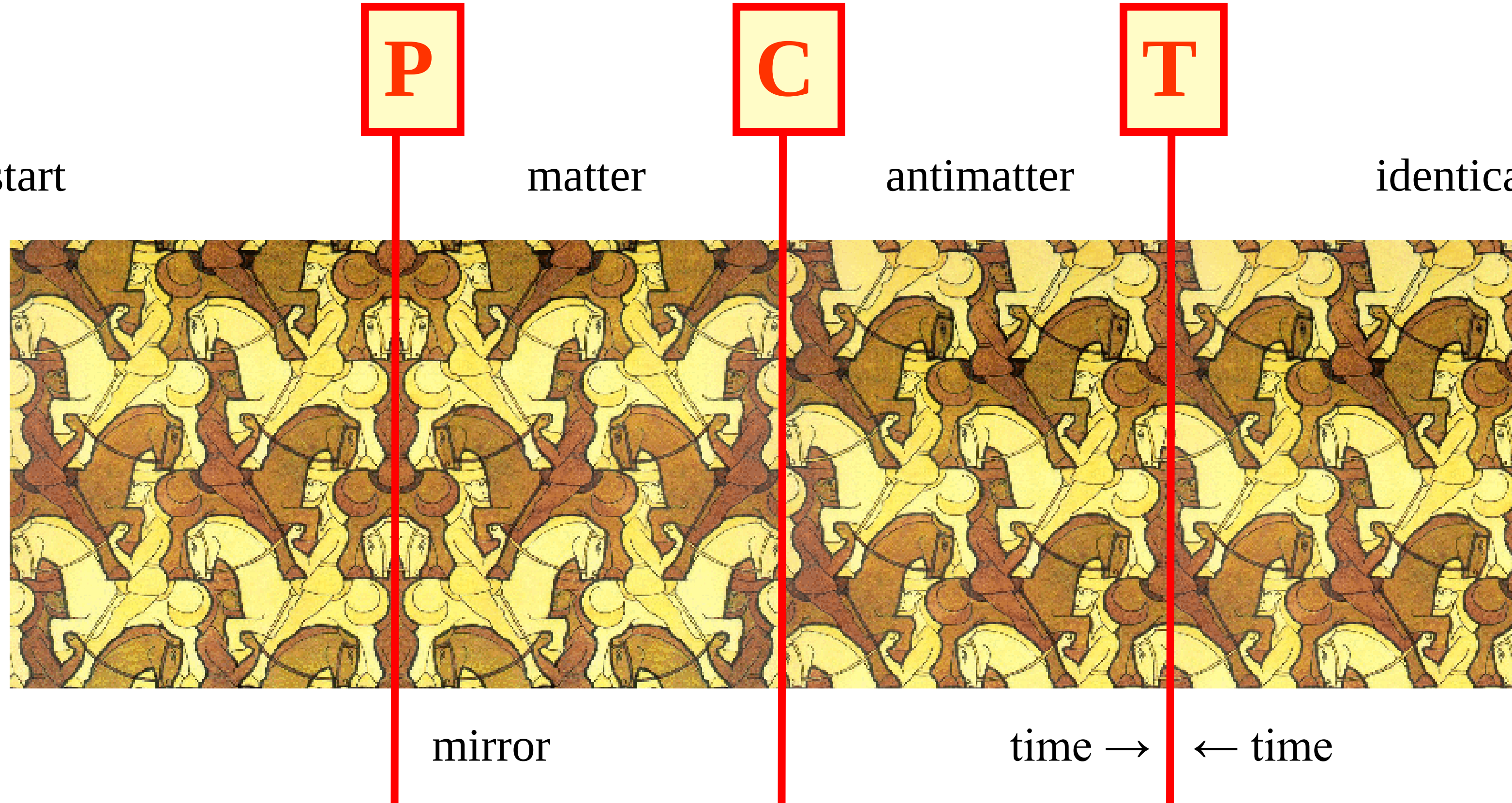
Particle physics: particle zoo

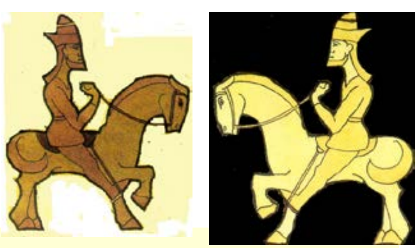


How can we bring order in the particle zoo ?

new symmetry ? new conservation quantities ? new order !

The world according to Escher/Pauli



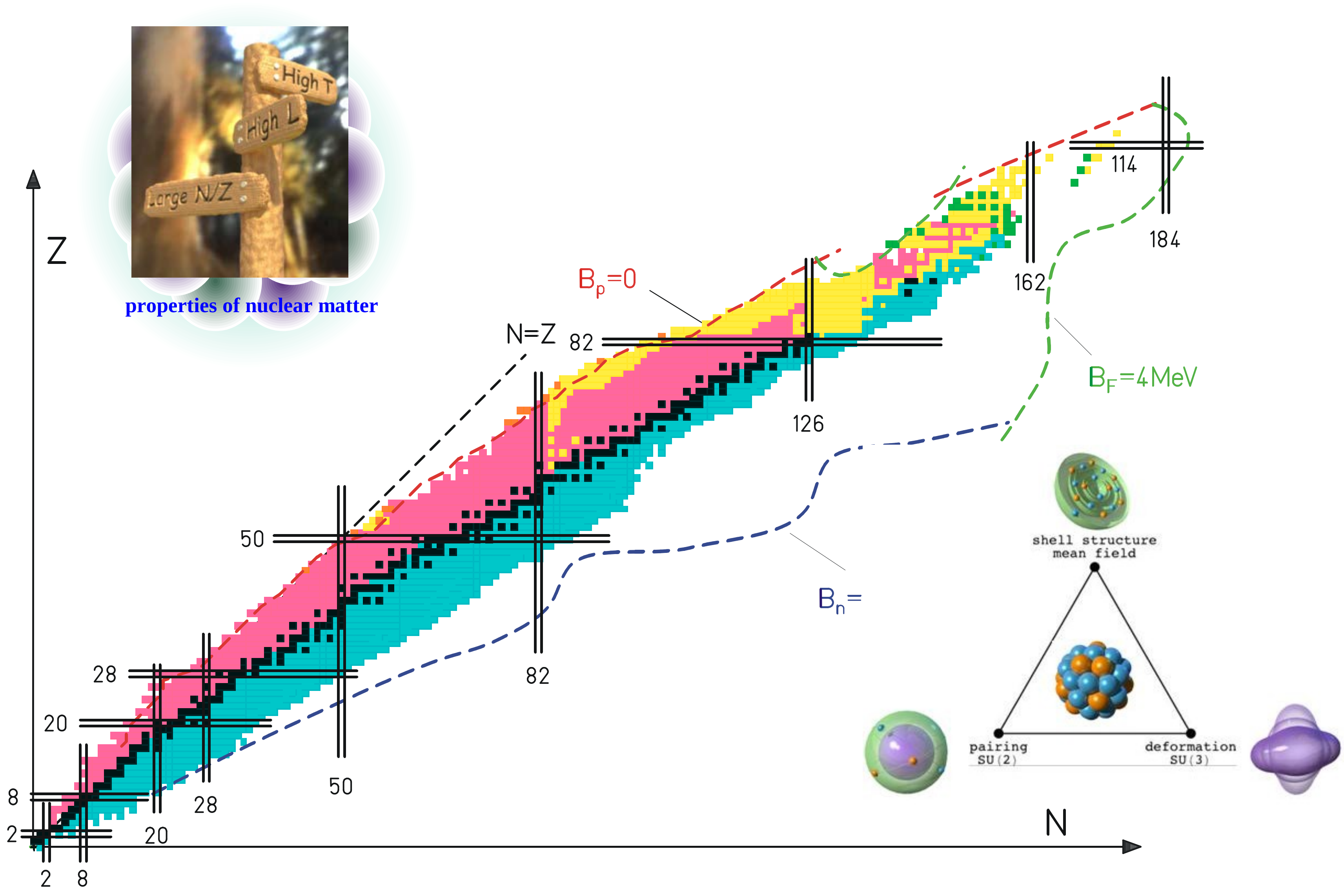


antiparticle particle

Holds on very general grounds:
Nature is local, causal & Lorentz invariant.
True for gauge theories!
Matter antimatter asymmetry not explained

P ≡ space inversion, **C** ≡ charge, **T** ≡ time reversal invariance

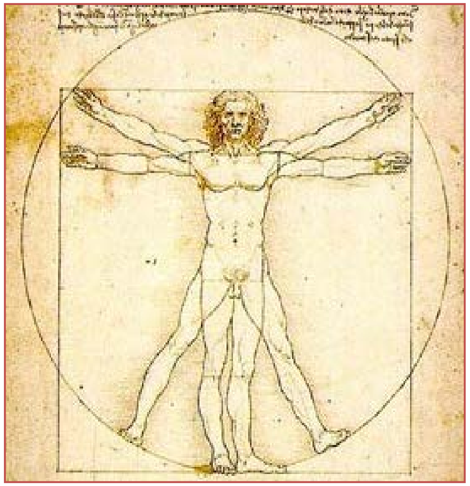
Chart of nuclei

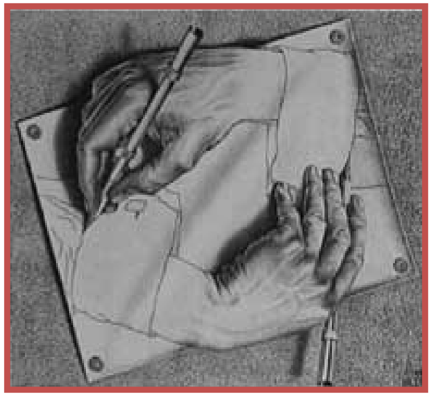


properties of nuclear matter

Symmetries help to understand nature

Investigation of fundamental symmetries:
a key-question in physics



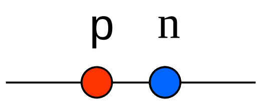


chirality - if an image in a plane mirror cannot be brought to coincide with itself

