Antiproton Stacking at FNAL :

Methodical Aspects of Simulations

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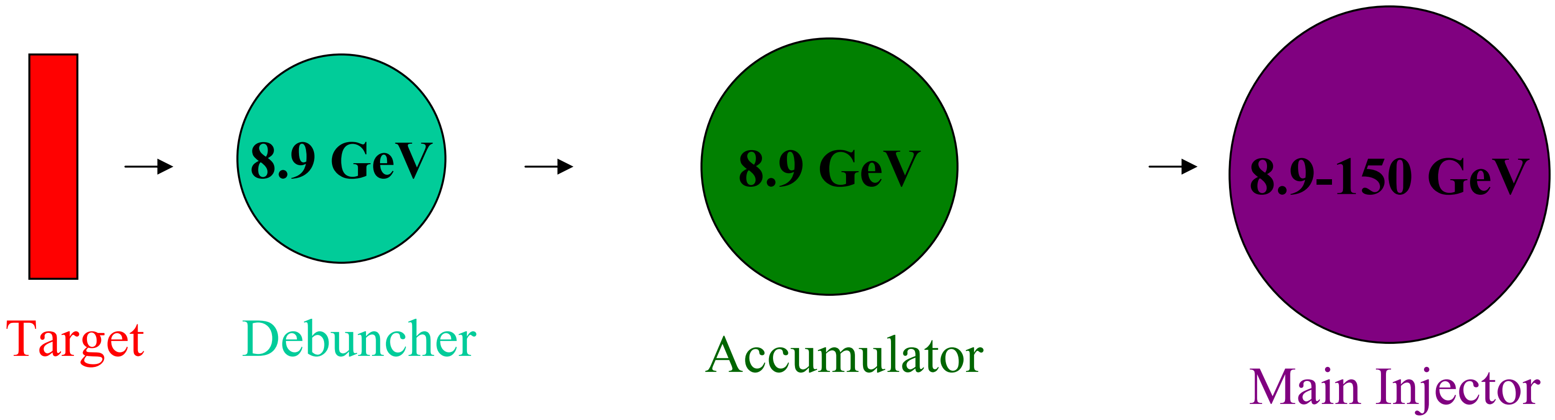
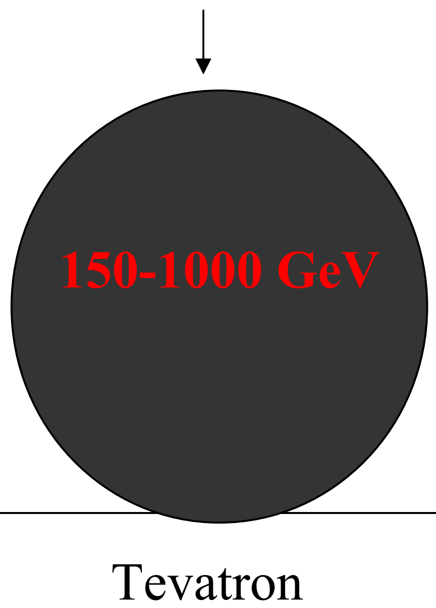
Antiprotons at FNAL: Past and Present



- Antiprotons are stochastically cooled in the Debuncher and Accumulator (8.9 GeV).
- After that, they are transferred to the Main Injector, accelerated to 150 GeV, injected in the Tevatron, and used in p-pbar collisions at full energy 1 TeV.

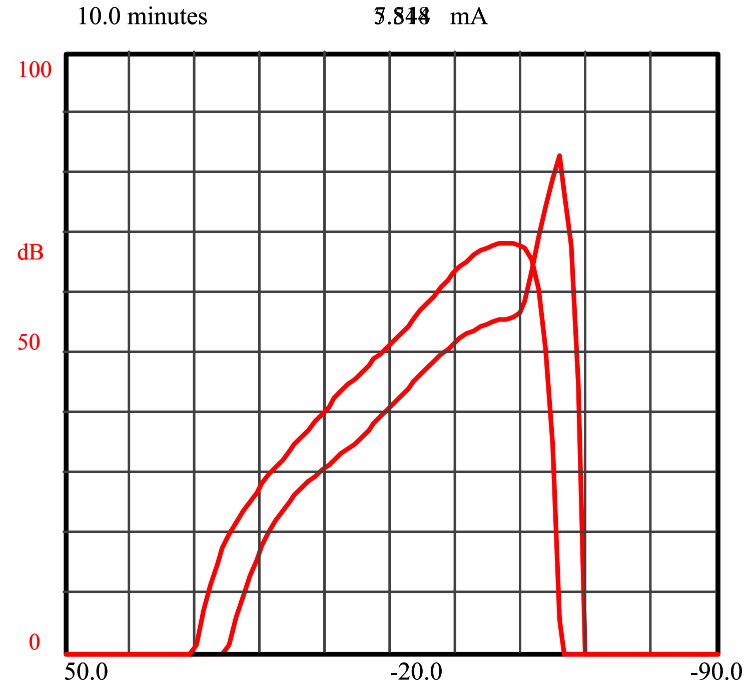


Limits of Accumulator Stochastic Cooling (SC)

