Perturbation of the HESR lattice due to the Electron Cooling Insertion

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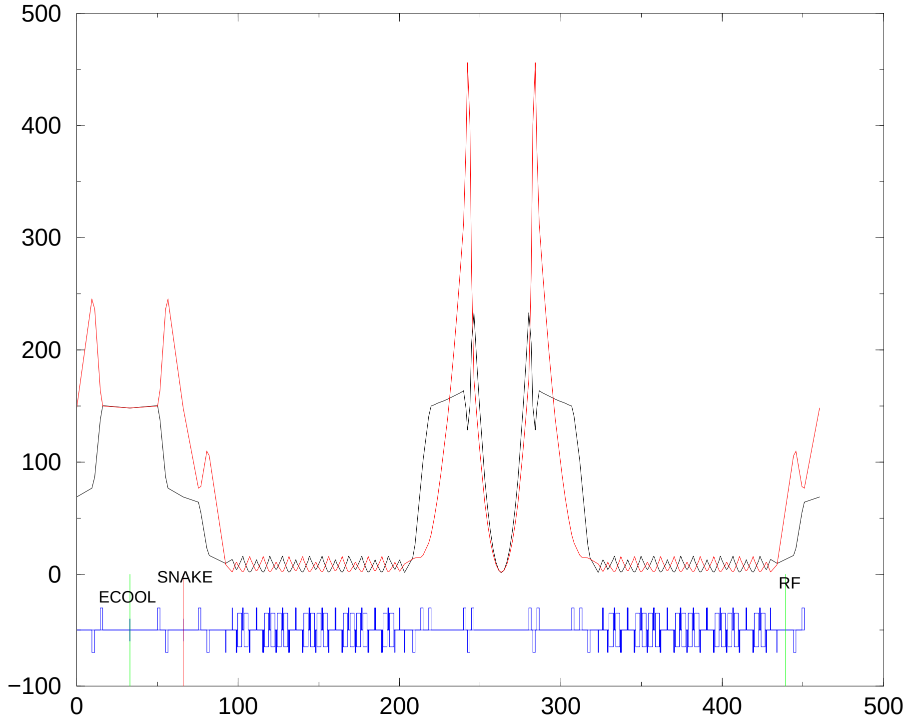
The antiproton beam in the 1-15 GeV storage ring HESR, in part designed at Jülich for the GSI laboratory, will contain a section with electron cooling.

The electron beam in this section generates a radial electric field that produces a marked defocusing of the antiprotons in both transverse planes.

This note presents a model of the cooling beam and describes the corresponding perturbation of the optics.

Electron beam: $I = 0.1$ to $1\ A$, energy $\gamma = 0.4$ to 14.5. Unperturbed lattice of HESR. (Y.Senitchev)



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Electron cooling is done in a 30m long straight section. The (round) beam envelope in this section varies from $\approx$ 150 m at both ends to $\approx$ 148 m in its center. The electron beam shape should match the gaussian profile of the beam, with an r.m.s cross section. σ_L = Raleigh length.

$$\sigma_\perp(z) = \sigma_T(0)\left[1+\left(\frac{z}{\sigma_L}\right)^2\right]$$

The electron beam is assumed to have a transverse Gaussian density. The antiproton beam is initially larger than the e-bem. Its transverse size decreases during cooling, to ideally become equal to the e-beam size.

The static electric potential of the electron beam has a Gaussian profile

$$\Phi(x,y,z)=\frac{\mu_0 c}{2\pi\beta}I\ \exp\left(-\frac{x^2+y^2}{2\sigma_T^2}\right)$$

$$\frac{\mu_0 c}{2\pi}\approx 60[V/A]$$

E-beam electric field

$$\vec{E}=-\vec{\Delta}\Phi:\begin{cases}E_x=-\frac{\partial\Phi}{\partial x}=-\frac{x}{\sigma_\perp^2}\Phi\\E_y=-\frac{\partial\Phi}{\partial y}=-\frac{y}{\sigma_\perp^2}\Phi\\E_z=-\frac{\partial\Phi}{\partial z}=-\frac{1}{\sigma_L^3}\frac{\partial\sigma_L}{\partial z}\Phi\end{cases}$$

