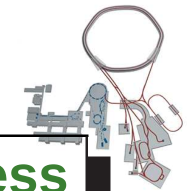

Longitudinal and Transverse Impedances and Shielding Effectiveness of a Resistive Beam Pipe for Arbitrary Energy and Frequency

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- **Introduction – Motivation-Problems-Importance**
- **Electromagnetic fields in a cylindrical pipe**
  - Solution with Leontovich boundary condition
- **Longitudinal impedance of a conducting pipe**
- **Longitudinal coupling impedance between beam and pipe**
- **Transmission coefficient**
- **Shielding effectiveness**
  - Shielding effectiveness of SIS 18, SIS 100
- **Transverse resistive wall impedance**
  - Hollow ring distribution (RHIC)
  - Uniform distribution
- **Summary**

Motivation

Coupling impedance between beam and pipe in accelerators

$\longrightarrow$ instabilities in the ISR at CERN

Design of accelerators

$\longrightarrow$ reduce the coupling impedance of the beam to its environment

Prevent longitudinal and transverse beam instabilities

Longitudinal coupling impedance includes the space-charge and resistive-wall impedances

Modeling of the longitudinal dynamics of charged particle beams

Longitudinal (transverse) beam instabilities from longitudinal (transverse) impedances

Not perfectly conducting wall

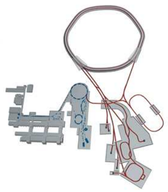
$\longrightarrow$ electromagnetic field strays into and behind pipe

Penetration depth is given by the skin depth δ_s

Current induced in the wall leads to heating of pipe

Zotter, Zimmermann and Oide, Gluckstern and Zotter, Al-khateeb et al

Exact solutions – no approximations



Problems

Superconducting ⟶ cool walls stay cool

⟶reduce Eddy currents

⟶ thin wall of 0.3 mm (SIS 18), 0.1 mm (SIS 100)

Skin depth about 1 mm ⟶ compromise

Perturbation of electronics just outside pipe!

In reality: impedance depends on structures outside pipe

Shielding by beam pipe is of relevance for the SIS 18 and new SIS 100

analytical closed form expressions compare with approximations

Apply to GSI SIS18 and future SIS200/300

Importance

Also of importance for other fast ramping synchrotron projects

$\longrightarrow$ rapid cycling 3 GeV synchrotron for J-PARC

Need closed form expression for design of high current ring machines

for resistive wall impedance and for shielding effectiveness

in relevant range of frequencies, beam energies and wall thicknesses

Skin depth

Skin (penetration) depth for the frequency ω :