In Accumulator, pbars are stacked by means of stochastic cooling.
Efficiency is limited by

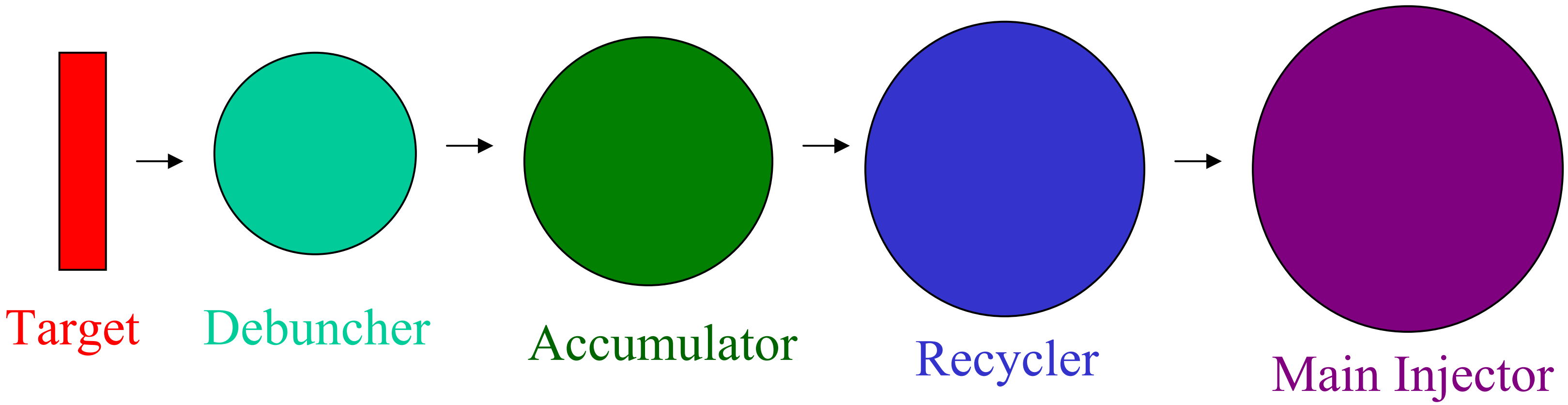
- SC: More stack – less flux
- IBS: hor. emit. growth

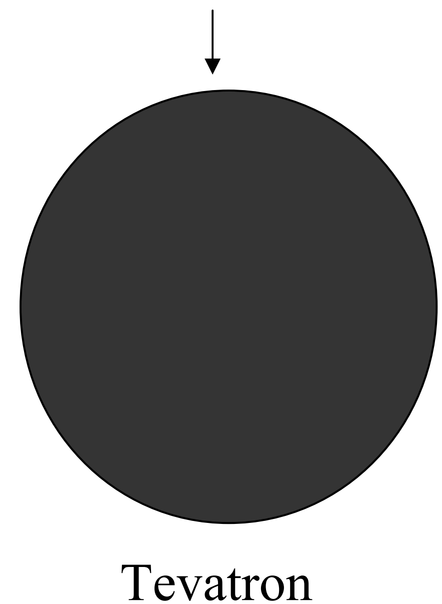
All that limits stack at
N=(200-300)E10



Stack-tail in AA (by P.Derwent)

Antiprotons at FNAL: +Recycler



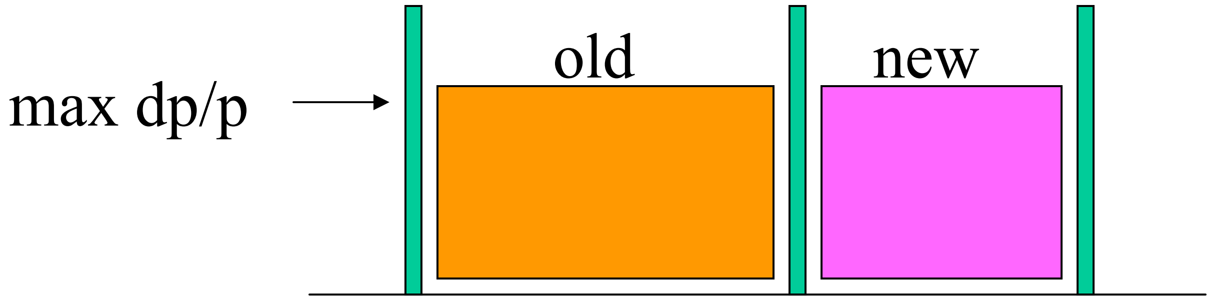


Recycler is 3.3 km long ring, based on permanent magnets and equipped with 4 wide band cavities, allowing any shape of the || potential well.

Recycler, brief description

Originally, Recycler was supposed to be used for utilizing the remnants of the pbar beam after the Tevatron store, which explains its name. Later this scenario was dropped after V. Lebedev showed its inefficiency in his modeling.

An only mission of the Recycler seen now is to serve as an additional volume for fresh pbars cooling and stacking.



