Outline: Deformed (Nilsson) shell model

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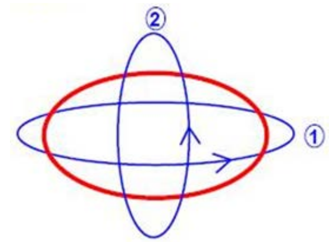
web-page: https://web-docs.gsi.de/~wolle/ and click on

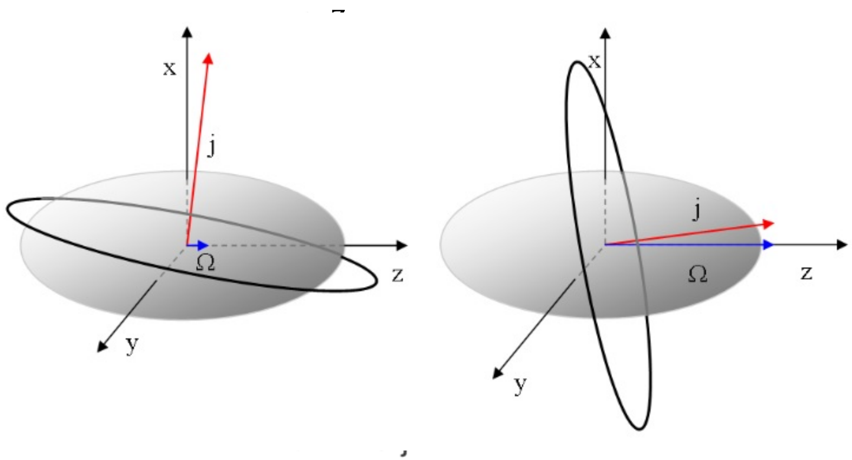
1. deformed shell model
2. Nilsson model for small deformation
3. shell closure for large deformation
4. K-isomers e.g. ^{178}Hf
5. structure of super heavy elements

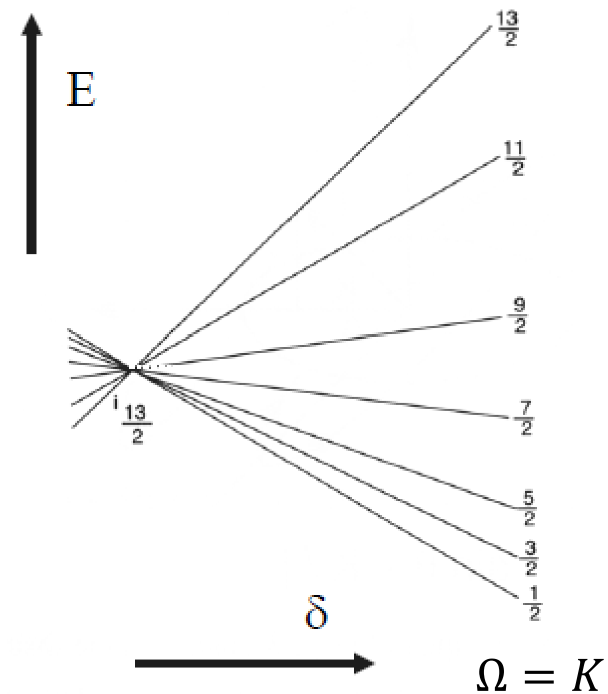
Deformed (Nilsson) shell model

- spherical nucleus: $R = R_0$
- deformed nucleus: $R = R_0 \cdot \left[1 + \sum_{\lambda=2,\mu} \alpha_{2\mu} \cdot Y_{2\mu}\right]$
  → can rotate



orbit 1 is closer to the center of gravity than orbit 2.
The energy of orbit 1 is the lowest.

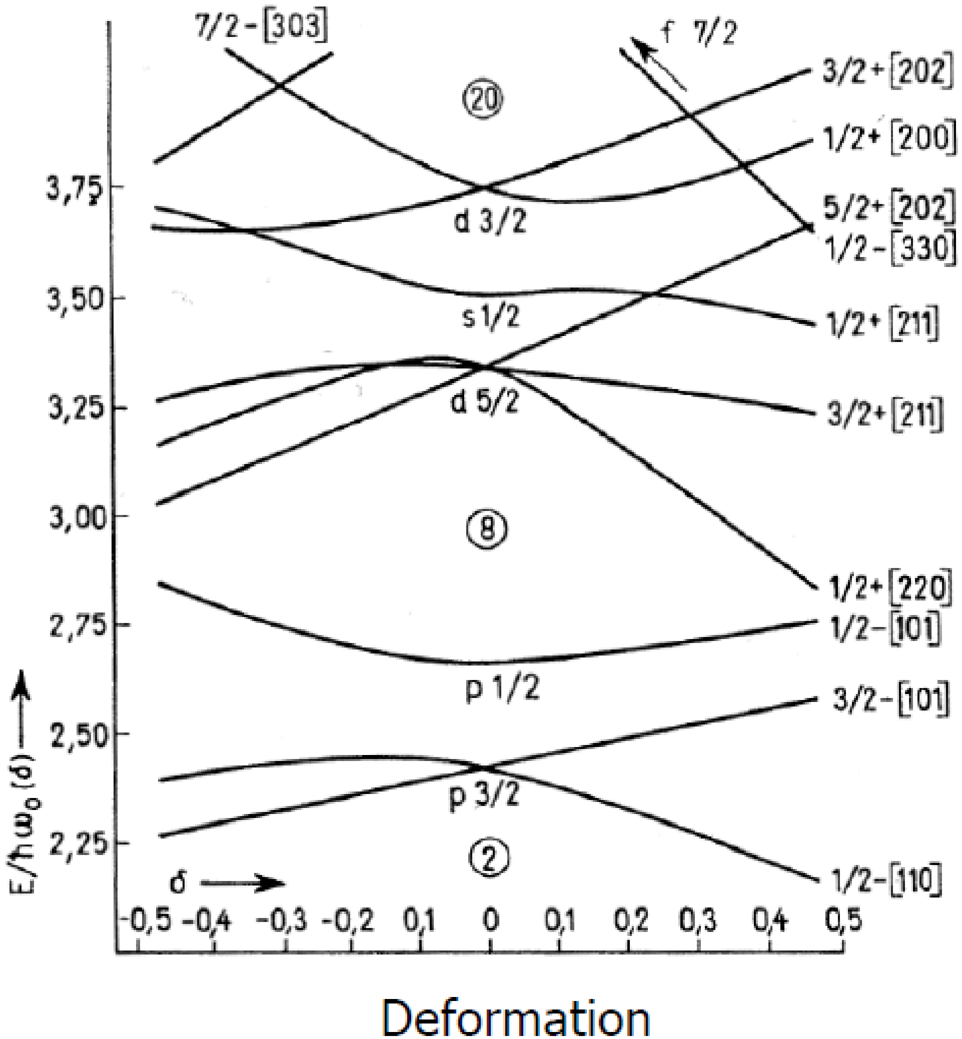




- Separation of laboratory system and body-fixed (intrinsic) system
- Ω = K projection of the single-particle angular momentum onto the symmetry axis
- Rotation perpendicular to the symmetry axis will not change the Ω-quantum number

Nilsson model (quadrupole interaction)

Sven Gösta Nilsson



Nilsson Model is a single-particle model for deformed nuclei.

$$H = \frac{p^2}{2m} + \frac{m \cdot \left[\omega_x^2\left(x^2 + y^2\right) + \omega_z^2 \cdot z^2\right]}{2} + C \cdot \mathbf{l} \cdot s + D \cdot \mathbf{l}^2$$

with $\omega_x^2 = \omega_0^2 \cdot \left(1 + \frac{2}{3} \cdot \delta\right)$ $\omega_z^2 = \omega_0^2 \cdot \left(1 - \frac{4}{3} \cdot \delta\right)$

The labelling of the Eigen-states is: $\Omega^\pi[Nn_z\Lambda]$

Ω projection of the total particle angular momentum on the symmetry axis

π parity of the wave function $\pi = (-1)^N$

N the principal quantum number of the major oscillator shells

n_z the number of quanta associated with the wave function moving along the z-direction

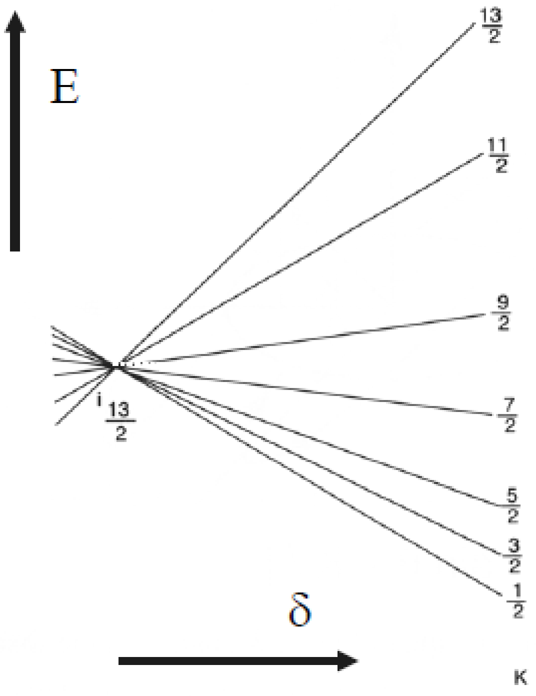
$\Lambda = m_\ell$ projection of the orbital angular momentum onto the z-axis