Procedures we followed for this problem

1. Integrate the eq. of motion of an antiproton through the e-cooling section for several rays, to calculate the transformation map of the section

2. insert the map in the *MAD* description of the lattice

3. run *MAD* to find the perturbation to the lattice

4. run the tracking code *SIMBAD* in this lattice

Relativistic Eqs of Motion with z as indep. coordinate

$$\frac{d}{dt} = \beta_z c \frac{d}{dz},\ \vec{\beta} = \frac{\vec{v}}{c}, \beta^2 = \beta_x^2 + \beta_y^2 + \beta_z^2,\ \gamma = \frac{1}{1-\beta^2}$$

Numerically integrate five 1.st order diff. eqs. in $x, \beta_x, y, \beta_y, \beta_z$

$$\frac{d\vec{p}}{dt} = e\vec{E} : \begin{cases} dx/dz & = \beta_x/\beta_z \\ d\beta_x/dz & = A/\gamma^2 \left[(1-\beta_x^2)E_x - \beta_x\beta_y E_y - \beta_x\beta_z E_z\right] \\ dy/dz & = \beta_y/\beta_z \\ d\beta_y/dz & = A/\gamma^2 \left[-\beta_y\beta_x E_x + (1-\beta_y^2)E_y - \beta_y\beta_z E_z\right] \\ d\beta_z/dz & = A \left[-\beta_z\beta_y E_x - \beta_z\beta_y E_y + (1-\beta_z^2)E_z\right] \end{cases}$$

$1/\gamma^2$ for partial compensation of electric and magnetic forces.

$$A = \frac{e}{m_p c^2}\frac{1}{\beta_z\gamma}$$

The Jacobian

Transform maps for a machine insertion can be found via the Jacobian, for a small a-beam near the axis

$$R = \begin{pmatrix} \frac{\partial x}{\partial \widehat{x}} & \frac{\partial x}{\partial \widehat{p_x}} & \frac{\partial x}{\partial \widehat{y}} & \frac{\partial x}{\partial \widehat{p_y}} \\ \frac{\partial p_x}{\partial \widehat{x}} & \frac{\partial p_x}{\partial \widehat{p_x}} & \frac{\partial p_x}{\partial \widehat{y}} & \frac{\partial p_x}{\partial \widehat{p_y}} \\ \frac{\partial y}{\partial \widehat{x}} & \frac{\partial y}{\partial \widehat{p_x}} & \frac{\partial y}{\partial \widehat{y}} & \frac{\partial y}{\partial \widehat{p_y}} \\ \frac{\partial p_y}{\partial \widehat{x}} & \frac{\partial p_y}{\partial \widehat{p_x}} & \frac{\partial p_y}{\partial \widehat{y}} & \frac{\partial p_y}{\partial \widehat{p_y}} \end{pmatrix}, T_x = \begin{pmatrix} \frac{\partial^2 x}{2\partial \widehat{x}^2} & \frac{\partial^2 x}{\partial \widehat{x} \partial \widehat{p_x}} & \frac{\partial^2 x}{\partial \widehat{x} \partial \widehat{y}} & \frac{\partial^2 x}{\partial \widehat{x} \partial \widehat{p_y}} \\ & \frac{\partial^2 x}{2\partial \widehat{p_x}^2} & \frac{\partial^2 x}{\partial \widehat{p_x} \partial \widehat{y}} & \frac{\partial^2 x}{\partial \widehat{p_x} \partial \widehat{p_y}} \\ & & \frac{\partial^2 x}{2\partial \widehat{y}^2} & \frac{\partial^2 x}{\partial \widehat{y} \partial \widehat{p_y}} \\ & & & \frac{\partial^2 x}{2\partial \widehat{p_y}^2} \end{pmatrix}$$

