Nonlinear stopping power of ions in plasmas

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The study of the nonlinear stopping power of ions in plasmas is of fundamental importance for various applications. One example is the energy loss of heavy ions passing through a plasma. Due to the high non-equilibrium charge states specific to heavy ions, the plasma regime with coupling parameters $1/N_D < 1$ and $Z_p/N_D \gtrsim 1$ ($N_D \sim$ number of electrons in a Debye sphere, Z_p charge of the ion) is of interest. In this regime, the Vlasov-Poisson system cannot be linearized, rather a fully nonlinear treatment is required. In the present paper, the Vlasov-Poisson system is solved numerically by using the capability of the new generation of massively parallel supercomputers. The results are compared with the standard dielectric theory and a recent binary collision approach. It is demonstrated that nonlinear effects lead to a strongly reduced Bragg-peak for $Z_p/N_D \gtrsim 1$. In the nonlinear regime, the scaling of the stopping power is close to a $Z_p^{3/2}$ law, which is found to be characteristic for the nonlinear stopping power, if the influence of close collisions on the induced potential is treated properly. © *1996 American Institute of Physics.* [S1070-664X(96)03405-1]

I. INTRODUCTION

The stopping power of charged particles in plasmas is an important quantity used in a variety of disciplines. One example is the heating of matter with heavy ion beams.[1] Heavy ion beams are potential candidates to serve as drivers for inertial confinement fusion (ICF)[2] and to generate dense plasmas in parameter regions not accessible before. The attainable target temperature is determined by the specific energy $(dE/dz)(N/\pi r_0^2)$, with (dE/dz) the stopping power, N the number of particles and r_0 the focal spot radius on the target. At Gesellschaft für Schwerionenforschung (GSI) in Darmstadt, the key parameters of matter heated by heavy-ion beams are evaluated experimentally. Recent results concerning the energy loss enhancement, relative to a cold target, of heavy ions passing through a fully ionized hydrogen plasma ($n_e \leqslant 7\times 10^{16}$ cm^{-3}) were reported in Ref. 3. The energy loss in denser plasmas $n_e > 10^{20}$ cm^{-3}, relevant for the ICF scenarios, will be measured in future experiments.

Another point of interest for the experimental work at GSI, and elsewhere, is the electron cooling of ion beams,[4] a technique used in accelerator physics to reduce the phase space volume of these beams. The cooling process is based on the energy loss due to Coulomb interaction of the ions in a superimposed cold electron beam moving with the same average velocity and guided by a longitudinal magnetic field. Recent experimental results[5] of the longitudinal cooling force for medium and heavy ions show deviations from the well known linearized theories. For low relative velocities, the experiments show a scaling of the longitudinal cooling force in the charge state close to a $Z_p^{3/2}$ law, in contrast to the standard dielectric theory (DT) described, for example, in Ref. 6. Understanding the origin of the modified scaling in Z_p is of interest.

A last example is the internal α-particle heating of fusion pellets. Here the interaction becomes quite strong due to the high density of the compressed pellet ($0.1 \lesssim Z_p/N_D \lesssim 10$).[7,8]

Usually the stopping power is calculated in the framework of the dielectric theory. Characteristic for the obtained stopping power in hot plasmas is the maximum approximate projectile velocities compared to the electron thermal speed (Bragg-peak). The aim of the analysis presented here is to calculate the exact stopping power for a given charge state by solving the Vlasov-Poisson system numerically. The exact solution of the Vlasov-Poisson system is of fundamental interest, because the standard DT is limited to cases with $Z_p/N_D \ll 1$. Due to the high non-equilibrium charge states specific to heavy ions and high plasma densities, this condition is violated in the applications.

