Multiple Coulomb Ordered Strings of Ions in a Storage Ring

Rainer W. Hasse*
GSI Darmstadt, D-64291 Darmstadt, Germany
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We explain that the anomalous frequency shifts of very close masses obtained in the high precision mass measurement experiments in the ESR storage ring result from the locking of Coulomb interacting strings of ions. Here two concentric strings which run horizontally close to each other are captured into a single string if their thermal clouds overlap and give up their identity.

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In a recent paper, Radon *et al.* [1] reported on high precision mass measurements with Schottky mass spectrometry of about 100 new isotopes in the region $57 \leq Z \leq 84$ with a high resolving power of 3.3×10^5. This value is not only limited by the stability of the bending and focusing magnets (see Ref. [1]), but also by the effect of anomalous frequency shifts discussed below. In this experiment at GSI fully stripped isotopes were produced by fragmentation of a $^{209}\mathrm{Bi}^{67+}$ beam at 930 MeV/u, separated in the fragment separator FRS, and stored and cooled in the storage ring ESR. The electron cooler cooled down the remaining 2000–5000 isotopes to a longitudinal velocity spread as low as $\delta v/v = 7 \times 10^{-7}$, where $v = \beta c = 0.685c$ is the beam velocity. The corresponding momentum spread was $\delta p/p = \gamma^2 \delta v/v = 1.3 \times 10^{-6}$, where $\gamma = 1.37$ is the relativistic factor.

For fixed magnetic rigidity $B\rho$ around 7 Tm each species of the beam has its own frequency $f_i = v/C_i$, where C_i is the length of its trajectory close to the circumference of the ESR, $C = 108.4$ m. Neighboring masses Δm by their slightly different trajectories due to different rigidities, thus have different frequencies

$$\frac{\Delta f}{f} = -\alpha_p \frac{\Delta(m/q)}{(m/q)} \pm \left(1 - \frac{\gamma^2}{\gamma_t^2}\right)\frac{\delta v}{v}. \tag{1}$$

Here q is the charge and $\gamma_t = 2.67$ is the transition energy which is connected to the slip factor $\eta = \gamma^{-2} - \gamma_t^{-2} = 0.39$ and to the momentum compaction factor

$$\alpha_p = \frac{1}{\gamma_t^2} = \int_0^C \frac{D(z)}{r(z)} \frac{dz}{C} = 0.14, \tag{2}$$

where $D(z)$ is the dispersion function of Fig. 1 and $r(z)$ is the local radius of the central orbit. Note that without the effect of dispersion from the bending magnets, $\alpha_p = 0$, the average length of the trajectories and, hence, all frequencies would be the same.

The measurements were done at the 16th harmonic of the revolution frequency 1.9 MHz; hence $f = 30.4$ MHz and the thermal frequency resolution from the last term of Eq. (1) is about 15 Hz. At present, experiments are under way [2] at the transition energy, $\gamma = \gamma_t$, in which these thermal effects are eliminated although no cooling is needed. Each spectrum obtained was averaged up to 10 000 times, and the masses were identified and determined with sophisticated analysis on the basis of Eq. (1).

In the results typical deviations were found from the measured frequencies of known isotopes as compared to the known calibration curve. As shown in Fig. 2, if there are at least two masses so close to each other that their difference in frequencies is smaller than about 80–90 Hz, on the average the lower (higher) one is shifted characteristically to a higher (lower) value. According to Eq. (1) this anomalous effect limits the mass resolution to about 20 μu and such masses then have to be discarded from the results. They are the major reason for the systematic errors. The authors believe that *"due to a high number of ions circulating with close distances next to each other in the ESR, Coulomb interactions may lead to collective coupled motion."*

In what follows we explain and make a model for the origin of these anomalous frequency shifts on the basis of the anticipation that the ions run on strongly correlated trajectories. The situation here is similar to the one of the machine experiments on the anomalous jump to very low momentum spreads in the ESR [3] and in the SIS [4]. There it has been shown [5] that under the experimental conditions intrabeam scattering is strongly suppressed and that the ions cannot pass each other any more and, thus, run on strings.

With the parameters given above and a mean tune of $Q = 2.3$ the Wigner-Seitz radius is $a_{\mathrm{WS}} = (3q^2/2m\omega_\beta^2)^{1/3} = 14$ μm, where the betatron frequency $\omega_\beta = Q\omega_{\mathrm{rev}}$ is the Q *fold* of the revolution frequency. The

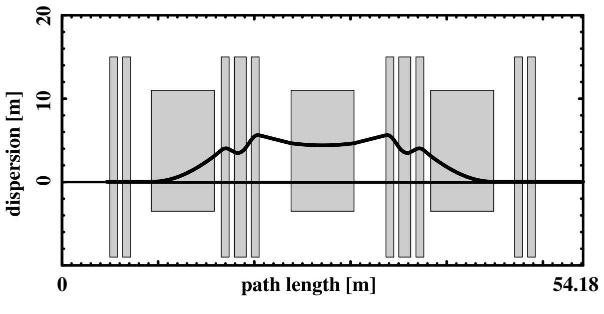

FIG. 1. Dispersion function of half of the ESR. The magnets in the lattice (pairs and triplets of quadrupoles, single dipoles) are indicated schematically.