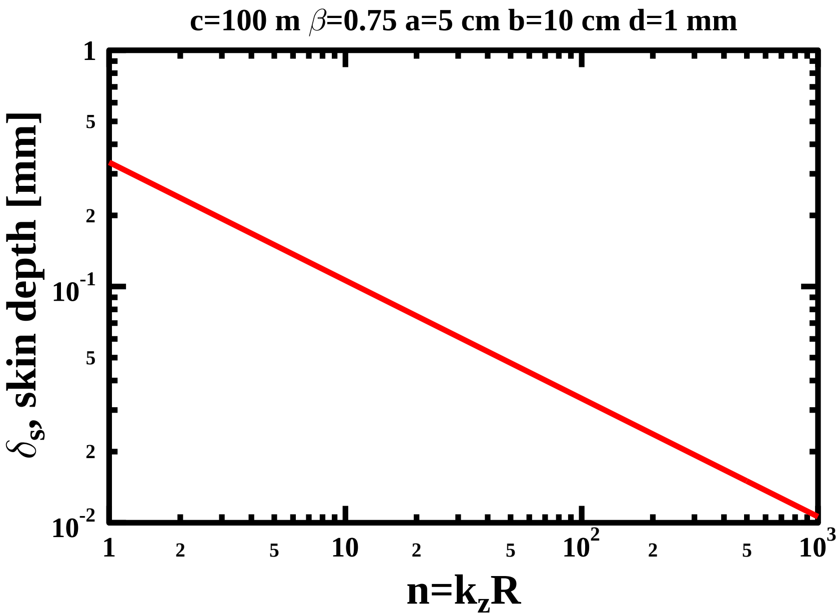
$$\delta_s = \sqrt{2/\mu_0 S\omega}$$

$$\omega = k_z \beta c$$

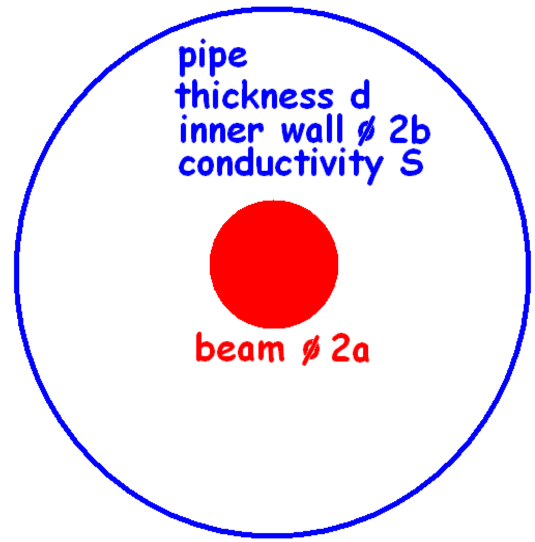
$$\delta_s^* = \sqrt{2/\mu_0 S\omega_0}$$

skin depth at the revolution frequency

$$\omega_0 = \beta c/R$$



Longitudinal calculations



Finite conductivity
Finite wall thickness
conductivity of stainless steel $S = 10^6(\Omega\mathrm{m})^{-1}$

Faradays and Ampères laws:

$$\nabla^2 \vec{B}(\vec{r},t) - \mu_0\epsilon_0 \frac{\partial^2 \vec{B}(\vec{r},t)}{\partial t^2} - \mu_0 S\,\frac{\partial \vec{B}(\vec{r},t)}{\partial t} = -\mu_0 \vec{\nabla}\times\vec{j}(\vec{r},t)\ ,$$

$$\nabla^2 \vec{E}(\vec{r},t) - \mu_0\epsilon_0 \frac{\partial^2 \vec{E}(\vec{r},t)}{\partial t^2} - \mu_0 S\,\frac{\partial \vec{E}(\vec{r},t)}{\partial t} = \mu_0 \frac{\partial \vec{j}(\vec{r},t)}{\partial t} + \frac{\vec{\nabla}\rho_c(\vec{r},t)}{\epsilon_0}$$

ρ_c and $\vec{j}$ are the external (free) charge and current densities
continuity equation:

$$\frac{\partial \rho_c(\vec{r},t)}{\partial t} + \vec{\nabla}\cdot\vec{j}(\vec{r},t) = 0$$

Boundary condition

Simple nonconducting wall:
tangential component vanishes: $\mathbf{E} \times \hat{\mathbf{n}} = \mathbf{0}$ *on the surface*
$\hat{n}$ is a unit normal to the surface pointing into the metallic wall

Conducting: more general Leontovich (impedance) boundary condition:
Accounts for finite surface currents within the wall
to find the fields outside the metallic wall:

$\mathbf{H} \times \hat{\mathbf{n}} - \lambda(\mathbf{E} \times \hat{\mathbf{n}}) \times \hat{\mathbf{n}} = \mathbf{0}$ *on the surface*
with a positive function λ so that $\mathbf{B}(\mathbf{E}, \mathbf{H}) = \mathbf{H} \times \hat{\mathbf{n}} - \lambda(\mathbf{E} \times \hat{\mathbf{n}}) \times \hat{\mathbf{n}}$ *everywhere*

Solution: $E_{\mathrm{t}} = Z_m H_{\mathrm{t}}$

$$Z_m = \sqrt{\mu_0 \omega / iS} = (1 - i)/S\delta_s$$

Z_m is the complex surface impedance of the metallic surface.
Since E_t and H_t are continuous, their values outside the metal near the surface must be related in the same way
Determines the fields outside the metal without considering the fields inside.

