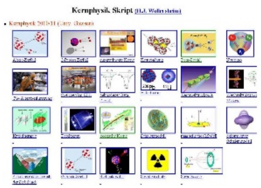
Outline: Nuclear shell model

Lecturer: Hans-Jürgen Wollersheim

e-mail: h.j.wollersheim@gsi.de

web-page: https://web-docs.gsi.de/~wolle/ and click on 

1. structure of nuclear force
2. Fermi gas model
3. shell structure in nuclei
4. Woods-Saxon potential with ℓs-coupling
5. success of extreme single particle shell model

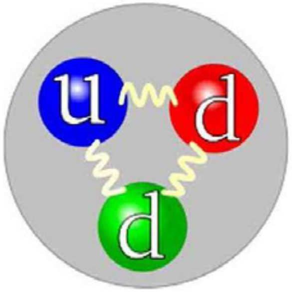
Nuclear shell model

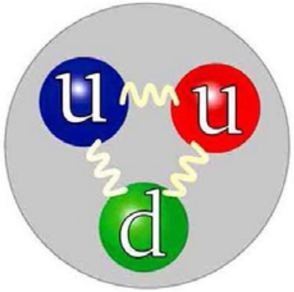
The nucleon building blocks...

Name	up quark (u)	down quark (d)
mass (MeV)	1.7 – 3.1	4.1 – 5.7
charge (e)	+2/3	-1/3
spin	1/2	1/2

The nuclear building blocks...

Name	neutron	proton
mass (MeV)	939.565378(21)	938.272046(21)
charge (e)	0	1
constituents	$2d + 1u$	$2u + 1d$
I^π	$1/2^+$	$1/2^+$



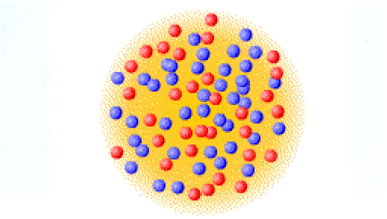


Themes and challenges in modern science

➢ Complexity out of simplicity – Microscopic

How the world, with all its apparent complexity and diversity can be constructed out of a few elementary building blocks and their interactions

individual excitations of nucleons

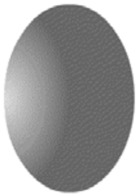


➢ Simplicity out of complexity – Macroscopic

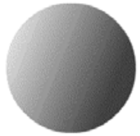
How the world of complex systems can display such remarkable regularity and simplicity

vibration

rotation



fission

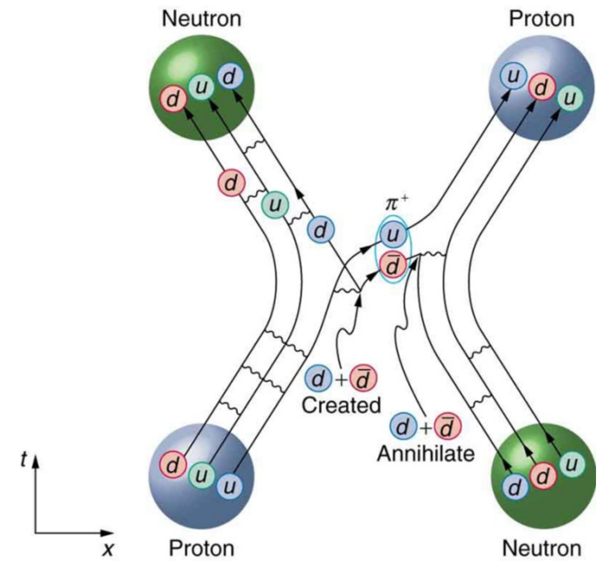


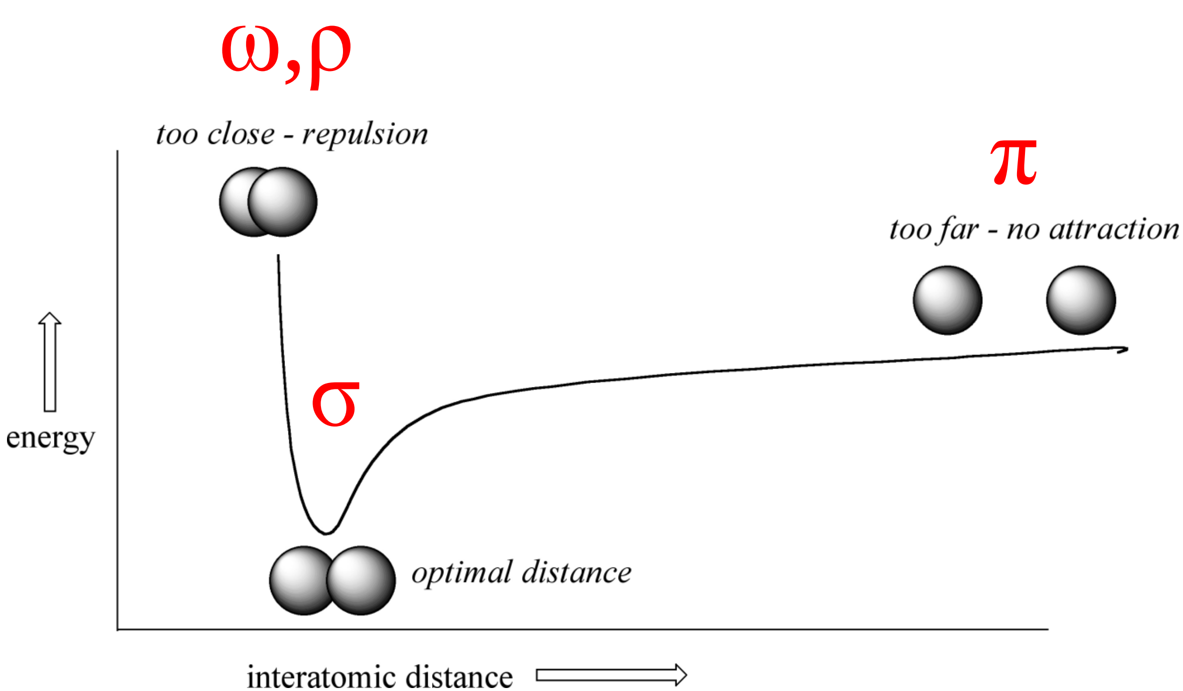
The nuclear force

The nuclear force is short-range (nuclear mass), but does not allow for compression of nuclear matter.

Yukawa – potential:

$$V_0(r) = g_s \cdot \frac{1}{r} \cdot e^{-\left(\frac{m_\pi c}{\hbar}\right) \cdot r}$$



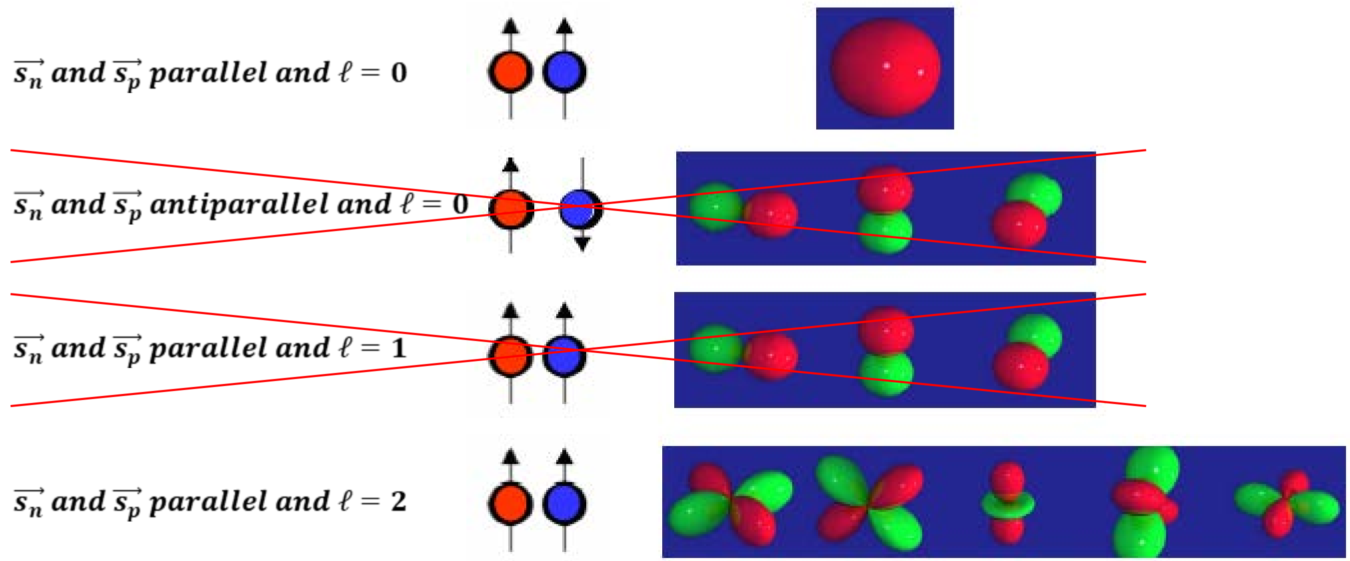


m(π) ≈ 140 MeV/c²
m(σ) ≈ 500-600 MeV/c²
m(ω) ≈ 784 MeV/c²

Deuteron

Possible combinations of the spins and the relative orbital angular momenta:

The nuclear force is spin dependent !



The measured nuclear spin of the deuteron is J=1 $\langle \vec{J}^2 \rangle = J \cdot (J+1)\hbar^2$ ***experiment !***

Parity of the deuteron:
Properties of the emitted gamma radiation following neutron capture on proton results, that the parity of the deuteron is positive ($\pi = +1$).
Properties of the spherical harmonics for the parity are $(-1)^\ell = +1$ and hence ***experiment !***
only even angular momenta of $\ell = 0$ and $\ell = 2$ can occur.

Deuteron: magnetic moment

- The measured ***nuclear spin*** of the deuteron is ***J = 1***
- The ***parity*** of the deuteron is ***positive***, only even ***angular momenta*** $\ell = 0$ and $\ell = 2$.
- The ***magnetic moment*** of the deuteron, which can be determined by e.g. nuclear magnetic resonance (NMR), results to: $\mu = 0.8574 \cdot \mu_K$

The ***gyromagnetic factor g*** is the proportionality constant between the ***magnetic moment*** of a particle and the ***spin*** (in case of the deuteron angular momentum g = 1):

$$\vec{\mu} = g \cdot \frac{e \cdot \hbar}{2 \cdot m_p \cdot c} \cdot \vec{S} \equiv g \cdot \mu_K \cdot \vec{S}$$

$\vec{S}$ denotes the spin operator and $\mu_K = \frac{e \cdot \hbar}{2 \cdot m_p \cdot c}$ the nuclear magneton

For a point-like proton (s=1/2) one expects g = 2. The inner structure of a proton (uud) and neutron (udd) shows in the experimental values

$$g_s^{proton} = 5.5857, \quad g_s^{neutron} = -3.8261$$

For a parallel alignment of the nucleon spin ***S = 1*** and an assumed angular momentum of $\ell = 0$ or $\ell = 2$ one obtains for the sum of the magnetic moment for proton and neutron

$$\mu_{deuteron}^{J,\ell,S} = \frac{\mu_N}{4 \cdot J \cdot (J+1)} \{ (g_s^{proton} + g_s^{neutron}) \cdot (J \cdot (J+1) - \ell \cdot (\ell+1) + S \cdot (S+1)) + J \cdot (J+1) + \ell \cdot (\ell+1) - S \cdot (S+1) \}$$

$$\mu_{deuteron}^{J=1,\ell=0,S=1} = 1/2 \cdot (g_s^{proton} + g_s^{neutron}) = 0.88$$

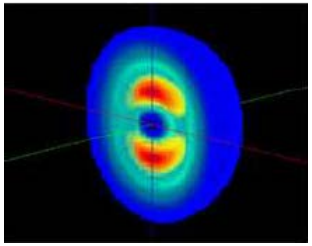
$$\mu_{deuteron}^{J=1,\ell=2,S=1} = 1/4 \cdot (3 - g_s^{proton} - g_s^{neutron}) = 0.31$$

The wave function of the deuteron consists of
96% of the $\ell = 0$ state and
4% of the $\ell = 2$ state

Deuteron: quadrupole moment

- The measured ***nuclear spin*** of the deuteron is ***J = 1***
- The ***parity*** of the deuteron is ***positive*, only even *angular momenta ℓ = 0*** and ***ℓ = 2*** .
- The ***magnetic moment*** of the deuteron yields to $\mu = 0.8574 \cdot \mu_K$ The angular momentum has to **4%** the value of ***ℓ = 2***
- The deuteron is ***not spherical***.