Nilsson model for small deformations

$$H = \underbrace{-\frac{\hbar^2}{2m}\Delta + \frac{m}{2}\omega_0^2 r^2 + C\cdot\vec{L}\bullet\vec{S} + D\cdot\vec{L}^2}_{\text{shell model with H.O. potential}} \underbrace{- m\omega_0^2 r^2 \delta \frac{4}{3}\sqrt{\frac{4\pi}{5}} Y_{20}(\theta,\Phi)}_{H_{def}}$$

shell model with H.O. potential

H_{def}



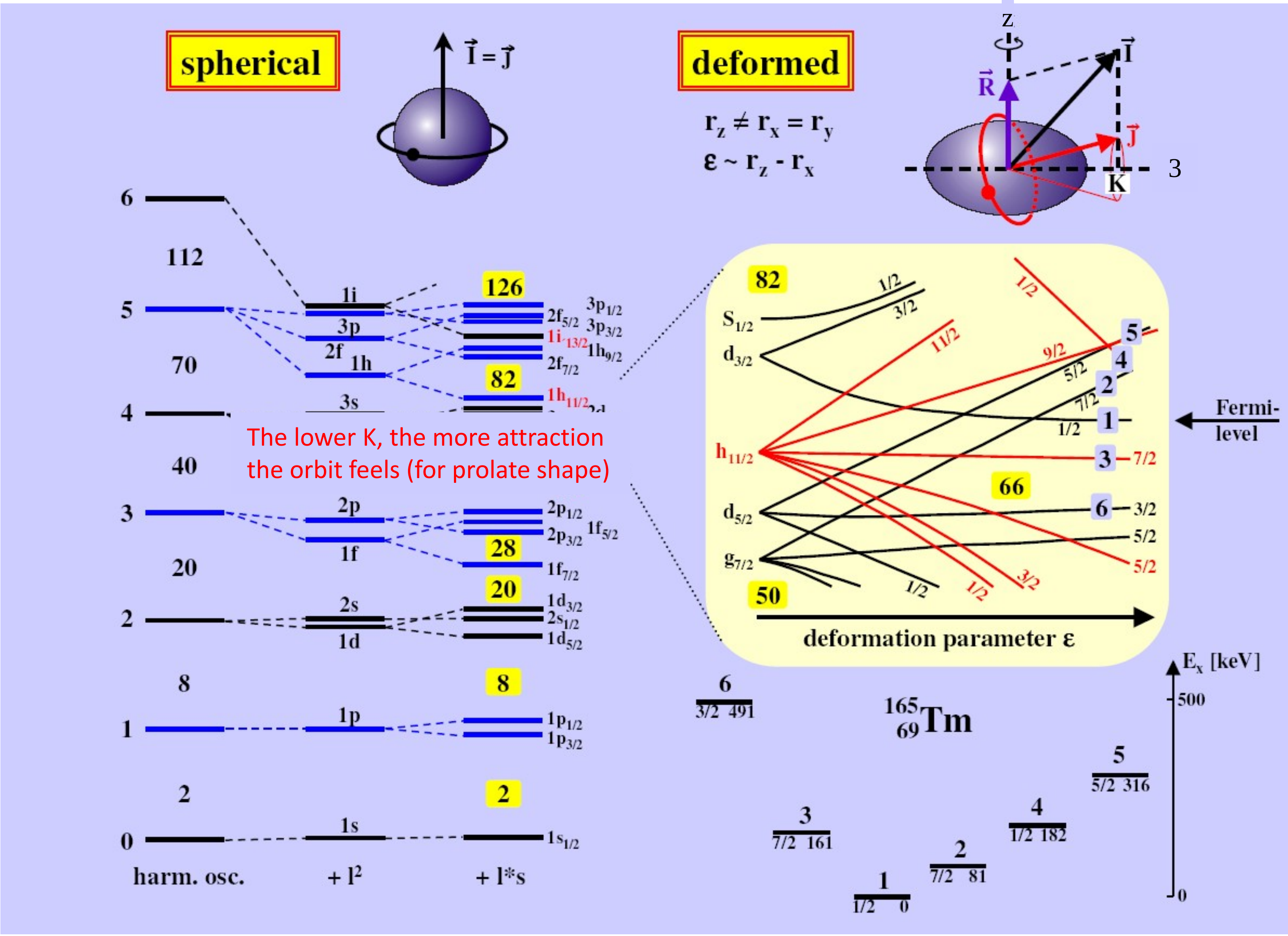
$$\Delta E(N\ell jK) = -\frac{4}{3}\sqrt{\frac{4\pi}{5}} m\omega_0^2 \cdot \delta \cdot \underbrace{\langle N\ell jm | r^2 Y_{20} | N\ell jm \rangle}$$

$$\Delta E(N\ell jK) = -\frac{2}{3}\hbar\omega_0\left(N + \frac{3}{2}\right)\cdot\delta\cdot\frac{[3K^2 - j(j+1)]\cdot\left[\frac{3}{4} - j(j+1)\right]}{(2j-1)j(j+1)(2j+3)}$$

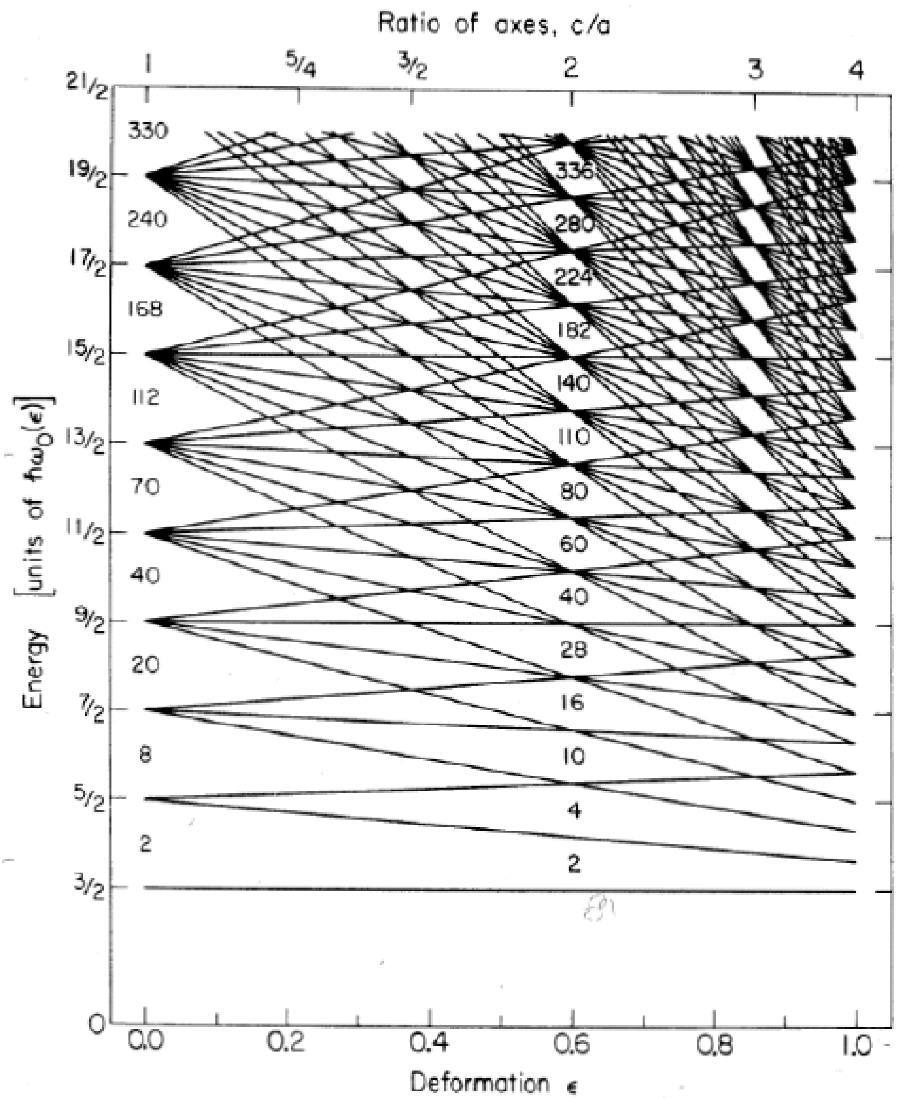
results for small deformations δ:

- $\Delta E \propto \delta \approx \beta$
- $\Delta E \propto K^2 - j(j+1)$
- $\Delta E \propto N$

Spherical shell model → Nilsson model



Shell closures for large deformations

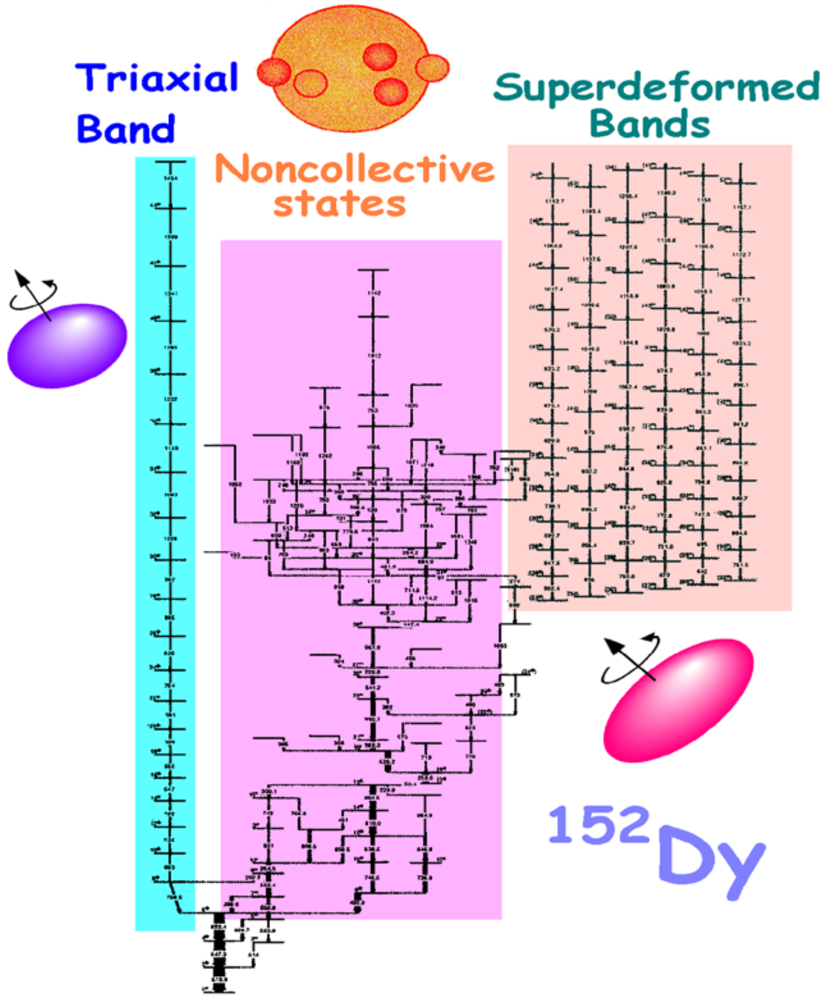


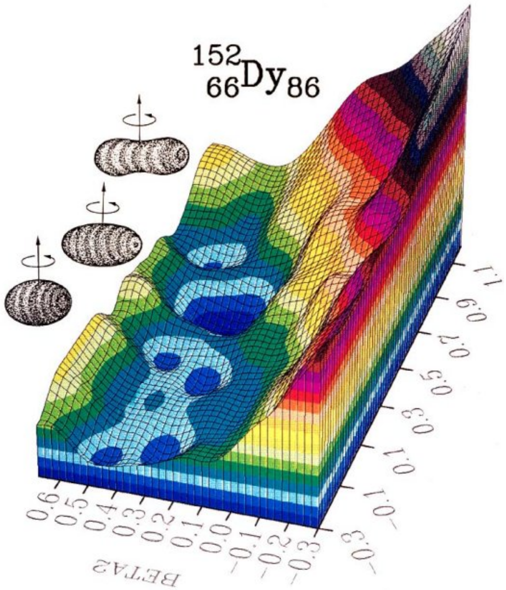
For large deformation the $\vec{\ell} \cdot \vec{s}$ and $\vec{\ell}^2$ terms can be neglected.