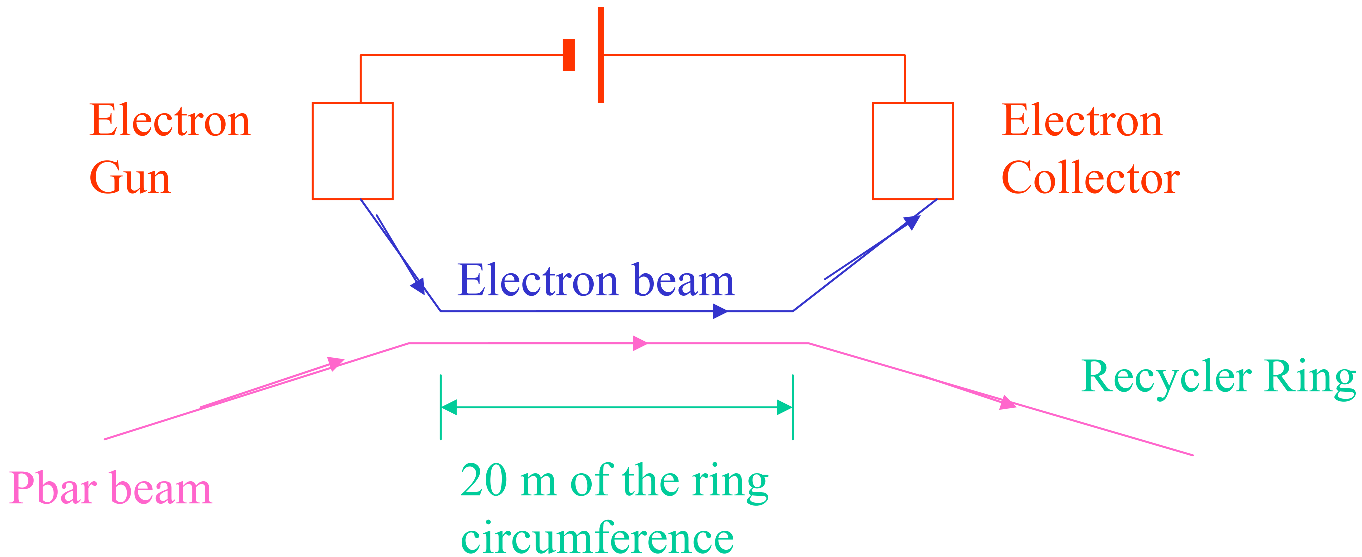
SC, installed: 0.5-2 GHz of || filter cooling 2-4 GHz of ⊥ cooling.

EC, under installation: 20 m cooling section with ~0.5 A of DC beam, parallel in the cooling section.

Electron Cooling, briefly

A stored pbar beam is overlapped with a nearly monochromatic and parallel electron beam (DC, ~0.5 – 1 A) in a straight section of the storage ring.

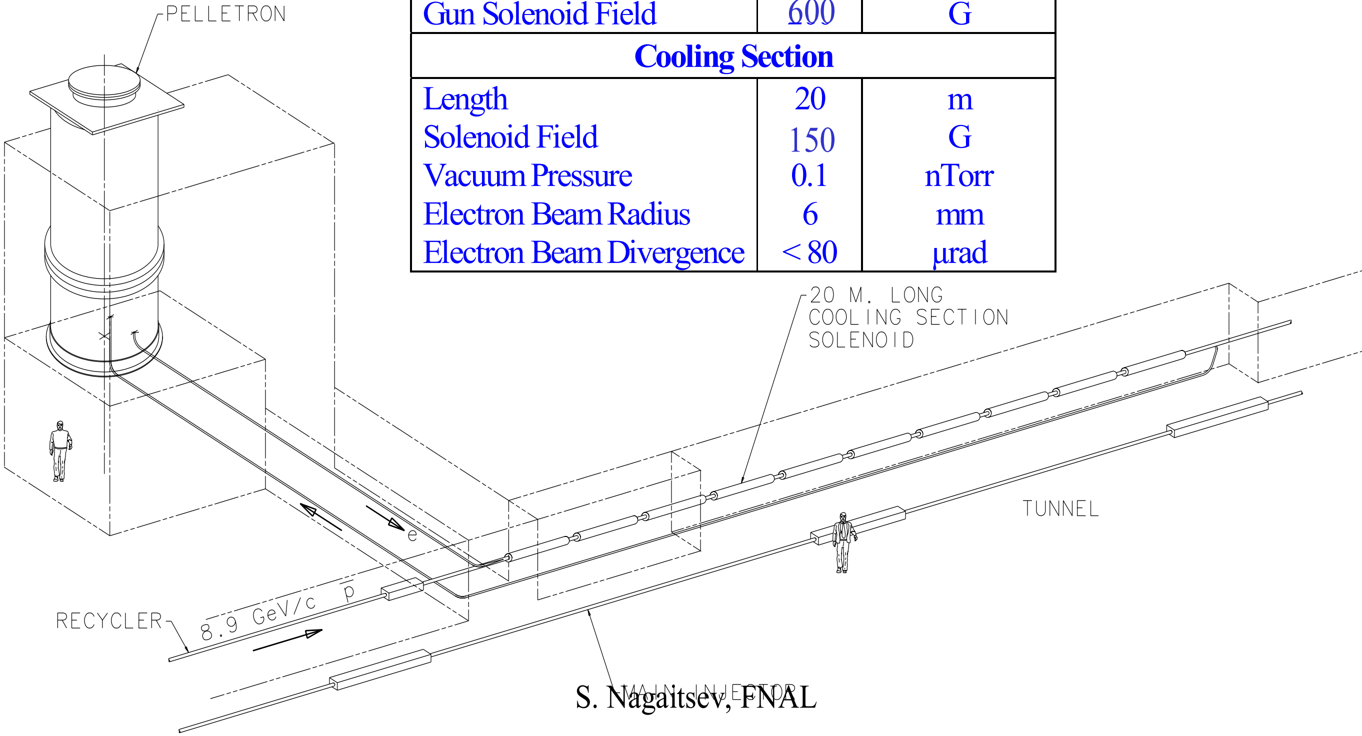
Cooling takes place due to a thermal heat transfer from pbars to electrons.