conservation laws

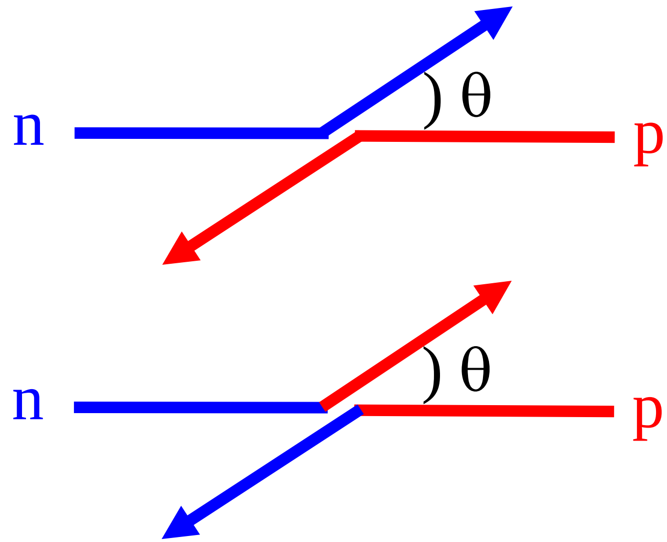
good quantum numbers

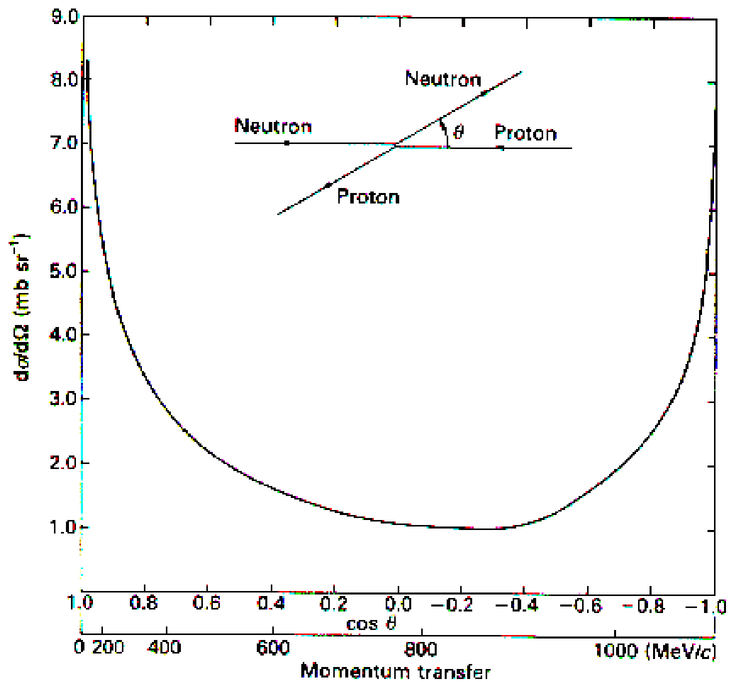
Symmetries in nuclear physics



Isospin symmetry: 1932 Heisenberg SU(2)

1901-1976
Nobel prize 1932





exchange forces

$m_p = 938.3$ MeV $m_n = 939.5$ MeV

⇨ Strong interaction can not distinguish between protons and neutrons

⇨ Proton and neutron are for strong interaction states of one particle (nucleon) → **Isospin**

Isospin

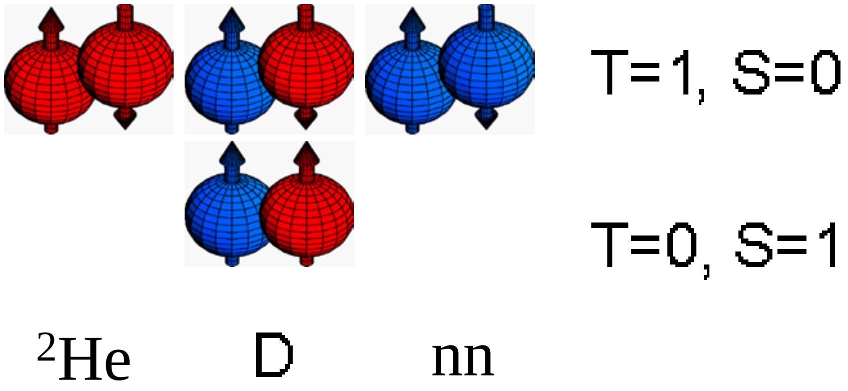
$$T_z = 1/2(Z-N)$$

$$T = |T_z|$$

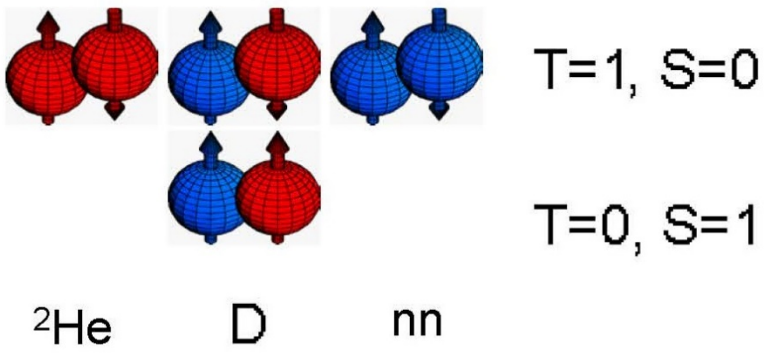
proton: $T_z(p) = +1/2$

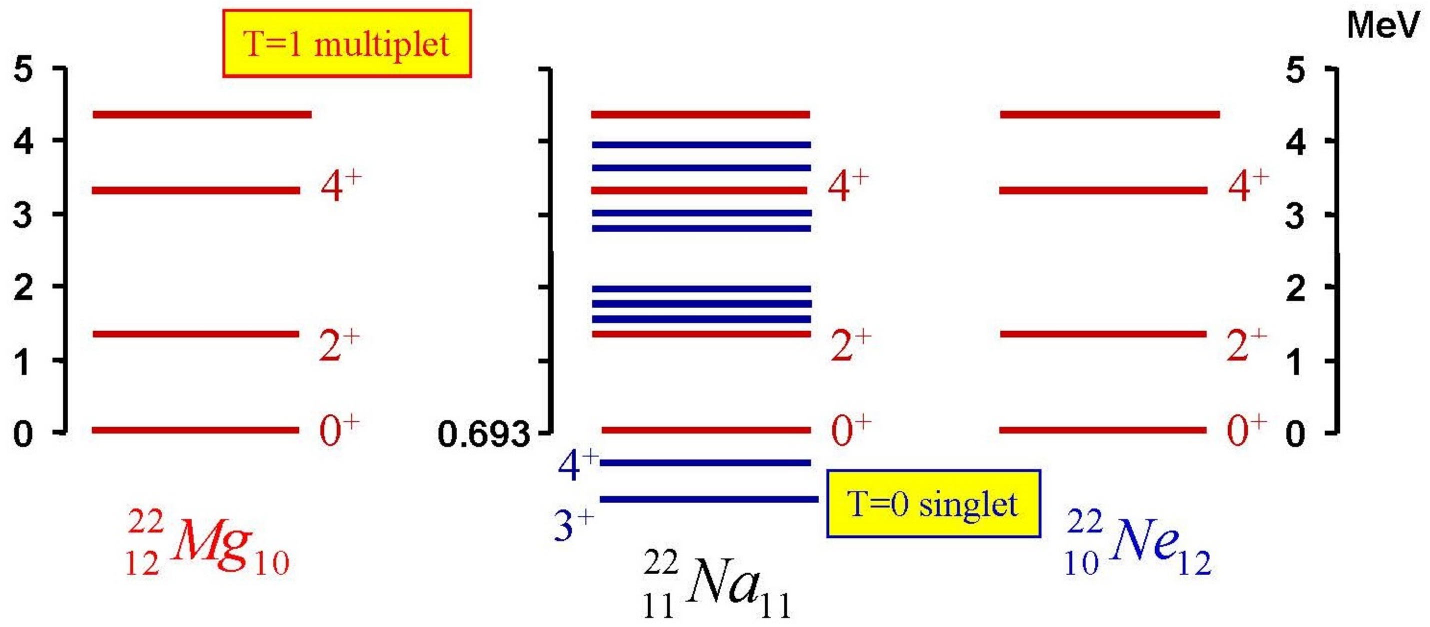
neutron: $T_z(n) = -1/2$

- Proton and neutron are 2 states of the same particle.
- Pauli principle forbids T=0 states for nn und ^{2}He
- Deuteron (T=0,S=1) is the only A=2 bound system



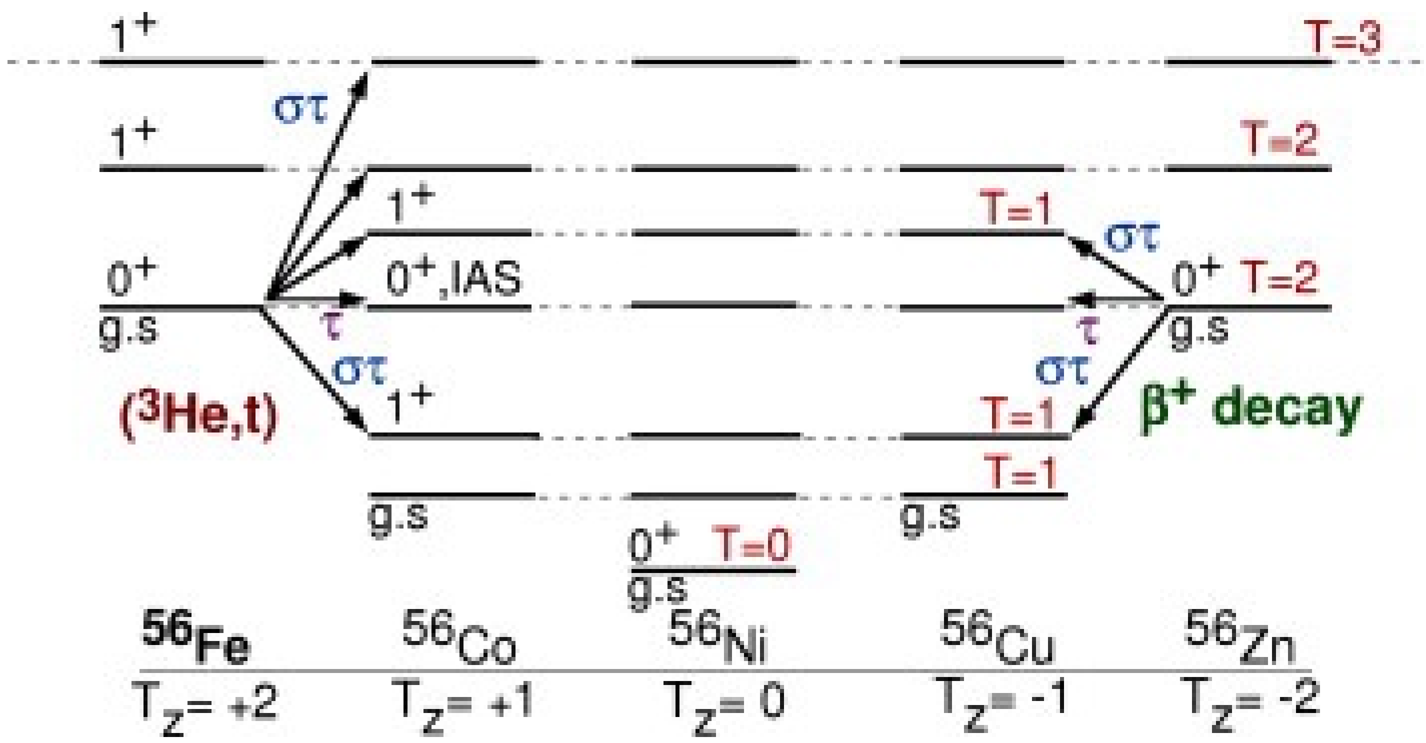
- Is V_{np} interaction equal to the V_{nn} and V_{pp}?
- Compare the energy levels for nuclei with constant A.
- Equal spin / parity states have the same energy.
- $V_{np}=V_{nn}=V_{pp}$