spectrum of a prolate deformed harmonic oscillator as a function of the deformation parameter ε

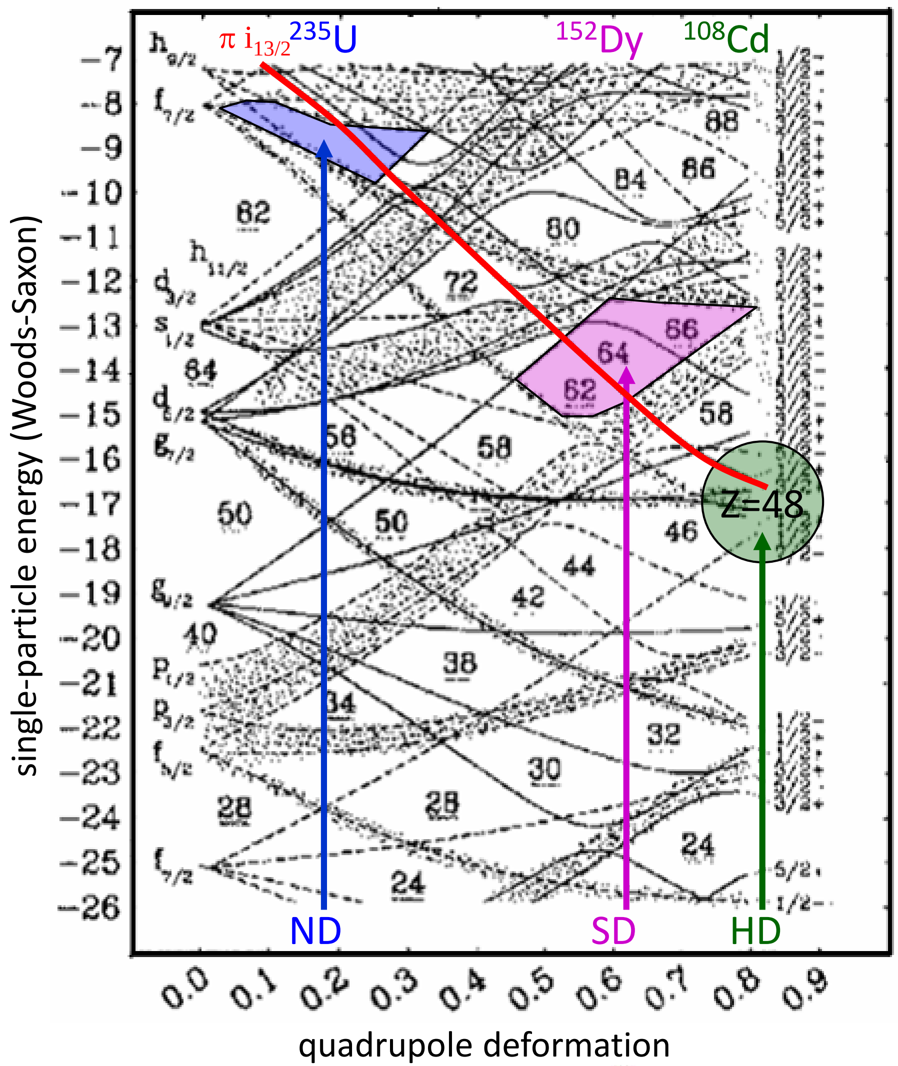
Nuclear structure of ^{152}Dy: hadronic field theory in nuclei

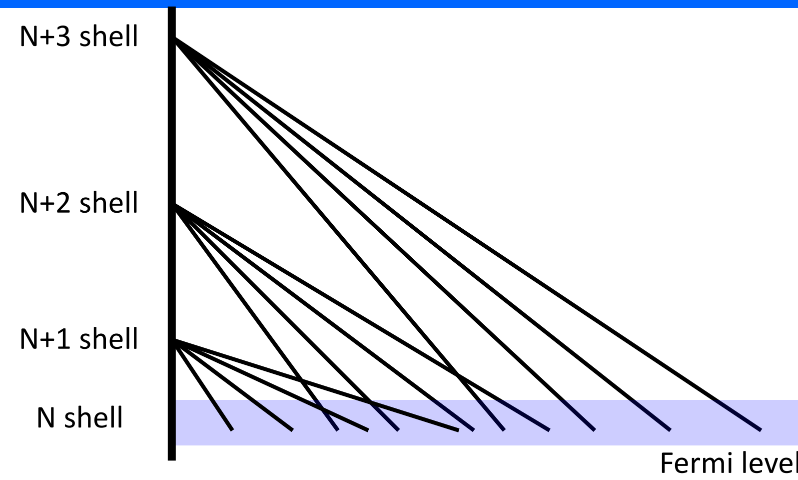
Coexistence of collective and noncollective motion

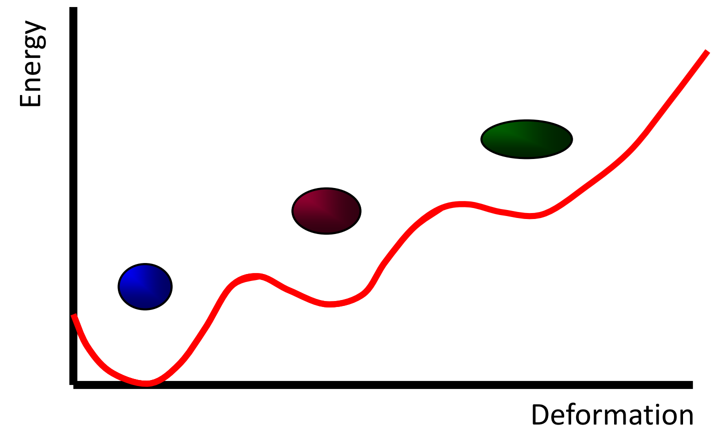




Nuclear structure and intruder states

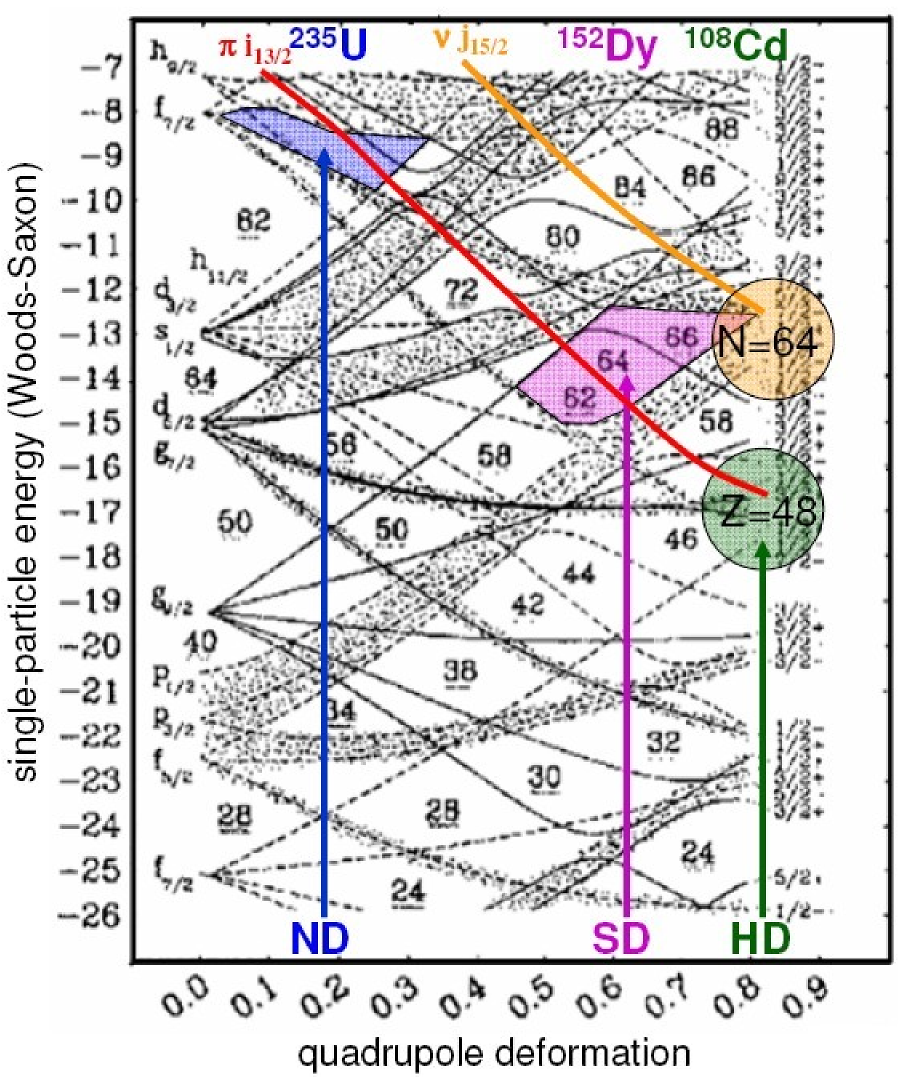






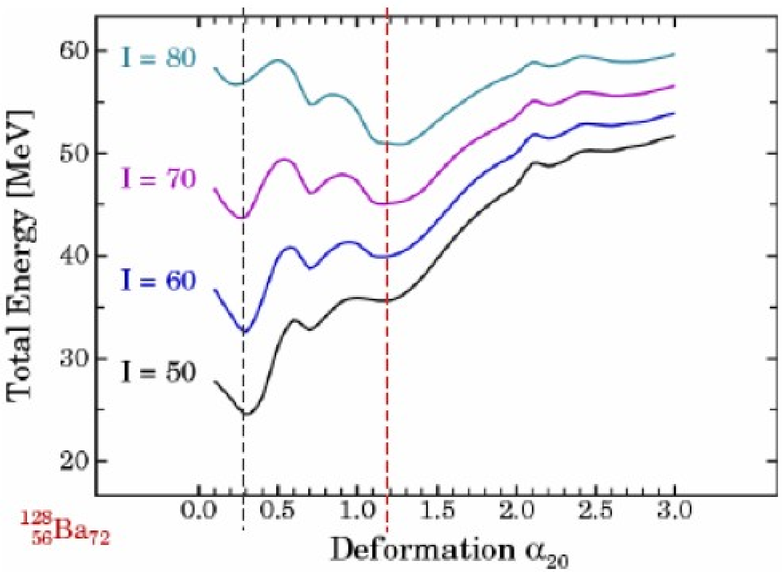
- (N+1) intruder
  $\Rightarrow$ normal deformed, e.g. ^{235}U
- (N+2) super-intruder
  $\Rightarrow$ super deformation, e.g. ^{152}Dy, ^{80}Zr
- (N+3) hyper-intruder
  $\Rightarrow$ hyper deformation in ^{108}Cd, ?