Schematic Layout of the Fermilab's Recycler Electron Cooling

Electron Cooling System Parameters

Parameter	Value	Units
Electrostatic Accelerator		
Terminal Voltage	4.3	MV
Electron Beam Current	0.5	A
Terminal Voltage Ripple	500	V (FWHM)
Cathode Radius	2.5	mm
Gun Solenoid Field	600	G
Cooling Section		
Length	20	m
Solenoid Field	150	G
Vacuum Pressure	0.1	nTorr
Electron Beam Radius	6	mm
Electron Beam Divergence	< 80	µrad



S. Nagaitsev, FNAL

Run II: Current and Target

.		Now	Goal	Ratio
Peak Luminosity	E32	1.0	3.0-5.0	3-5
Total pbars, extracted	E10	~150.0	600.0	4
Pbar bunches		36	36	1
Pbar prod. rate	E10/hr	15-7	40	3-6
MI long. emitt. per bunch	eVs	3.5	2.5	0.7
Proton 95% n. emit., coll.	mm mrad	20	20	1
Pbar 95% n. emit., coll	mm mrad	14	14	1

Simulations: what to account

Optimization of the pbar cooling / stacking in Recycler requires fast and reliable simulations which have to include:

- Periodic injections of new pbar batches from the Accumulator;
- Stochastic cooling, || and ⊥ , for the stack and batch separately (gated);
- IBS (|| and ⊥, , stack), which might be extremely detrimental;
- Electron cooling, || and ⊥, stack ;
- Gas scattering - ⊥ diffusion for stack and batch;

Simulations: Stochastic Cooling

Longitudinal stochastic cooling is described by a non-linear Fokker-Planck equation (FPE) on the distribution function $f(v,t)$. The friction and diffusion terms contain dispersion integrals

$$\chi(u) \equiv \int \frac{f'(v)}{u-v} dv$$

Calculation of these integrals takes significant time. This time is reduced orders of magnitude, if to fit the integral by some combination of elementary functions, dependent on several free parameters:

$$\chi(u) = F(u,a,b,c,...)$$

The integral parameters ***a, b, c, …*** can be numerically fitted for 20-30 arbitrary "representative" functions.

The monotone normalized distribution functions $f(v)$ can be characterized by their own parameters, such as

– rms width δv ,

– maximum of derivative $f'_{\max}$,

– argument where the derivative is maximal v_0 .

After that, the table of the integral parameters ***a, b, c*** vs function parameters $\delta v, \quad f'_{\max}, \quad v_0$ can be interpolated into a functional dependence:

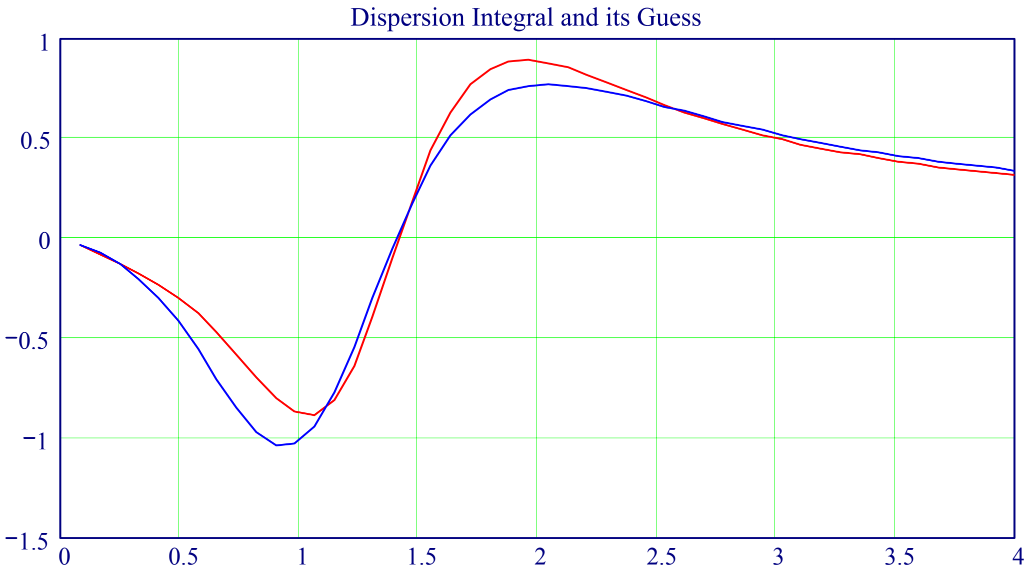
$$a = a(f'_{\max}\delta v^2, v_0/\delta v), \quad b = b(f'_{\max}\delta v^2, v_0/\delta v), \quad \ldots$$