  It has an experimentally determined ***quadrupole moment*** of **Q = 0.00282 eb.**



A free neutron and a free proton have no electric quadrupole moment.

The deuteron can only possess a quadrupole moment because of its angular momentum of ***ℓ = 2***.

$$Q_{zz} = \int \rho(\vec{r}) \cdot r^2 \cdot (3 \cdot cos^2\theta - 1) d\tau$$

A pure $\ell = 0$ wave function has a vanishing quadrupole moment, because of its rotational symmetry.

The nuclear force is spin dependent !

The nuclear forces must raise a torsional moment, that depend on the radius r and the angle θ .

If the nuclear force depends on r and θ , then there is a non-central force component a Tensor force

Structure of the nuclear force

Structure of the nuclear force is more complex than e.g. Coulomb force. It results from its structure as residual interaction of the colorless nucleons.

central force $V_0(r)$

results from deuteron properties (96% $^3S_1$ state) $^{2S+1}L_J$

spin dependent central force

results from neutron-proton scattering (spin-spin interaction)

not central tensor force

results from deuteron properties (4% $^3D_1$ state) $^{2S+1}L_J$

spin-orbit (ℓ·s) term

results from scattering of polarized protons (left/right asymmetry)

$$V(r) = V_0(r)$$ central potential

$$+V_{ss}(r) \cdot \vec{s_1} \cdot \vec{s_2} \cdot \frac{1}{\hbar^2}$$ spin-spin interaction

$$+V_T(r) \cdot \frac{3}{\hbar^2} \frac{(\vec{s_1} \cdot \vec{x})(\vec{s_2} \cdot \vec{x})}{r^2} - \vec{s_1} \cdot \vec{s_2}$$ tensor force

$$+V_{\ell s}(r) \cdot (\vec{s_1} + \vec{s_2}) \cdot \vec{\ell} \cdot \frac{1}{\hbar^2}$$ spin-orbit interaction

Structure of the nuclear force

- spin-spin force:

$$\sim V_{ss}(r) \cdot \vec{s_1} \cdot \vec{s_2} \,/\hbar^2$$

different eigenvalues for triplet and singlet states

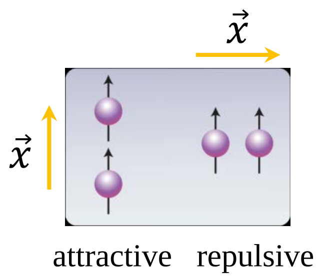
$$\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \qquad s = 0,\ \ell = 1$$

$$|\uparrow\uparrow\rangle \qquad \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right) \qquad |\downarrow\downarrow\rangle \qquad s = 1,\ \ell = 0$$

- tensor force:

$$\sim V_T(r) \cdot \frac{3}{\hbar^2} \frac{(\vec{s_1} \cdot \vec{x})(\vec{s_2} \cdot \vec{x})}{r^2} - \vec{s_1} \cdot \vec{s_2}$$

small deformation of deuterium
maximum magnetic dipole moments

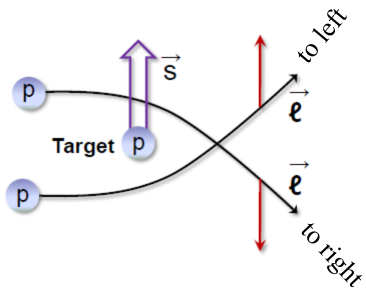


- $\ell \cdot s$ coupling:

$$\sim V_{\ell s}(r) \cdot (\vec{\ell} \cdot \vec{s})$$

scattering of protons on polarized protons
asymmetry of counting rates

- left scattering: $\vec{\ell} \cdot \vec{s} > 0$
- right scattering: $\vec{\ell} \cdot \vec{s} < 0$

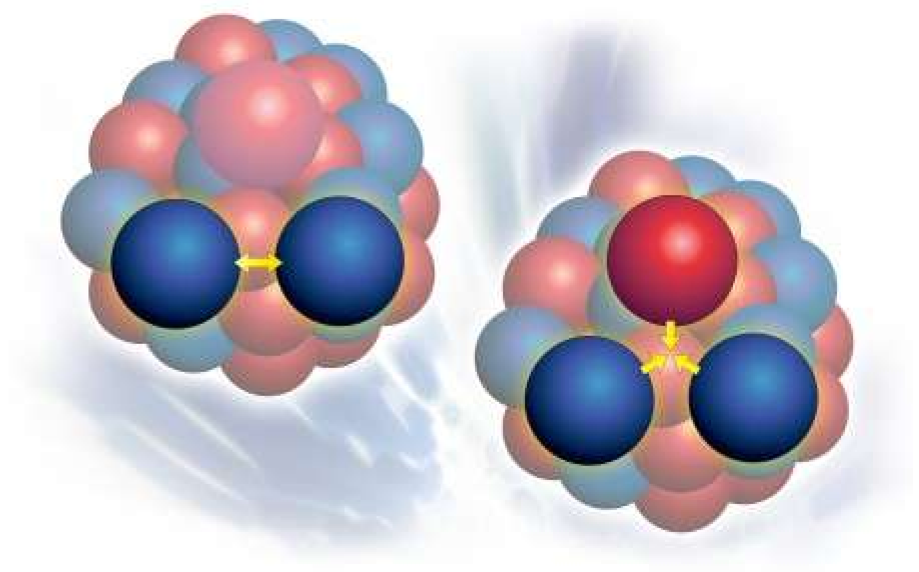


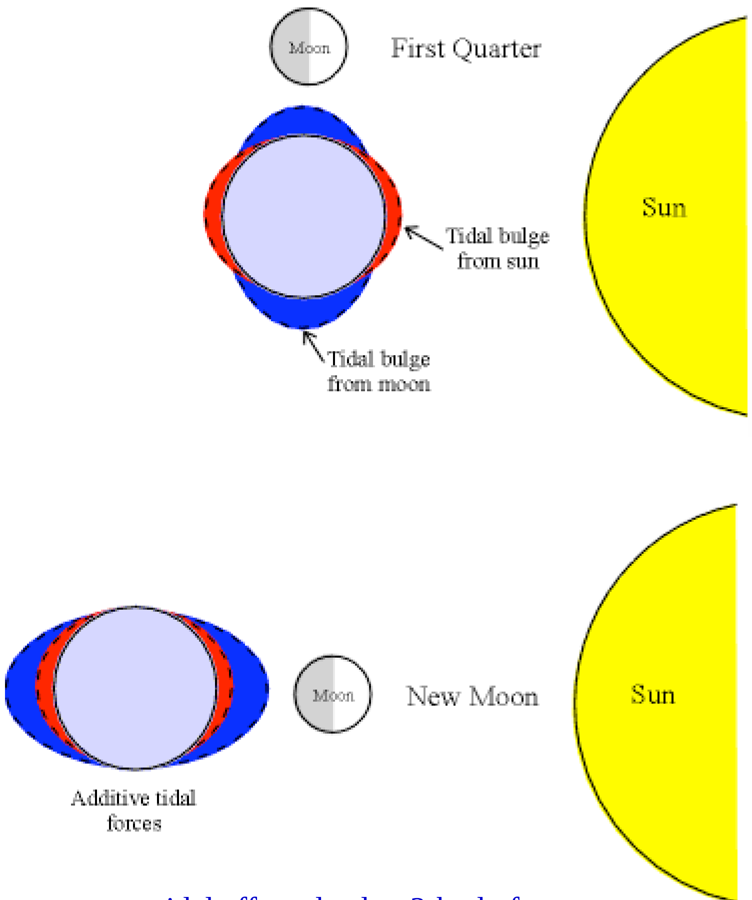
$\ell \cdot s$ coupling:

- no net contribution in the center of nucleus
- radial dependence at the surface of the nucleus

$$V_{\ell s}(r) \propto \frac{1}{r} \cdot \frac{d\rho}{dr}$$

Many-body forces



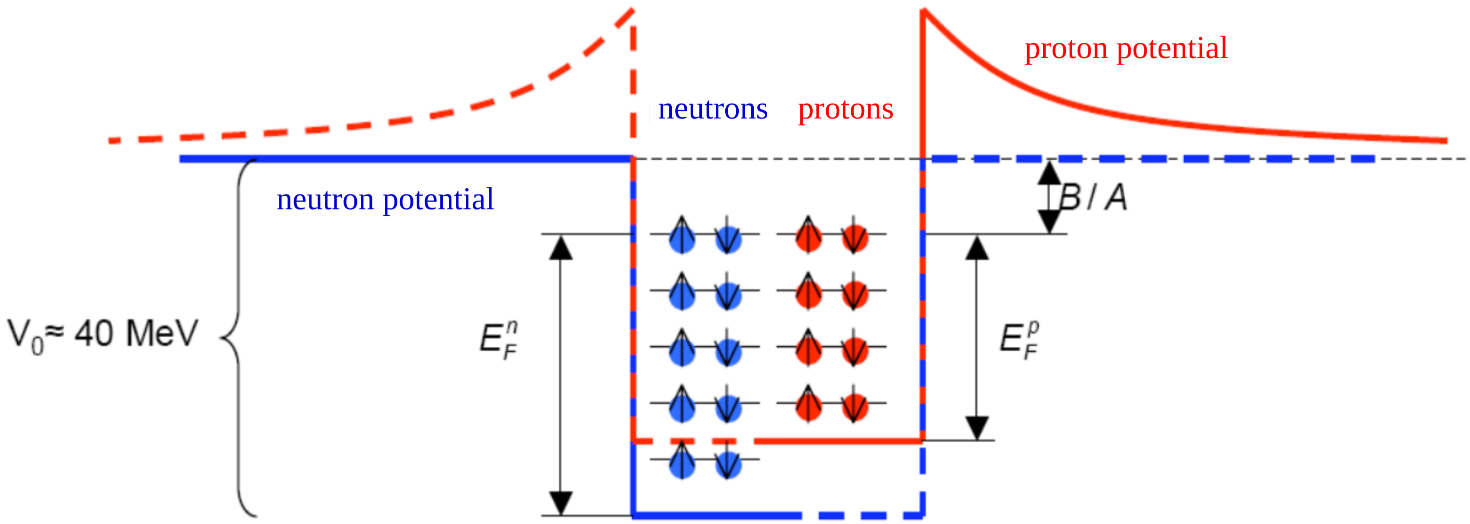


tidal effects lead to 3-body forces
in earth-sun-moon system

The force on one nucleon does not only depend on the position of the other nucleons, but also on the distance between the other nucleons! These are called many-body forces.

<u>Remember:</u> Nucleons are finite-mass composite particles, can be excited to resonances. Dominant contribution Δ(1232 MeV)

The Fermi gas model



- The Fermi gas model assumes that protons and neutrons are moving freely within the nuclear volume. They are distinguishable fermions (s = ½) filling two separate potential wells obeying the Pauli principle (↑↓-pair).
- The model assumes that all fermions occupy the lowest energy states available to them to the highest occupied state (Fermi energy), and that there is no excitation across the Fermi energy (i.e. zero temperature).
- The Fermi energy is common for protons and neutrons in stable nuclei.
- If the Fermi energy for protons and neutrons are different then the β-decay transforms one type of nucleons into the other until the common Fermi energy (stability) is reached.