Isospin independence in nuclei

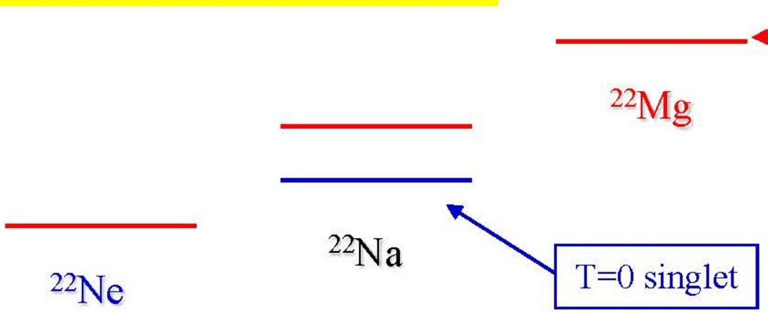
- Nuclei with the same isospin, T, show nearly identical structure
- Total energy is changed by the Coulomb force (+ other small differences)
- Nuclei with varying T_z are called members of a multiplet



Isospin symmetry in T=1 nuclei

(apart from the Coulomb energy)

- **naturally, Vpp ≠ Vnn ≠ Vpn**



- Difference in the binding energy for mirror nuclei.
  (Bethe-Weizsäcker formula)

$$BE\left({}^{22}_{10}Ne\right) - BE\left({}^{22}_{12}Mg\right) = -a_{Coul} \cdot \frac{10 \cdot 9 - 12 \cdot 11}{22^{1/3}} \cong 10.6\ MeV$$

- Isobaric-Multiplet-Mass equation

$$BE\,(i, T_z = T) - BE\,(i, T_z = -T) = 2b_i T$$

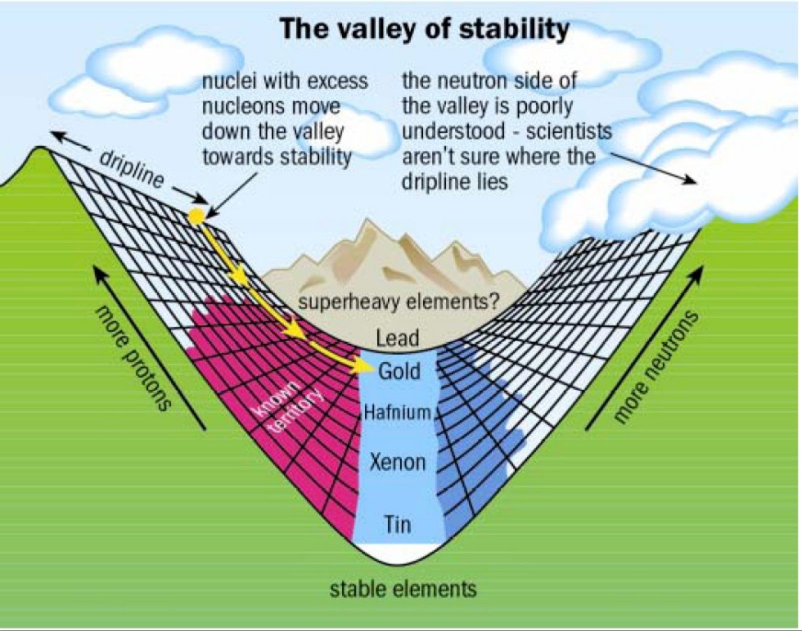
$$E_{Coul} = \frac{3}{5}\frac{(Ze)^2}{R} \Rightarrow \Delta E_{Coul} \approx \frac{3}{5}\frac{e^2}{r_0} A^{2/3} \cdot (2T)$$

$$BE(i, T, T_z) = a_i + b_i T_z + c_i T_z^2$$

Isovector
~ 3-15 MeV (~ $A^{2/3}$)

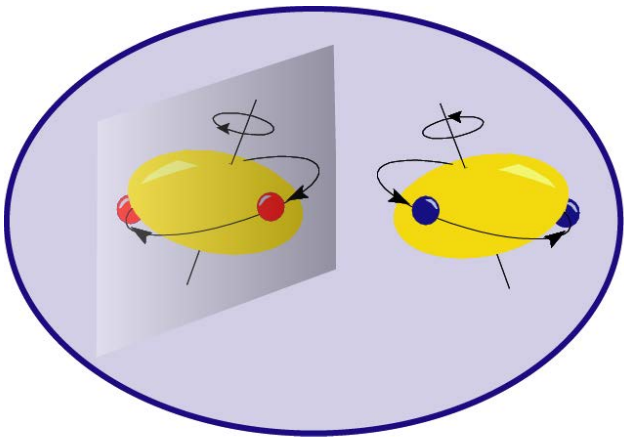
Isoscalar
Dominated by the strong interaction
~ 100's MeV

Isotensor
~200-300 keV

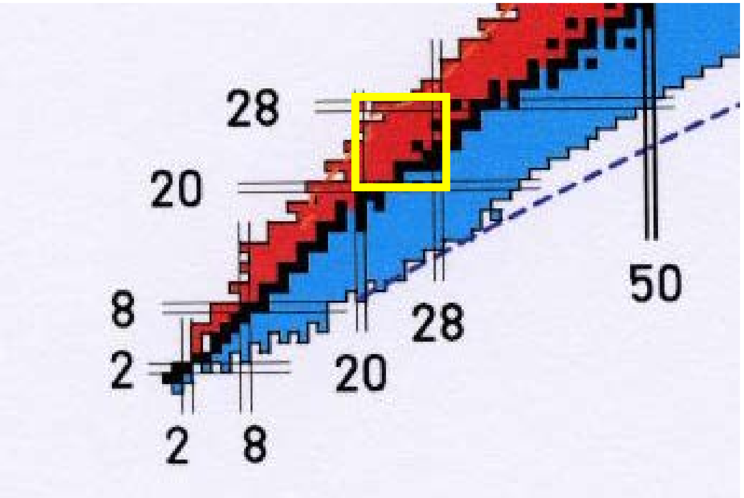


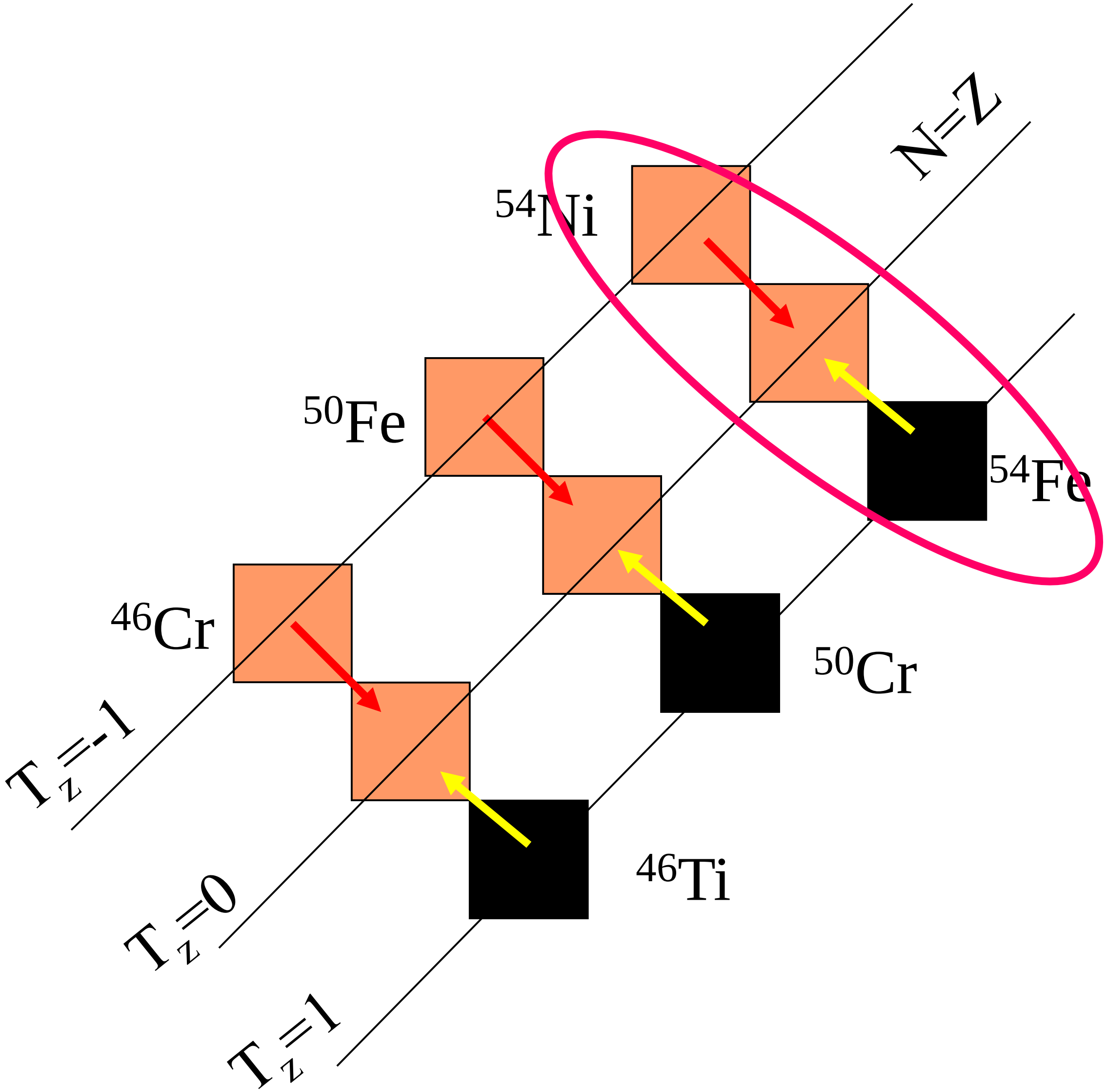
Search for deviations from isospin symmetry