Then, you have to calculate only the function parameters $\delta v, \quad f'_{\max}, \quad v_0$, use the interpolation formulas and get the dispersion integral without its actual calculation.

Precision of this integral guess value was never worse than ~ 10-15%.

Guess for Dispersion Integral

An example of a typical agreement between actual dispersion integral and its guess is presented in the Figure below.



This accuracy should be sufficient for all practical purposes.
Application of this trick makes MathCad based SC program run taking ~10-15 s.

Simulations: IBS

Stochastic Cooling only:

If the stack occupies the entire circumference, || IBS emittance growth is intolerable. To reduce it, the beam could be stacked at ~ thermal equilibrium, where its average || rms velocity is equal to the ⊥ one. This requires strong bunching of the stack (to the bunching factor 0.2 – 0.3) and keeping it at as low emittance as ~ 3 mm mrad (95%, norm).

Conventionally, IBS is considered as a pure diffusion. In reality, it contains a friction as well. The two factors altogether drive the beam to a thermal equilibrium, and they both have to be taken into account if $\boldsymbol{v}_{\parallel} \cong \boldsymbol{v}_{\perp}$. Thus, the Bjorken-Mtingwa-Piwinski approach (BMP) is not sufficient – it tells you nothing about IBS shaping the distribution.

At the same time, general Landau collision integrals would take too much time.

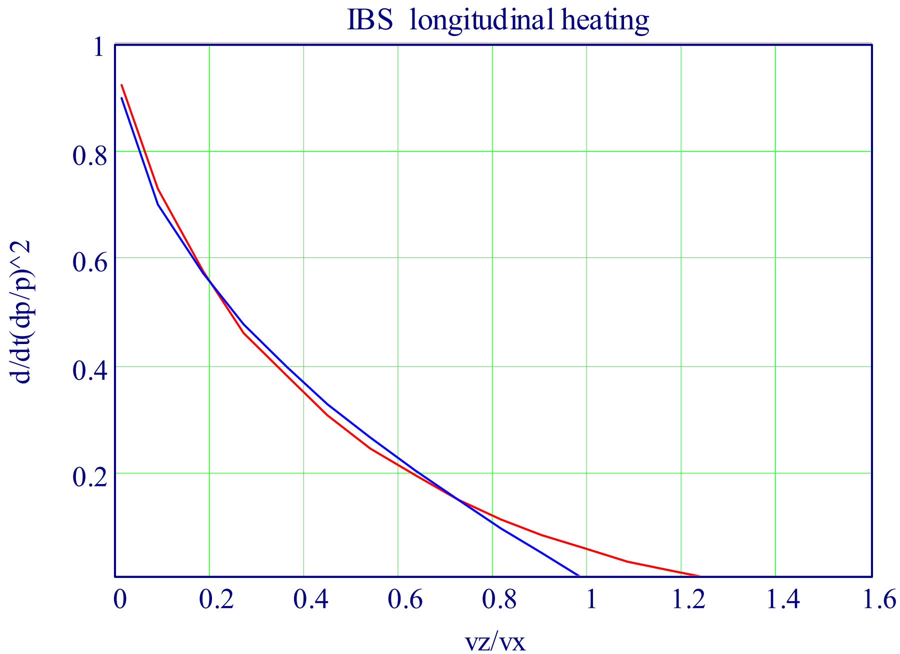
We need smth better than BMP, but not as heavy as Landau integrals.

IBS with Stochastic Cooling

- Solution: $$\frac{\partial F}{\partial t} = \frac{\partial}{\partial x}\left(\lambda F + \frac{D}{2}\frac{\partial F}{\partial x}\right) + \frac{\partial \phi_{\text{ibs}}}{\partial x}$$

$$\phi_{\text{ibs}}(x) = g_{\text{ibs}} x F(x) + \frac{D_{\text{ibs}}}{2}\frac{\partial F(x)}{\partial x} \qquad D_{\text{ibs}} = D_0\,, \quad g_{\text{ibs}} = \frac{D_0}{2\langle x^2\rangle}\left[1 - Q\left(v_{\parallel}/v_{\perp}\right)\right]$$

- where D_0 is the diffusion coefficient at zero longitudinal temperature and Q has to be taken to agree with the BM result, due to $d\langle x^2\rangle / d\tau = D_0 Q(v_{\parallel}/v_{\perp})$



IBS factor Q: BM result (red)

and a fit $Q = 1 - \sqrt{v_{\parallel}/v_{\perp}}$

Pluses:

- Gaussian Equilibrium
- Agreement with BM

However: tail IBS is overestimated, to be improved in near future.