Similar expressions for T_{px}, T_y, T_{py}. x at end, $\widehat{x}$ at beginning of track, respectively

To obtain the first order R, one tracks five particles: a reference particle and four more, each with only one component of the phase space changeded by a small amount in turn. If particle '0' starts with $(\widehat{x}, \widehat{p_x}, \widehat{y}, \widehat{p_y})$, particle '1' will start with $(\widehat{x}+\delta, \widehat{p_x}, \widehat{y}, \widehat{p_y})$, etc. Using two points, it is

$$\frac{\partial x_1}{\partial \widehat{x}} \approx \frac{x_1^{(1)} - x_1^{(0)}}{\delta},$$

and so on.

For the second order T, one needs the function calculated in three points. We will use 1+8 particles and the expression of one of the second derivatives is

$$\frac{\partial^2 x_1}{\partial \widehat{x} \partial \widehat{y}} \approx \frac{x_1^{(1)} - 2x_1^{(0)} + x_1^{(7)}}{2\delta^2}.$$

Thin beam

In a first study, transport maps were calculated for a thin beam as a Jacobian. For an electron current $I = 0.27$ and a beam energy $\gamma = 16$, obtain a matrix

$$R = \begin{pmatrix} 1.01016 & 30.49740 & 0.01667 & 0.15831 \\ 0.00068 & 1.01024 & 0.00111 & 0.01682 \\ 0.01667 & 0.15832 & 1.01627 & 30.55544 \\ 0.00111 & 0.01682 & 0.00108 & 1.01640 \end{pmatrix}.$$