mirror nuclei

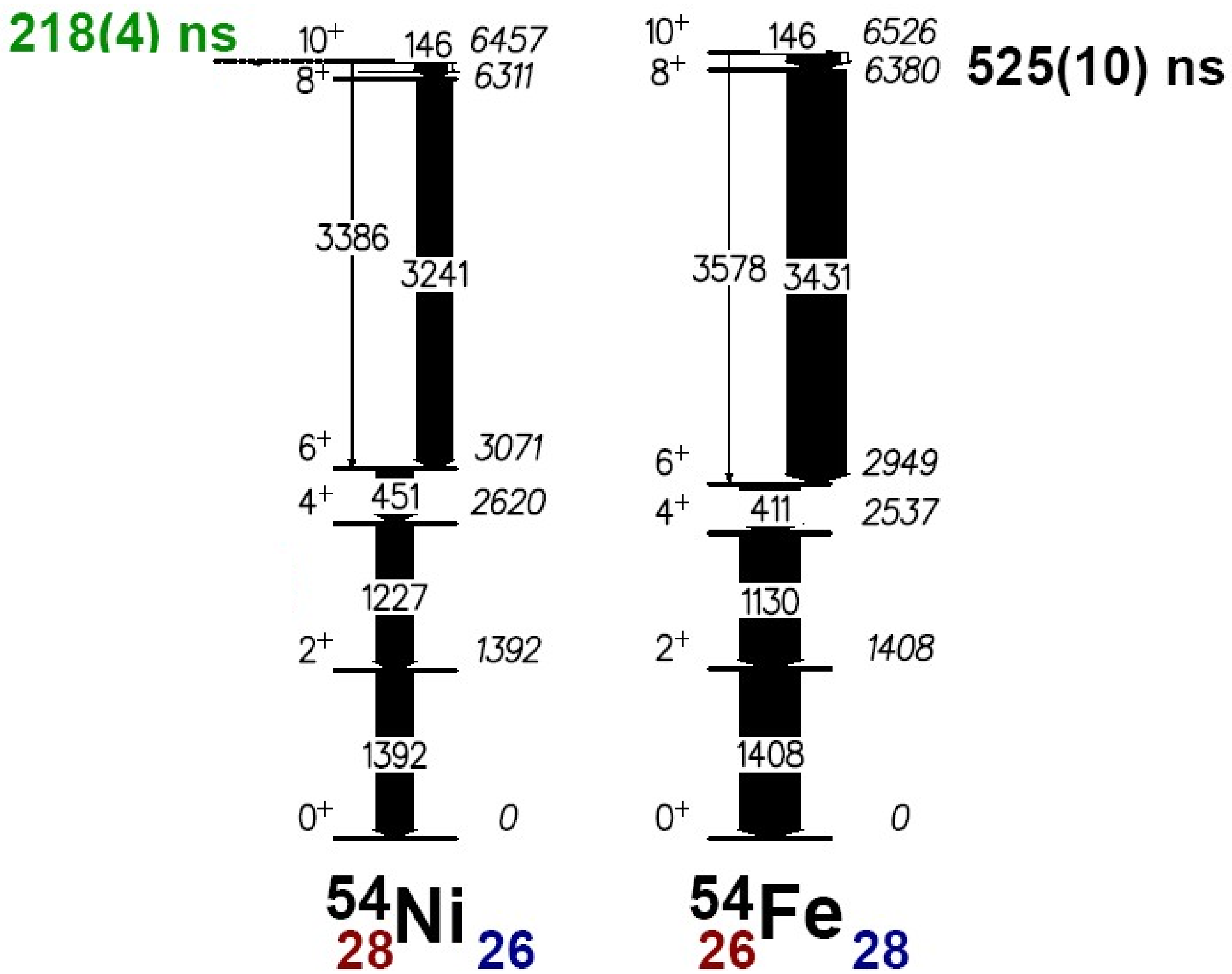


$^{54}_{28}Ni_{26}$ $^{54}_{26}Fe_{28}$



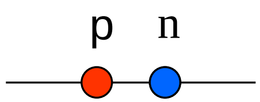


Proton radioactivity – decay of the $I^{\pi}=10^{+}$ isomer in ^{54}Ni

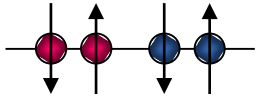


D. Rudolph, R. Hoischen et al., Phys.Rev.C78 (2008), 021301

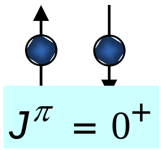
Symmetries in nuclear physics



Isospin Symmetry: 1932 Heisenberg SU(2)



Spin-Isospin Symmetry: 1936 Wigner SU(4)

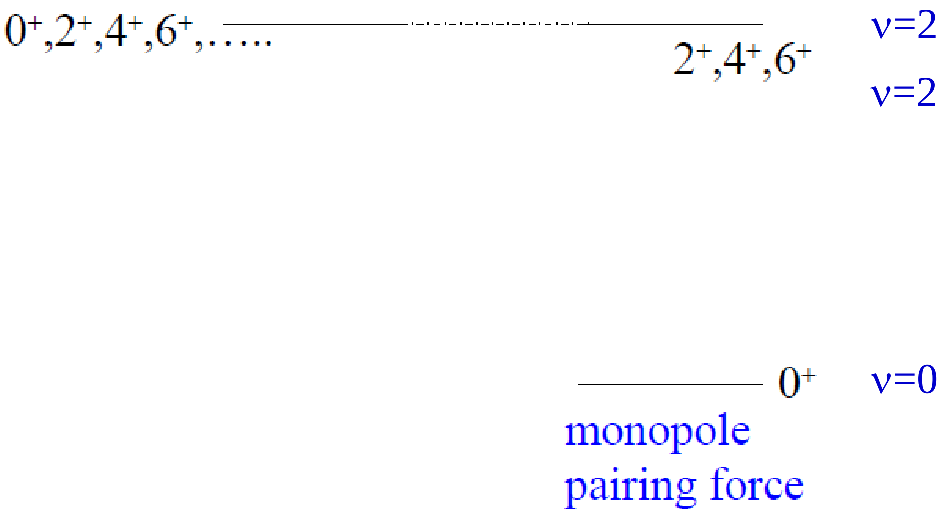


Seniority-Pairing: 1943 Racah

Pairing force: Seniority

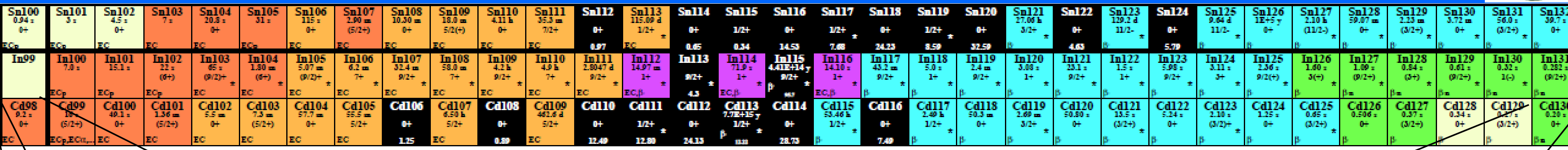
Seniority ν is a number of unpaired nucleons. (pairs are usually coupled to J=0)

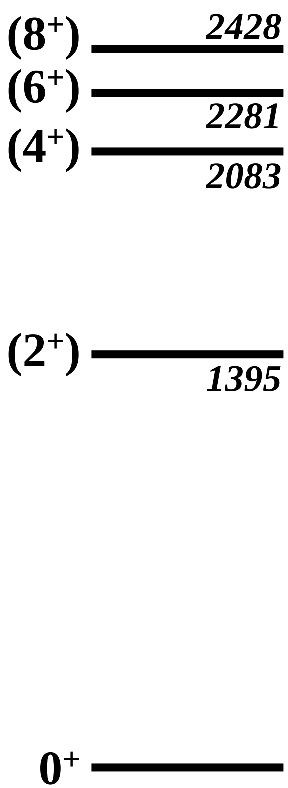
- A large spin-orbit splitting (***magic nuclei***) leads to a jj-coupling scheme.
- The pairing interaction between two nucleons in a j-subshell is only for ν=0 and J=0 different from zero.

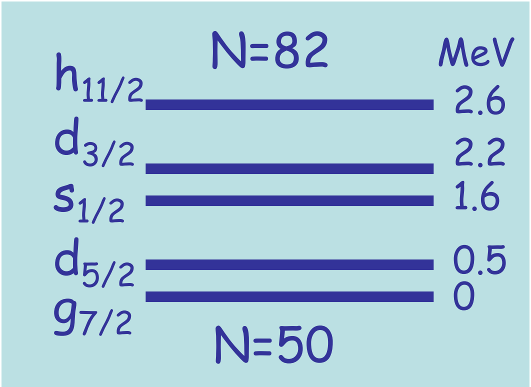


- The δ-interaction explains the resulting seniority-spectra in a simple geometrical picture.

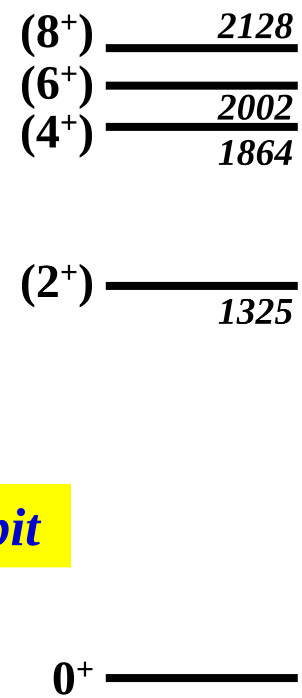
$8^+(g_{9/2})^{-2}$ seniority isomers in ^{98}Cd and ^{130}Cd







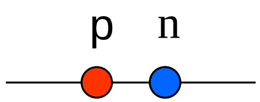
participating N-orbitals



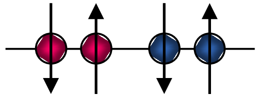
two proton holes in the $g_{9/2}$ orbit

No dramatic shell quenching!