Stochastic Cooling Only: Conclusion

Simulations and the last year experience show that for SC-only scenario, Recycler is only marginally useful.

To be effective, RR has to be equipped with Electron Cooler.

Electron Cooling (EC)

- The main purpose for EC is pbars || cool & stack, keeping the $\perp$ emittance at desired level. The cooling rates have to be sufficient for
  - Digesting phase spaces of pbar batches periodically arriving from the Accumulator;
  - Counteracting IBS and gas diffusion.
- EC rates Λ_{EC} are extremely sensitive to relative e-p angles, $\Lambda_{EC} \propto 1/\theta_{ep}^3$.
- EC requires :
  - Pbar emittances to be sufficiently low;
  - E-beam current high enough;
  - Parallel e-beam in the cooler, all e-angles $\leq$ pbar angles;
  - E-beam radius 2-3 times wider pbar-beam rms size;
  - For e-beam focusing, magnetic field in the cooler ~ 100 G is needed.
- The cooler is inside a solenoid at 100 – 200 G. However, the cooling is essentially un-magnetized due to
  - Electron thermal angles do not exceed angles of the pbars (tail)
  - Un-magnetized Log >> Magnetized Log

EC vs IBS

- How strong is IBS compared with EC ? A simplified estimation follows from the main scaling factors of the two similar phenomena:

$$\frac{\Lambda_{EC}}{\Lambda_{IBS}} \cong \frac{m_p}{m_e}\frac{n_e}{n_p}\frac{l}{C}\frac{L_{Ce}}{L_{Cp}}\left(\frac{\theta_p}{\theta_{ep}}\right)^3$$

- The most uncertain factor here is the electron beam effective angle. Assuming $I_e = 0.5$ A of e-current and $N_p = 6 \cdot 10^{12}$ pbars it gives

$$\frac{\Lambda_{EC}}{\Lambda_{IBS}} \cong 6\left(\frac{\theta_p}{\theta_{ep}}\right)^3$$

- Thus, to survive IBS, it seems to be needed $\theta_e \leq \theta_p \cong 50\,\mu\text{rad}$.

- This requirement is extremely tough. Perhaps, it's too tough to be achievable.

- However, there is a way to overcome it !

The Solution: Thermal Equilibrium

- IBS rate above is actually an IBS relaxation rate. It shows how fast the beam distribution evolves toward 3D thermal equilibrium (TE).

$$\langle v_z^2 \rangle \approx \langle v_x^2 \rangle \approx \langle v_y^2 \rangle$$

- But what if the beam is already at the thermal equilibrium, or close to that? Beam state close to thermal equilibrium is possible in Recycler because
  - The optics is rather smooth;
  - The RR operates below transition, the dispersion is small.
- Even at "equilibrium", there is some 6D emittance growth Λ^0_{IBS} . But for RR this growth is very small:

$$\Lambda^0_{IBS} \cong 0.05\, \Lambda_{IBS}$$

- Thus, the solution to overcome IBS is to keep the beam at TE.
- Both RF and SC (still needed) limit ***dp/p* < 0.002** . From here, the emittance is limited from the top:

$$\varepsilon_{n,95\%} \leq 3 \text{ mm mrad}$$

Some More Consequences

- Small || emittance + Thermal Equilibrium $\Longrightarrow$ strong || compression.

- For the longitudinal phase space as small as $\varepsilon_{\|,95\%} = \mathbf{30}\ \mathrm{eV\ s}$, the stack bunching factor = 0.2 .

- Particles kicked far out of the beam core by the single gas scattering are not cooled by EC. To get them back, SC at stack is needed.

- Thus, SC has to be gated: one gain for the batch, another for the stack.

- To prevent stack core over-cooling, regulated transverse noise can be applied to e-beam.

Simulations: EC, SC, IBS, stacking, gating

- EC rates strongly depend on the 3 pbar actions, there are no such numbers as "EC rates of the beam"! Different pbars have very different rates.

- EC rates are calculated by averaging over (2 or 3) pbar phases and 2 electron velocities. So, they are 4D – 5D integrals.

- Good fit for these integrals by standard functions is absolutely necessary to make simulation time reasonable. The fitting formulae accurate with ~ 10-20% were constructed (see Appendix).

- IBS is also a strong function of pbar actions; it has to be taken as functions of the pbar actions, or "detailed", not as "averaged" BMP approach. Fit formulae for IBS diffusion and friction terms are constructed at first approximation, and are in process of improving.