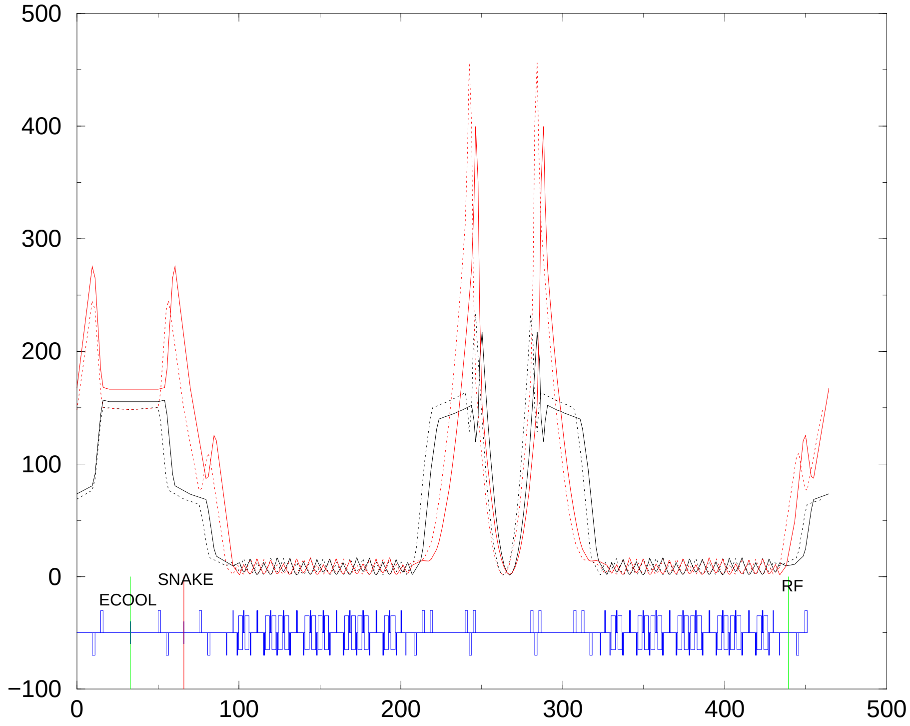
The second order map for this case is

$$T = \begin{pmatrix} 0.62866 & -7.3287(7) & 2.5161(6) & 2.8622(6) \\ & 79.918 & 7.5803(7) & 7.6149(7) \\ & & 3.0398 & 3.4616(5) \\ & & & 4.3906(2) \\ 2.9766(-2) & -2.4769(6) & 1.0494(3) & 3.8953(4) \\ & 9.3205 & 2.4780(6) & 2.5159(6) \\ & & 0.1905 & 3.7903(4) \\ & & & 6.0153(1) \\ 2.0810 & 3.4613(5) & -2.5011(6) & -7.5661(7) \\ & 3.2951(2) & -2.8470(6) & -7.6007(7) \\ & & 1.7689 & -7.3160(7) \\ & & & 2.9338(2) \\ 0.14239 & 3.7899(4) & -7.1186(1) & -2.4631(6) \\ & 44.951 & -3.7948(4) & -2.5010(6) \\ & & 0.12188 & -2.4630(6) \end{pmatrix}.$$

We inserted the map shown above in *MAD*, as element type 'MATRIX' and run. The results are shown in the following comparative Table.

	ν_x	ν_y	ν'_x	ν'_y	$\beta_{x,max}$	$\beta_{y,max}$	$D_{x,rms}$	$D_{y,rms}$
w/o	8.1899	8.1367	0.3578	-0.0357	229.65	448.02	1.9844	0.0000
w	8.1820	8.1234	0.6479	0.6753	217.12	399.49	1.9845	0.0006

Performing some correction in the lattice we could exactly match the e-cool section. This could be achieved by fine tuning (with additional trim coils) the existing quadrupole doublets existing in this lattice across the e-cool section.



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HESR lattice modified by the e-cool. Bare: dotted

Phase space figure matching

The Jacobian has here a very limited validity. The a-proton beam is initially much larger than the e-beam, ideally to shrink to the same transverse size.

In the process, an increasing larger fraction of the a-beam is subjected to defocusing in both planes, producing distortions of the phase space figure. Also, the process is not conservative and the emittance of the beam increases.

A complete treatment would involve high order non symplectic maps. Here, we will limit ourselves to first order, to allow a first evaluation with *MAD* of the effects of the e-lens on the optics of the HESR.

To calculate the transfer map we did the following:

1. propagate a number of particles through the structure,

2. match the starting and final phase space with an equivalent r.m.s. ellipse,

3. find the appropriate linear transformation of the starting ellipse (rotation and dilation) to bring it to coincide with the final ellipse

The r.m.s. ellipse representing the phase space distribution in each transverse plane is (in $x - x'$)

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon$$

The Courant-Snyder parameters $\gamma, \alpha, \beta, \epsilon$ can be calculated from the covariance dispersion matrix of the transverse distribution (in this note, we disregard coupling)