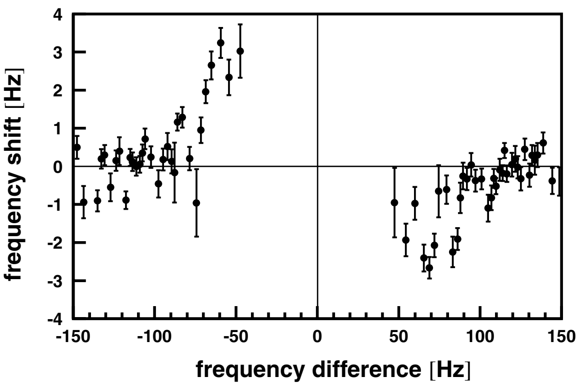

FIG. 2. Average anomalous frequency shifts of neighboring isotopes vs difference between their measured frequencies. Each point is the average of about 100 data (after Ref. [1]).

Wigner-Seitz radius roughly is the closest distance which ions can attain in infinite Coulomb matter at zero temperature. With 2000 (5000) ions in the ring the particles have an average distance in the beam direction of $d = 5(2)$ cm and a linear density of $\lambda = a_{\mathrm{WS}}/d = 0.000\,25(0.0007)$, which is well within the region of stable strings, $\lambda < 0.7$ [6]. The average Coulomb energy between two particles at distance d is $e_{\mathrm{C}} = q^2\gamma/d = 0.19$ (0.47) meV. With the methods of Ref. [5] we derive a longitudinal kinetic energy $T_{\mathrm{long}} = m(c\beta\delta v/v)^2/(8\ln 2) = 3$ meV and take the same transverse kinetic energy as in [3], $T_{\mathrm{trans}} = 1.5$ eV. The FWHM transverse thermal diameter of the projection of the ion cloud onto the transverse plane calculated in the harmonic oscillator potential generated by the betatron frequency $\omega_\beta = 2\pi Q\beta c/C$ becomes $2\delta x = 4d\sqrt{\lambda^3 T_{\mathrm{trans}}/3e_{\mathrm{C}}} = 46\ \mu$m [7]. An estimate for the transverse emittance from $\epsilon_{\mathrm{trans}} = (\delta x)^2/\beta_{\mathrm{trans}}$ with $\beta_{\mathrm{trans}} \approx 15$ m [3] being the average transverse beta function in the free sections yields the extremely low value $4 \times 10^{-5}\pi$ mm mrad. All these parameters are of the same order of magnitude as in the experiments of Ref. [3]. Consequently in the present experiment the ions have to run on strings as well.

Nuclides with different masses but with the same velocity run on different trajectories which are horizontally displaced so that, apart from thermal fluctuations, $v = Cf$ remains constant; thus

$$\frac{\Delta C}{C} = -\frac{\Delta f}{f}\,. \tag{3}$$

This difference in length of trajectory transforms into the horizontal displacement $-\Delta x = (C/2\pi)\Delta f/f = 17\ \mathrm{m} \times \Delta f/f$ which attains the value 43–48 μm for the experimental value of Fig. 2 of the onset of the anomalous frequency shifts at $\Delta f = 80$–90 Hz. In addition, the dispersion by the bending magnets also causes a horizontal displacement due to the remaining thermal fluctuations. With the dispersion function of Fig. 1, the average dispersion is about $\bar{D} \approx 2.4$ m. Hence the maximum thermal displacement $\bar{D}\delta v/v \approx 1.7\ \mu$m should be small as compared to the displacement of Eq. (3). An additional displacement from the centrifugal force can be neglected since this effect would be smaller by a factor of $\Delta x/C \approx 4 \times 10^{-7}$.

This fact leads to the model of Fig. 3, namely that if the displacement is larger than the thermal diameter, the two (or more) strings run on well separated trajectories yielding two (or more) distinct peaks in the Schottky spectrum which lie at the correct positions. However, in the opposite case when the clouds overlap, the strings lose their identity and lock partially or fully into the same frequency; thus smaller frequencies apparently become larger and vice versa.