M. A. Leontovich, Issledovanya po Raspostraneniyu Radiovoln
(USSR Acad. Press, Moscow, 1948)
Zimmermann and Oide, PRSTAB **7**, 044201 (2004)

Shielding effectiveness

R. L. Gluckstern and B. Zotter, Phys. Rev. STAB **4** (2001), 024402

$$\mathrm{SE} = 10\,\log_{10}(\tau^{-2}) = 20\,\log_{10}(E_{\mathrm{incident}}/E_{\mathrm{transmitted}})$$

τ : ratio of transmitted to incident electric (or magnetic) field

very general quantity for RF shielding)

commercial, automotive applications 40 dB, military, aeronautical 80...100 dB

Bunch compression in proposed SIS 100:
$\Rightarrow$ peak currents of 100 A
$\Rightarrow$ with 40 dB shielding effectiveness of beam pipe
$\Rightarrow$ still 1 % of the induced field or 1 A of the peak image current leaks through the pipe

Results: longitudinal resistive wall impedance

Resistive wall impedance =
total impedance minus the impedance of a perfectly conducting beam pipe

$$Z^{\mathrm{wall}}(\omega) = \frac{nZ_0\beta\delta_s^*}{\sqrt{inb}} \frac{4I_1^2(\sigma_0 a)}{\sigma_0^2 a^2 I_0^2(\sigma_0 b)} \times \frac{1+\eta\frac{K_1(\sigma_0 h)}{K_0(\sigma_0 h)}\tanh\underline{\sigma}d}{\tanh\underline{\sigma}d+\eta\left(\frac{K_1(\sigma_0 h)}{K_0(\sigma_0 h)}+\frac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)}\right)+\eta^2\frac{K_1(\sigma_0 h)}{K_0(\sigma_0 h)}\frac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)}\tanh\underline{\sigma}d}$$

Limiting case

thick very well conducting pipe $d \gg b$ and $k\delta_s \ll 1$:

$$Z^{(\mathrm{wall})}(\omega) \approx \frac{nZ_0\beta\delta_s^*}{\sqrt{inb}} \frac{4I_1^2(\sigma_0 a)}{\sigma_0^2 a^2 I_0^2(\sigma_0 b)} \coth\underline{\sigma}d$$

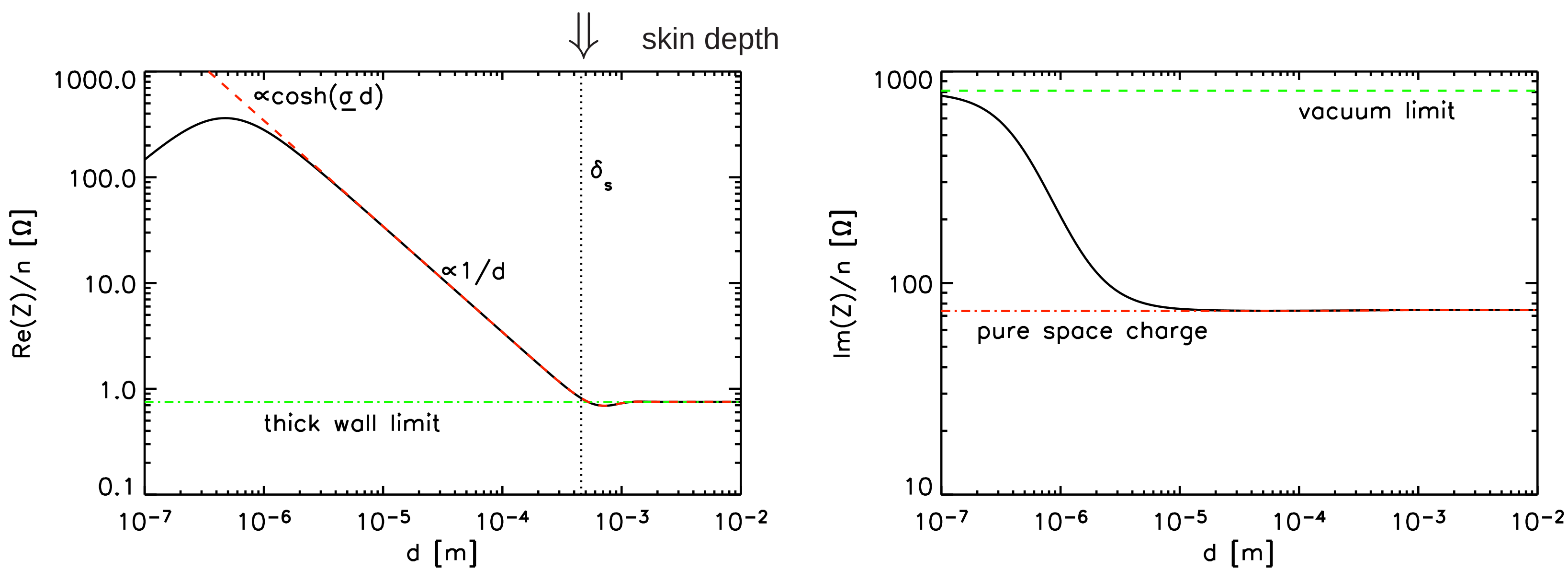
where $\sigma_0 = k_z/\gamma_0$ and $\underline{\sigma} = k_z/\underline{\gamma}$ and $\underline{\gamma}^{-2} = \gamma_0^{-2} - i\mu_0 S\omega/k_z^2$.