Symmetries in nuclear physics



Isospin Symmetry: 1932 Heisenberg SU(2)



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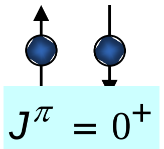
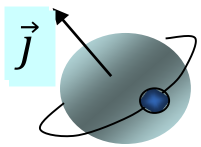
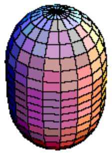
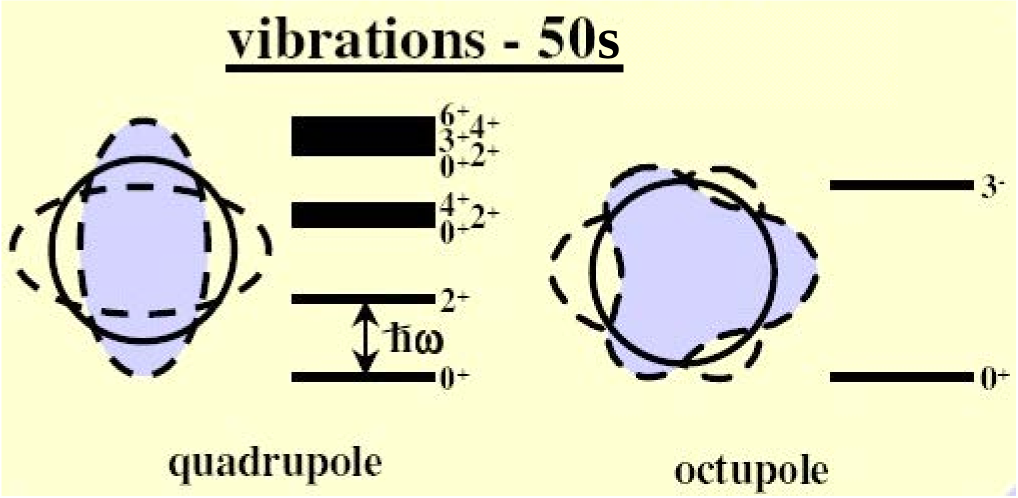


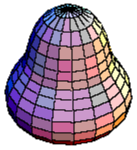
Seniority-Pairing: 1943 Racah

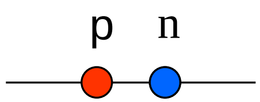


Spherical Symmetry: 1949 Mayer









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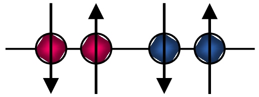

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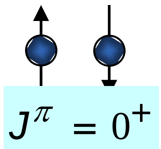

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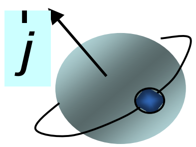

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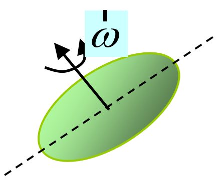


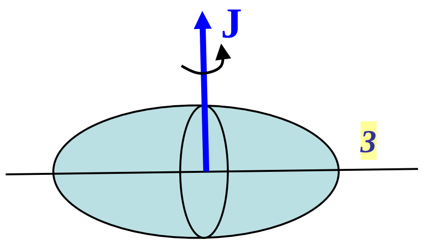
Deformed nuclear field (spontaneous symmetry breaking)
symmetry restoration → rotational spectra:
1952 Bohr-Mottelson

SU(3) dynamical Symmetry: 1958 Elliott
bridge between the spherical shell model and the liquid drop model

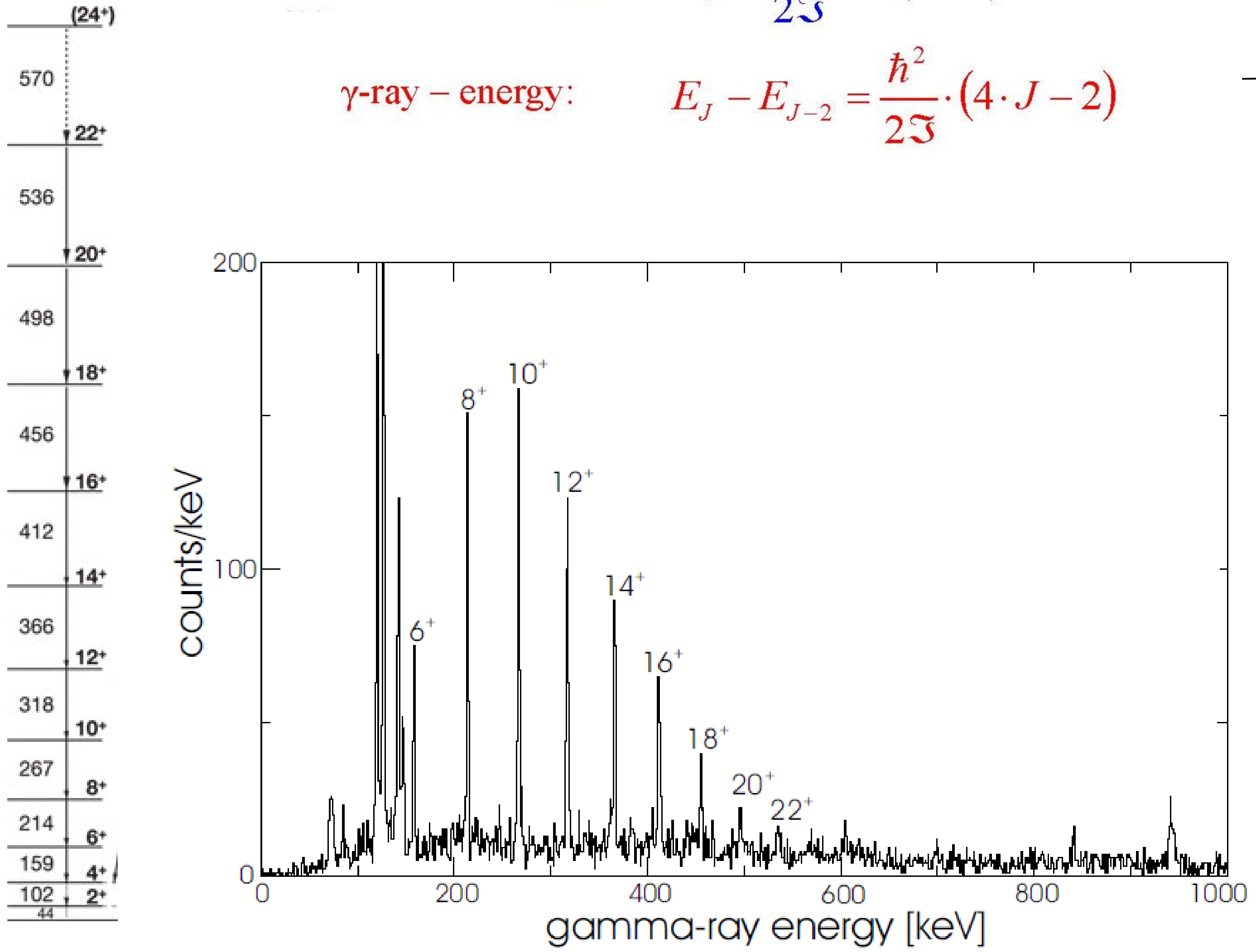
Rotational spectrum of ^{254}No

rotational energy: $$E_J = \frac{\hbar^2}{2\Im} \cdot J \cdot (J+1)$$

γ-ray – energy: $$E_J - E_{J-2} = \frac{\hbar^2}{2\Im} \cdot (4 \cdot J - 2)$$

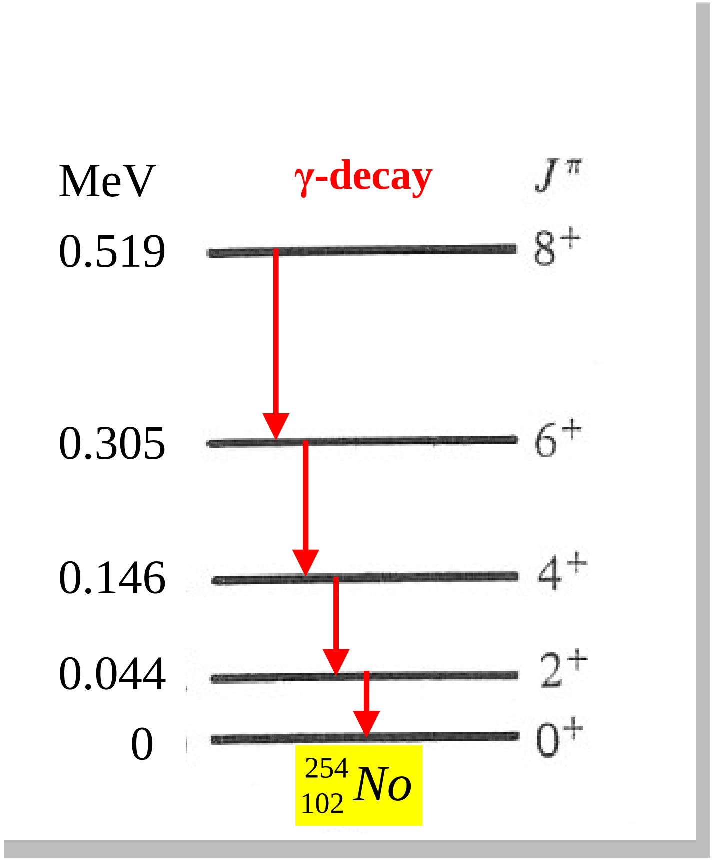


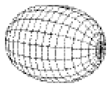
states with projections
K and –K are degenerated

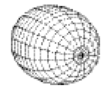


S. Eeckhaudt et al., Eur. Phys. J. A 26, 227 (2005)

Rotational invariance

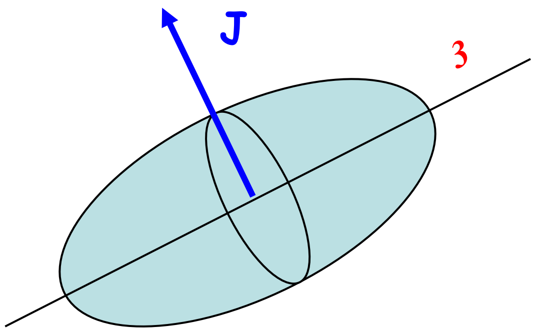


$|\Psi\rangle = |$  $\rangle$

$R(\omega)|\Psi\rangle = |$  $\rangle$

$$R(\omega)|\Psi\rangle \neq |\Psi\rangle$$

Broken symmetries are restored for the wave function in the laboratory frame.

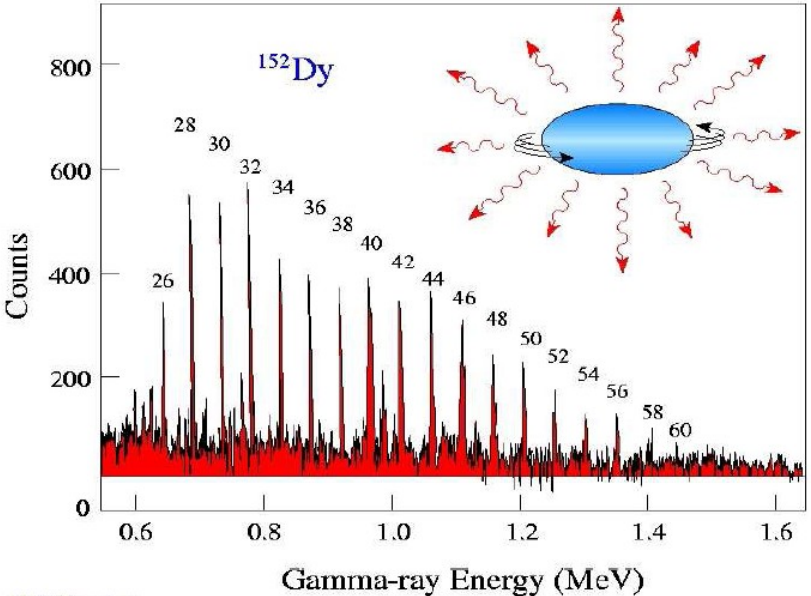


Notice – larger $\Im$ means smaller distances between the energy levels!