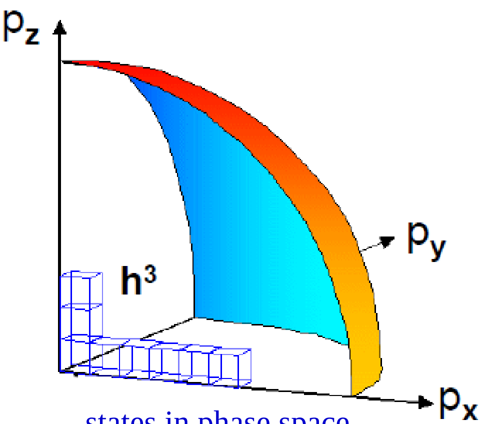
Number of nucleon states

Heisenberg Uncertainty Principle: $\Delta x \cdot \Delta p \geq \frac{1}{2}\hbar$

The volume of one particle in phase space: $2\pi \cdot \hbar$

The number of nucleon states in a volume V:

$$n = \frac{\iint d^3r\, d^3p}{(2\pi \cdot \hbar)^3} = \frac{V \cdot 4\pi \int_0^{p_{max}} p^2\, dp}{(2\pi \cdot \hbar)^3}$$



states in phase space

At temperature T = 0, i.e. for the nucleus in its ground state, the lowest states will be filled up to the maximum momentum, called the Fermi momentum p_F. The number of these states follows from integration from 0 to $p_{max} = p_F$.

$$n = \frac{V \cdot 4\pi \int_0^{p_F} p^2 dp}{(2\pi \cdot \hbar)^3} = \frac{V \cdot 4\pi \cdot p_F^3}{(2\pi \cdot \hbar)^3 \cdot 3} \quad \rightarrow \quad \boxed{n = \frac{V \cdot p_F^3}{6\pi^2\hbar^3}}$$

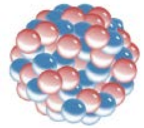
Since an energy state can contain two fermions of the same species, we can have

$$\textit{neutrons}: \quad N = \frac{V \cdot (p_F^n)^3}{3\pi^2\hbar^3} \qquad \textit{protons}: \quad Z = \frac{V \cdot \left(p_F^p\right)^3}{3\pi^2\hbar^3}$$

p_F^n is the Fermi momentum for neutrons, p_F^p for protons

Fermi momentum

Use $R = r_0 \cdot A^{1/3}\ fm$

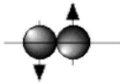


$$V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3} r_0^3 \cdot A$$

The density of nucleons in a nucleus = number of nucleons in a volume V:

$$n = 2 \cdot \frac{V \cdot p_F^3}{6\pi^2\hbar^3} = 2 \cdot \frac{4\pi}{3} r_0^3 \cdot A \cdot \frac{p_F^3}{6\pi^2\hbar^3} = \frac{4A}{9\pi} \frac{r_0^3 \cdot p_F^3}{\hbar^3}$$

two spin states



Fermi momentum p_F:

$$p_F = \left(\frac{6\pi^2\hbar^3 n}{2V}\right)^{1/3} = \left(\frac{9\pi\hbar^3}{4A}\frac{n}{r_0^3}\right)^{1/3} = \left(\frac{9\pi \cdot n}{4A}\right)^{1/3} \cdot \frac{\hbar}{r_0}$$

After assuming that the proton and neutron potential wells have the same radius, we find for a nucleus with n = Z = N = A/2 the Fermi momentum p_F.

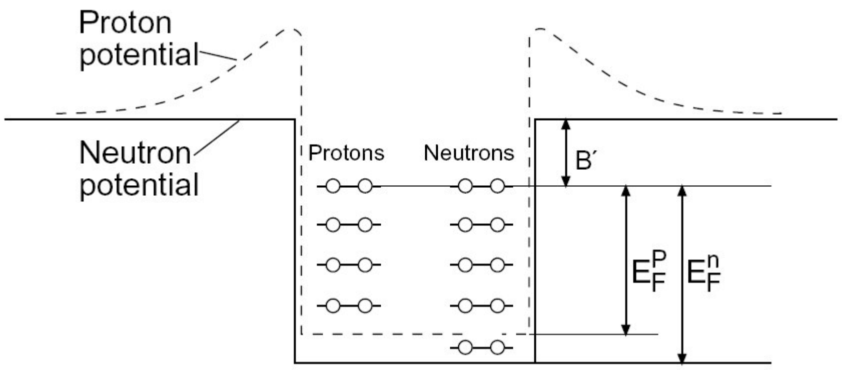
$$p_F = p_F^n = p_F^p = \left(\frac{9\pi}{8}\right)^{1/3} \cdot \frac{\hbar}{r_0} \approx 250\ MeV/c$$

The nucleons move freely inside the nucleus with large momenta

Fermi energy: $E_F = \frac{p_F^2}{2m_N} \approx 33\ MeV$

m_N = 938 MeV/c^2 – the nucleon mass

Nucleon potential



The difference B´ between the top of the well and the Fermi level is the average binding energy per nucleon B/A = 7 – 8 MeV.

→ The depth of the potential V_0 and the Fermi energy are independent of the mass number A:

$$V_0 = E_F + B' \approx 40\ MeV$$

Heavy nuclei have a surplus of neutrons. Since the Fermi level of the protons and neutrons in a stable nucleus have to be equal (otherwise the nucleus would enter a more energetically favorable state through β-decay) this implies that the depth of the potential well as it is experienced by the neutron gas has to be larger than of the proton gas.

Protons are therefore on average less strongly bound in nuclei than neutrons. This may be understood as a consequence of the Coulomb repulsion of the charged protons and leads to an extra term in the potential:

$$V_C = (Z-1)\frac{\alpha \cdot \hbar c}{R}$$

Protonen: 33MeV + 7MeV, Neutronen: 43MeV + 7 MeV

The Fermi gas model and the neutron star

Assumption: neutron star as cold neutron gas with constant density

- 1.5 sun masses: M = $3\cdot10^{30}$ kg ($m_N = 1.67\cdot10^{-27}$ kg), number of neutrons: $n = 1.8\cdot10^{57}$

Fermi momentum p_F for cold neutron gas:

$$p_F = \left(\frac{9\pi\cdot n}{4}\right)^{1/3}\cdot\frac{\hbar}{R}$$

R is the radius of the neutron star

Average kinetic energy per neutron:

$$\left\langle E_{kin}/N\right\rangle = \frac{3}{5}\cdot\frac{p_F^2}{2m_N} = \left(\frac{9\pi\cdot n}{4}\right)^{2/3}\cdot\frac{3\hbar^2}{10\cdot m_N}\cdot\frac{1}{R^2} = \frac{C}{R^2}$$

Gravitational energy of a star with constant density has an average potential energy per neutron:

$$\left\langle E_{pot}/N\right\rangle = -\frac{3}{5}\cdot\frac{G\cdot n\cdot m_n^2}{R} = -\frac{D}{R}$$

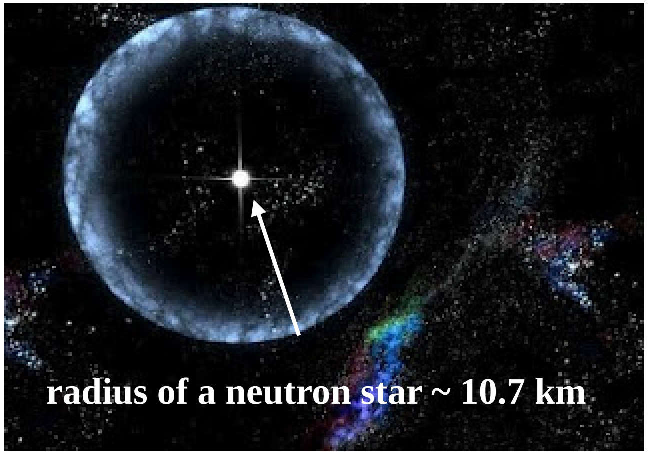
$$G = 6.67\cdot10^{-11}\ \frac{m^3}{kg\cdot s^2}$$

Minimum total energy per neutron:

$$\frac{d}{dR}\langle E/N\rangle = \frac{d}{dR}\left[\langle E_{kin}/N\rangle + \langle E_{pot}/N\rangle\right] = 0$$

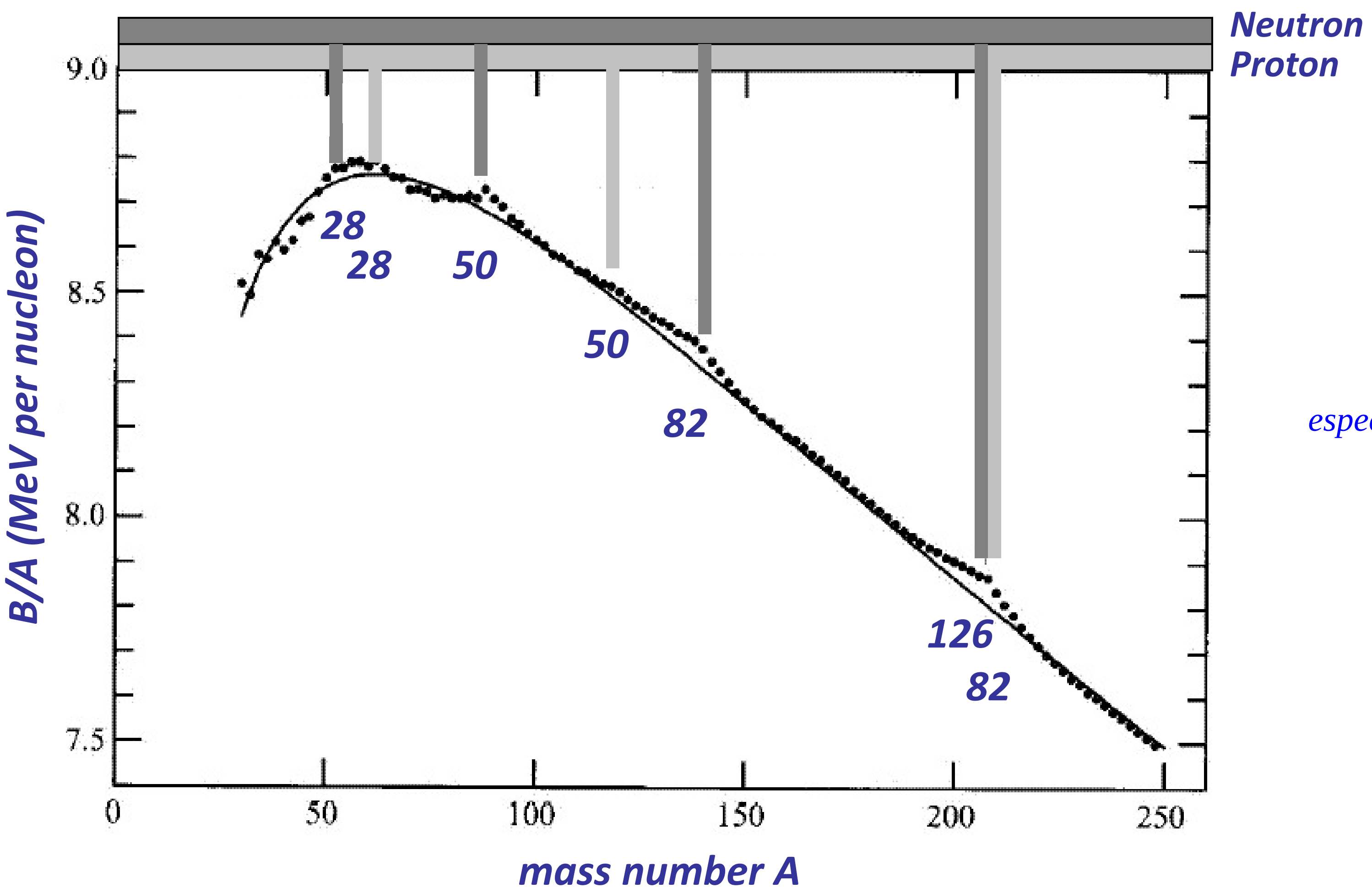
$$\frac{d}{dR}\left[\frac{C}{R^2}-\frac{D}{R}\right] = -\frac{2C}{R^3}+\frac{D}{R^2} = 0$$

$$R = \frac{2C}{D} \quad\rightarrow\quad R = \frac{\hbar^2\cdot(9\pi/4)^{2/3}}{G\cdot m_N^3\cdot n^{1/3}}$$



Shell structure in nuclei

Deviations from the Bethe-Weizsäcker mass formula:



especially stable:

$^{4}_{2}He_{2}$

$^{16}_{8}O_{8}$

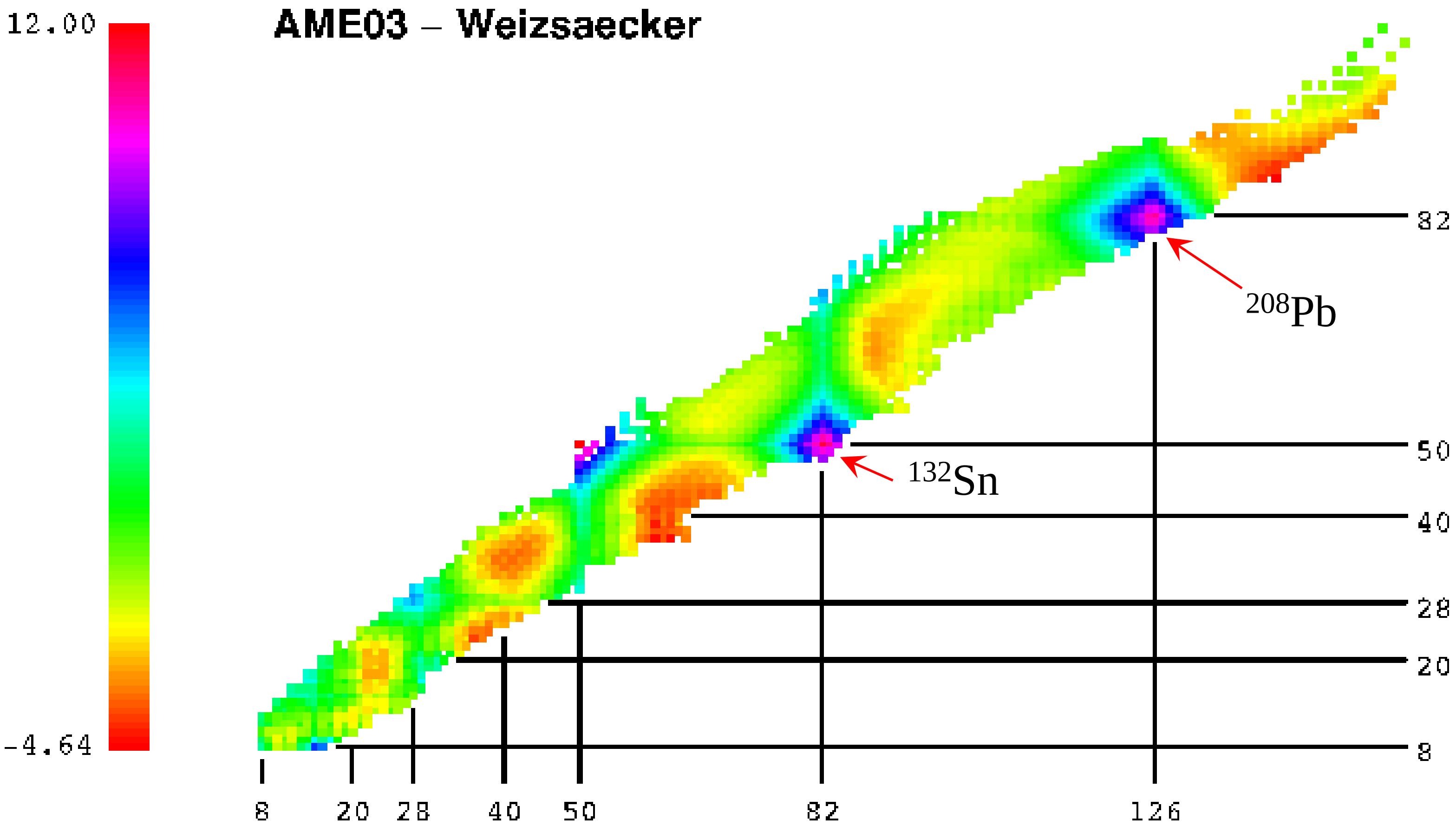
$^{40}_{20}Ca_{20}$

$^{48}_{20}Ca_{28}$

$^{208}_{82}Pb_{126}$

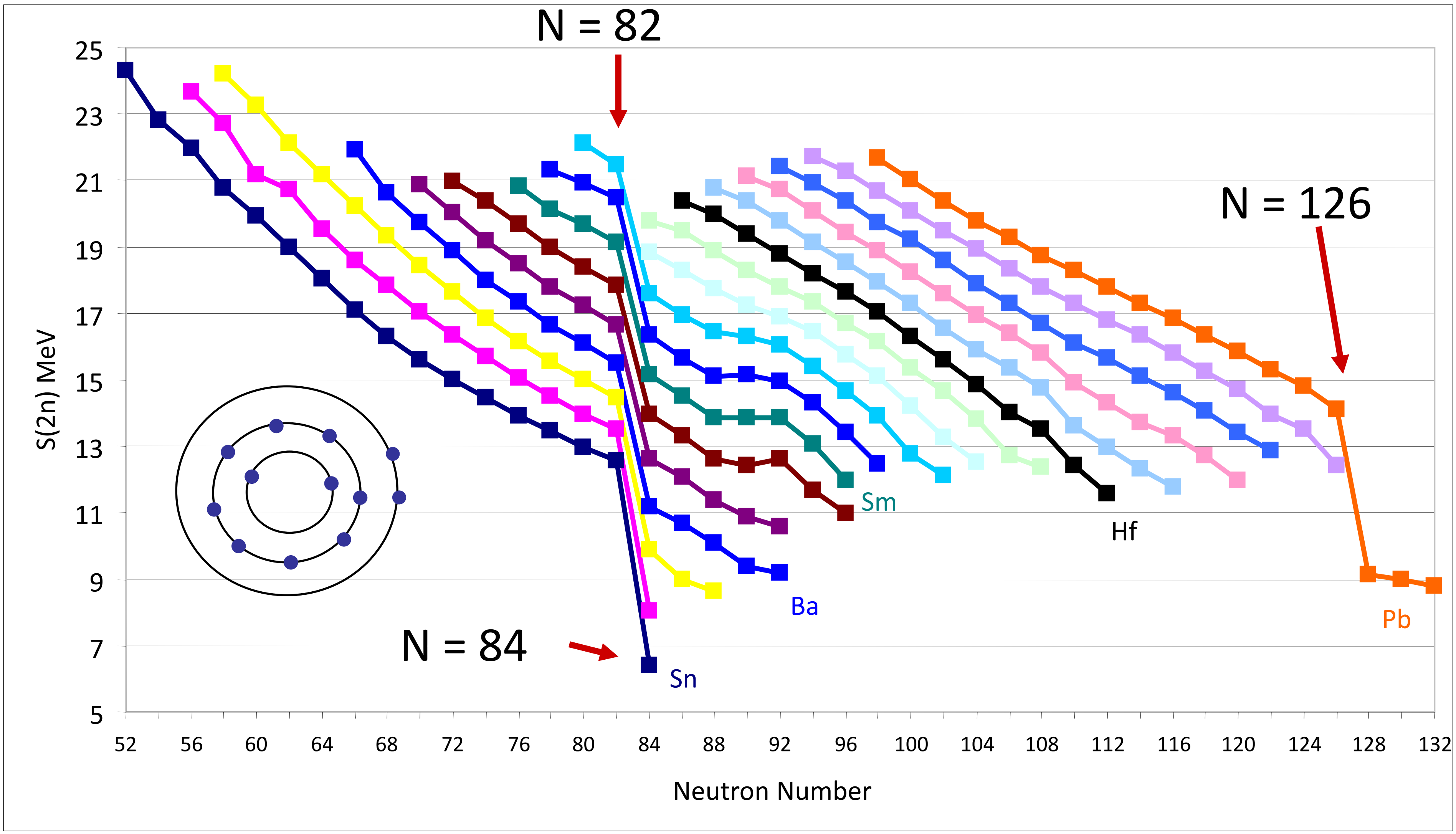
Shell structure in nuclei

- *deviations from the Bethe-Weizsäcker mass formula:* ***large binding energies***

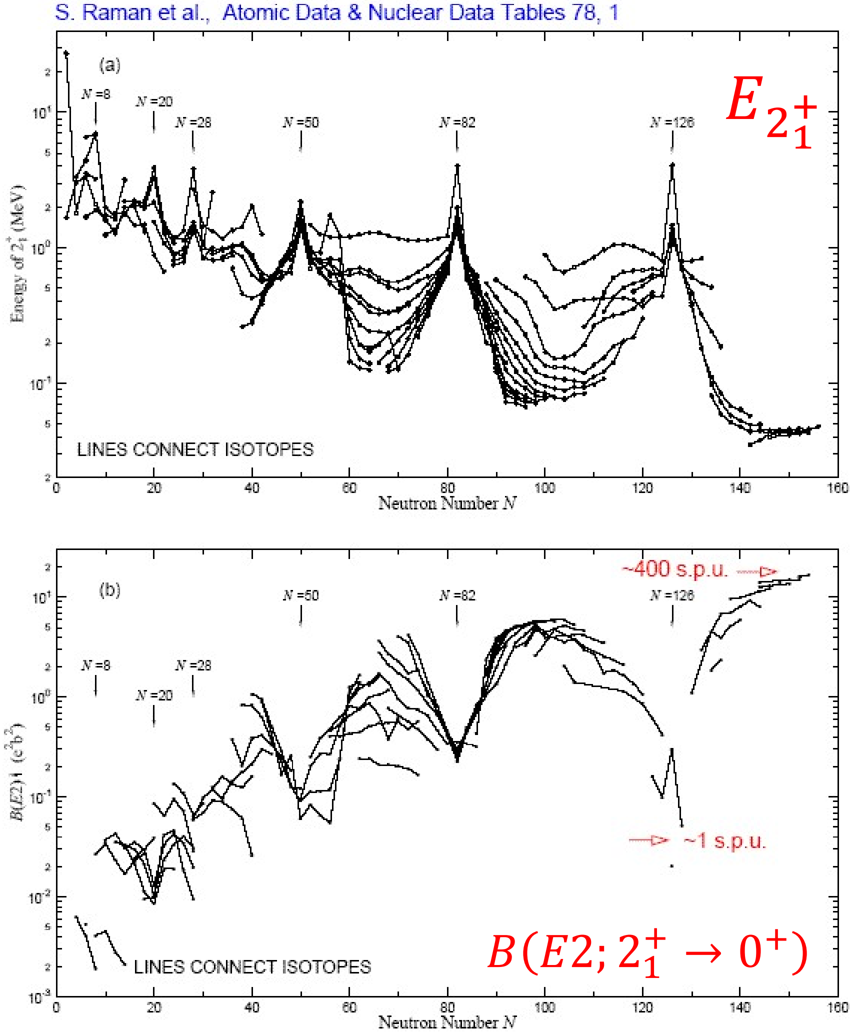


2-neutron binding energies = 2-neutron ´separation´ energies

$$S_{2n} = BE(N,Z) - BE(N-2,Z)$$



Shell structure in nuclei



Nuclei with magic numbers
of neutrons/protons

- ***high energies of the first excited 2^+ state***

- ***small nuclear deformations***

  transition probabilities measured in single particle units (spu)

Shell structure in nuclei

S. Raman et al., Atomic Data & Nuclear Data Tables 78, 1

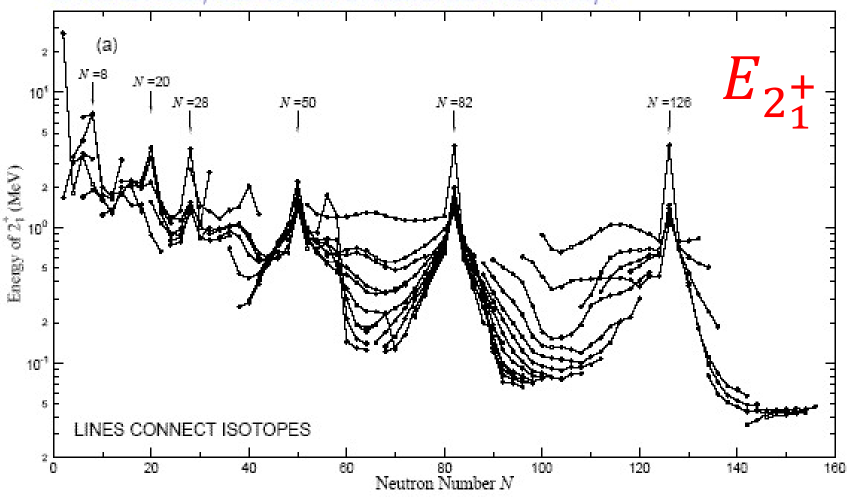


Table 1 -- Nuclear Shell Structure (from *Elementary Theory of Nuclear Shell Structure*, Maria Goeppert Mayer & J. Hans D. Jensen, John Wiley & Sons, Inc., New York, 1955.)