First the DT will be described briefly. Then the failure of this theory for slow ions is demonstrated and a binary collision approach for a Debye-shielded projectile is presented. The DT and the new approach are then compared with the exact numerical solution of the Vlasov-Poisson system, which is the basis of the analysis presented here, since only collective plasma effects will be treated ($N_D > 1$). The standard tool for the simulation of collective processes in plasmas are particle-in-cell (PIC) codes. Here Vlasov simulations are performed, which are more laborious, but provide a much better phase-space resolution. In order to also, study the stopping power PIC simulations were performed at GSI.[9] The results are difficult to analyze due to the inherent strong noise, absent in Vlasov simulations. Owing to the noise in the PIC results, it was not possible to obtain results for $Z_p/N_D < 5$, for example. The comparison of PIC results with results obtained by using a Vlasov simulation was performed in Ref. 10, in the case of ion stopping along a strong magnetic field.

To describe the process of ion stopping in a classical thermal plasma, it is useful to introduce the following dimensionless variables: $r \rightarrow r/\lambda_D$, $v = v_e/v_{th}$, $t \rightarrow \omega_P t$, $f \rightarrow v_{th}^3 f/n_0$, $\phi \rightarrow e\phi/kT$, $n = n_e/n_0$, $Z = Z_p/N_D$, where n_0 is the initial electron density, $\lambda_D^2 = kT/4\pi n_0 e^2$ the Debye length, $N_D = n_0\lambda_D^3$ the number of particles in the Debye

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sphere, $v_{th}^2 = kT/m_e$ the electron thermal velocity, T the electron plasma temperature, k the Boltzmann constant and e and m_e the electron charge and mass, respectively. $Z = Z_p/N_D$ characterizes the coupling strength between the projectile ion and the plasma. A handy formula is $Z \approx 10^{-8} Z_p \sqrt{n_e[\text{ cm}^{-3}]}/(4T^{3/2}[\text{ eV}])$. Of importance is the impact parameter for 90^o degree deflection in the bare Coulomb potential $b_\perp = Z/(4\pi u^2)$, with the relative velocity $u^2 \approx v_p^2 + 2$. Note that for the mean quadratic electron speed, $\langle v_e^2 \rangle = 2v_{th}^2 = 2$ holds in the chosen units. Often the Coulomb logarithm $\Lambda = \ln(\lambda_D/b_\perp)$, which can easily be related to the coupling parameter Z, is used as measure of small angle collisions relative to large angle scattering of the electrons passing the projectile. In this work, the coupling parameter region $Z \leqslant 10$ ($\Lambda \gtrsim 0.92$), relevant for the applications, will be treated based on the Vlasov equation for the distribution function $f(\mathbf{r},\mathbf{v},t)$ of the plasma electrons

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + \frac{\partial \phi(\mathbf{r})}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{v}} = 0. \tag{1}$$

The Poisson equation is used to describe the electrostatic field

$$-\frac{\partial^2 \phi}{\partial \mathbf{r}^2} = 1 - n(\mathbf{r}) + Z\delta(\mathbf{r} - v_p \mathbf{e}_z t), \tag{2}$$

$$n(\mathbf{r}) = \int d^3 v f(\mathbf{r},\mathbf{v}). \tag{3}$$

The stopping power is defined as

$$-\frac{dE}{dz} = ZN_D \frac{\partial \phi_{\text{ind}}(\mathbf{r} = v_p \mathbf{e}_z t)}{\partial z}, \tag{4}$$

where v_p is the velocity of the ion. $\phi_{\text{ind}} = \phi - Z/(4\pi r)$, the potential induced by the ion. The plasma ions are treated as a homogeneous background, because it is assumed that v_p is much larger than the thermal speed of the plasma ions.

II. ANALYTICAL APPROACHES

A. Dielectric theory

In the standard dielectric theory (DT), it is assumed that the ion can be treated as a small perturbation ($Z \ll 1$). The stopping power resulting from the linearization of the Vlasov-Poisson-system is[6]

$$-\frac{dE}{dz} = \frac{Z^2 N_D}{2\pi^2 v_p^2} \int_0^{v_p} dw\, w G(w, k_{\max}), \tag{5}$$