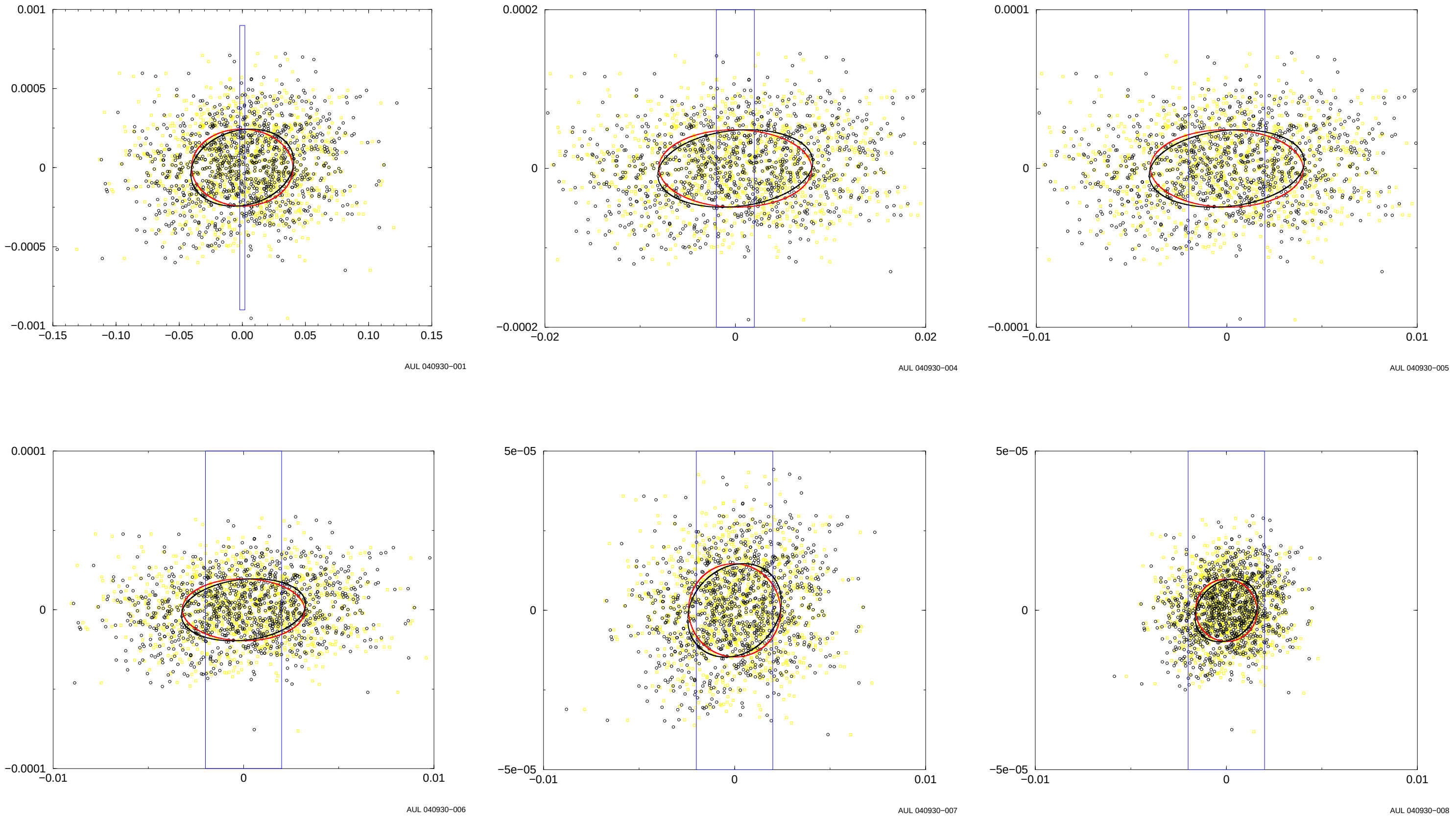
$$\left(\begin{array}{cccc} \beta_x \epsilon_x & -\alpha_x \epsilon_x & < xy > & < xy' > \\ -\alpha_x \epsilon_x & \gamma_x \epsilon_x & < x'y > & < x'y' > \\ < yx > & < yx' > & \beta_y \epsilon_y & -\alpha_y \epsilon_y \\ < y'x > & < y'x' > & -\alpha_y \epsilon_y & \gamma_y \epsilon_y \end{array} \right).$$

The emittance ϵ is the square root of the determinant of the 2×2 minor matrices on the diagonal, for x and y, respectively.

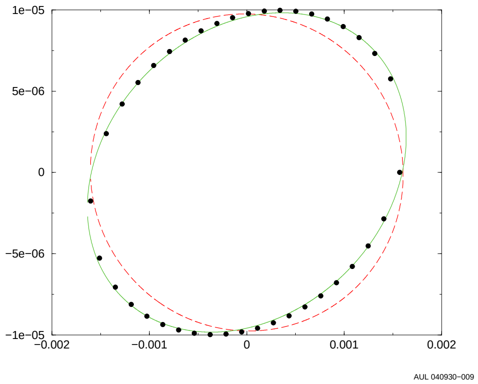
High Energy

Variable defocusing during e-cool is illustrated by a series of images created by tracking random a-protons through the e beam. In the simulation, the modelling beam consisted of 1000 random coordinates on a Gaussian distribution.