Angular Momentum ($h\Omega/2\pi$)		Spin-Orbit Coupling (1/2, 3/2, 5/2, 7/2...)	Number of Nucleons Shell	Total	Magic Number
7	1j	1j 15/2	16	[184]	{184}
		3d 3/2	4	[168]	
6	4s	4s 1/2	2	[164]	
6	3d	2g 7/2	8	[162]	
		1i 11/2	12	[154]	
6	2g	3d 5/2	6	[142]	
		2g 9/2	10	[136]	
6	1i				
		1i 13/2	14	[126]	{126}
		3p 1/2	2	[112]	
5	3p	3p 3/2	4	[110]	
		2f 5/2	6	[106]	
5	2f	2f 7/2	8	[100]	
		1h 9/2	10	[92]	
5	1h				
		1h 11/2	12	[82]	{82}
4	3s	3s 1/2	2	[70]	
		2d 3/2	4	[68]	
4	2d	2d 5/2	6	[64]	
		1g 7/2	8	[58]	
4	1g				
		1g 9/2	10	[50]	{50}
		2p 1/2	2	[40]	{40}
3	2p	1f 5/2	6	[38]	
		2p 3/2	4	[32]	
3	1f				
		1f 7/2	8	[28]	{28}
		1d 3/2	4	[20]	{20}
2	2s	2s 1/2	2	[16]	
2	1d	1d 5/2	6	[14]	
		1p 1/2	2	[8]	{8}
1	1p	1p 3/2	4	[6]	
0	1s	1s 1/2	2	[2]	{2}

Maria Goeppert-Mayer

J. Hans D. Jensen

Nuclear potential

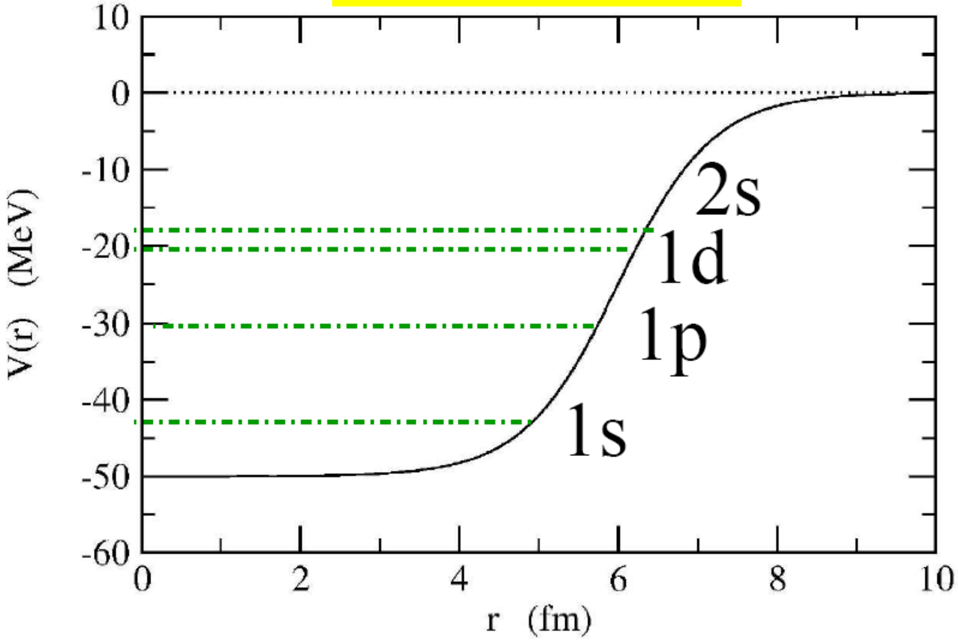
$$\hat{H} = \sum_{i=1}^{A} \frac{\hat{p}_i^2}{2m_i} + \sum_{i<j}^{A} \hat{V}(r_i, r_j)$$

$$\hat{H} = \sum_{i=1}^{A} \left[\frac{\hat{p}_i^2}{2m_i} + \hat{V}(r_i) \right] + \left[\sum_{i<j}^{A} \hat{V}(r_i, r_j) - \sum_{i=1}^{A} \hat{V}(r_i) \right]$$

$$\left[-\frac{\hbar^2}{2m} \nabla^2 + V(r) - \varepsilon \right] \Psi(r) = 0$$

$$\Psi(r) = \frac{u_\ell(r)}{r} \cdot Y_{\ell m}(\vartheta, \varphi) \cdot X_{m_s}$$

$$V(r) = \frac{-V_0}{1 + e^{(r-R_0)/a}}$$



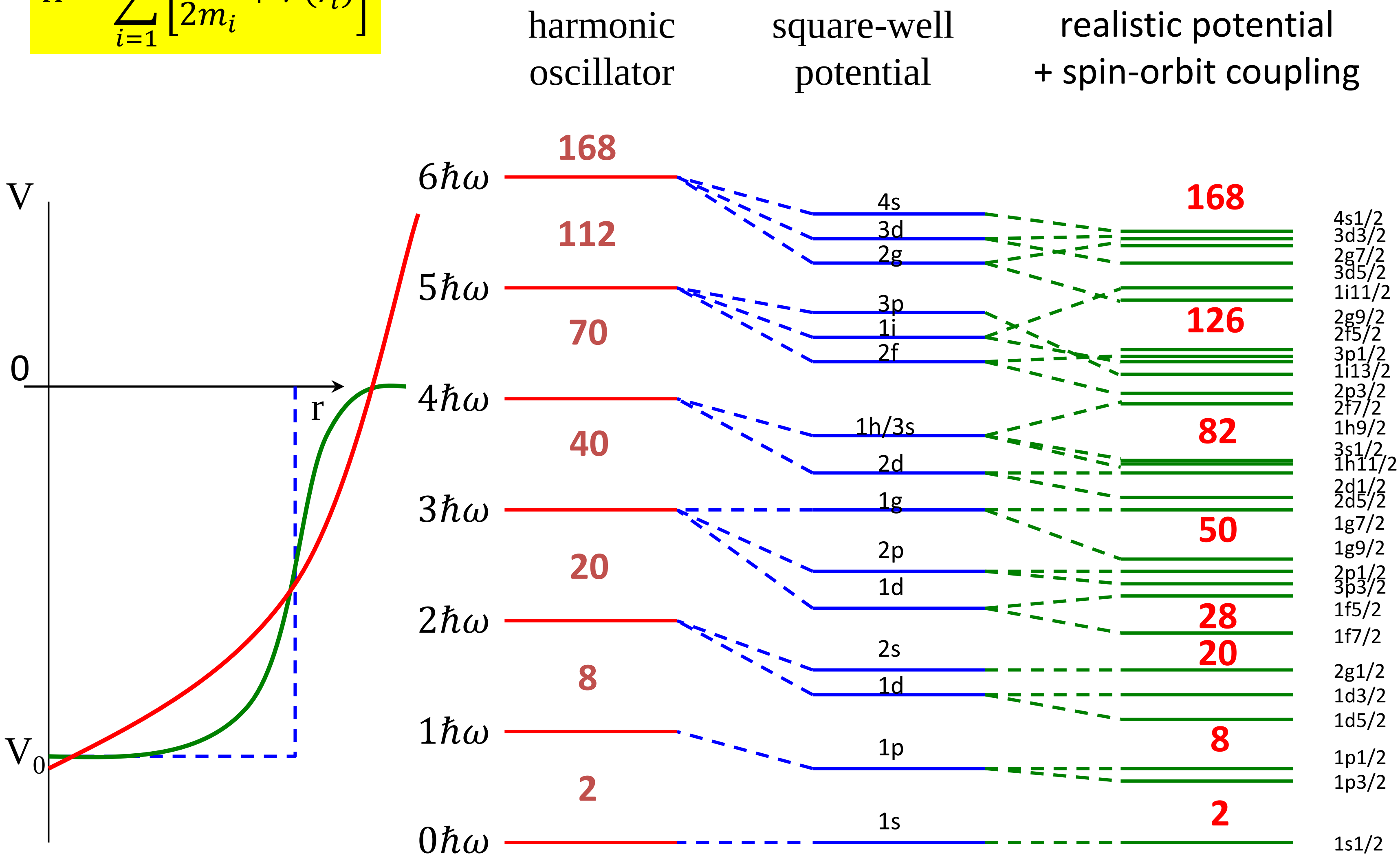
In the average nuclear potential V(r):

a) harmonic oscillator
b) square well potential
c) Woods-Saxon potential

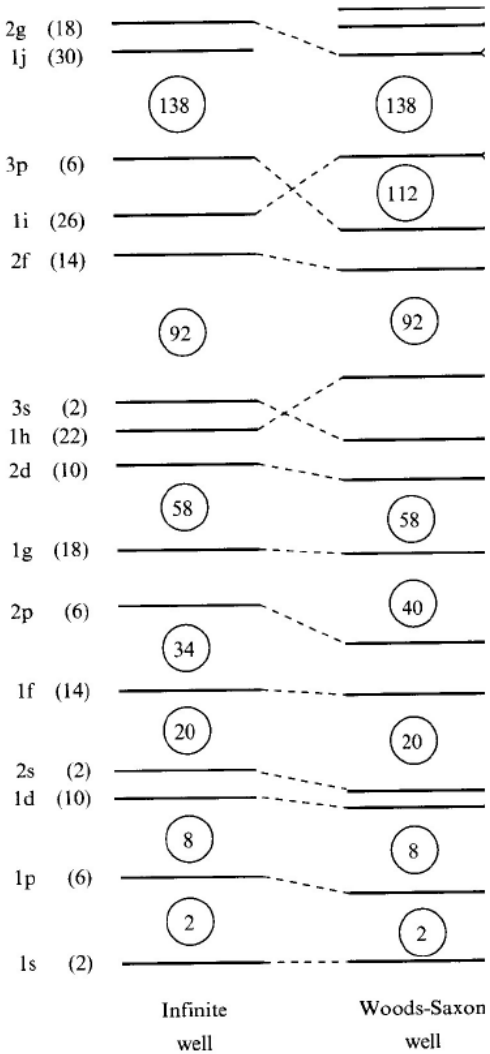
the nucleons move freely

Nuclear shell model

$$\hat{H} = \sum_{i=1}^{A} \left[\frac{\hat{p}_i^2}{2m_i} + \hat{V}(r_i) \right]$$



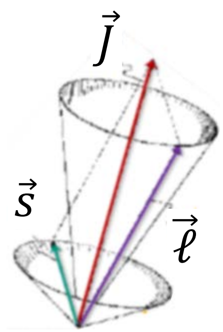
Woods-Saxon potential

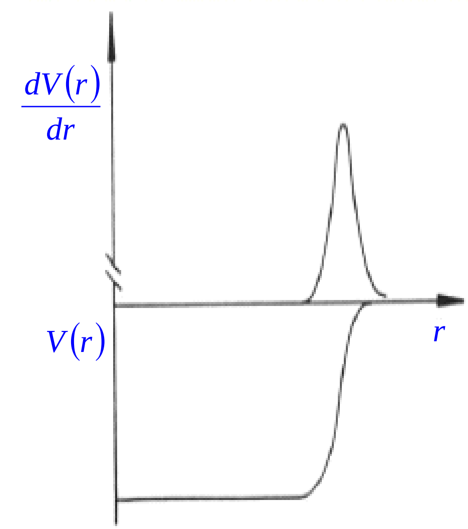


- ***Woods-Saxon does not reproduce the correct magic numbers***
  (2, 8, 20, 40, 70, 112, 168)$_{WS}$ ***(2, 8, 20, 28, 50, 82, 126)$_{exp}$***
- ***Meyer und Jensen (1949): strong spin-orbit interaction***

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{\ell s}(r)\cdot\vec{\ell}\cdot\vec{s} - \varepsilon\right]\Psi(r) = 0$$

