Emission of Electromagnetic Radiation

The interaction of the nucleus with an external electromagnetic field is very well known, and is therefore a very direct way of obtaining information about a nucleus.

A nuclear state $|i>$ decays to a lower excited state $|f>$ or to the ground state $|gs>$ by emitting a photon (γ-ray) with energy

$$E_\gamma = \hbar\omega = E_i - E_f \tag{1}$$

and wavelength

$$\lambda\!\!\!^{-} = \frac{\hbar c}{E_\gamma} = \frac{197.3}{E_\gamma(MeV)} \quad (fm) \tag{2}$$

Since each nuclear state has a definite nuclear spin I, its components M (M=-I,...,I) and parity π, a photon must take out angular momentum $\lambda \geq 1$ (component μ) and parity π in accordance with the conservation laws

$$|I_i - I_f| \leq \lambda \leq I_i + I_f \quad and \quad \mu = M_i - M_f \tag{3}$$

$$\pi_i \pi_f = (-1)^\lambda \quad for\ electric\ multipole\ radiation \tag{4}$$

$$\pi_i \pi_f = (-1)^{\lambda-1} \quad for\ magnetic\ multipole\ radiation \tag{5}$$

The probability for γ-ray emission of angular momentum λ from an excited nuclear state I_i into a lower-lying state I_f is expressed by

$$T(^E_M\lambda; I_i \rightarrow I_f) = \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\frac{1}{\hbar}(\frac{E_\gamma}{\hbar c})^{2\lambda+1}B(^E_M\lambda; I_i \rightarrow I_f) \tag{6}$$

for electric (Eλ) and magnetic (Mλ) multipole quanta. The angular momentum of the photon, λ, is called the multipole order of radiation. For the most important cases of λ, the expression (Eq. 6) gives, for the decay rate per second,

$$T(E1; I_i \rightarrow I_f) = 1.590\ 10^{17}\ E_\gamma^3\ B(E1; I_i \rightarrow I_f) \tag{7}$$

$$T(E2; I_i \rightarrow I_f) = 1.225\ 10^{13}\ E_\gamma^5\ B(E2; I_i \rightarrow I_f) \tag{8}$$

$$T(E3; I_i \rightarrow I_f) = 5.709\ 10^{8}\ E_\gamma^7\ B(E3; I_i \rightarrow I_f) \tag{9}$$

$$T(E4; I_i \rightarrow I_f) = 1.697\ 10^{4}\ E_\gamma^9\ B(E4; I_i \rightarrow I_f) \tag{10}$$

$$T(M1; I_i \rightarrow I_f) = 1.758\ 10^{13}\ E_\gamma^3\ B(M1; I_i \rightarrow I_f) \tag{11}$$

$$T(M2; I_i \rightarrow I_f) = 1.355\ 10^{7}\ E_\gamma^5\ B(M2; I_i \rightarrow I_f) \tag{12}$$

$$T(M3; I_i \rightarrow I_f) = 6.313\ 10^{0}\ E_\gamma^7\ B(M3; I_i \rightarrow I_f) \tag{13}$$

$$T(M4; I_i \rightarrow I_f) = 1.877\ 10^{-6}\ E_\gamma^9\ B(M4; I_i \rightarrow I_f) \tag{14}$$

where $E_\gamma = E_i - E_f$ is the energy of the emitted γ-quantum in MeV (E_i, E_f are the nuclear level energies of the initial and final states,respectively), and the reduced transition probabilities $B(E\lambda)$ in units of $e^2(barn)^\lambda$, and $B(M\lambda)$ in units of $\mu_N^2 = (e\hbar/2Mc)^2(fm)^{2\lambda-2}$ with $(e\hbar/2Mc)^2 = 0.01592\ MeV\ fm^3$.

The number of multipole transitions, which actually contribute to the transition probability, is rather restricted. Since the size of the radiation source (nuclear radius) is much smaller than the wavelength $\lambdabar \sim \hbar c/E_\gamma$ of a photon, the emission of high multipole radiation is strongly suppressed. In case the lowest allowed multipolarity is of electric type, all higher possible multipoles can usually be neglected while in case it is of magnetic type the next higher electric multipole E(λ+1) might have to be taken into account. For the most important case of an M1-E2 mixture, the ratio of the E2/M1 transition rate amplitude or the so-called E2/M1 mixing ratio is given by

$$\begin{aligned} \delta_{if}(E2, M1) = \frac{\delta_{if}(E2)}{\delta_{if}(M1)} &= -0.835\ E_\gamma \frac{< I_f||i^2 M(E2)||I_i >}{< I_f||i^0 M(M1)||I_i >} \\ &= 0.835\ E_\gamma \frac{< I_f||M(E2)||I_i >}{< I_f||M(M1)||I_i >} \end{aligned} \tag{15}$$

if E_γ is measured in MeV and the electric and magnetic reduced matrix elements are inserted in units of $e(barn)$ and μ_N, respectively. This definition of the E2/M1 mixing ratio is phase consistent with the definition of Steffen and Alder [Ste75]. The transition rate amplitude is defined by

$$\delta_{if}({}^E_M\lambda) = (-1)^{\Lambda({}^E_M)} \sqrt{\frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \frac{1}{\hbar} (\frac{E_\gamma}{\hbar c})^{2\lambda+1}} \frac{< I_f||i^{\lambda-\Lambda({}^E_M)} M({}^E_M\lambda)||I_i >}{\sqrt{2I_i+1}} \tag{16}$$

where we use $\Lambda(E) = 0$ for electric and $\Lambda(M) = 1$ for magnetic transitions. The absolute square of the transition rate amplitude (Eq. 16) is equal to the partial transition probability (Eq. 6) per unit time.

For the lifetime $\tau(= T_{1/2}/\ln 2)$ of an excited nuclear state I_i we obtain

$$\tau(I_i) = \{\sum_{I_f}\sum_{\lambda} T({}^{E}_{M}\lambda; I_i \rightarrow I_f)[1 + \alpha_T(\lambda)]\}^{-1} \tag{17}$$

where we have summed over all final states I_f into which level I_i can decay and all multipoles λ in the transition $I_i \rightarrow I_f$. The quantity $\alpha_T(\lambda)$ is the usual total λ-pole conversion coefficient.

In the special case of a pure E2 transition from the first excited state 2^+_1 to the ground state 0^+_{gs}, the lifetime of the 2^+_1-state is given by

$$\tau(2^+_1) = \{T(E2; 2^+_1 \rightarrow 0^+_{gs})[1 + \alpha_T(E2)]\}^{-1} \tag{18}$$

which yields the relation between the lifetime and the reduced transition probability $B(E2; 2^+_1 \rightarrow 0^+_{gs})$

$$\tau(2^+_1) = 8.16 * 10^{-14}\{[1 + \alpha_T(E2)]E^5_{\gamma}B(E2; 2^+_1 \rightarrow 0^+_{gs})\}^{-1} \tag{19}$$

with τ in sec, E_γ in MeV and B(E2) in $e^2 barn^2$.

References

[Ste75] R.M. Steffen and K. Alder: Angular Distribution and Correlation of Gamma Rays, in: W.D. Hamilton, ed., The Electromagnetic Interaction in Nuclear Spectroscopy (1975) North-Holland Publishing Company, Amsterdam, p. 505