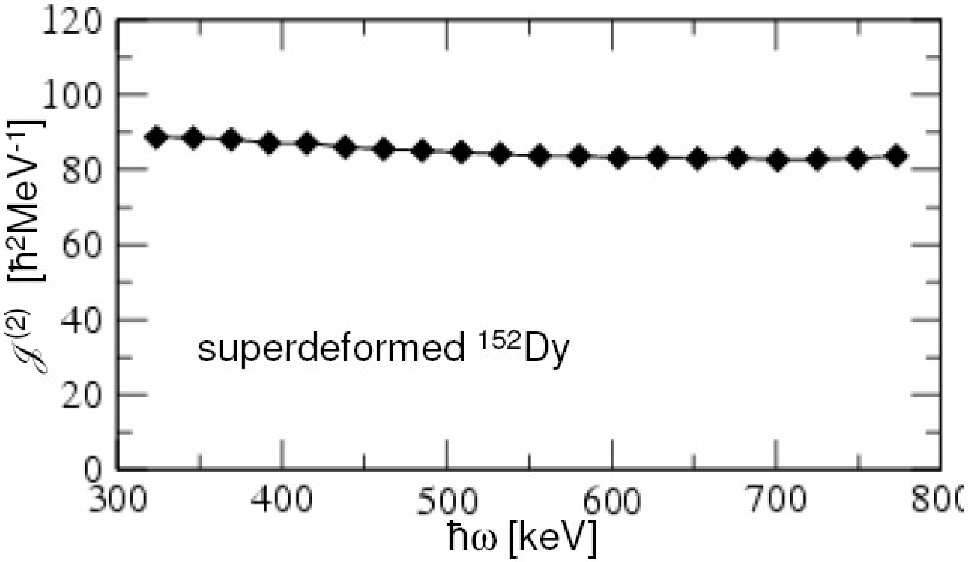
Notice - rotations around the symmetry axis 3 are indistinguishable;
the angular momentum has to be perpendicular to the symmetry axis 3.

$$\Im = \int r^2 \, dm$$

Superdeformation of ^{152}Dy

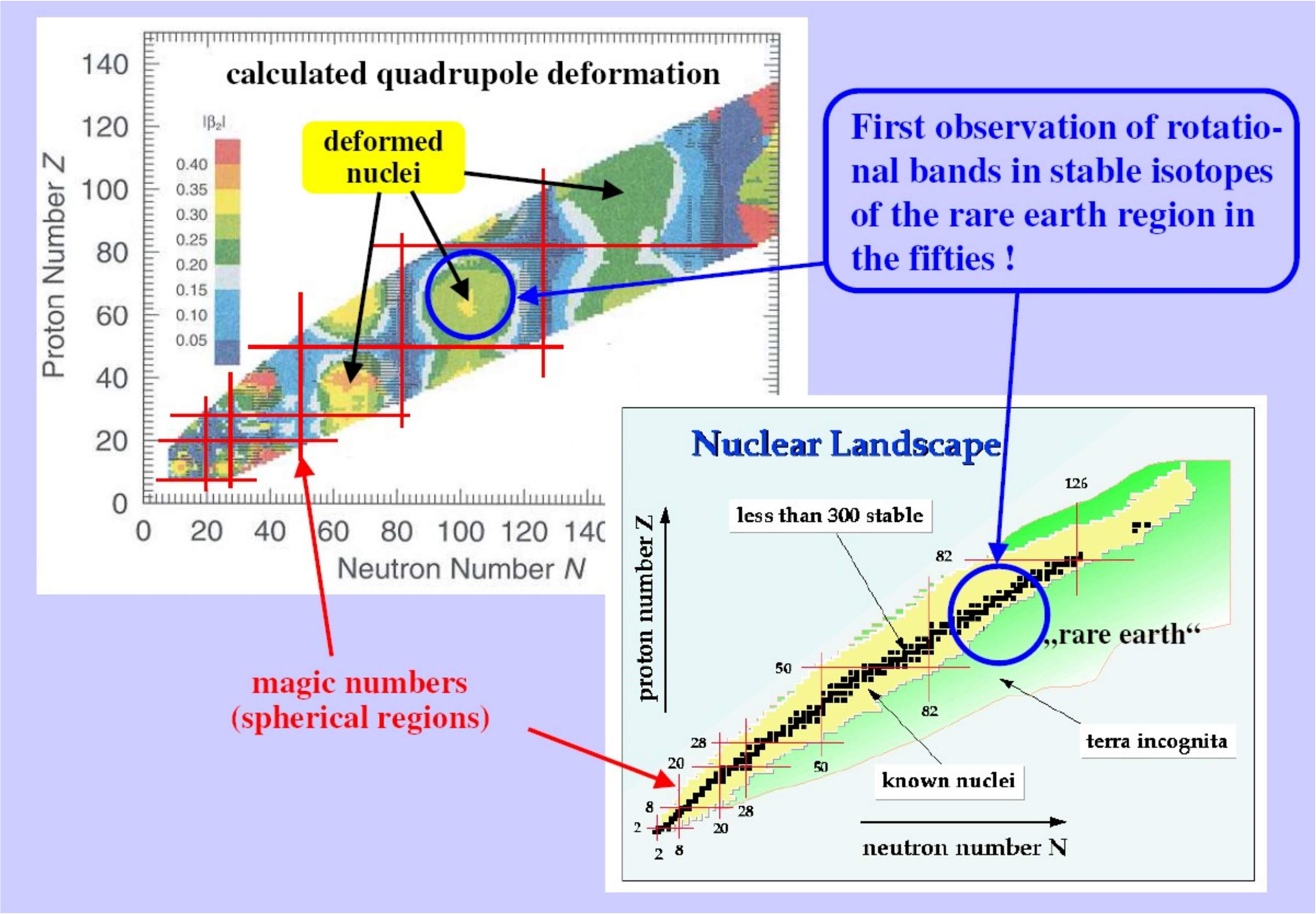


P. Twin et. al
Phys. Rev. Lett. 57 (1986)

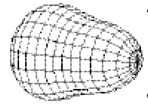


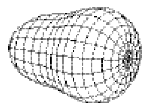
moment of inertia → deformation β=0.6
axis ratio 2:1