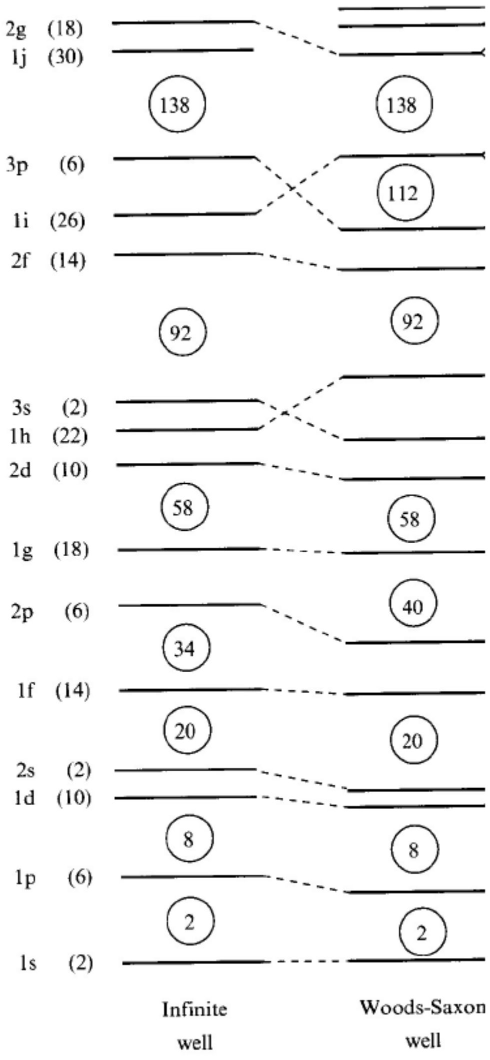
$$V_{\ell s}(r) \sim -\lambda\cdot\frac{1}{r}\cdot\frac{dV}{dr} \quad mit \quad \lambda > 0$$





The spin-orbit term has its origin in the relativistic description of the single particle motion inside the nucleus

Woods-Saxon potential (jj-coupling)



$$\vec{j} = \vec{\ell} + \vec{s} \quad \Rightarrow \quad \langle \ell \cdot s \rangle = \frac{1}{2} \cdot [\langle j^2 \rangle - \langle \ell^2 \rangle - \langle s^2 \rangle] \cdot \hbar^2$$

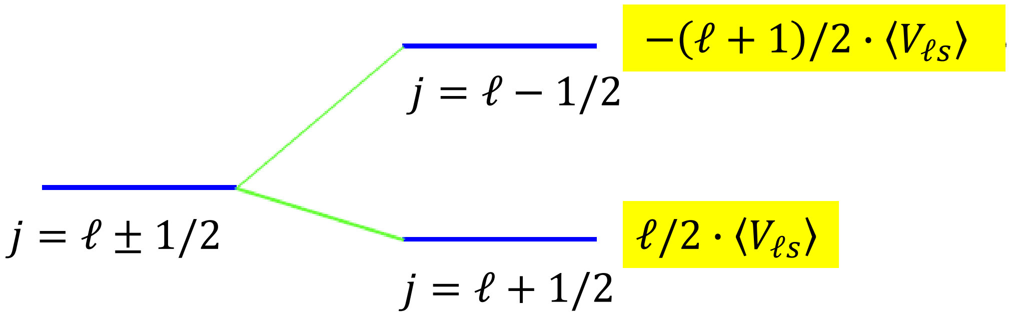
$$= \frac{1}{2}[j(j+1) - \ell(\ell+1) - s(s+1)] \cdot \hbar^2$$

The nuclear potential with spin-orbit term:

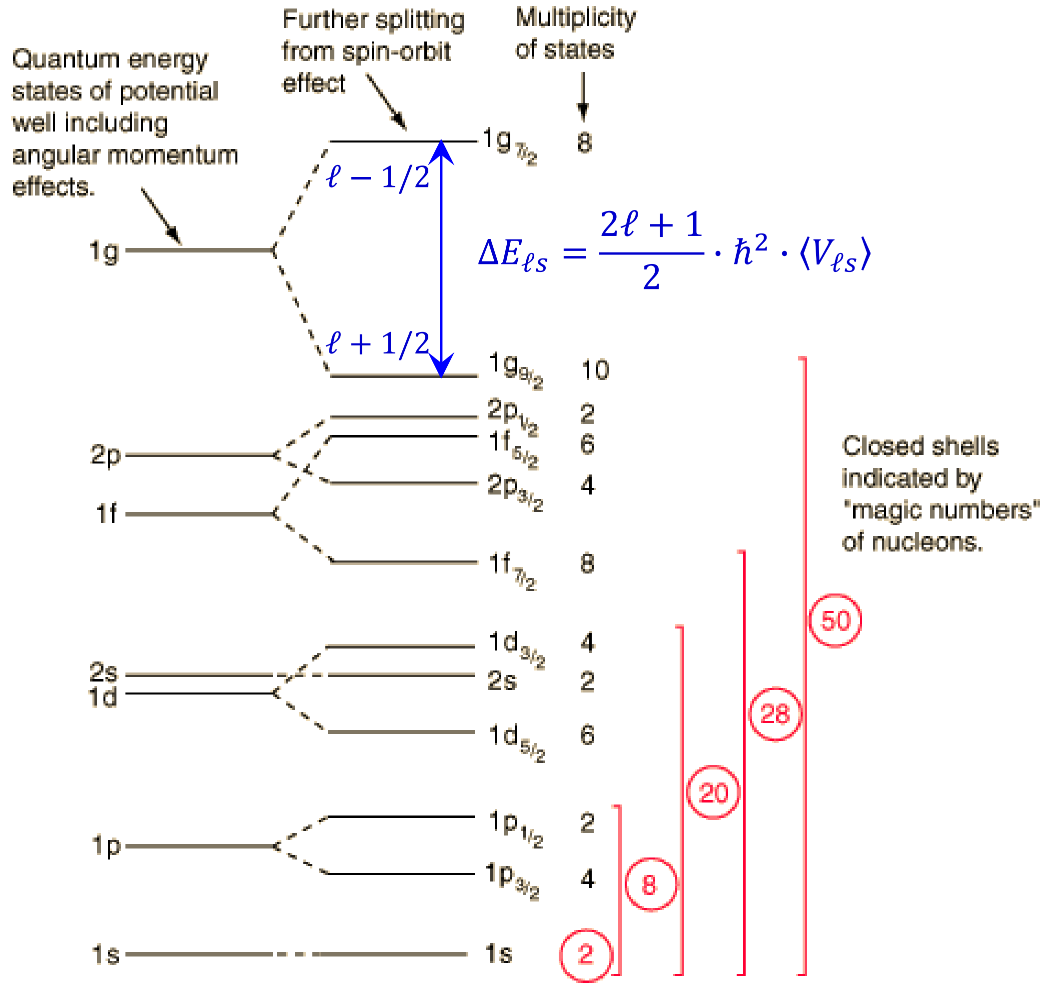
$$V(r) + \frac{\ell}{2} \cdot V_{\ell s} \quad for \quad j = \ell + 1/2$$

$$V(r) - \frac{\ell+1}{2} V_{\ell s} \quad for \quad j = \ell - 1/2$$

spin-orbit interaction leads to a large splitting for large ℓ.



Woods-Saxon potential



$$\Delta E_{\ell s} = \frac{2\ell + 1}{2} \cdot \hbar^2 \cdot \langle V_{\ell s} \rangle$$

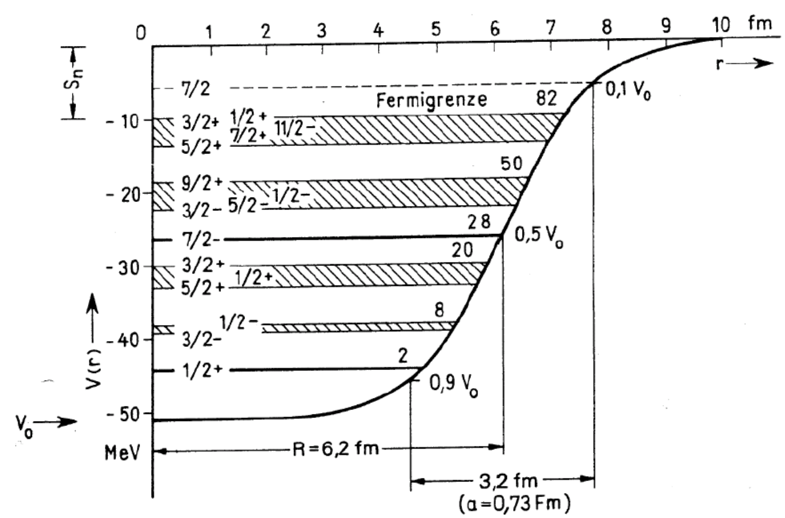
The spin-orbit term

- lowers the j = ℓ+1/2 orbital from the higher oscillator shell (intruder states)
- reproduces the magic numbers

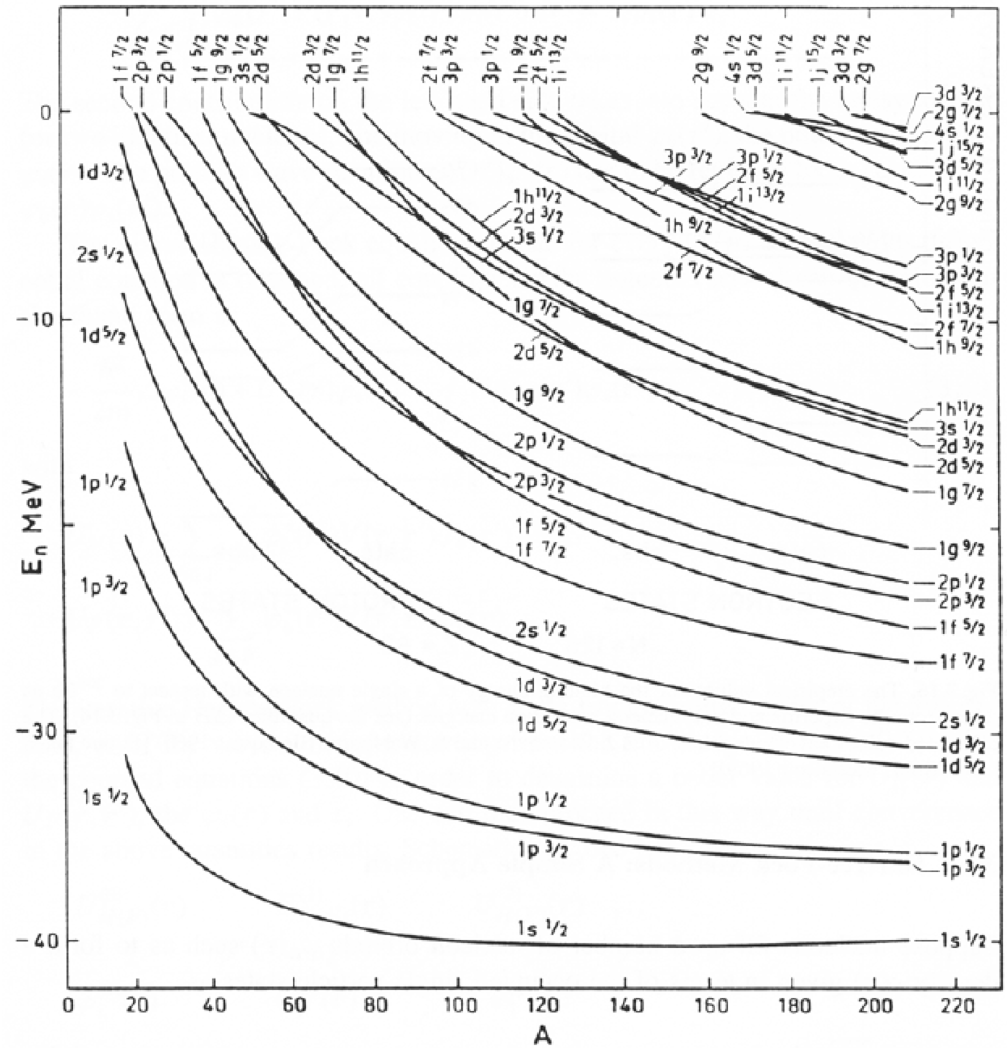
  large energy gaps → very stable nuclei

Important consequences:

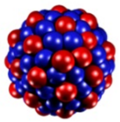
- lowering orbitals from higher lying N+1 shell having different parity than orbitals from the N shell
- strong interaction preserves the parity. The lowered orbitals with different parity are rather pure states and do not mix within the shell