$\delta_s^* = \sqrt{2/\mu_0 S\omega_0}$: skin depth at the revolution frequency $\omega_0 = \beta c/R$

$\delta_s = \sqrt{2/\mu_0 S\omega}$

Results: Impedance vs. wall thickness

SIS 18: C=216 m, a=1 cm, b=10 cm



Real and imaginary parts (solid lines) of the total (space charge and resistive wall) impedance for $n = 1$ and $\gamma_0 = 2$ as a function of the wall thickness d δ_s is the penetration depth

Results: Transmission coefficient vs. energy

$$\tau^{-1} = \sqrt{\frac{b+d}{b}}\sigma_0 b K_1(\sigma_0 h) I_0(\sigma_0 b)$$

$$\times \left[\cosh \underline{\sigma} d + \frac{K_0(\sigma_0 h)}{\eta K_1(\sigma_0 h)} \sinh \underline{\sigma} d + \frac{I_1(\sigma_0 b)}{I_0(\sigma_0 b)} \left(\eta \sinh \underline{\sigma} d + \frac{K_0(\sigma_0 h)}{K_1(\sigma_0 h)} \cosh \underline{\sigma} d\right)\right]$$

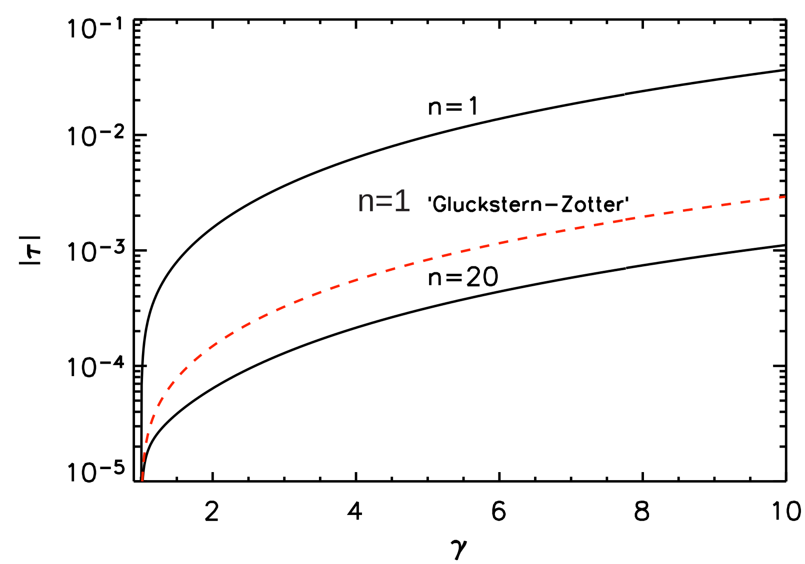
Limiting case

$\sigma_0 b \ll 1$ and $\sigma_0 h \ll 1$ and $d \ll b$

i.e. not too high frequencies

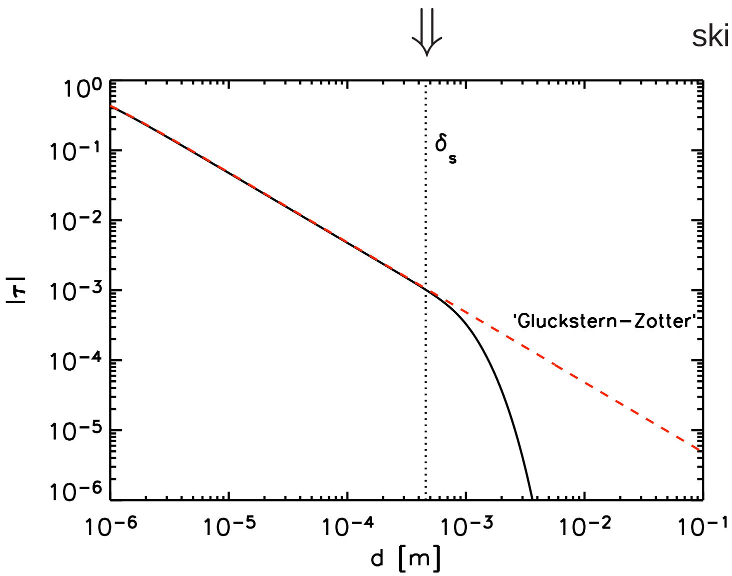
or ultrarelativistic beam energies

$$\tau^{-1} \approx 1 + \frac{\beta^2 k_z^2 b}{2} d + i \frac{2bd}{\beta^2 \gamma_0^2 \delta_s^2} \ln \frac{k_z b}{\gamma_0}$$

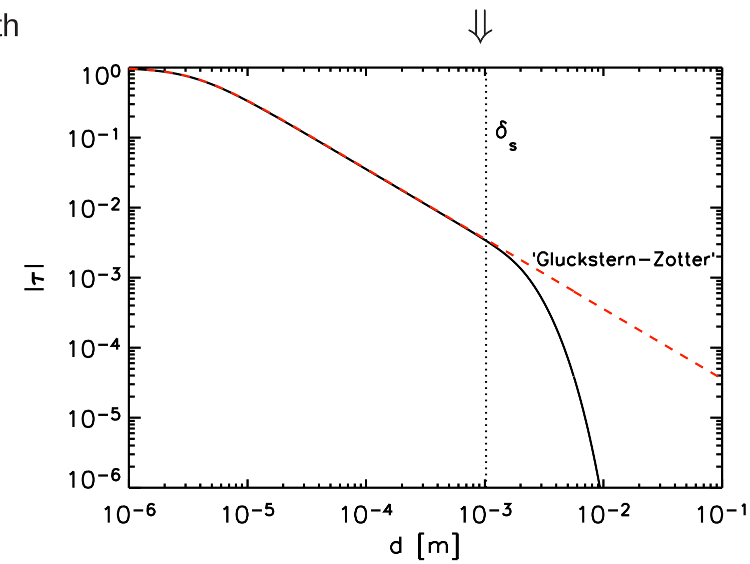


Transmission coefficient vs. wall thickness

skin depth