$$\begin{aligned} G(w,k_{\max}) = Y(w)\Bigg(& \ln(k_{\max}) \\ & + \frac{1}{4} \ln \frac{(1 + X(w)/k_{\max}^2)^2 + Y(w)^2/k_{\max}^4}{X^2(w) + Y^2(w)}\Bigg) \\ & - \frac{X(w)}{2Y(w)} \left(\arctan \frac{k_{\max}^2 + X(w)}{Y(w)} - \arctan \frac{X(w)}{Y(w)} \right), \end{aligned}$$

$$X(s) = 1 - s \exp\left(\frac{-s^2}{2}\right) \int_0^s dy \exp\left(\frac{y^2}{2}\right),$$

$$Y(s) = \sqrt{\frac{\pi}{2}} s \exp\left(-\frac{s^2}{2}\right).$$

The scaling of the DT in Z is approximately $\sim Z^2 \ln(k_{\max})$.[6] In the framework of the DT close collisions with $b \lesssim b_\perp$ cannot be treated.[11] In order to exclude these collisions, one usually sets $k_{\max} = 1/b_\perp$ with $b_\perp \approx Z/(4\pi(v_p^2 + 2))$. This procedure is only appropriate if the total energy transfer in close collisions is negligible.

B. Nonlinear screening of an ion at rest

For an ion with $v_p \lesssim 1$ one can get the approximate stopping power by using the definition [Eq. (4)] of the stopping power together with the static induced electric field $F_{\text{ind}} = -\partial \phi_{\text{ind}}/\partial z$ near the core (ϕ_{ind} is the static induced potential),

$$-\frac{dE}{dz} = -ZN_D F_{\text{ind}}(\mathbf{r} = \lambda \mathbf{e}_z). \tag{6}$$

It is assumed that the electron cloud around the ion is shifted, independent on Z, by the characteristic length $\lambda = v_p \lesssim 1$.[12] This polarization effect causes the stopping force. It is worth noticing that $\lambda \approx v_p$ is a crude estimate. Using the Debye potential in Eq. (6) results in a Z^2 scaling of the stopping power. In order to obtain the exact static potential of an ion at rest, the distribution function of the free electrons $f(r,v) = 1/(2\pi)^{3/2} \exp(-v^2/2 + \phi)$ for $v^2 \geqslant 2\phi$ is used. The trapped electrons cause no polarization effect if the ion is moving. For the resulting free electron density (3) follows:

$$n(\phi) = \exp(\phi)(1 - \operatorname{erf}(\sqrt{\phi})) + \frac{2}{\sqrt{\pi}}\sqrt{\phi}. \tag{7}$$

The Poisson equation describes the nonlinear coupling between the bare ion potential and the electron density (7),

$$-\frac{\partial^2 \phi_{\text{ind}}}{\partial r^2} = 1 - n\left(\phi = \phi_{\text{ind}} + \frac{Z}{4\pi r}\right). \tag{8}$$

This can also be formulated as an integral equation for ϕ_{ind},

$$\phi_{\text{ind}}(r) = \int_0^\infty dr' q(\phi(r')) \frac{(r')^2}{r_>}, \tag{9}$$

with $r_> = \max(r', r)$ and the total charge density $q(r) = 1 - n(r)$. Equation (8) can be solved numerically by using a successive-over-relaxation (SOR) scheme.[13] In the numerical scheme, a smoothed Coulomb potential is used $Z/(4\pi r) \to Z/(4\pi r) \operatorname{erf}(r/\epsilon)$, with a sufficiently low $\epsilon \ll 1$. The results for different Z are shown in Fig. 1. It is seen that even for $Z < 1$ the static induced potential, which causes the stopping force on slow ions, is different from the well-known Debye result. In the following, an exact analytical result will be obtained for the induced electric field near the core. For $r \ll 1$ the pure Coulomb potential dominates $\phi(r) \to Z/(4\pi r)$. From (7) we get in the limit $\phi \gg 1$,

$$q(r) = -\frac{2}{\sqrt{\pi}}\sqrt{\phi} = -\frac{1}{\pi}\sqrt{\frac{Z}{r}}, \tag{10}$$