Assuming that this locking is not *ad hoc* but by virtue of the averaging procedure over the experimental data that there is a smooth transition with a probability proportional to the overlap region of the clouds, $P = [\alpha - \sin(\alpha)]/\pi$, where α is the opening angle of Fig. 3, and folding this probability together with the experimental resolution of 15 Hz into the sawtooth curve $-f\Theta(|90\ \mathrm{Hz}| - f)$ of shifts (the dashed lines in Fig. 4), where Θ is the Heaviside function, yields the result of Fig. 4. The dashed sawtooth line would result if the strings were always captured if their thermal radii start to overlap. Note that apart from the uncertainty coming from the assumed capture probability this model has no free parameters and that the result agrees nicely with the experiment.

In order to substantiate this we performed classical 3D molecular dynamics calculations with the methods of Refs. [6,7] in the geometry of an infinite straight cylinder. Here 30 particles are distributed in a disk over two strings with different occupation. Their initial positions and velocities are distributed randomly according to their available longitudinal and transverse kinetic energies (the number of particles in the simulation is limited by the computer memory for the fast Fourier transform). The small disk of the cylinder has repeating boundary conditions, thus summing up all Coulomb interactions by Ewald

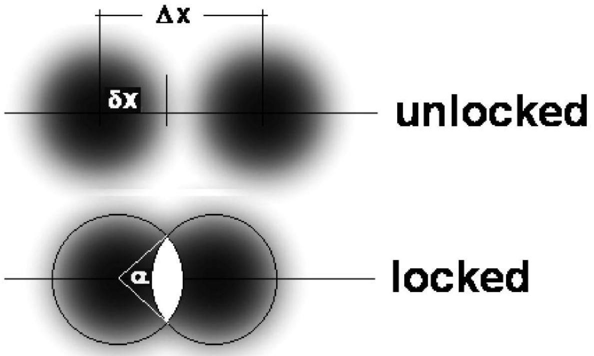

FIG. 3. Schematic drawing of the projection of the ion clouds onto the transverse plane in the unlocked and locked cases.

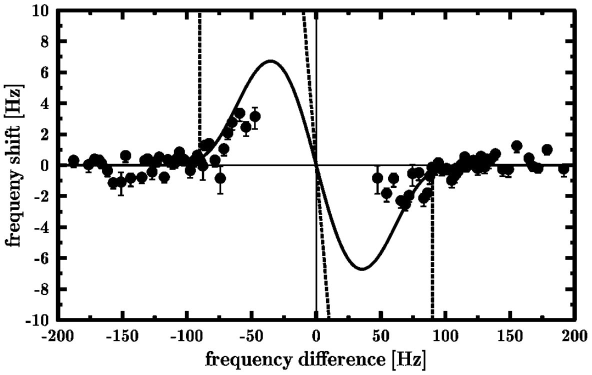

FIG. 4. Same as Fig. 2 but with the result of the model calculation included (full line). The sawtooth curve (dashed line, only partly in the frame) would result without folding.

summation; see Ref. [6]. The effect of different masses or frequencies in this straight geometry is transformed to different velocities of the strings according to Eq. (1), i.e., a relative velocity in the rest frame which corresponds to a frequency difference of 75 Hz. No cooling mechanism is incorporated. All positions over 5000 betatron oscillations in a lattice with solenoidal focusing with the mean tune of the ESR are then Fourier transformed and each spectrum has been averaged over 11 runs with different random initial conditions to yield Fig. 5. Here the occupation numbers of the strings are written at the bases and the relative temperatures with respect to the average Coulomb interaction energy between two ions are indicated in the key.

The upper frame treats the unlocked case where the distance between the strings is much larger than the thermal diameter. Here the difference of 75 Hz between the frequencies of the two central peaks in the lower curve with equal occupation corresponds to the relative velocity between the strings. Although the strengths of the peaks vary with the occupation, their distance remains constant. In the bottom frame, the locked case, the particles have lost their identity resulting in a broad random distribution. The partially locked case is intermediate.

There is indication that the effect of locking or capture is not restricted to the existence of ultracold chains. As pointed out in Ref. [8], during the preparation of the precision mass experiments similar anomalous frequency shifts have been observed in an experiment with a bare $^{197}\mathrm{Au}^{79+}$ beam at 295 MeV/u. Its isotopes $^{196,195,194}\mathrm{Au}^{79+}$ and other strongly populated fragments had velocity spreads of about $\delta v/v = 10^{-5}$ and thermal widths of about 4 mm. By scraping the primary beam down to a width of 600 μm and under steady electron cooling the velocity spread decreased by 1 order of magnitude. Nearby fragment nuclides with slightly larger masses then acquired anomalous negative frequency shifts of the order of a few Hertz; see Fig. 6. This effect remained if the primary beam was further scraped to a width of 50 μm.