Nuclear deformation



For large spins: interaction between

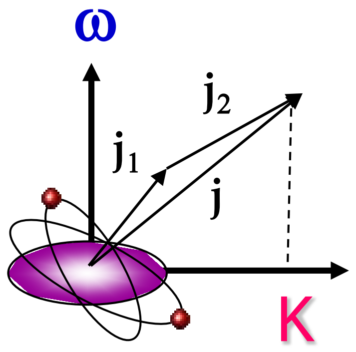
- macroscopic effects: liquid drop
- microscopic effects: shell structure



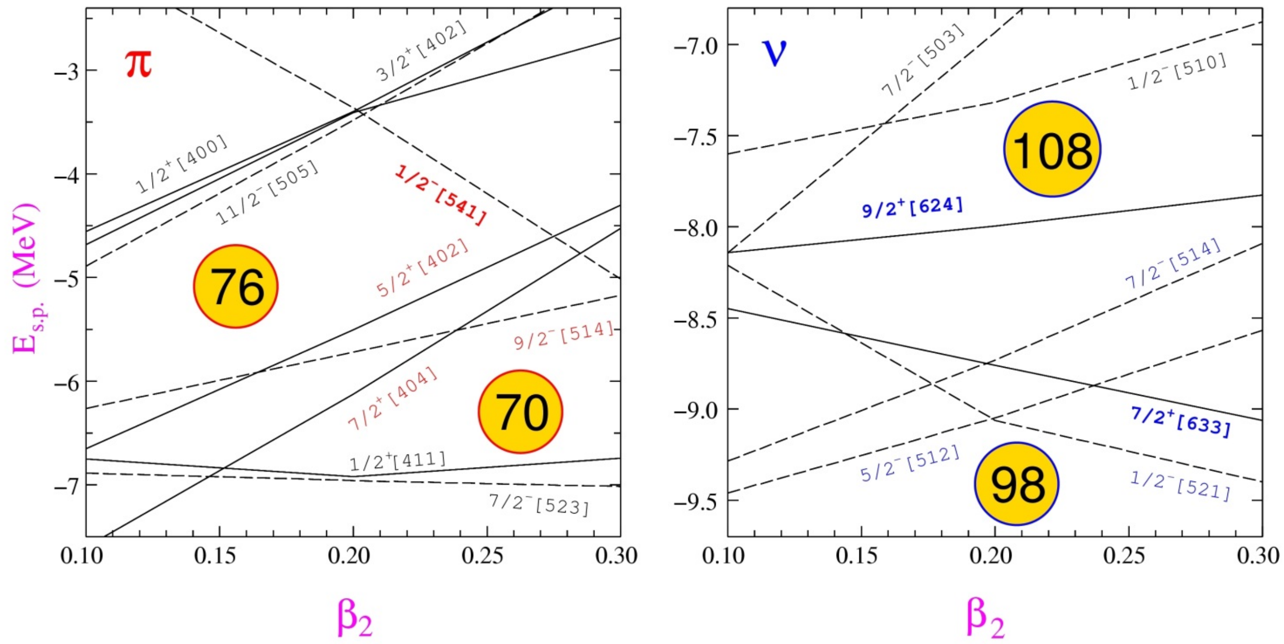
The different slopes of the single-particle states create different minima for the same number of nucleons

K-Isomers

❑ Deformed nuclei with axially-symmetric shape



Mass 180 region : Yb (Z=70) ***- Ir*** (Z=77)



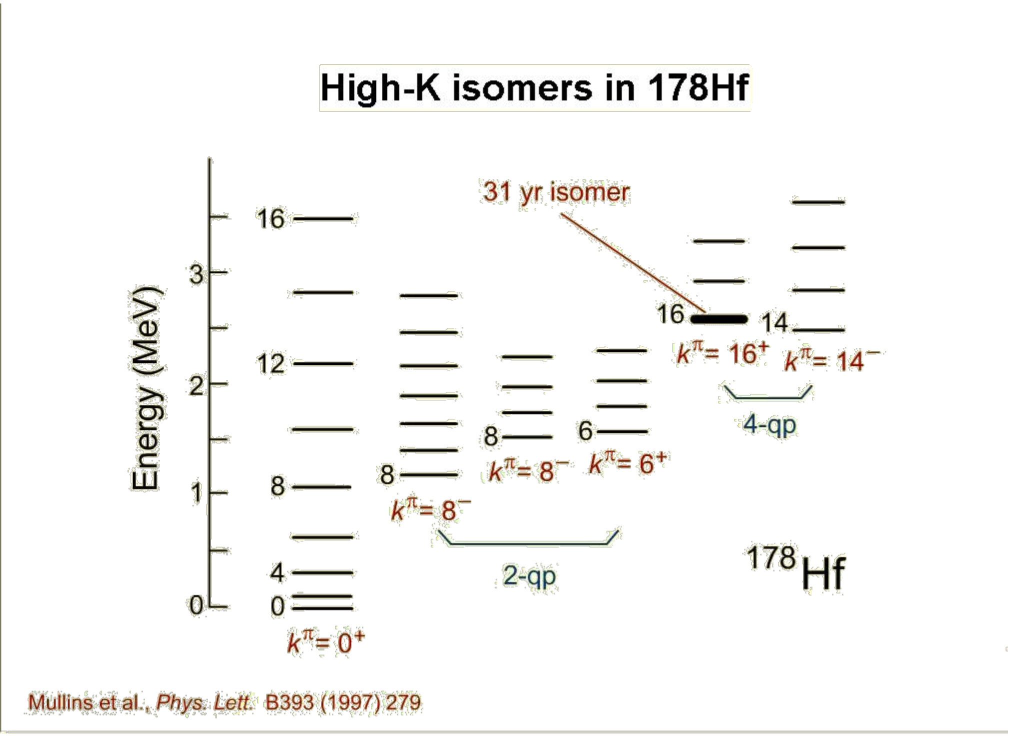
❑ High-K orbitals near the Fermi surface

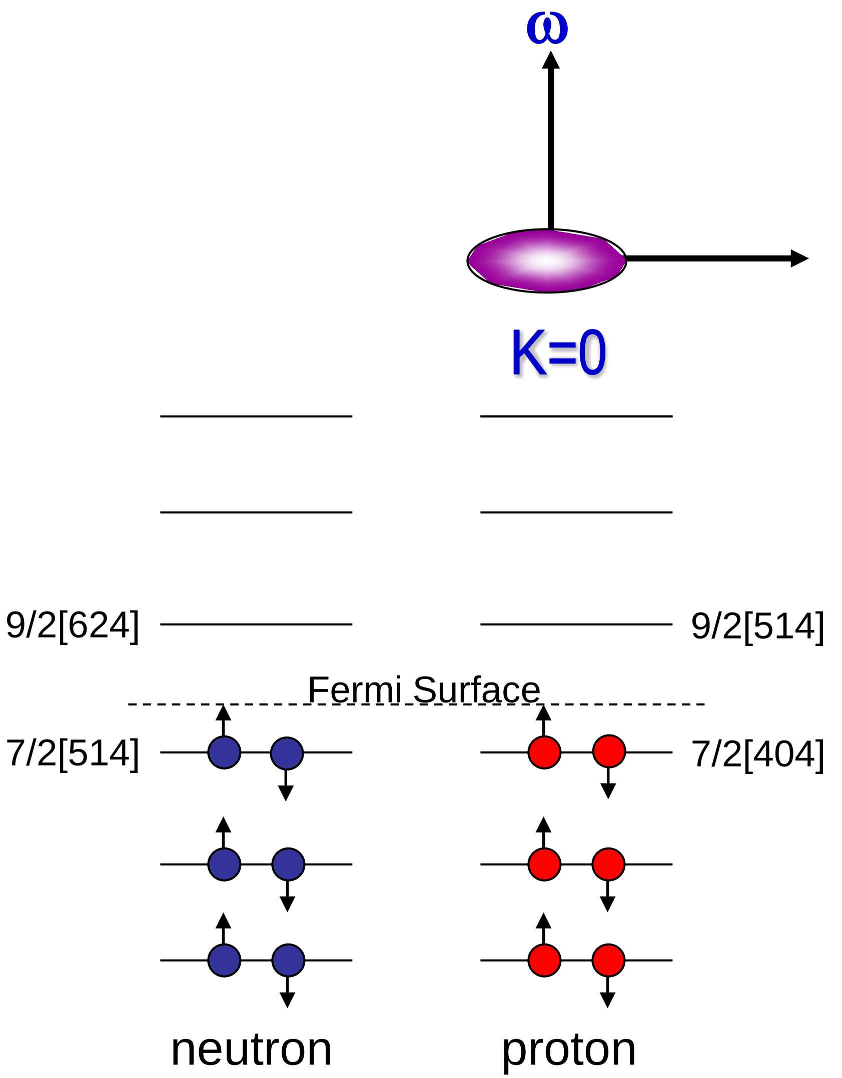
π: 7/2[404], 9/2[514], 5/2[402]

ν: 7/2[514], 9/2[624], 5/2[512], 7/2[633]

K-Isomers

- A well-known example:





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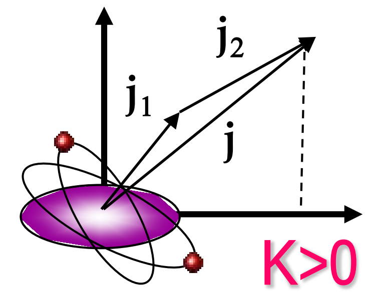

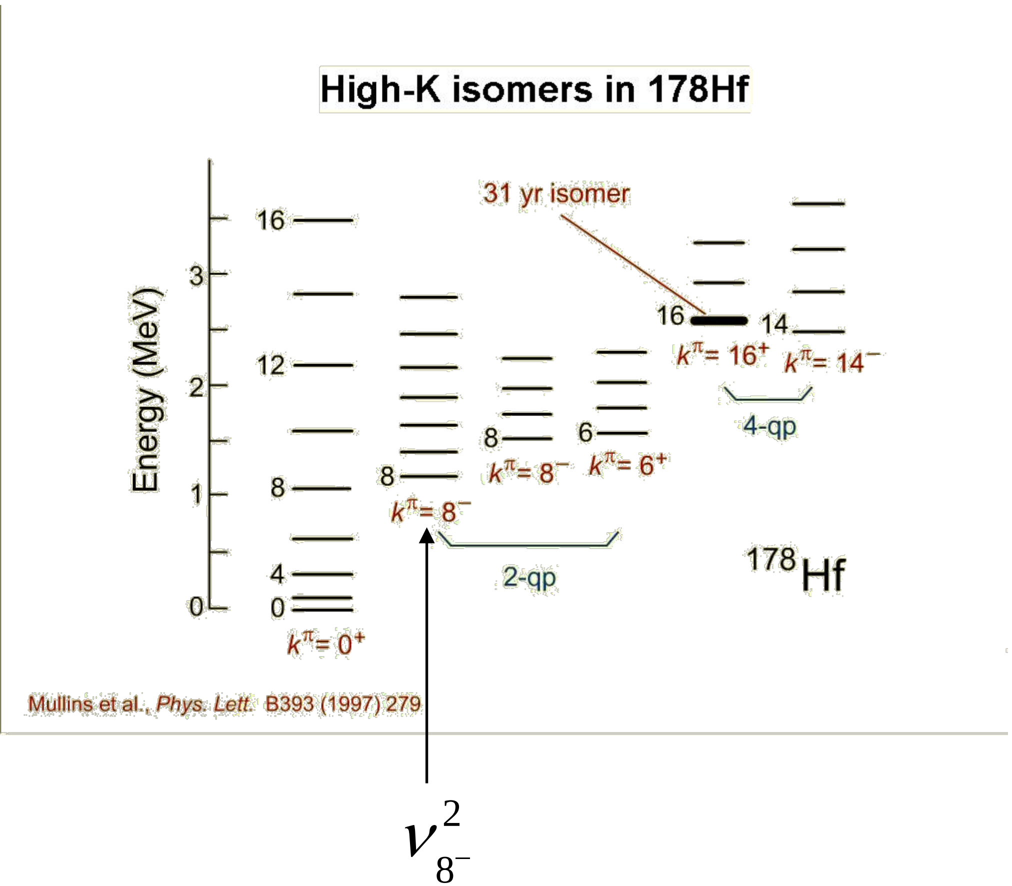

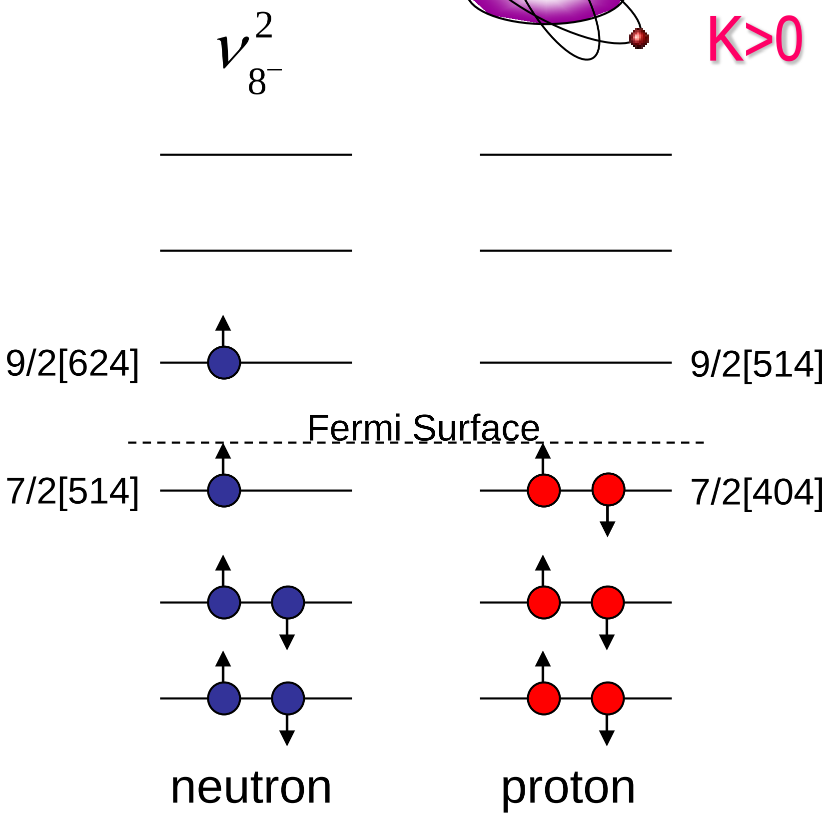

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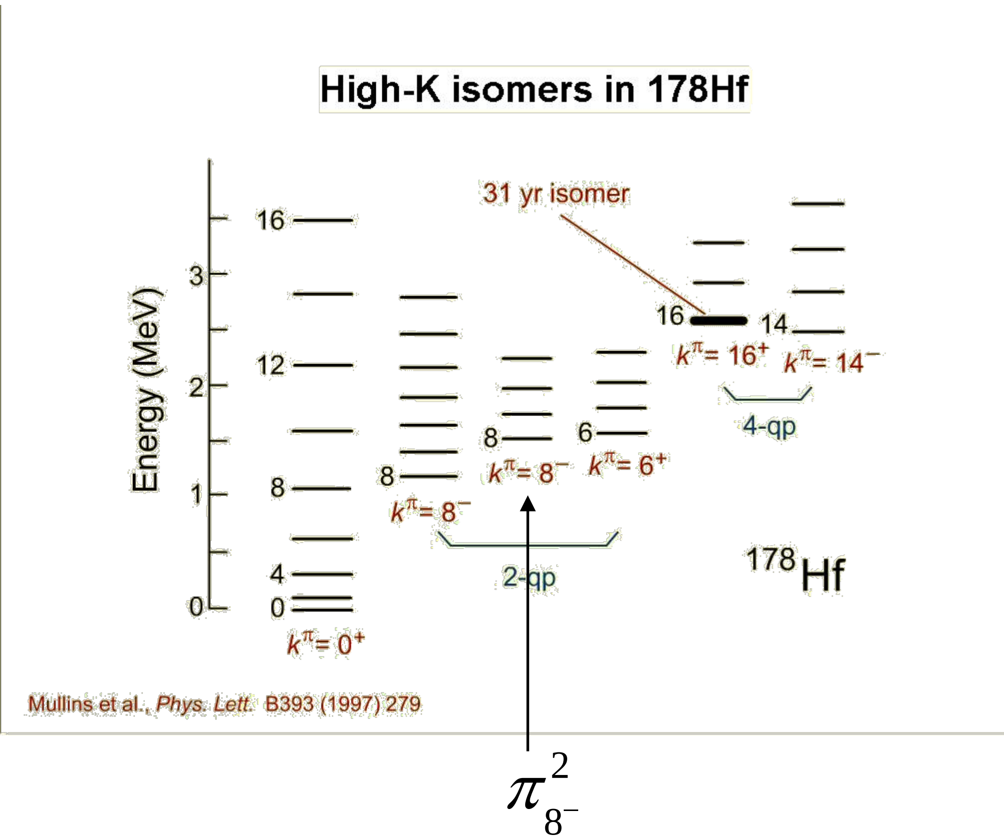
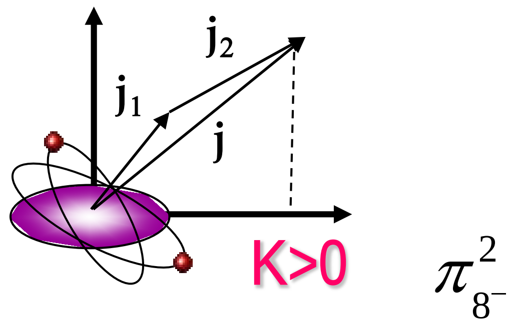
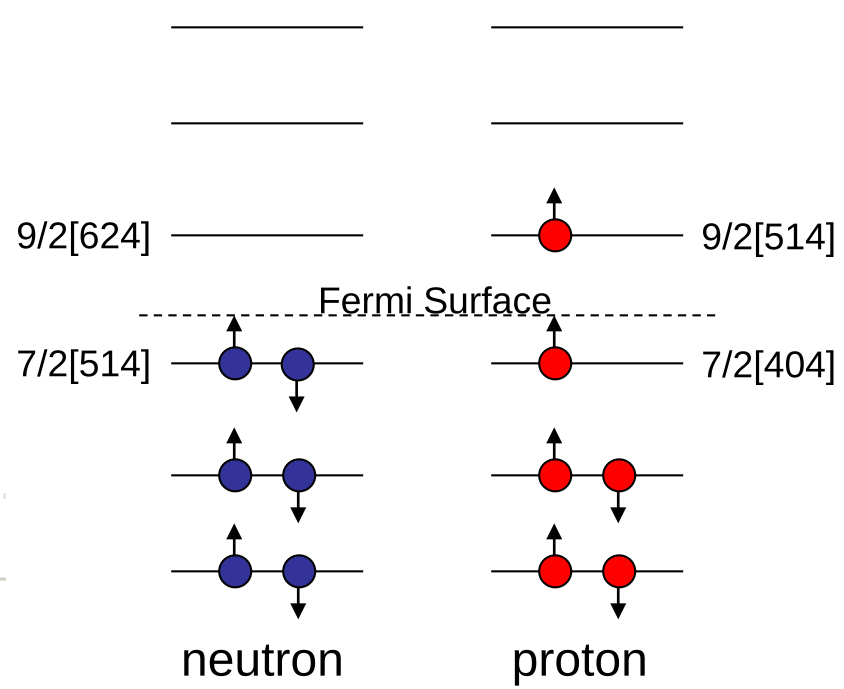
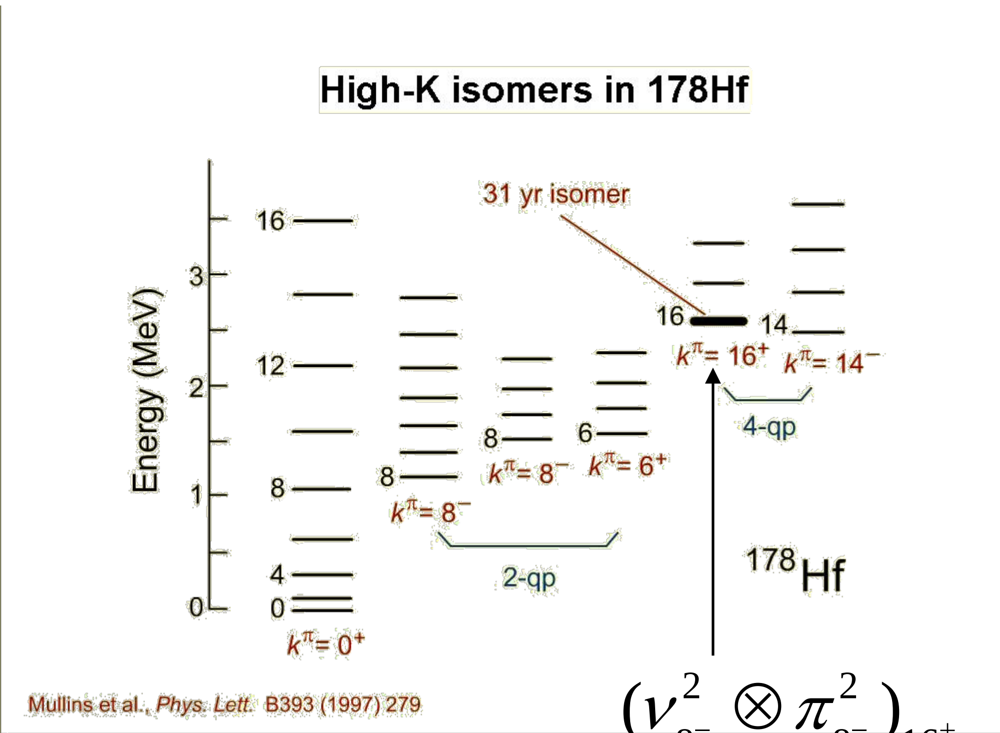