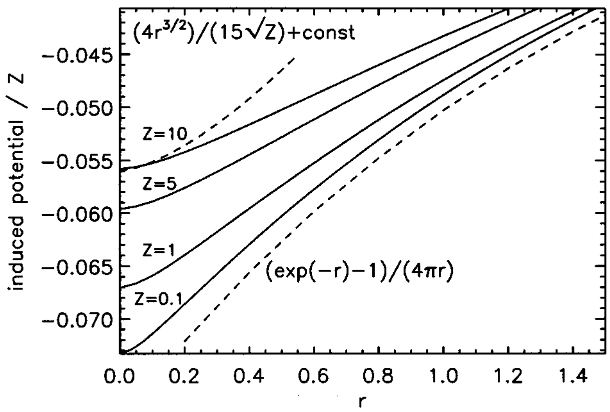

FIG. 1. The induced potential divided by Z calculated numerically (solid lines). The dashed lines represent the approximative expression presented in the text and the Debye result.

in contrast to the Debye result for small r, assuming $\phi \ll 1$,

$$q(r) = -\phi = -\frac{Z}{4\pi r}. \tag{11}$$

This lowering of the electron density n for small r, relative to the Debye result, is caused by the strong deflection and the high velocity of the electrons near the core. Electrons passing the ion with impact parameters $b < b_\perp \approx Z/(8\pi)$ cannot be treated correctly in the linear dielectric description, which is valid only for nearly straight line trajectories. Using the exact charge density near the core (10) together with Gauss's law, it is possible to obtain the static induced electric field:

$$F_{\rm ind}(r) = \frac{1}{r^2}\int_0^r q(r')(r')^2 dr' = -\frac{2}{5\pi}\sqrt{rZ}. \tag{12}$$

Again, in contrast to the Debye result, $F_{\rm ind}(r) = -Z/(8\pi)$ for small r. In Fig. 1, it can be seen that the induced potential $\phi_{\rm ind}(r) = 4/(15\pi)\sqrt{Z} r^{3/2} + \text{const}$ resulting from Eq. (12) is valid for approximately $r \lesssim b_\perp \approx Z/(8\pi)$. The approximate stopping power for slow ions can now be obtained by using Eq. (6) together with Eq. (12). This leads to a $Z^{3/2}$ scaling of the stopping power.

The direct relation to the $Z_p^{3/2}$ scaling of the longitudinal cooling force for slow ions, measured in recent experiments, is difficult, because a strong magnetic field is present in the electron cooler and should be considered in the calculations. This is quite a complicated task. Nevertheless, for low ion velocities and high charge states, it can be assumed that the magnetic field plays only a minor role for the determination of the scaling law. Note that $F_{\rm ind}(\lambda)$ resulting from Eq. (12) only depends on the electrons in the region $r < \lambda$. The scattering angle of the electrons passing the ion with $b < \lambda \lesssim b_\perp$ will be unaffected by the magnetic field, because the electrons are strongly accelerated near the core and so the interaction time is short in comparison to the cyclotron period.

For relative velocities $r_L \ll v_p$ (r_L is the Larmor radius), it becomes more appropriate to calculate the longitudinal cooling force in the infinitely strong magnetic field approximation.[10]

In conclusion, it was demonstrated that for $Z \gtrsim 0.1$ the standard dielectric or Debye theory is inadequate to calculate the exact potential of an ion at rest on distances $r < 1$ from the core. Only the full nonlinear theory can treat the influence of the close collisions on the shielding sufficiently and reproduce the correct $\sqrt{Z_p}$ scaling of the induced electric field near the core, responsible for the stopping power. Due to this fact, it could be expected that for $Z \gtrsim 0.1$ the stopping power of slow ions cannot be treated accurately in the framework of the dielectric theory.