SIS 18: C=216 m, b=10 cm



SIS 100: C=1080 m, b=5 cm.

Transmission coefficient *vs.* d for $\gamma_0 = 2$ and $n = 1$
dashed line: approximate transmission coefficient.

Transverse calculations

Electric dipole moments from transverse displacements $\Longrightarrow$ transverse coupling impedance

Definition of transverse coupling impedance:

The first multipoles of the beam charge distribution $\sigma(r, \theta)$ give average electric dipole moment:

$$\vec{P} = P_r \hat{r} = \int_0^{2\pi} \int_0^a (r \cos\theta \ \hat{x} + r \sin\theta \ \hat{y}) \ \ \sigma(r, \theta) \ r dr d\theta$$

Define $Z_r = \frac{i}{P_r} \int_{-\infty}^{\infty} dz \ \ [E_r - \beta c \mu_0 H_\theta]$

Faraday law $\longrightarrow$ $\mu_0 H_\theta = \frac{i}{\omega} \ \left[\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r}\right]$

Transverse equations

The z components E_z, B_z produced by the dipole sources P_{r} :

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{k_z^2}{\gamma_0^2} - \frac{1}{r^2}\right] E_z(r,\omega) = i\frac{k_z}{\epsilon_0\gamma_0^2\beta c}\;\frac{P_{\mathrm{r}}\;\delta\,(r-a)}{\pi a^2}\;,$$

$$\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{k_z^2}{\gamma_0^2} - \frac{1}{r^2}\right] B_z(r,\omega) = 0$$

Solutions are again modified Bessel functions, etc.