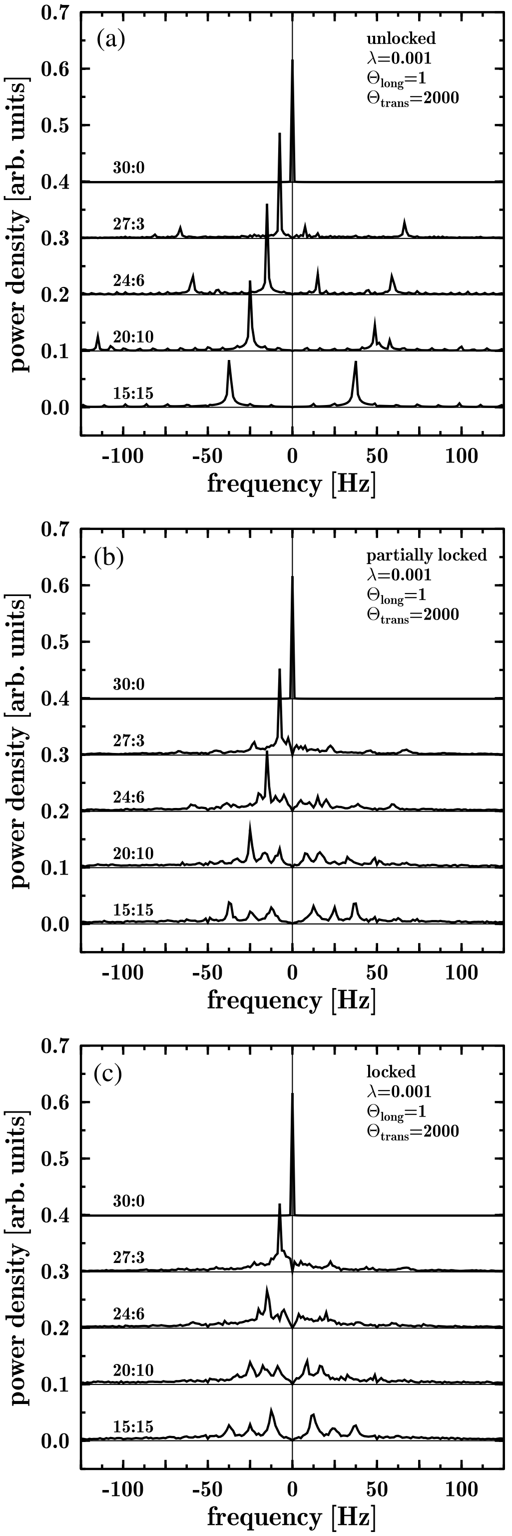

FIG. 5. Theoretical averaged Fourier spectra around the third harmonic from molecular dynamics calculations with 30 particles in a repeating boundary box distributed over two strings with the occupation numbers written at the bases. Unlocked (a), partially locked (b), and locked (c) means that the displacements of the strings are larger, in between, and smaller than the thermal diameter. Θ_{long} and Θ_{trans} are the longitudinal and transverse kinetic energies in units of the Coulomb interaction energy of two particles at distance d and λ is the linear density.

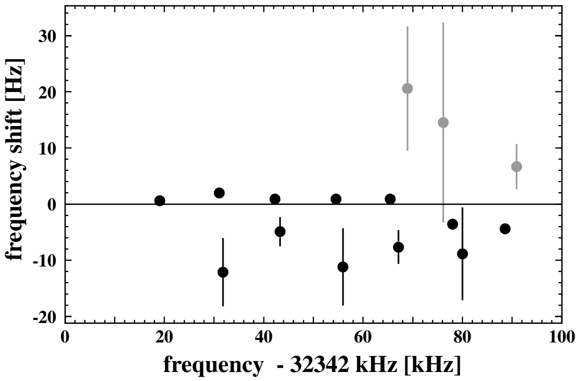

FIG. 6. Measured anomalous frequency shifts in a relatively warm beam. The dots with nearly zero shifts belong to isotopes of the primary beam, and others are nearby fragment nuclides. The gray dots are questionable by the lack of intensity (after Ref. [8]).

Evidently, the low intensity fragment isotopes (with negative shifts and error bars) and masses slightly larger than the mass of the primary beam (with nearly zero shifts and no error bars) have been absorbed into the cloud of the primary beam itself, thereby reducing their frequencies slightly. Unfortunately, there are no isotopes with masses slightly less than the primary beam masses with sufficient population. Hence in Fig. 6 the gray dots should be discarded because the signals came from very few ions only.

In summary, we have shown that with the high precision mass measurements in the ESR very cold multiple strings of ions have been produced which either run parallel to each other or lock into a single string if their displacement becomes very small.

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*Electronic address: r.hasse@gsi.de

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