C. Binary collision theory

In principle, the exact stopping power can be calculated in the framework of the binary collision theory[14] if the energy transfer cross section for the dynamically shielded potential is known,

$$-\frac{dE}{dz} = \int d^3v f(\mathbf{v}) \frac{u}{v_p} \int d\sigma \Delta E, \tag{13}$$

where $\Delta E = 2v_p(v_p - v_\parallel)\sin^2(\Theta_c/2) - v_p v_\perp \sin\ \Theta_c$ is the energy transfer due to a single electron with the initial velocity $\mathbf{v} = (v_\perp, v_\parallel)$.[15] Θ_c is the scattering angle in the frame of the ion. For a slow ion ($v_p \lesssim 1$), it can be assumed that the total potential is nearly spherical. Then Eq. (13) can be evaluated further,

$$-\frac{dE}{dz} = \frac{Z^2 N_D}{4\pi}\int_{-\infty}^{\infty}\int_0^{\infty} dv_\parallel dv_\perp \times f(v_\perp, v_\parallel)\frac{\lambda(u)}{u^3}(v_p - v_\parallel), \tag{14}$$

with the generalized Coulomb logarithm,

$$\lambda(u) = \frac{1}{b_\perp^2}\int_0^\infty b\, db \sin^2(\Theta_c(b)/2), \tag{15}$$

and $b_\perp = Z/(4\pi u^2)$. For a Debye potential a numerical fit for $\lambda(u)$ is given by the authors of Ref. 15 ($s = 1.17/b_\perp$),

$$\lambda(s) = \frac{(1+s)^2\ \ln(1+s)}{(2+s)^2} - \frac{s}{2(2+s)} + 0.15s\ \exp(-0.5s), \tag{16}$$

which can be used in Eq. (14) to obtain the stopping power in the framework of the binary collision theory for a Debye shielded potential (BCTD).

In Ref. 15, the authors used Eq. (13) without the u/v_p term in order to calculate the stopping power (see the comment[16] on Ref. 15). Here the correct binary collision theory[14] is used together with Eq. (16). Note that in contrast to the standard DT, the BCTD treats the energy transfer in close collisions exactly.

For the determination of Eq. (16) it is assumed that the ion is Debye-shielded. This assumption fails if dynamical shielding becomes important at higher projectile velocities. For practical purposes, a stopping power formula covering

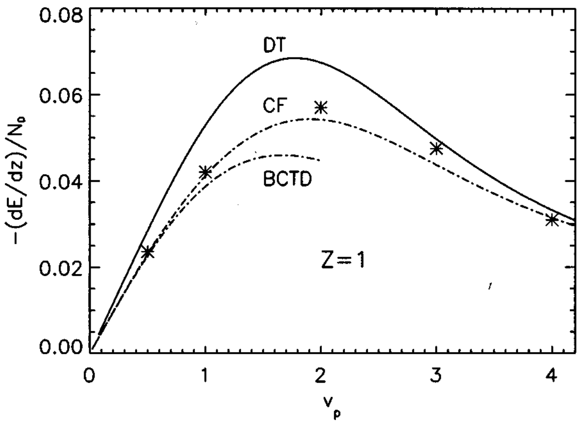

FIG. 2. Comparison of the numerical results (stars) with the CF, the DT and the BCTD for $Z=1$ as a function of v_p.

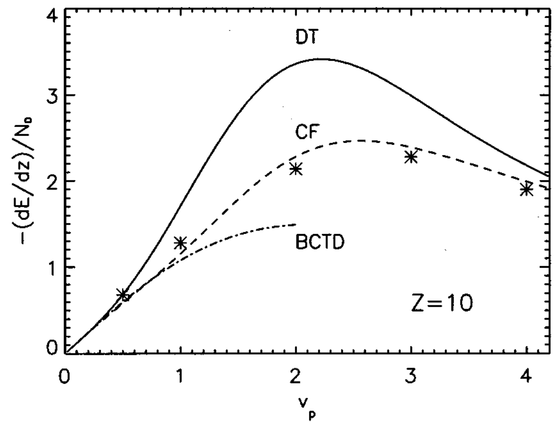

FIG. 4. Comparison of the numerical results (stars) with the CF, the DT and the BCTD for $Z=10$ as a function of v_p.

all projectile velocities is necessary. This can be done by simply combining the BCTD with the DT [combined formula (CF)],

$$\frac{dE}{dz}=\frac{(v_{\max}-v_p)}{v_{\max}}\left.\frac{dE}{dz}\right|_{\mathrm{BCTD}}+\frac{v_p}{v_{\max}}\left.\frac{dE}{dz}\right|_{\mathrm{DT}},$$

$$v_p<v_{\max},\quad \frac{dE}{dz}=\left.\frac{dE}{dz}\right|_{\mathrm{DT}},\quad v_p\geqslant v_{\max}.$$

Here the BCTD is used for $v_p \ll v_{\max}$ and the DT for $v_p > v_{\max}$, with $v_{\max}$ to be determined by the exact numerical solution of the Vlasov-Poisson system (see Sec. III).