Nuclear deformation and rotations

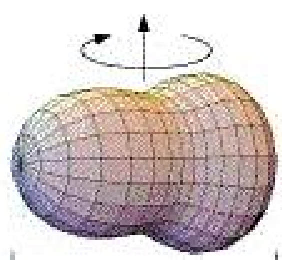


Space inversion invariance: octupole deformed nuclei

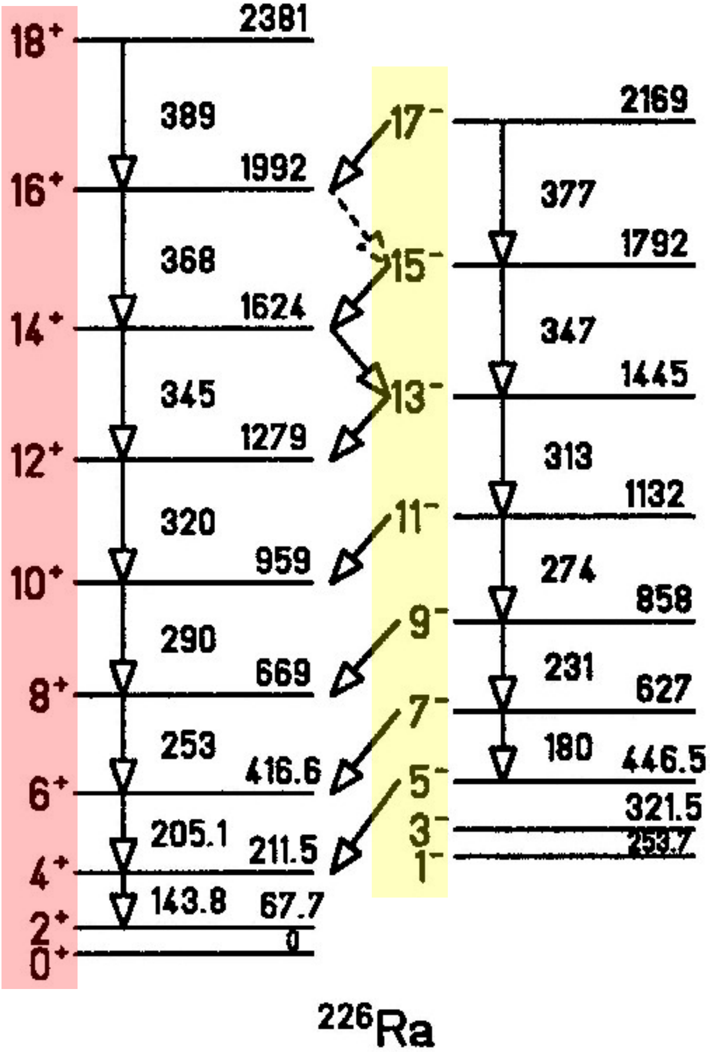
$|\Psi\rangle = |$  $\rangle$

$P|\Psi\rangle = |$  $\rangle$

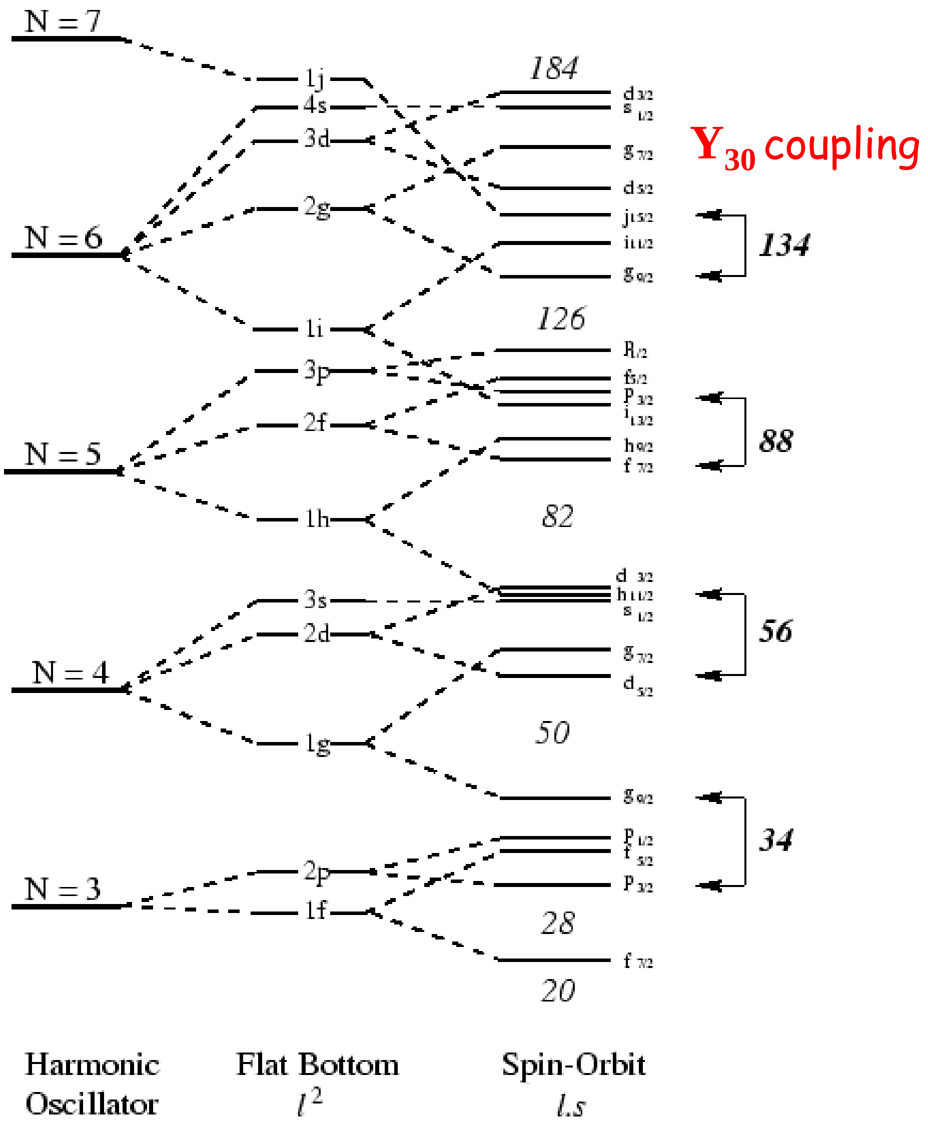
$$P|\Psi\rangle \neq |\Psi\rangle$$



Rotation

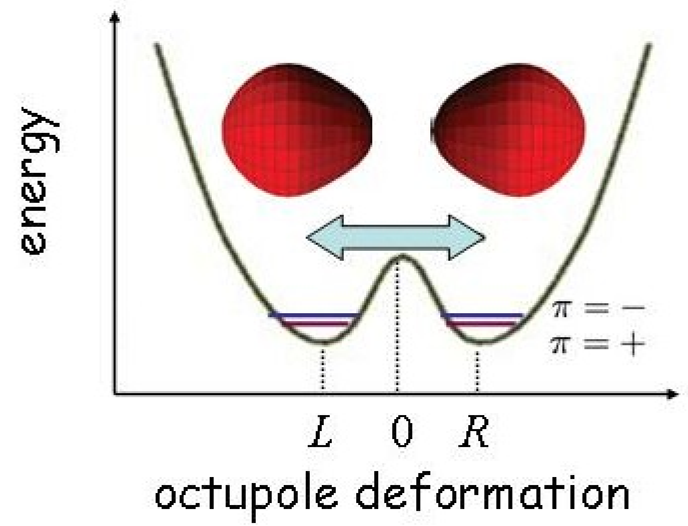


Space inversion invariance: octupole deformed nuclei



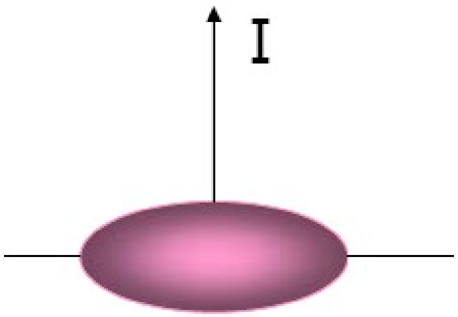
Search of electric dipole moments

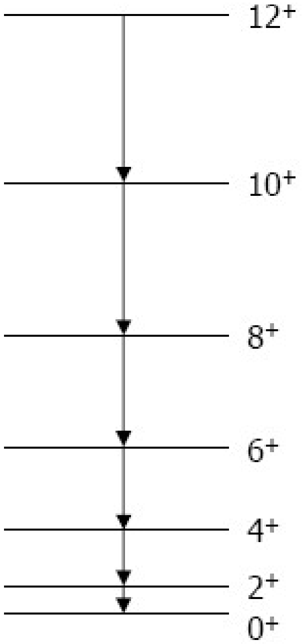
(violation of the time reversal)

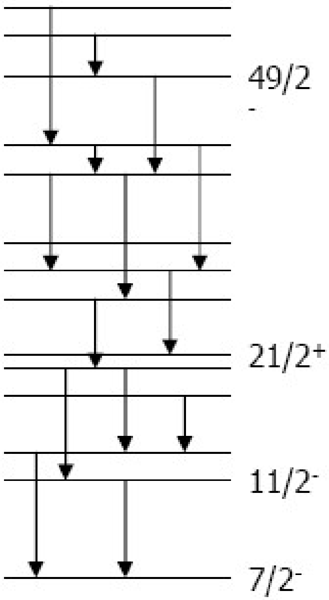


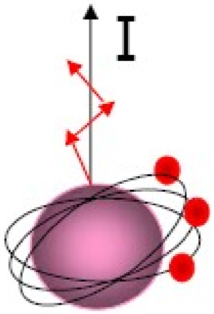
In **octupole deformed** nuclei the center of mass and charge are separated which yields a non-vanishing **electric dipole moment**.

Creation of angular momenta in nuclei









$I = J\omega$ $\quad {}^{156}\mathrm{Dy}$ $\quad {}^{147}\mathrm{Gd}$ $\quad I = \Sigma\, I_j$

$E_I = \hbar^2/2J\ I(I+1)$ $\quad E_I = \Sigma\, e_j + \Sigma\Sigma\, V_{jk}$

B(E2) ~ 200 W.U.

deformed nucleus $\quad$ spherical nucleus

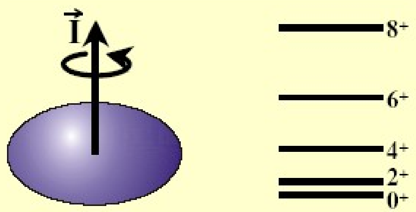
Creation of angular momenta in nuclei

collective models

single-particle models

SU(3)

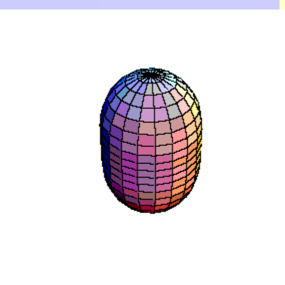
rotations – 50er Jahre



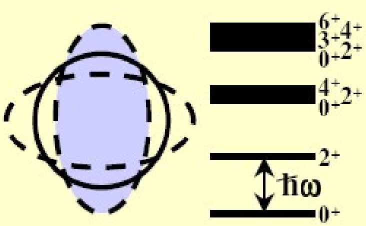
ee nucleus

$E_x = I(I+1)\frac{\hbar^2}{2J}$

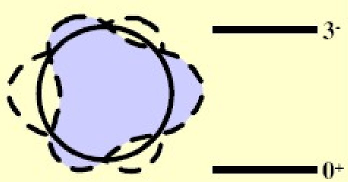
U(5)



vibrations - 50er Jahre

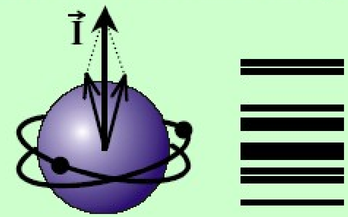


quadrupole



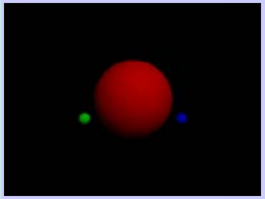
octupole

shell model - 1949



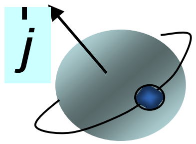
spins and magn. moments
of ground states

SU(2)

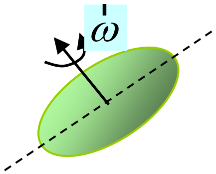


$\omega_{rot} \sim \omega_{vib} \sim \omega_{SP}$

Symmetries in nuclear physics

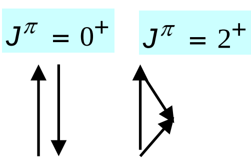


Spherical Symmetry: 1949 Mayer

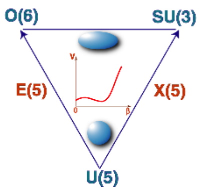


Deformed nuclear field (spontaneous symmetry breaking)
symmetry restoration → rotational spektra:
1952 Bohr-Mottelson

SU(3) dynamical Symmetry: 1958 Elliott

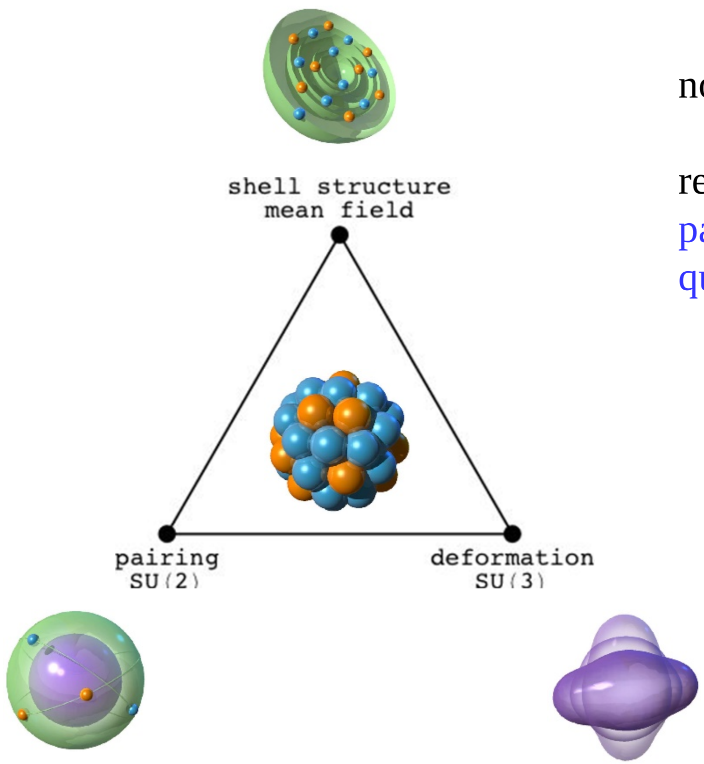


Interacting Boson Model (IBM dynamical symmetry):
1974 Arima and Iachello



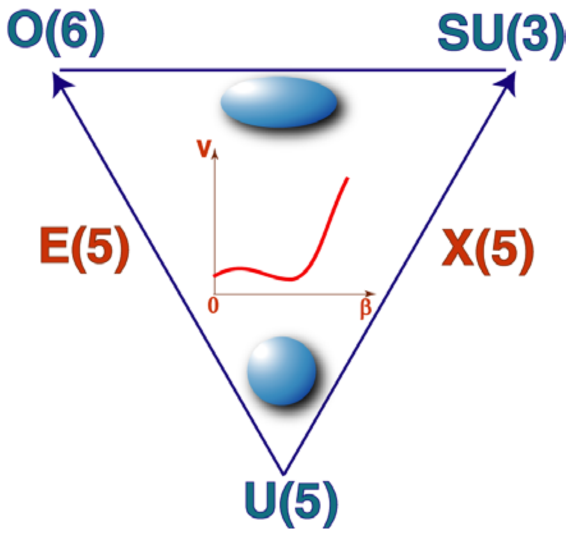
Critical point symmetry E(5), X(5) ….
2000… F. Iachello