K-Isomer

- A well-known example:



$$(\nu^2_{8^-} \otimes \pi^2_{8^-})_{16^+}$$

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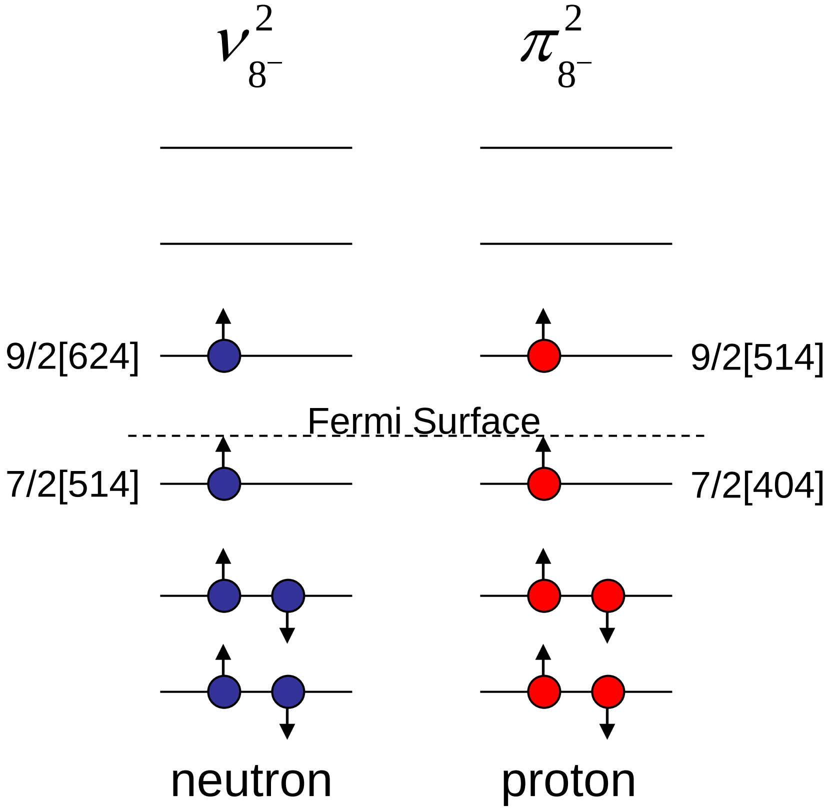


Magnetic moments in ^{178}Hf

$$g(j)=\begin{cases}\dfrac{2\cdot l\cdot g_l+g_s}{2\cdot l+1} & \text{for } j=l+1/2\\[2ex] \dfrac{2\cdot(l+1)\cdot g_l-g_s}{2\cdot l+1} & \text{for } j=l-1/2\end{cases}$$

proton $g_l=1 \quad g_s=5.59$

neutron $g_l=0 \quad g_s=-3.83$

$g(h_{11/2}) = 1.42$ $\quad g(g_{7/2}) = 0.49$ $\quad g(f_{7/2}) = -0.55$ $\quad g(i_{13/2})=-0.29$

$$g(j_1\times j_2;J)=\frac{1}{2}\cdot(g_1+g_2)+\frac{j_1\cdot(j_1+1)-j_2\cdot(j_2+1)}{2\cdot J\cdot(J+1)}\cdot(g_1-g_2)$$

$g(h_{11/2} \times g_{7/2}; 8^-) = 1.08$ $\quad g(f_{7/2} \times i_{13/2}) = -0.36$

$g(8^- \times 8^-; 16^+) = 0.36 \quad \rightarrow \quad \mu = g\cdot I = 5.76$ nm