Detailed IBS

Landau kinetic equation

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left(F_i f + \frac{D_{ij}}{2}\frac{\partial f}{\partial p_j} \right)$$

$$F_i(\mathrm{p}) = \frac{4\pi n e^4 L_C}{m}\int f(\mathrm{p}')\frac{u_i}{|\mathbf{u}|^3}d^3p' \qquad D_{ij}(\mathrm{p}) = 4\pi n e^4 L_C \int f(\mathrm{p}')\frac{\mathbf{u}^2\delta_{ij} - u_i u_j}{|\mathbf{u}|^3}d^3p'$$

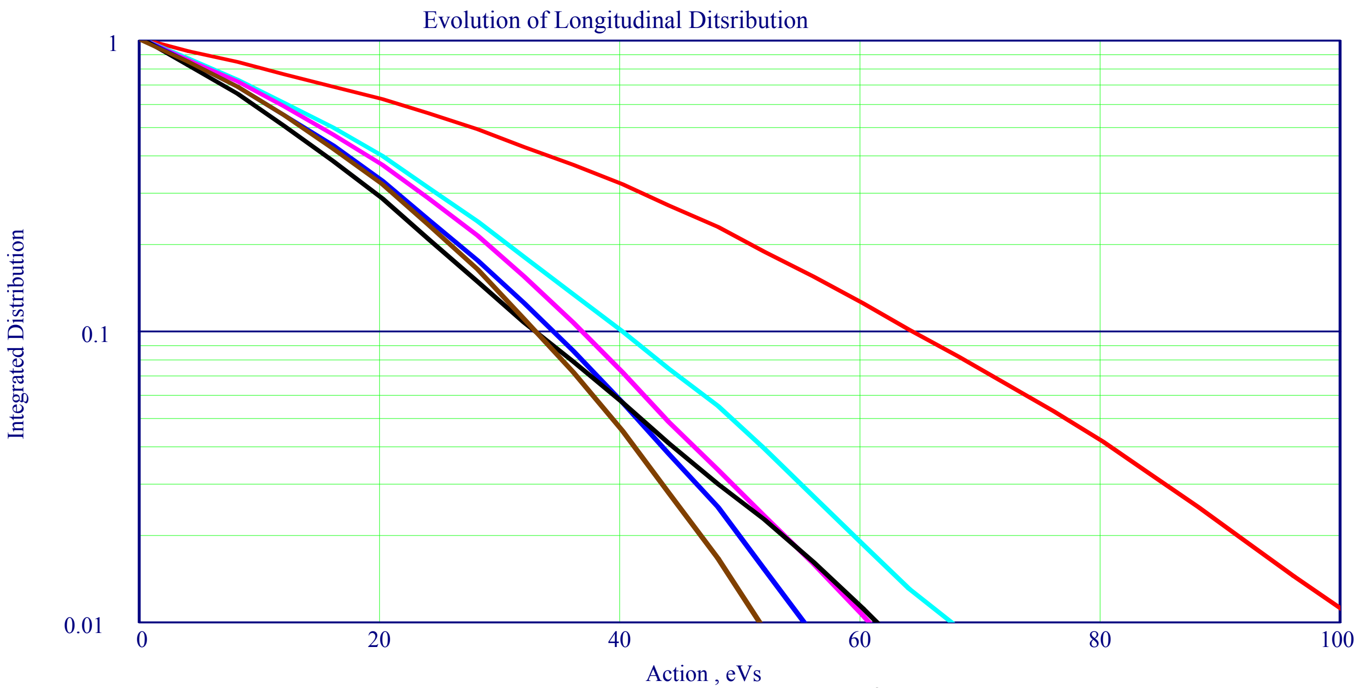
has to be averaged over phases (2 or 3 depending on the potential well). The resulting 5D – 6D integrals can be approximately calculated by the following way:

- Assume Gaussian distribution for every degree of freedom;
- Find good fit for the core;
- Calculate exactly for the far tails;
- Match these two results;
- Make sure the result is in reasonable agreement with BM.

EC Scenario for Recycler

- Every 30 min, new batch with 22E10 pbars injected from AA inside a separate barrier bucket.
- During 30 min, the batch is pre-cooled by gated ⊥ SC. To make it faster, batch || phase space is inflated to 40 eVs (95%). The ⊥ emittances cooled from 10 to 3 mm mrad. EC is not applied for the batch.
- After that, the stack of 40 eVs, 3 mm mrad is merged with 40 eVs, 3 mm mrad batch. Gated EC and ⊥ SC are applied. After 30 min, the || phase space is cooled from 80 to 40 eVs, getting ready for the next merge.

Evolution of Longitudinal distribution



Evolution of the longitudinal integral distribution $\int_{A}^{\infty} f(A')dA'$:

Brown – stack before the last merge, Red – Stack + Batch right after the merge,
All other show cooling evolution for the total repetition time of 30 min.
The stack is back to its initial phase space of 40 eVs.