The figures show how the defocusing lens (in both planes) equivalent to the e-beam is increasingly more powerful as the beam shrinks. This is not too evident at high particle energy (because of the factor $1/\gamma^3$ in the equation of motion.)



$I_e = 1A$, $\gamma = 16$. The rms of the electron beam is shown



Starting and final phase space x-ellipse. Dots represent the dashed start ellipe after rotation.

The lens can be represented by the coefficients of a simple rotation that brings the starting upright ellipse to coincide with the final rotated ellipse.

$$\begin{cases} \widehat{x} = (\cos\theta)x + (\sin\theta)x' \\ \widehat{x'} = (-\sin\theta)x + (\cos\theta)x' \end{cases}$$

Results for three cases. An unrealistic third case: a-beam smaller than the e-beam, has been added to show the effect of the e-beam on a a-beam totally immersed in it

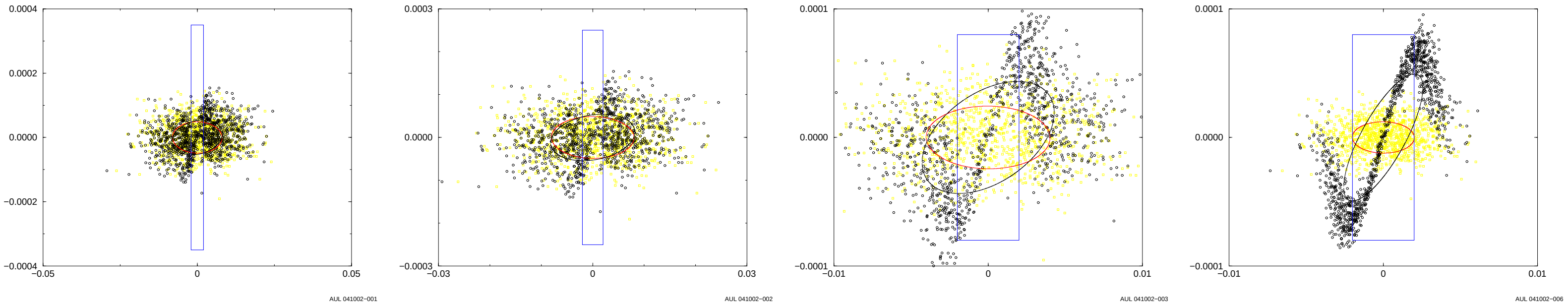
$$\begin{array}{cccc} \sigma_a/\sigma_e = & 1.6 & 1.2 & 0.8 \\ R = & \begin{pmatrix} 1 & -0.00105 \\ 0.00105 & 1 \end{pmatrix} & \begin{pmatrix} 1 & -0.00122 \\ 0.00122 & 1 \end{pmatrix} & \begin{pmatrix} 1 & -0.00140 \\ 0.00140 & 1 \end{pmatrix} \end{array}$$

The three transfer maps have been inserted in the lattice of the HESR for *MAD* runs. Results are compared with a case with no e-cooling

$$\begin{array}{ccccc} \sigma_a/\sigma_e = & \text{no/ecool} & 1.6 & 1.2 & 0.8 \\ \nu = & 8.1878/8.1346 & 8.159/8.106 & 8.157/8.104 & 8.155/8.101 \end{array}$$