Hollow ring beam (RHIC)

solution in closed form

radius a: $\sigma(r,\theta) = \frac{Q}{2\pi a}\delta(r-a)\sum_{\ell=0}^{\infty} A_\ell \mathrm{e}^{i\ell\theta}$

perfectly conducting wall at $r = b$.

Transverse radial space charge impedance R. Gluckstern, CERN 2000–011 :

$$Z_r = i\,\frac{L I_1^2(\sigma_0 a)}{\epsilon_0 \pi a^2 \gamma_0^2 \beta c}\left[\frac{K_1(\sigma_0 a)}{I_1(\sigma_0 a)} - \frac{K_1(\sigma_0 b)}{I_1(\sigma_0 b)}\right]$$

ultra relativistic limit $\Rightarrow \sigma_0 a \ll 1$ and $\sigma_0 b \ll 1$

$\Rightarrow$ approximation $Z_r = i\,\frac{Z_0 L}{2\pi\epsilon_0 \beta \gamma_0^2 c}\left[\frac{1}{a^2} - \frac{1}{b^2}\right]$

again, L=circumference,

In the relativistic limit, the resistive wall impedance reduces to:

$$Z_r(\omega) = Z_0(1-i)\frac{\omega_0 \delta_s^*}{c b^3 \sqrt{n}}$$

For very high wall conductivity: $Z_r(\omega) = \frac{8}{b^2 k_z}\, Z_{\|}^{\mathrm{res.wall}}(\omega)$

$Z_0 = 377\Omega$

Uniform beam

$\sigma(r,\theta) = \sigma_0 \sum_{\ell=0}^{\infty} A_\ell \mathrm{e}^{i\ell\theta}$

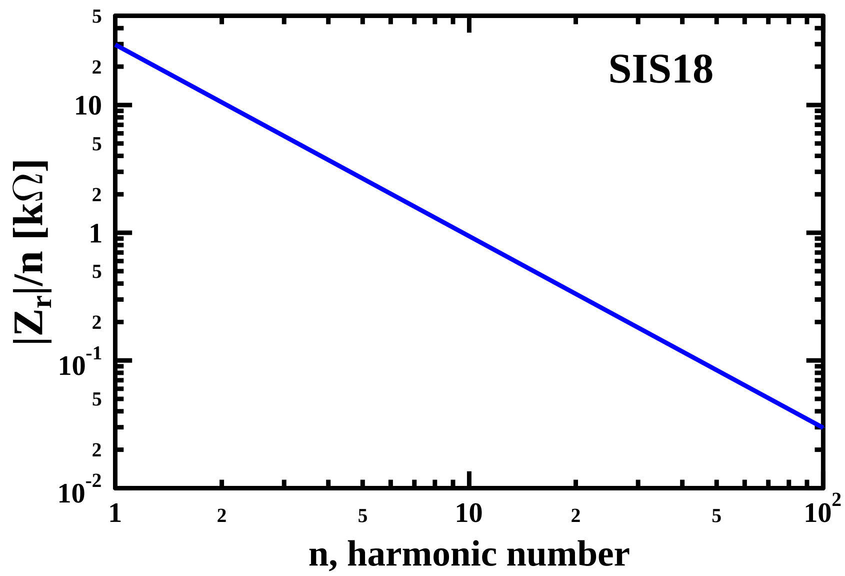
ultra relativistic limit $\Rightarrow \sigma_0 a \ll 1$ and $\sigma_0 b \ll 1$

$\Rightarrow$ approximation $Z_r = i \, \frac{Z_0 L}{2\pi\epsilon_0 \beta \gamma_0^2 c} \left[\frac{3}{2a^2} - \frac{1}{b^2}\right]$

Result: transverse radial resistive wall impedance of uniform beam

In the relativistic limit, the resistive wall impedance reduces to:

$Z_r(\omega) = 4Z_0(1-i)\frac{\omega_0 \delta_s^*}{cb^3\sqrt{n}}$ 4 times larger!



Summary

SIS 200/300 superconducting, so stay cool, reduce Eddy currents
Skin depth about 1 mm ⟶ compromise

Perturbation of electronics just outside pipe!

In reality: impedance depends on structures outside pipe

Shielding by beam pipe is of relevance
for the high-current SIS 18 upgrade and new SIS 100,200,300

Transverse impedance: hollow ⟵⟶ uniform charge distribution factor 4

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