Parameters for Recycler

Transverse stochastic cooling band	**2 – 4** GHz
Batch transverse emittances at injection, 95% norm	**10** π mm mrad
Batch longitudinal 95% phase area before pre-cooling	**40** eVs
Repetition time	**30** min
Pbars in the stack, up to	**6E12**
Stack emittance before merger, 95% norm	**3** π mm mrad
Stack long. 95% phase area before merger	40 eVs
Peak electron current	**0.5** A
E-cooling length	**20** m
Electron 1D rms angle in the cooler	**0.2** mrad
Electron beam radius	**2.7** mm
Beta-function in the e-cooler	**22** m
Gas diffusion (norm 95% emittance growth)	2 π mm mrad/hr

Summary

- A tool for pbar cooling-stacking simulations is prepared.
- SC, EC (detailed), IBS (detailed), gas scattering are taken into account in sufficiently accurate and efficient way.
- Main scenario is developed, variations are possible.
  - Stack transverse emittance = 3 mm mrad;
  - Stack has to be at thermal equilibrium;
  - For e-current ~ 0.5 A e-angles (eff. rms) could be as high as 200 μrad;
  - EC and SC (gated) are to be working together;
  - Broad-band transverse instability damper (up to ~ 300 MHz) is needed (Valeri is giving more about that).
- Run II goals are looking achievable.

Appendix 1: Transverse EC Rate (Un-magnetized)

$$\mathcal{A}(z) \equiv 2\pi z \exp(-2z) I_0(z)^2$$

$$\mathcal{G}(t) \equiv \frac{t}{t^2+1}\left[1+t+t\ln(t)\right]$$

$$\mathcal{F}_{\rm in}^{\perp}(x,y,z) \equiv \mathcal{G}\left(\sqrt{\frac{x^2+z^2}{y^2+z^2}}\right)(x^2+z^2)^{-3/2}$$

$$\mathcal{F}_{\rm out}^{\perp}(x,y,z) \equiv (2/\pi)(x^2+y^2+z^2)^{-3/2}(x^2+1)^{-1/2}(y^2+1/4)^{-1/2}$$

$$\mathcal{F}_{\rm tot}^{\perp}(x,y,z) \equiv \frac{\mathcal{F}_{\rm in}^{\perp}(x,y,z)}{0.7x^{7/2}+3.4y^6-1.8y^{7/2}+1} + \frac{\mathcal{F}_{\rm out}^{\perp}(x,y,z)(x^4+y^4)}{x^4+y^4+1}$$

$$\Gamma_e \equiv \frac{4(I_e/e)r_e r_p \eta_c L_\perp}{\gamma^{5/2}\varepsilon_{ea}^{5/2}\beta_c^{-1/2}}$$

$$\Lambda_x \equiv -\frac{1}{J_x}\frac{dJ_x}{dt} = \Gamma_e \mathcal{F}_{\rm tot}^{\perp}\left(\sqrt{\frac{2J_x}{\varepsilon_{ea}}}, \sqrt{\frac{2J_y}{\varepsilon_{ea}}}, \frac{\mathrm{v}_\parallel}{\mathrm{v}_x}\sqrt{\frac{\varepsilon_{nx}}{\varepsilon_{ea}}}\right)\left[\mathcal{A}\left(\frac{J_x+\beta_c \mathrm{v}_\parallel^2/(2\gamma)}{\varepsilon_{eT}}\right)\mathcal{A}\left(\frac{J_y+\beta_c \mathrm{v}_\parallel^2/(2\gamma)}{\varepsilon_{eT}}\right)\right]^{3/4}$$

Appendix 2: Longitudinal EC Rate (Un-magnetized)

$$\mathcal{F}_{\text{in}}^{\|}(x, y, z) \equiv \frac{2}{\pi z \sqrt{\left(x^2 + 2z^2/\pi\right)\left(y^2 + 2z^2/\pi\right)}}$$

$$\mathcal{F}_{\text{out}}^{\|}(x, y, z) \equiv \frac{1}{\pi x y \left(x^2 + y^2 + z^2\right)^{3/2}}$$

$$\mathcal{F}_{\text{tot}}^{\|}(x, y, z) \equiv \frac{\mathcal{F}_{\text{in}}^{\|}(x, y, z)}{2x^6 + 2y^6 + 1} + \frac{\mathcal{F}_{\text{out}}^{\|}(x, y, z)(x^3 + y^3)}{x^3 + y^3 + 1}$$

$$\Lambda_{\|} \equiv -\frac{1}{\mathrm{v}_{\|}}\frac{d\mathrm{v}_{\|}}{dt} = \Gamma_e \mathcal{F}_{\text{tot}}^{\|}\left(\sqrt{\frac{2J_x}{\varepsilon_{ea}}}, \sqrt{\frac{2J_y}{\varepsilon_{ea}}}, \frac{\mathrm{v}_{\|}}{\mathrm{v}_x}\sqrt{\frac{\varepsilon_{nx}}{\varepsilon_{ea}}}\right)\left[\mathcal{A}\left(\frac{J_x + \beta_c \mathrm{v}_{\|}^2/(2\gamma)}{\varepsilon_{eT}}\right)\mathcal{A}\left(\frac{J_y + \beta_c \mathrm{v}_{\|}^2/(2\gamma)}{\varepsilon_{eT}}\right)\right]^{1/2}$$