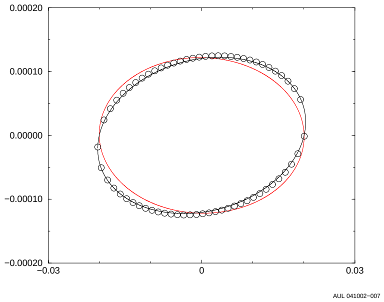
Low Energy

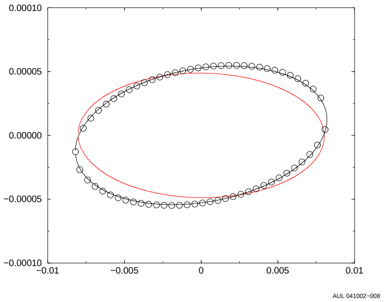
The a-beam phase space appears as composed by two parts, since only a part overlaps the e-beam

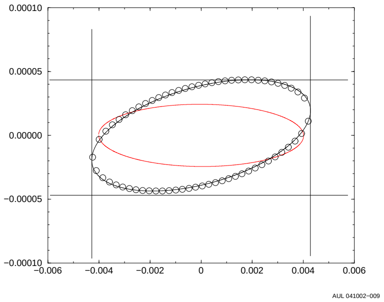


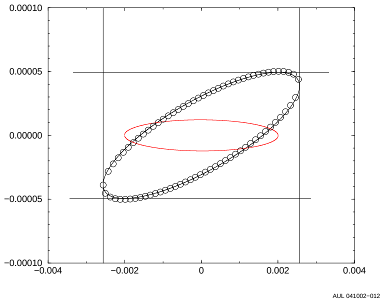
E-beam 1 A. $\gamma = 4$. $\sigma_a/\sigma_e = 10, 4, 2, 1$.

Matching diagrams for the same cases







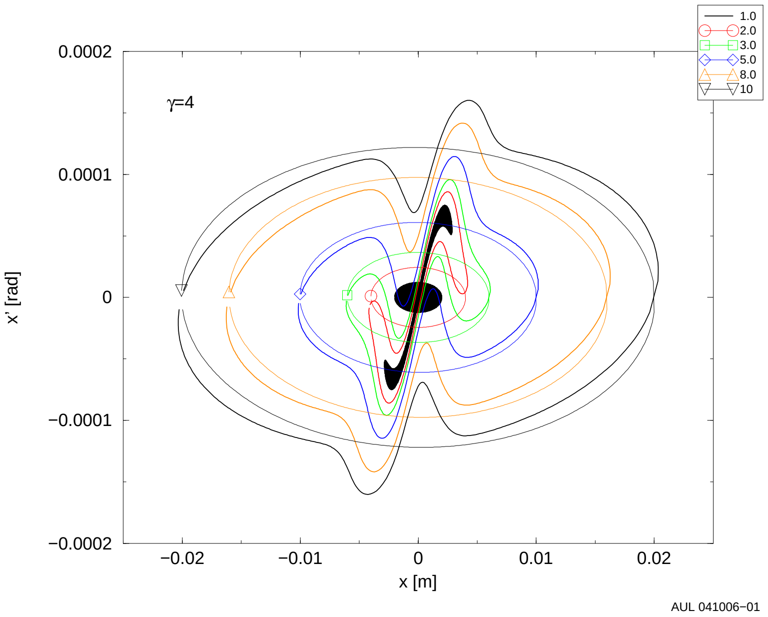


At this beam energy the HESR lattice had to be deeply retuned, because the defocusing in both plane brought the tune close to an integer (8), where the lattice was unstable.

Resulting tunes from *MAD* for ix cases including the four of the previous figures were