7.26±0.16 nm

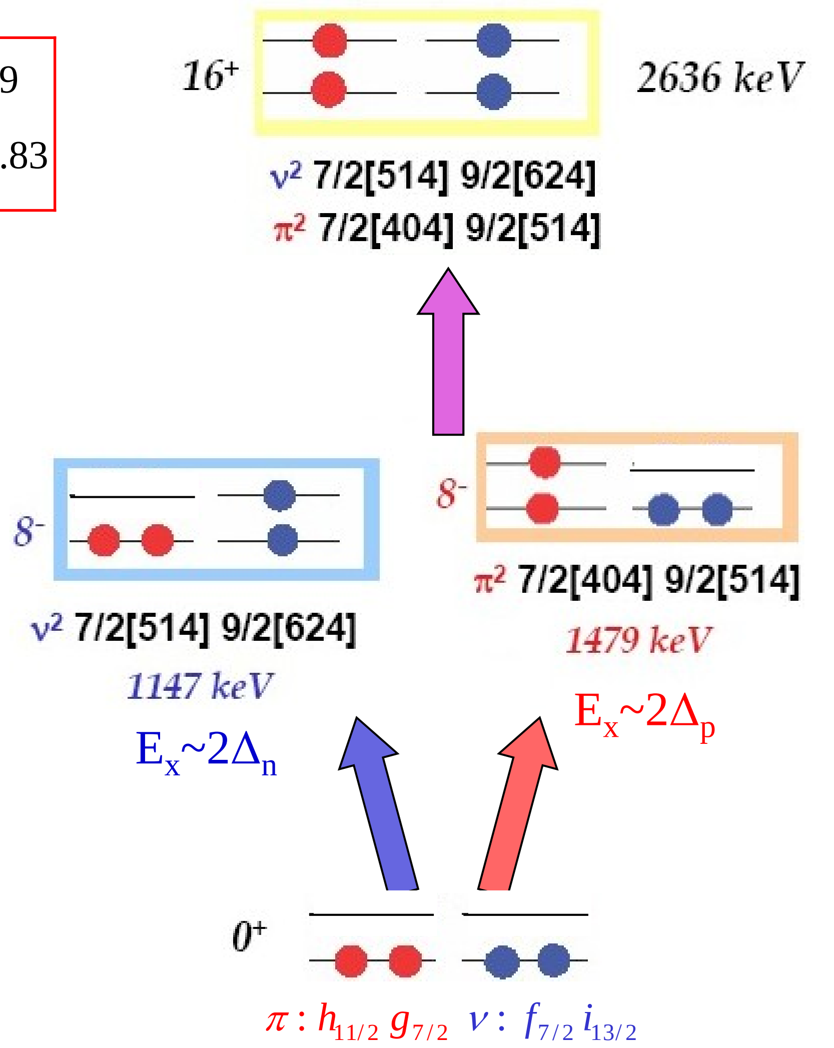
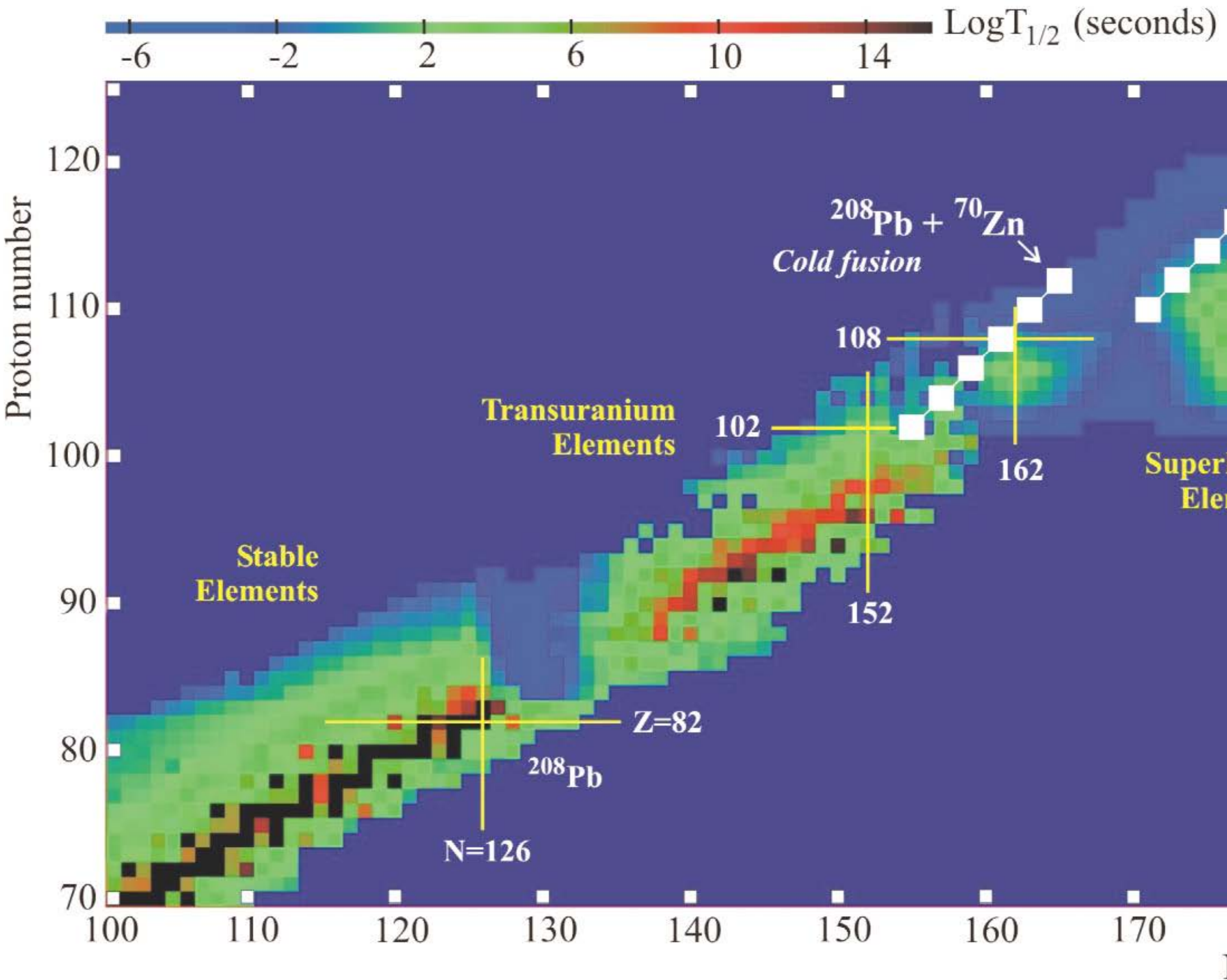
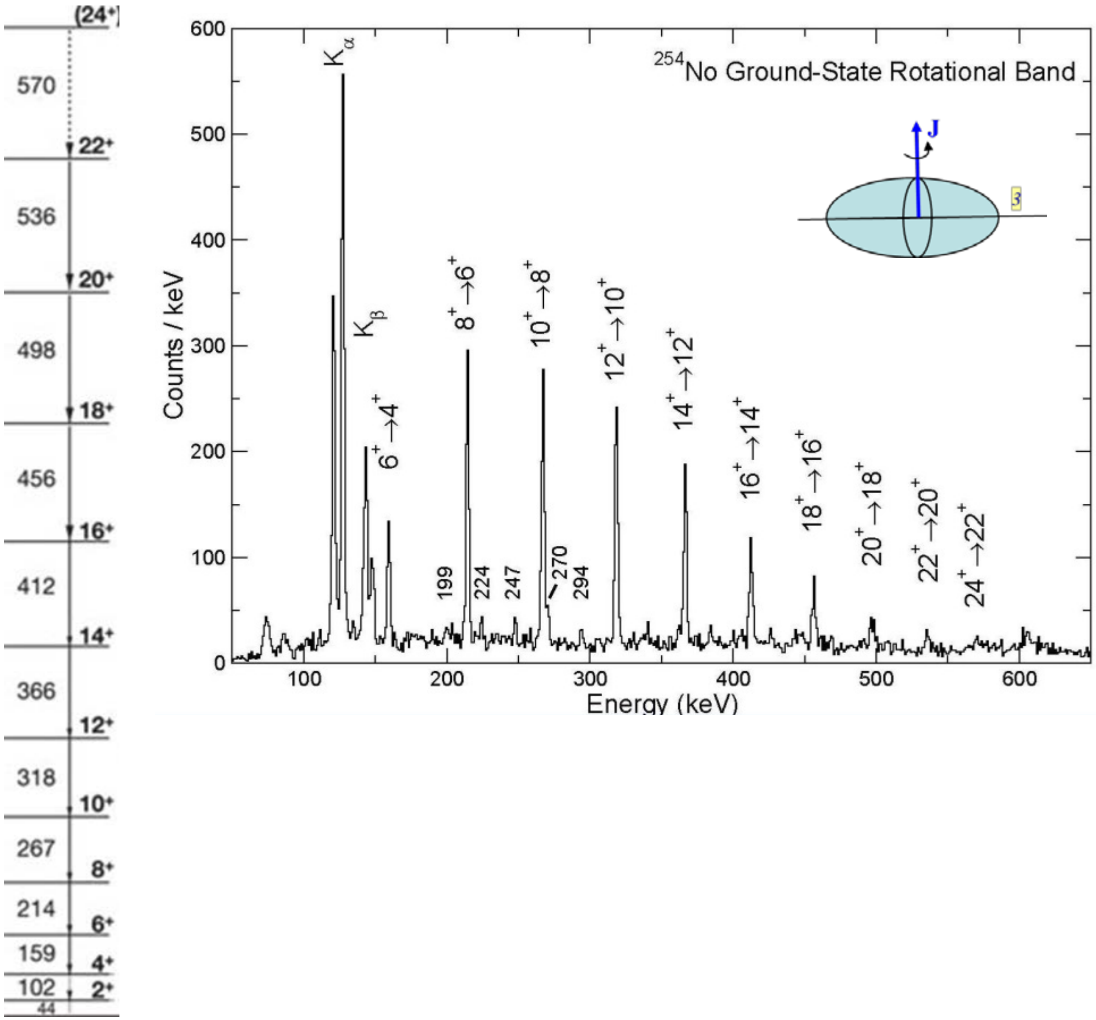


Chart of nuclides: the domain of heavy and super-heavy elements



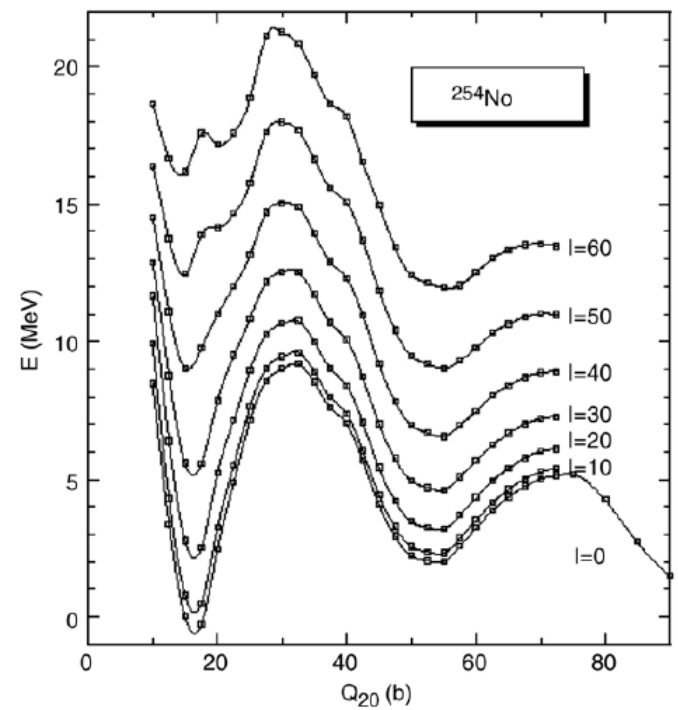
Spinning the heaviest elements



rotational energy: $E_I = \frac{\hbar^2}{2\Im} \cdot I(I+1)$

γ-ray energy: $E_I - E_{I-2} = \frac{\hbar^2}{2\Im} \cdot (4I-2)$

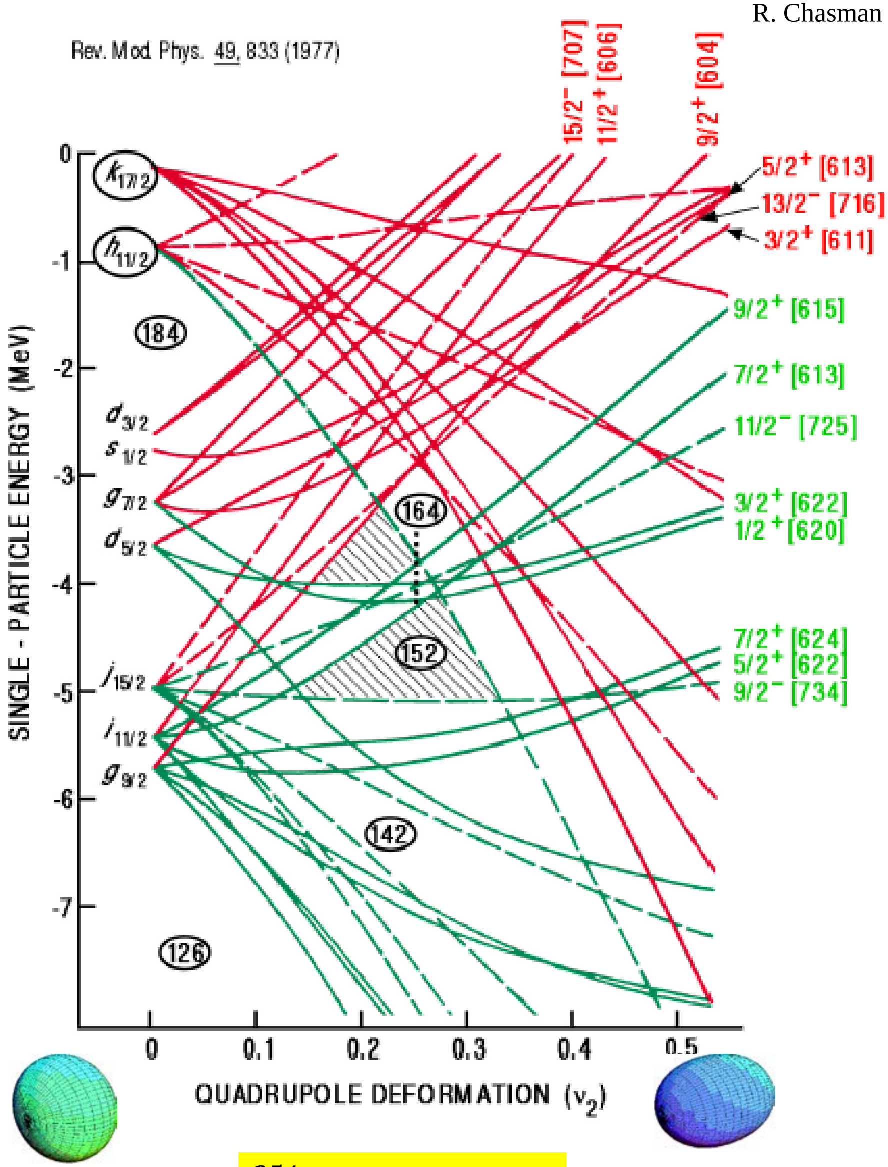
dependence of the fission barrier on spin I