For high Z, the linear Debye screening is lowered due to the strong acceleration of plasma electrons near the core (see Sec. II B). In this case, the BCTD will underestimate the stopping power and the generalized Coulomb logarithm for the exact potential (8) should be used instead of Eq. (16).

III. VLASOV SIMULATIONS

The time-dependent Vlasov-Poisson system is solved numerically by using a splitting scheme along the characteristics (see the Appendix). The code is implemented on massively parallel supercomputers in order to handle the large four-dimensional (4D) phase space.

Figure 2 shows the stopping power divided by N_D in the weak nonlinear case $(Z=1)$ as a function of v_p. In Figs. 3 and 4 the stopping power in the nonlinear case $(Z=5$ and $10)$ is plotted. It is seen that the standard DT (5) strongly overestimates the stopping power around the Bragg-peak for $Z\gtrsim 1$. As an approximation of the nonlinear stopping power, the presented combined formula (CF) (17) can be used if $v_{\max}=5$ is set.

In Figs. 5 and 6, the scaling of the stopping power in Z is shown. In Fig. 5 it is seen that for $Z\gtrsim 0.1$, the stopping power at $v_p=1$ is different from the DT result. For $Z>1$ the scaling becomes close to the $Z^{3/2}$ law obtained analytically above, in contrast to the DT. In Fig. 6, it is shown that for $Z>1$, the scaling of the Bragg-peak $(v_p\approx 2)$ is also close to a $Z^{3/2}$ law—again in contrast to the DT result.

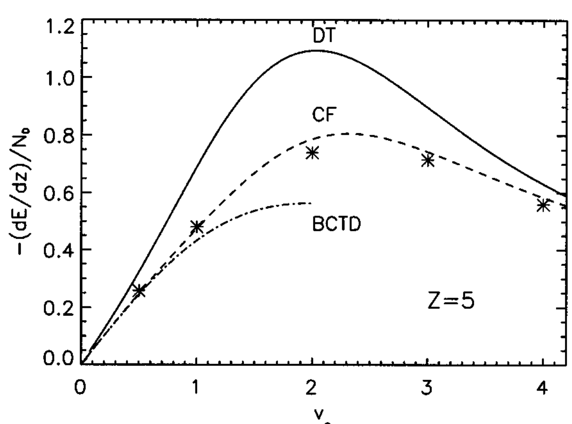

FIG. 3. Comparison of the numerical results (stars) with the CF, the DT and the BCTD for $Z=5$ as a function of v_p.

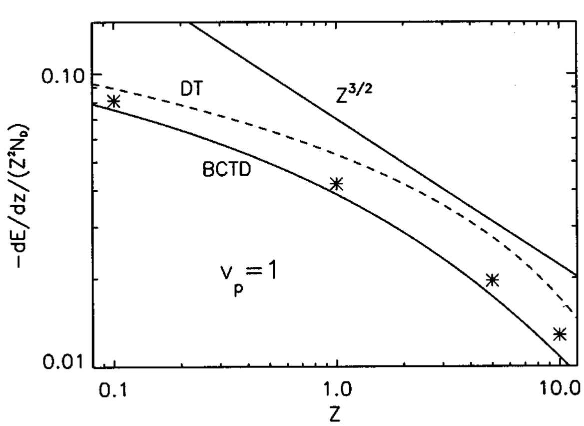

FIG. 5. Comparison of the DT, the BCTD, the numerical results (stars) and a $Z^{3/2}$ scaling law for $v_p=1$.