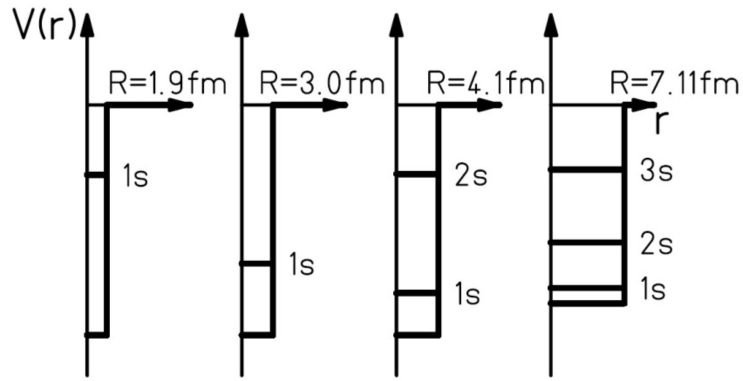


Shell model – mass dependence of single-particle energies



- ***Mass dependence of the neutron energies:*** $E \sim R^{-2}$
- ***number of neutrons in each level:*** $2 \cdot (2\ell + 1)$





Success of the extreme single-particle shell model

Z	Isotope	Observed J^{π}	Shell model nlj
3	^{9}Li	$(3/2^-)$	$1p_{3/2}$
5	^{13}B	$3/2^-$	$1p_{3/2}$
7	^{17}N	$1/2^-$	$1p_{1/2}$
9	^{21}F	$5/2^+$	$1d_{5/2}$
11	^{25}Na	$5/2^+$	$1d_{5/2}$
13	^{29}Al	$5/2^+$	$1d_{5/2}$
15	^{33}P	$1/2^+$	$2s_{1/2}$
17	^{37}Cl	$3/2^+$	$1d_{3/2}$
19	^{41}K	$3/2^+$	$1d_{3/2}$
21	^{45}Sc	$7/2^-$	$1f_{7/2}$
23	^{49}Va	$7/2^-$	$1f_{7/2}$
25	^{53}Mn	$7/2^-$	$1f_{7/2}$
27	^{57}Co	$7/2^-$	$1f_{7/2}$
29	^{61}Cu	$3/2^-$	$2p_{3/2}$
31	^{65}Ga	$3/2^-$	$2p_{3/2}$
33	^{69}As	$(5/2^-)$	$1f_{5/2}$
35	^{73}Br	$(3/2^-)$	$1f_{5/2}$

➢ ***Ground state spin and parity:***

Every orbital has 2j+1 magnetic sub-states, completely filled orbitals have spin J=0, they do not contribute to the nuclear spin.

For a nucleus with one nucleon outside a completely occupied orbital the nuclear spin is given by the single nucleon.

$$n\,\ell\,\boldsymbol{j} \rightarrow \boldsymbol{J}$$

$$(-)^{\ell} = \pi$$

Success of the extreme single-particle shell model

${}^{7}_{4}Be$

${}^{17}_{9}F$

${}^{63}_{28}Ni$

${}^{61}_{29}Cu$

${}^{91}_{40}Zr$

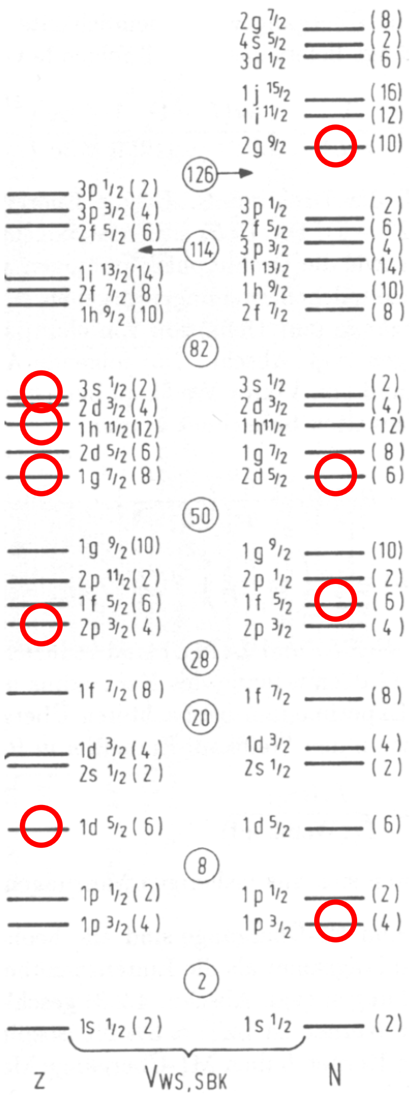
${}^{123}_{51}Sb$

${}^{159}_{65}Tb$

${}^{183}_{73}Ta$

${}^{199}_{81}Tl$

${}^{209}_{82}Pb$



Success of the extreme single-particle shell model

- ***Magnetic moments:***

  The g-factor g_j is given by:

$$\vec{\mu_j} = g_\ell \cdot \vec{\ell} + g_s \cdot \vec{s} = g_j \cdot \vec{j}$$

$$\Rightarrow \quad \vec{\mu_j} = \left[(g_\ell \cdot \vec{\ell} + g_s \cdot \vec{s}) \cdot \frac{\vec{j}}{|j|} \right] \cdot \frac{\vec{j}}{|j|}$$

with $\vec{\ell}^2 = (\vec{j} - \vec{s})^2 = \vec{j}^2 - 2 \cdot \vec{j} \cdot \vec{s} + \vec{s}^2$ $\qquad \vec{s}^2 = (\vec{j} - \vec{\ell})^2 = \vec{j}^2 - 2 \cdot \vec{j} \cdot \vec{\ell} + \vec{\ell}^2$

$$\vec{\mu}_j = \frac{g_\ell \cdot \{j(j+1) + \ell(\ell+1) - 3/4\} + g_s \cdot \{j(j+1) - \ell(\ell+1) + 3/4\}}{2 \cdot j(j+1)} \cdot \vec{j}$$

$$g_j = \frac{1}{2} \cdot (g_\ell + g_s) + \frac{1}{2} \cdot \frac{\ell(\ell+1) - s(s+1)}{2j(j+1)} \cdot (g_\ell - g_s)$$

Simple relation for the g-factor of single-particle states

$$\frac{\mu}{\mu_N} = g_{nucleus} = g_\ell \pm \frac{(g_s - g_\ell)}{2\ell + 1} \quad for \quad j = \ell \pm 1$$

nucleus	state	J^π	μ/μ_N model	μ/μ_N experiment
^{15}N	p-$1p_{1/2}^{-1}$	$1/2^-$	$-0,264$	$-0,283$
^{15}O	n-$1p_{1/2}^{-1}$	$1/2^-$	$+0,638$	$+0,719$
^{17}O	n-$1d_{5/2}$	$5/2^+$	$-1,913$	$-1,894$
^{17}F	p-$1d_{5/2}$	$5/2^+$	$+4,722$	$+4,793$

Success of the extreme single-particle shell model

- ***magnetic moments:***

$$\langle \mu_z \rangle = \begin{cases} \left[g_\ell \cdot \left(j - \frac{1}{2} \right) + \frac{1}{2} \cdot g_s \right] \cdot \mu_N & for \quad j = \ell + 1/2 \\ \frac{j}{j+1} \cdot \left[g_\ell \cdot \left(j + \frac{3}{2} \right) - \frac{1}{2} \cdot g_s \right] \cdot \mu_N & for \quad j = \ell - 1/2 \end{cases}$$

- ***g-factor of nukleons:***

proton: $g_\ell = 1$; $g_s = +5.585$
neutron: $g_\ell = 0$; $g_s = -3.82$

proton:

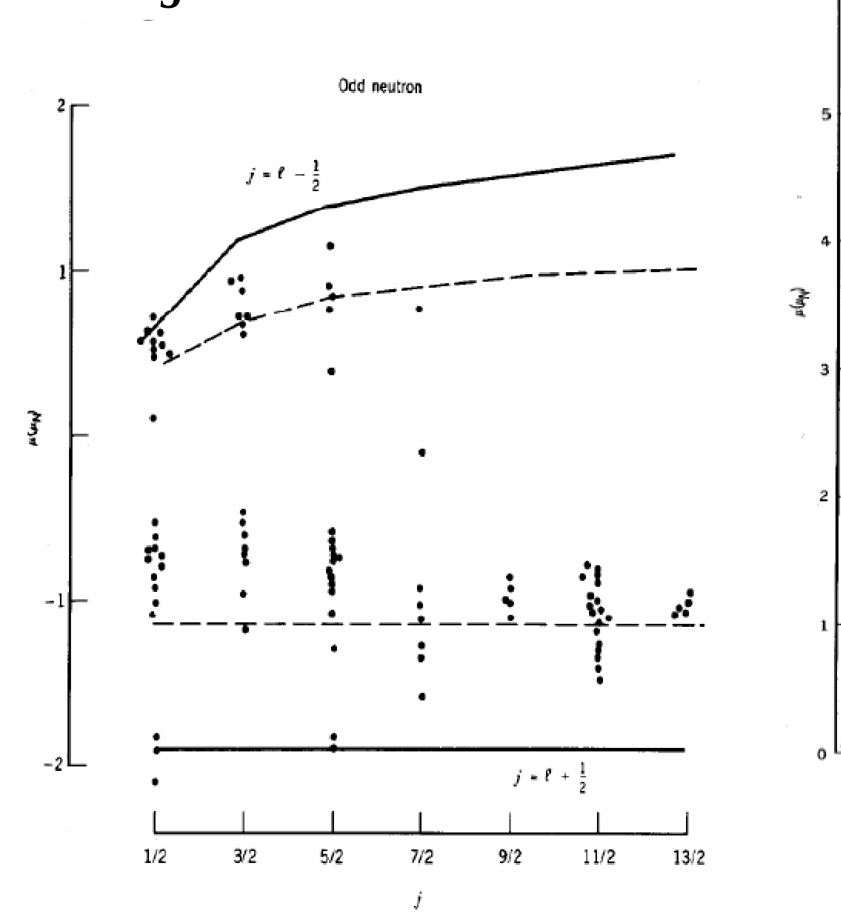
$$\langle \mu_z \rangle = \begin{cases} (j + 2.293) \cdot \mu_N & for \quad j = \ell + 1/2 \\ (j - 2.293) \cdot \frac{j}{j+1} \cdot \mu_N & for \quad j = \ell - 1/2 \end{cases}$$

neutron:

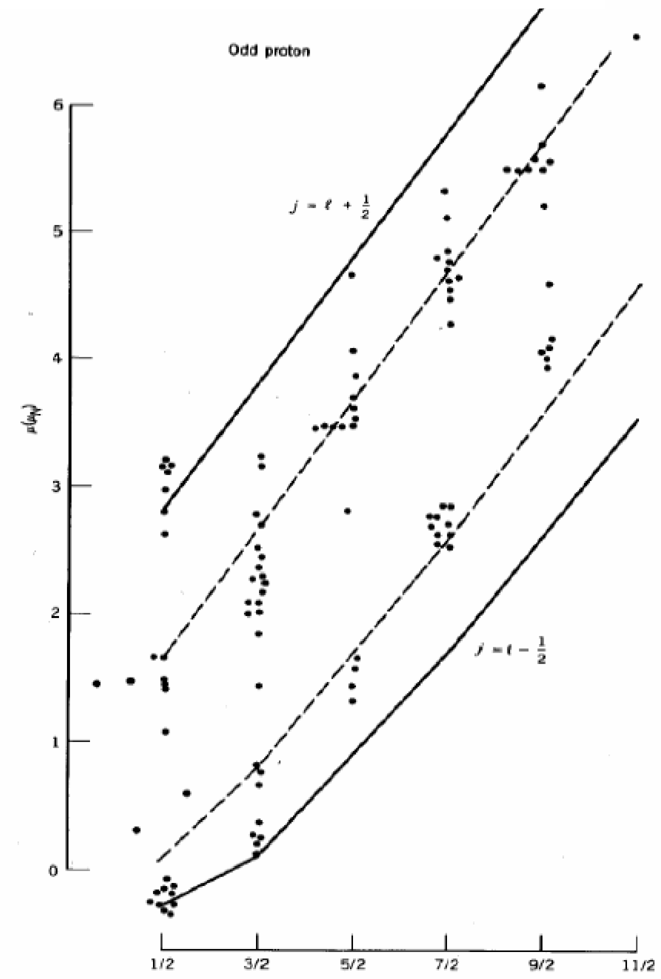
$$\langle \mu_z \rangle = \begin{cases} -1.91 \cdot \mu_N & for \quad j = \ell + 1/2 \\ +1.91 \cdot \frac{j}{j+1} \cdot \mu_N & for \quad j = \ell - 1/2 \end{cases}$$