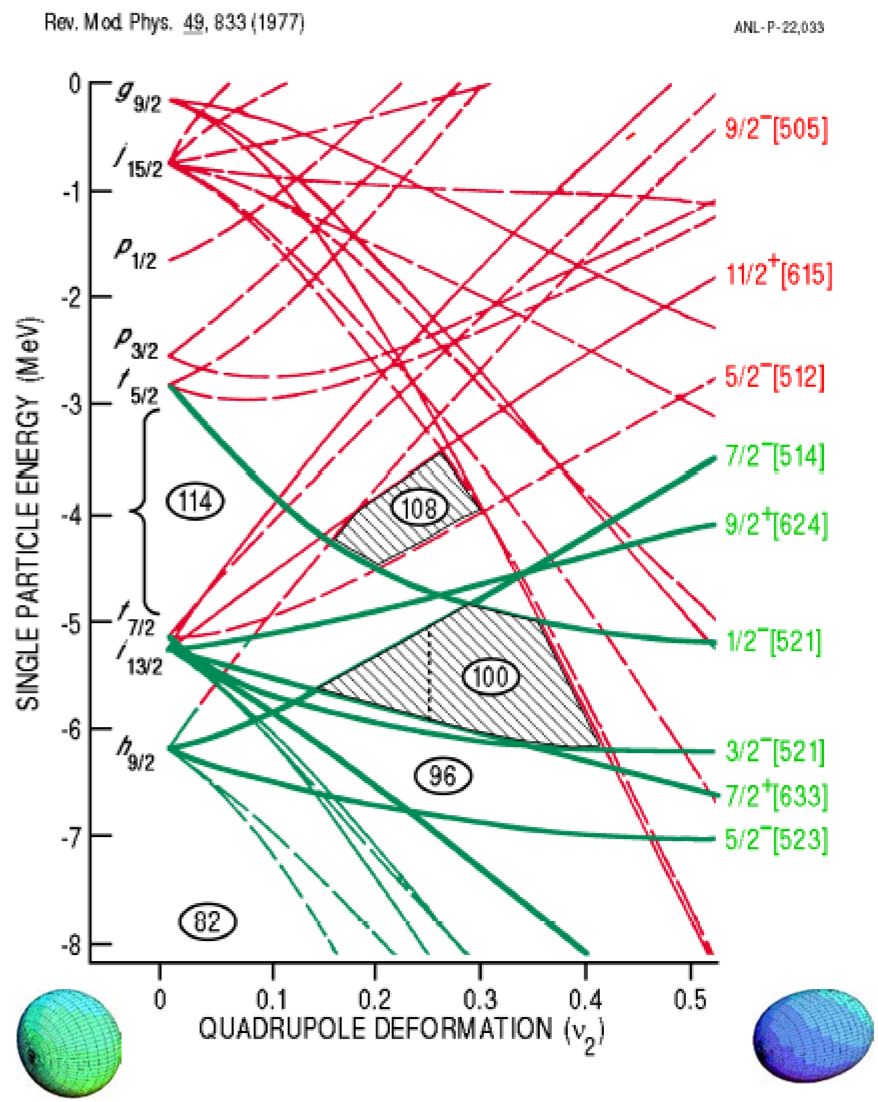


S. Eeckhaupt et al.; EPJA 26 (2005), 227

Single particle orbitals

R. Chasman et al. Rev. Mod. Phys. 49 (1977), 833



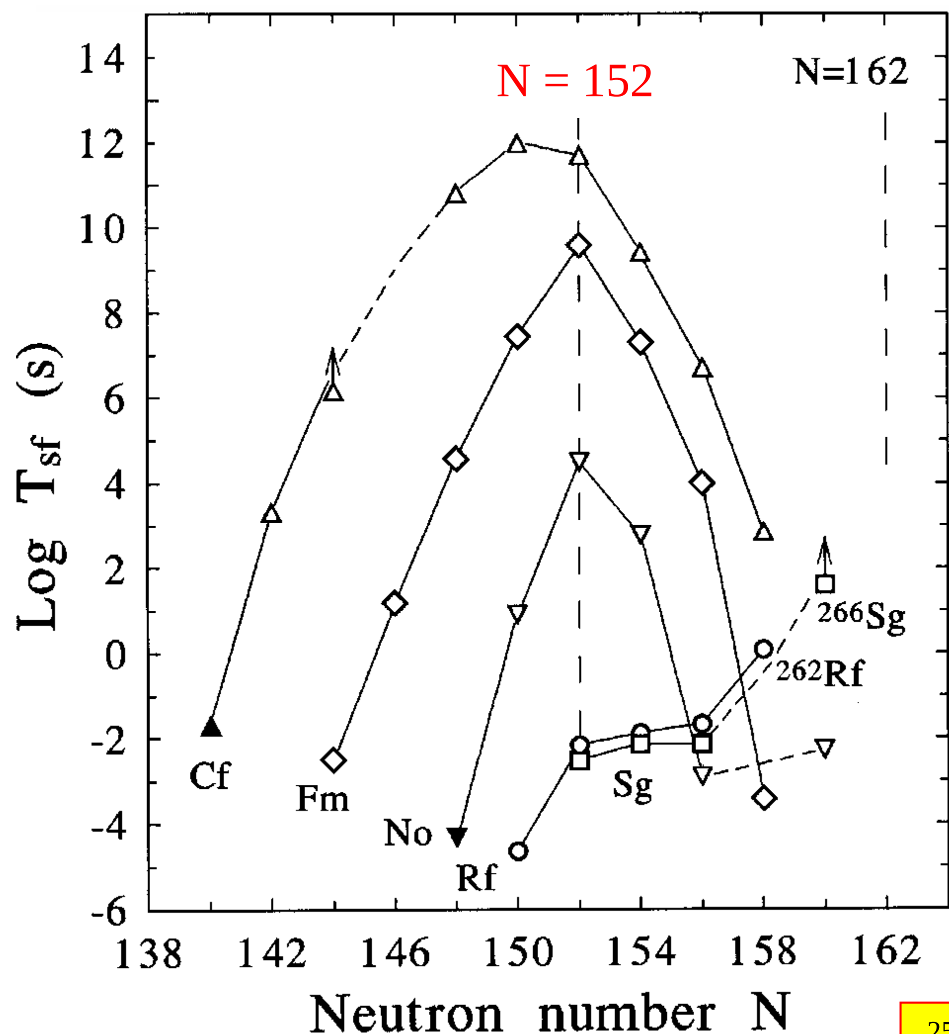


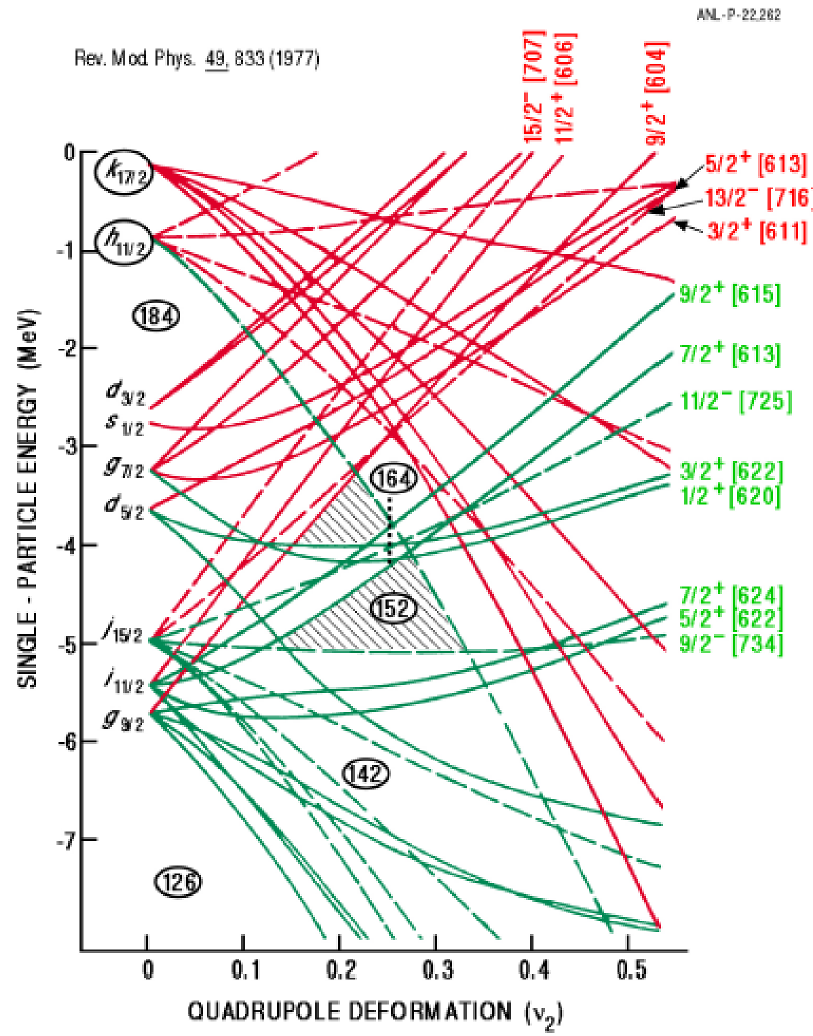
oblate

$^{254}_{102}No_{152}$ $\beta_2 \sim 0.28$

prolate

Stability of heavy elements – Nilsson level energy





^{254}No (Z=102), ^{252}Fm (Z=100) and ^{250}Cf (Z=98)
with N=152
seem to be more stable than their neighbors