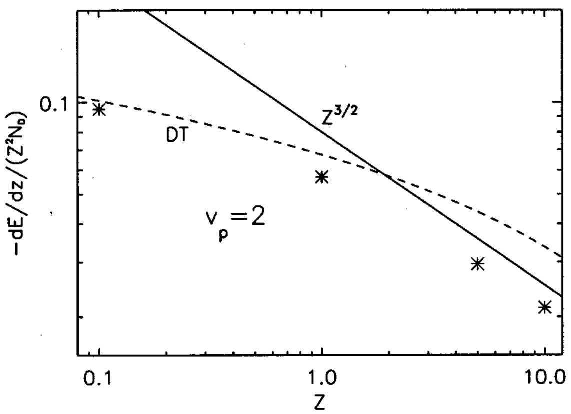

FIG. 6. Comparison of the DT, the numerical results (stars) and a $Z^{3/2}$ scaling law for $v_p = 2$.

Using the numerically obtained exact stopping power together with the BCTD, which is valid for low ion velocities and moderate high Z, it is possible to determine the ion velocity at which dynamical screening becomes important. At this velocity, the BCTD starts to underestimate the stopping power, because the ion is no longer Debye-shielded. This is the case for $v_p \gtrsim 1$, as can be seen in Figs. 2 and 3.

For $Z = 10$, the BCTD slightly underestimates the stopping power, even for $v_p = 0.5$ (see Fig. 4). At this high Z, the lowering of Debye-shielding, as described in Section II B, becomes important for the determination of the energy transfer.

In Fig. 7, the potential divided by Z is plotted on the z-axis for $Z = 1$, 5 and 10 and $v_p = 3$. It is seen that the scaling of excited wake field amplitudes in Z is lower than predicted by the dielectric theory $(\propto Z)$. This lowering is due to the fact that electrons are strongly deflected by the self-consistent potential humps excited behind the ion. The wake field in the linear case is excited by the coherent oscillations of the electrons passing the ion.[17] These oscillations are destroyed if these self-generated potential humps can no longer be treated as small perturbations with $\phi \ll 1$. This was verified by the numerically obtained exact electron trajectories, which show strong deflections by the induced wake field.

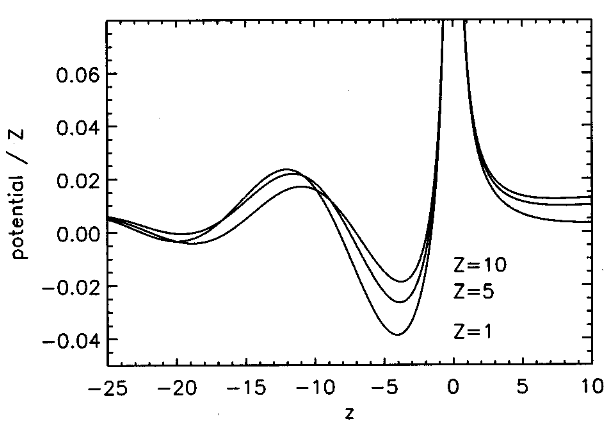

FIG. 7. Comparison of the total potentials divided by Z on the z-axis for $v_p = 3$ and $Z = 1$, 5 and 10.

IV. SUMMARY AND CONCLUSIONS

Based on the numerical solution of the Vlasov-Poisson system and a binary collision approach the nonlinear modifications of the dielectric theory are analyzed. It was demonstrated that for $Z \gtrsim 1$, due to nonlinear effects, the dielectric theory strongly overestimates the stopping power for slow ions. In the nonlinear regime, the Bragg-peak obeys a scaling law close to $Z_p^{3/2}$. For $Z_p/N_D \gtrsim 0.1$, the induced static potential near the ion core, which is responsible for the stopping power of slow ions, is different from the linear Debye result. The induced electric field near the core obeys a $\sqrt{Z_p}$ scaling law, in contrast to the well-known Debye result, which leads to the approximate $Z_p^{3/2}$ scaling of the stopping power for low velocities. The $Z_p^{3/2}$ scaling law is characteristic for the nonlinear stopping power due to the influence of close collisions on the induced potential. The presented binary collision theory for a Debye potential (BCTD) can be used for ion velocities $v_p < v_{th}$ and $Z_p/N_D < 10$. This theory is also relevant for the internal heating of high density fusion pellets, because for the typical velocity of charged-fusion products (α's, etc.), $v_p \lesssim v_{th}$ holds.[7] An expression combining the DT with the BCTD is able to fit the numerically obtained stopping power and should be used for the determination of the specific energy deposition of heavy ions in plasmas. Using the BCTD together with the exact numerical results, the onset of dynamical screening for $v_p \gtrsim v_{th}$ can be located. It is demonstrated that, in the nonlinear case, the wake field induced behind the ion is weaker than predicted by the dielectric theory. The structure of the wake field is important for the determination of the correlation in ion-clusters.[18]