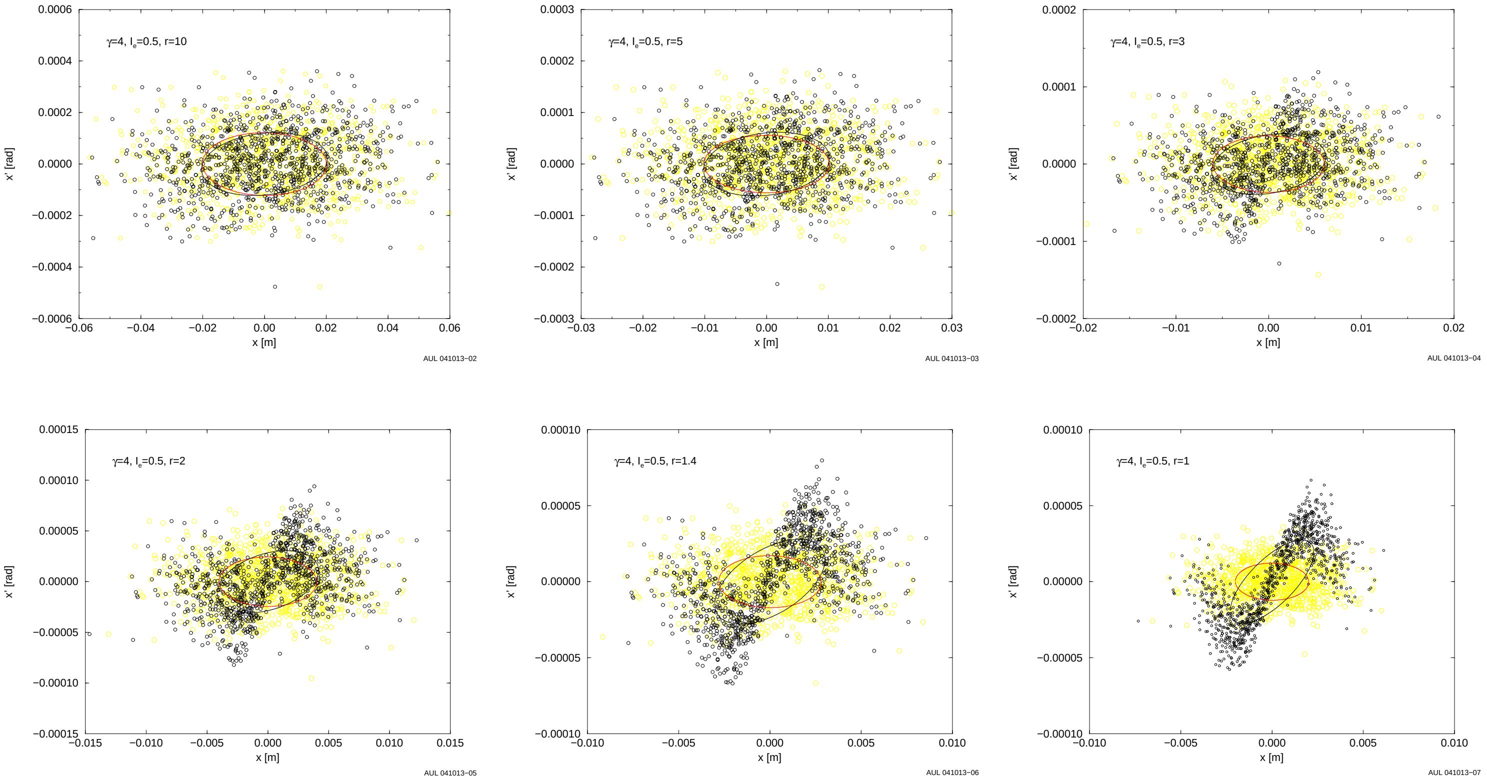
σ_a/σ_e	inf	10	4	2	1.5	1.2	1
ν_x	7.869	7.805	7.797	7.780	7.762	7.731	7.707
ν_y	8.178	8.215	8.207	8.185	8.162	8.116	8.070

Track ellipses instead



Labels are for the ratio a-beam/e-beam width

Low Energy. Low Electron Current



E-beam 500 mA. $\gamma = 4$. σ_a/σ_e = 10, 5, 3, 2, 1.4, 1

Conclusions

Preliminary results show that the effects of the e-cooling on the HESR lattice appear large at electron currents exceeding 1 A or at anti proton energies below, say, $\gamma = 10$.

To achieve a fast cooling at low energies may prove difficult