Magnetic moments: Schmidt lines

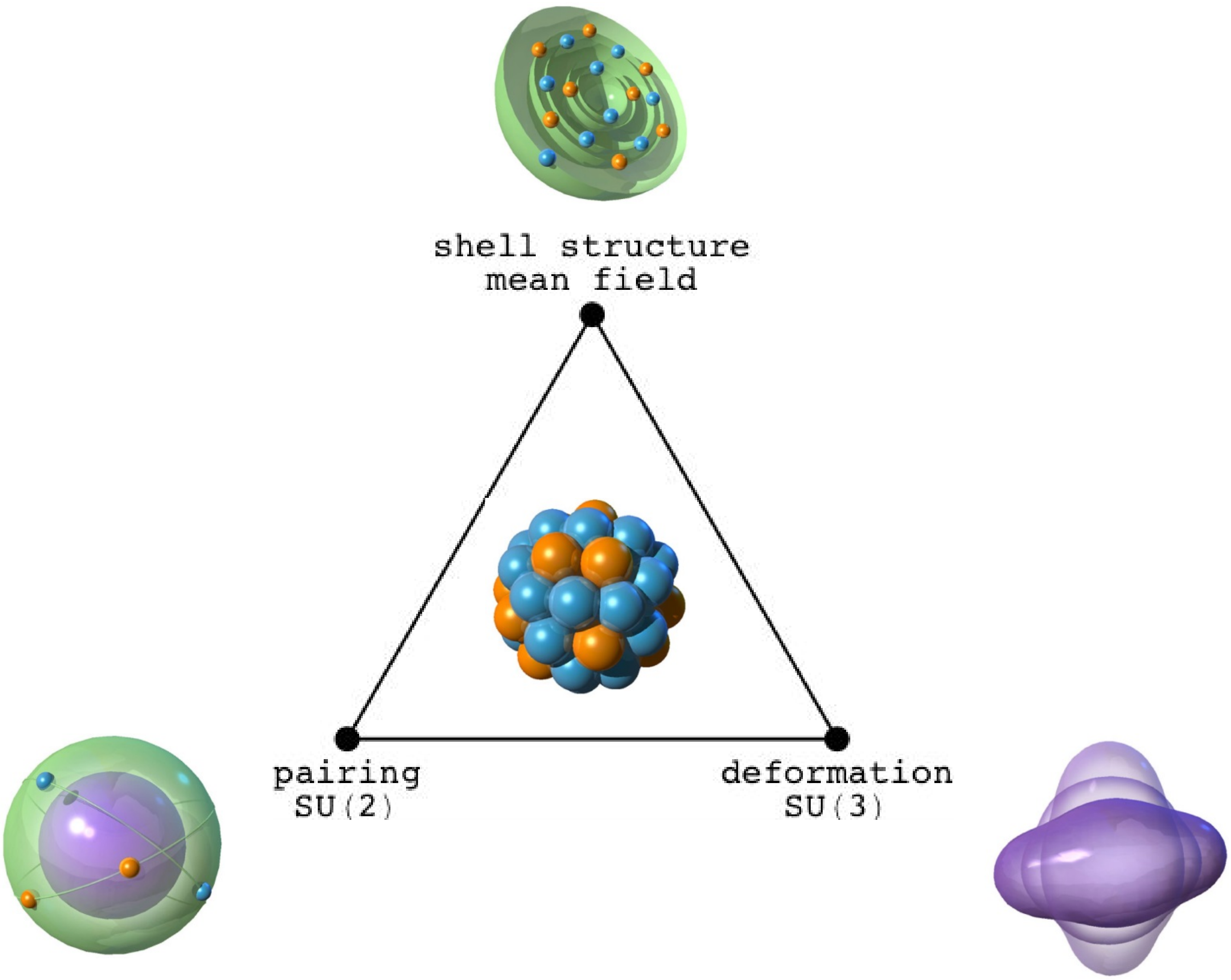
magnetic moments: neutron



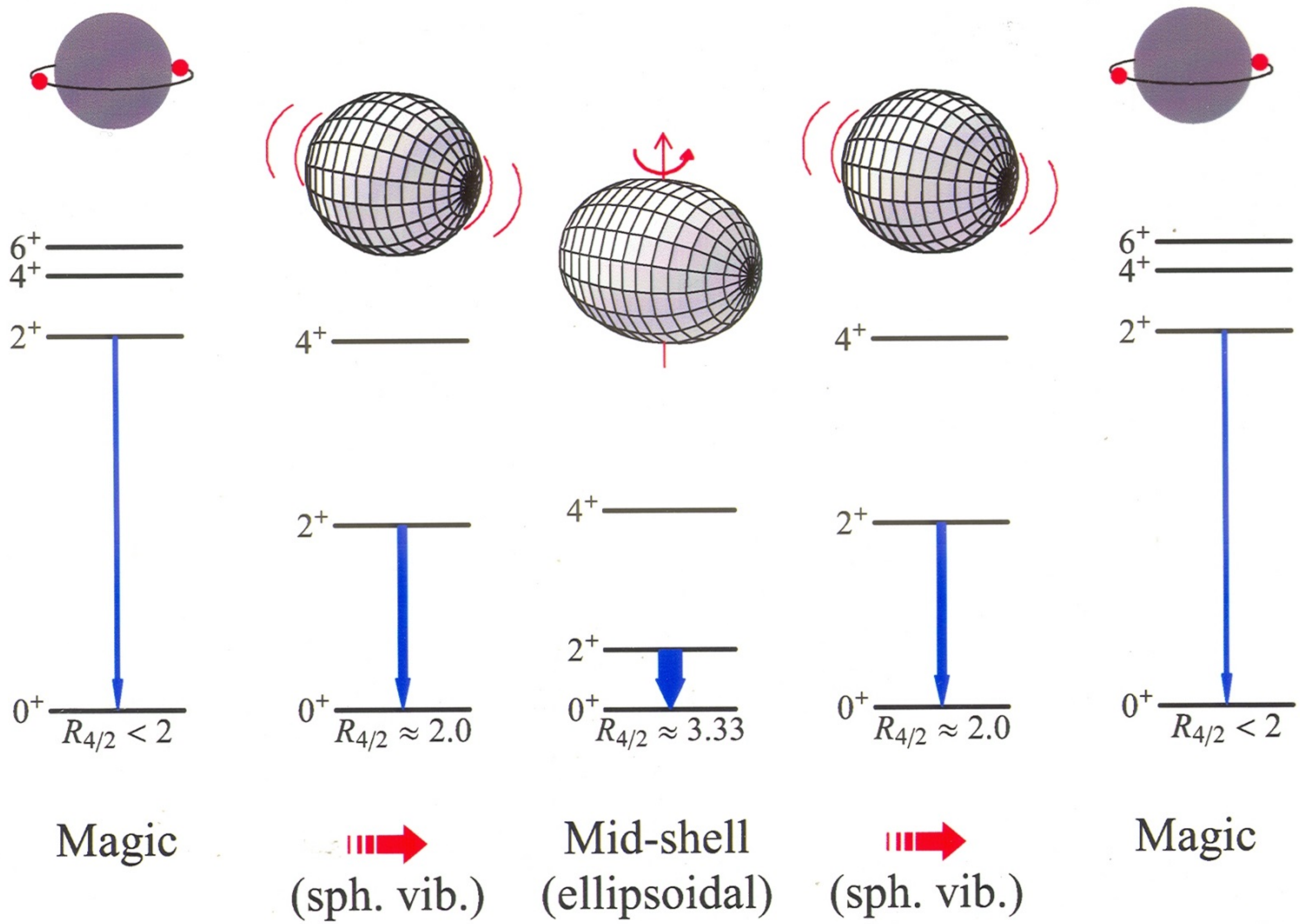
magnetic moments: proton



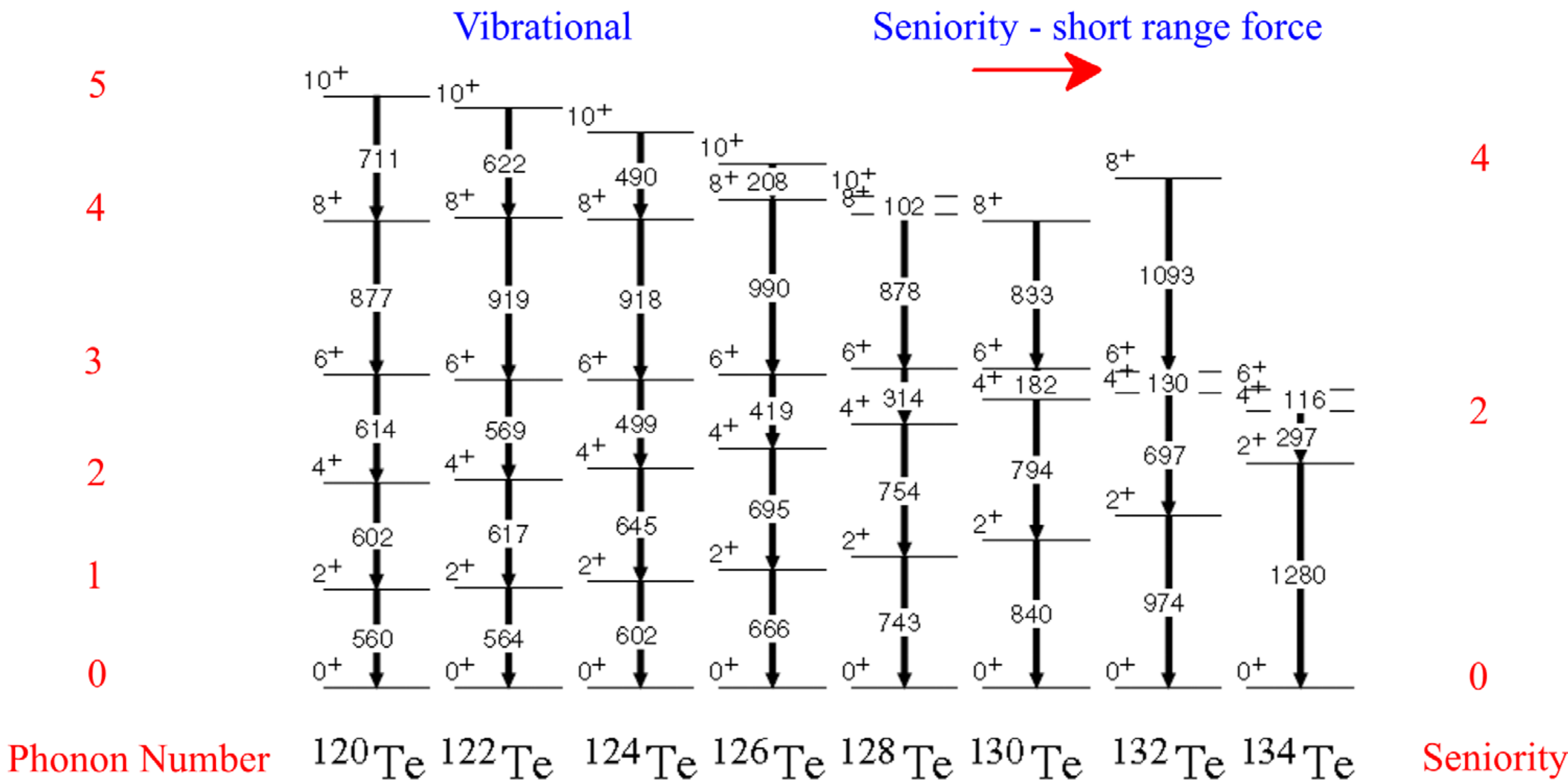
The three structures of the shell model



Evolution of nuclear structure
(as a function of nucleon number)



Systematics of the Te isotopes (Z=52)



Neutron number	68	70	72	74	76	78	80	82
Val. Neutr. number	14	12	10	8	6	4	2	0