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APPENDIX: NUMERICAL SCHEME

The time-dependent Vlasov-Poisson system is solved until a stationary stopping power is reached. The ion is located in the middle $z = L/2, r = 0$ of a cylindrical box with a total length L and radius R. In order to resolve the excited wake field, $L \gg v_p$ and $R \gg 1$ must hold. The energy transfer in close collisions determines dr, dz. Typical values chosen in the simulation are $L = 100$, $R = 20$, $dr, dz = 0.15$. They strongly depend on Z and v_p and have been found in extensive test runs. Initially, a Maxwellian distribution is chosen for the electrons. The Vlasov-Poisson system is advanced using the well-known method of fractional steps first described in Ref. 19. This method has been used successfully for the simulation of nonlinear plasma phenomena in two and three phase-space dimensions (see for example Refs. 20–22) and can easily be implemented on parallel computers. Here this scheme is used for the first time in four phase space dimensions. Let us define $t_n = n\Delta t$ and $t_{n+1/2} = (n + \frac{1}{2})\Delta t$.

Step 1:

$$f_1(r,z,v_r,v_z,t_{n+1/2}) = f\left(r - v_r \frac{\Delta t}{2}, z, v_r, v_z, t_n \right).$$

Step 2:

$$f_2(r,z,v_r,v_z,t_{n+1/2}) = f_1\left(r, z - v_z \frac{\Delta t}{2}, v_r, v_z, t_{n+1/2} \right).$$

Step 3:

$$f_3(r,z,v_r,v_z,t_{n+1/2}) = f_2(r,z,v_r + E_r(t_{n+1/2})\Delta t, v_z, t_{n+1/2}).$$

Step 4:

$$f_4(r,z,v_r,v_z,t_{n+1/2}) = f_3(r,z,v_r,v_z + E_z(t_{n+1/2})\Delta t, t_{n+1/2}).$$

Performing Step 2 and 1 a second time leads to the complete time step. The interpolation is done using an off-center three point interpolation following Ref. 23. The Poisson equation is solved at $t_{n+1/2}$ using a Fast Fourier Transform (FFT) in the z-direction, solving the resulting tridiagonal systems and performing the inverse FFT. Periodic boundary conditions are used at $z = 0, L$. At $r = 0, R$, reflecting boundary conditions are used for the Vlasov equation and for the Poisson equation, $\phi(R) = 0$ is set. The total number of grid points used is $N_r \times N_z \times N_{v_r} \times N_{v_z} = 100 \times 500 \times 70 \times 70 (\approx 1 \text{ GB})$.

The code is implemented on massively parallel supercomputers (96 Node Parsytec GC/PP, 140 Node Intel Paragon). This is done by using a master-slave scheme. The phase space is divided into equal partitions along the z-axis. Every slave process handles only one such partition. Before Step 3 the charge density is calculated locally and sent to the master process, which solves the Poisson equation. The master sends the calculated potential back to each slave. Before the shift in the z-direction is performed (Step 4), every slave gets a phase space strip of the thickness $v_z \Delta t + l \Delta z$ from his neighbors, where l is the number of points used for the interpolation. The scheme presented here is only applicable if the interpolation is done locally. The public domain software PVM (Parallel Virtual Machine) is used as a message passing library.

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