Symmetries in nuclear physics

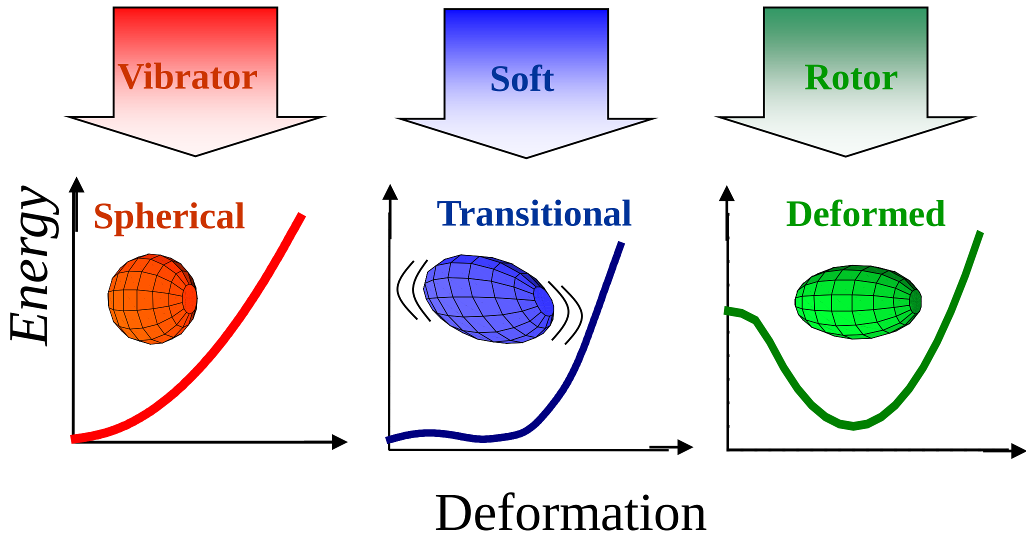


no residual interaction ⇒ independent particle shell model

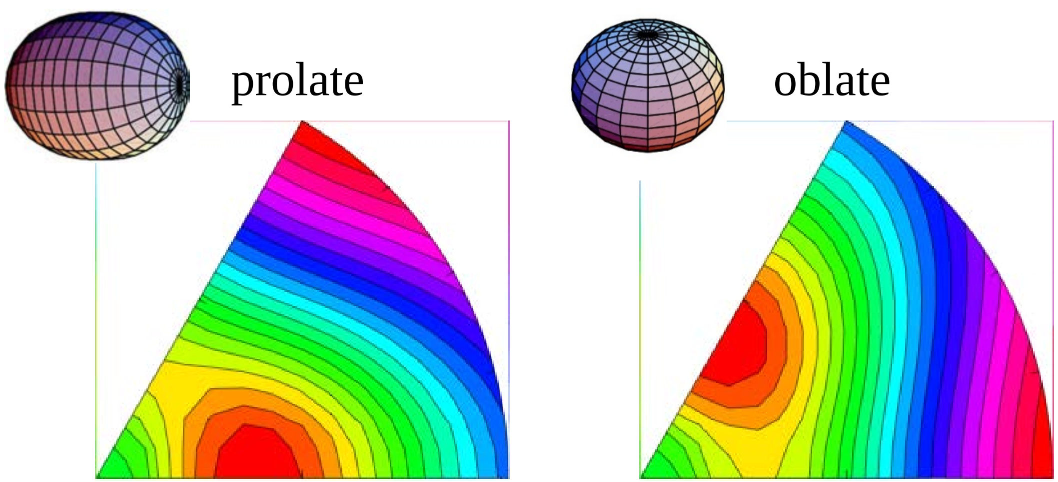
residual interaction:
pairing interaction (jj coupling) ⇒ Racah´s SU(2)
quadrupole interaction (LS coupling) ⇒ Elliott´s SU(3)

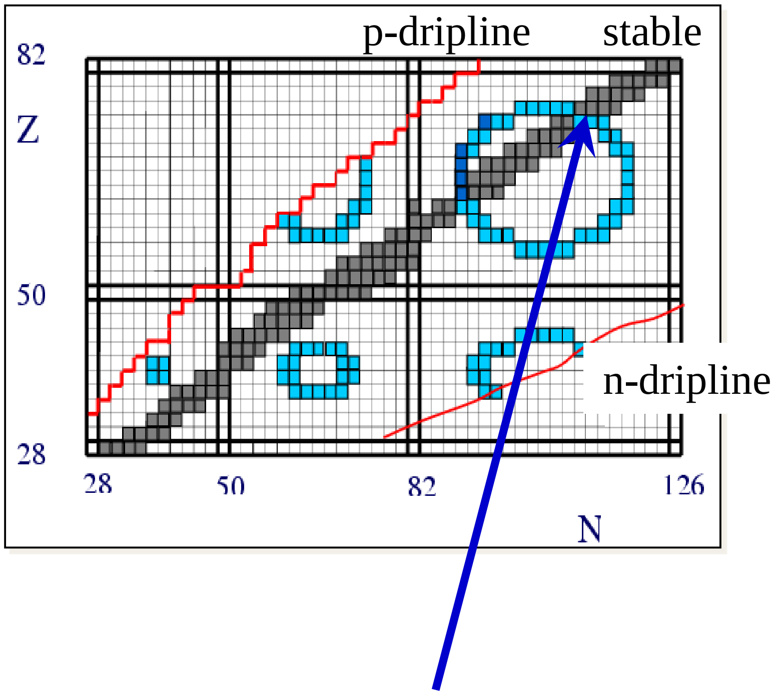


Nuclear shapes and symmetries



nuclei with X(5) symmetry: $P = \frac{N_p \cdot N_n}{N_p + N_n} \sim 5$





Transitional nuclei

R. F. Casten Nature Physics 2 (2006) 811

Dynamical symmetries in nuclear physics

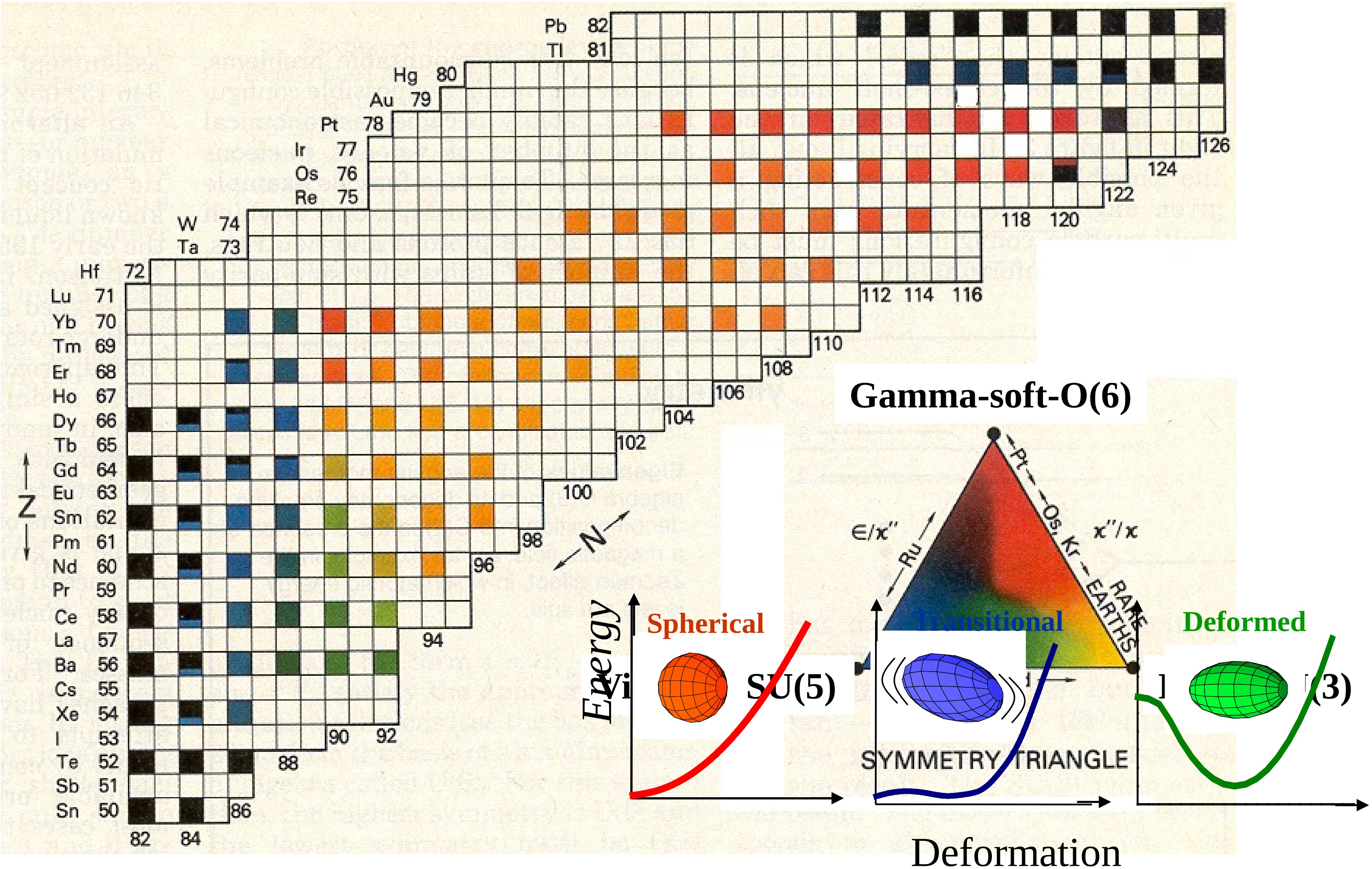


Chart of the Nuclei mirror nuclei and the nuclear shell model