

# overlap - nuclear overlap calculation

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## 1 Introduction

The nuclear overlap model, as described by Eskola in Nucl. Phys. B323(1989)37, expresses a nucleus-nucleus collision in terms of binary collisions between nucleons. The mass distributions within the two colliding nuclei is used to calculate the number of participating nucleons and the number of binary nucleon-nucleon (NN) collisions as a function of the impact parameter, as well as the total geometric cross section.

This note describes the program `overlap`. The source code is `overlap.f`. The input, the calculation, and the output are described below. The lengths are expressed in fm and cross sections in barn, mb, or fm<sup>2</sup>. Please note that

$$\begin{aligned} 1 \text{ barn} &= 1000 \text{ mb} \\ 1 \text{ barn} &= 100 \text{ fm}^2 \\ 1 \text{ fm}^2 &= 10 \text{ mb} \\ 1 \text{ fm}^{-2} &= 0.1 \text{ mb}^{-1} \end{aligned}$$

## 2 Input

- A – mass number of the projectile nucleus
- B – mass number of the target nucleus
- denflag – density profile selection, 1 for sharp sphere or 2 for Woods-Saxon
- sigma\_NN – inelastic NN cross section in mb, recommended values are 30 for 10-200 GeV  
LAB, 37, 41, 42, 60 for  $\sqrt{s}=56, 130, 200, 5500$  GeV, respectively
- statistics – number of trials per integral, 1000 is good for quick tests

Please note that the description of the density becomes unrealistic for low A. The systematic error related to this can be estimated by running the code with B=1, and comparing the obtained TAB(0) value with TA(0), the latter being the relevant quantity for pA. The result is shown below. The discrepancy is largest for Woods-Saxon profile with low A.

profile	A	B	TAB(0)/TA(0)
Woods-Saxon	1	236	0.9
Woods-Saxon	1	12	0.6
sharp sphere	1	236	1.0
sharp sphere	1	12	0.9

### 3 Calculation

#### 3.1 Density

Density is the number of nucleons per unit volume. The user has a choice between two density profiles, sharp sphere

$$n_A(r) = \begin{cases} n_0, & r \leq R \\ 0, & r > R \end{cases} \quad (1)$$

with

$$\begin{aligned} n_0 &= 0.17 \text{ fm}^{-3}, \\ R &= \left( \frac{3A}{4\pi \cdot n_0} \right)^{1/3} \end{aligned} \quad (2)$$

and Woods-Saxon

$$n_A(r) = \frac{n_0}{1 + \exp(\frac{r-R}{d})}. \quad (3)$$

with

$$\begin{aligned} n_0 &= 0.17 \text{ fm}^{-3}, \\ R &= (1.12 A^{1/3} - 0.86 A^{-1/3}) \text{ fm}, \\ d &= 0.54 \text{ fm}. \end{aligned} \quad (4)$$

Both densities are normalized to the number of nucleons:

$$\int d^3r n_A(r) = 4\pi \int_0^\infty r^2 dr n_A(r) = A. \quad (5)$$

#### 3.2 Thickness function

Thickness function is the density integrated along the beam axis  $z$ :

$$T_A(b) = \int_{-\infty}^{\infty} dz n_A(\sqrt{b^2 + z^2}). \quad (6)$$

The integral of the thickness function is

$$\int d^2b T_A(b) = A. \quad (7)$$

In the sharp sphere case the thickness function is the product of the density and the path length:

$$\begin{aligned} T_A(b) &= 2n_0\sqrt{R^2 - b^2} && \xrightarrow{b \rightarrow 0} 2n_0R \\ T_A(b) &= 3/2 \frac{A}{\pi R^3} \sqrt{R^2 - b^2} && \rightarrow \frac{3A}{2\pi R^2} \\ T_A(b) &= 2 n_0 \sqrt{\left(\frac{3A}{4\pi n_0}\right)^{2/3} - b^2} && \rightarrow \left(\frac{6An_0^2}{\pi}\right)^{1/3} \approx 0.38 A^{1/3}. \end{aligned} \quad (8)$$

Physically  $T_A(b) \sigma_{\text{NN}}$  can be interpreted as the number of NN collisions encountered by a nucleon passing through a nucleus at impact parameter  $b$ .  $\sigma_{\text{NN}}$  is the total inelastic nucleon-nucleon cross section.

Hard processes like D production scale with the number of NN collisions. If we denote the D production cross section in nucleon-nucleon collision by  $\sigma_{\text{NN}}^D$ , then the D meson multiplicity in a pA collision at impact parameter  $b$  is given by  $T_A(b) \sigma_{\text{NN}}^D$ . The inclusive D meson production cross section in pA, which can be obtained by integrating the D multiplicity over  $b$ , is then equal to  $A \sigma_{\text{NN}}^D$ .

### 3.3 Overlap function

For a given impact parameter  $\vec{b} := \langle \vec{r}_B \rangle - \langle \vec{r}_A \rangle$ , the overlap function is defined as the product of the thickness functions of the colliding nuclei A and B, integrated over the two transverse dimensions:

$$T_{AB}(b) = \int d^2s T_A(\vec{s}) T_B(\vec{s} - \vec{b}) \quad (9)$$

with  $\vec{b}$  and  $\vec{s}$  being perpendicular to the beam direction  $z$ . The overlap function can also be calculated directly from densities by a four-dimensional integration:

$$T_{AB}(b) = \int d^2s dz_A dz_B n_A(\vec{s}, z_A) n_B(\vec{s} - \vec{b}, z_B). \quad (10)$$

The integral of the overlap function is

$$\int d^2b T_{AB}(b) = AB. \quad (11)$$

With the sharp-sphere density description, the overlap function for the central A+A collision is

$$\begin{aligned} T_{AB}(0) &= 2\pi R^4 n_0^2 \\ T_{AB}(0) &= \frac{9A^2}{8\pi R^2} \\ T_{AB}(0) &= \left( \frac{81A^4 n_0^2}{32\pi} \right)^{1/3} \approx 0.29A^{4/3}. \end{aligned} \quad (12)$$

Physically,  $T_{AB}(b) \sigma_{\text{NN}}$  can be interpreted as the number of NN collisions in a A+B collision at impact parameter  $b$ .  $\sigma_{\text{NN}}$  is the total inelastic nucleon-nucleon cross section.

Hard processes like D production scale with the number of NN collisions. If we denote the D production cross section in nucleon-nucleon collision by  $\sigma_{\text{NN}}^D$ , then the D meson multiplicity in a A+B collision at impact parameter  $b$  is given by  $T_{AB}(b) \sigma_{\text{NN}}^D$ . The inclusive D meson production cross section in A+B, which can be obtained by integrating the D multiplicity over  $b$ , is then equal to  $AB \sigma_{\text{NN}}^D$ .

### 3.4 Number of participants

Participants are nucleons which have encountered at least one collision with another nucleon. The mean number of participants in pA and A+B collisions at an impact parameter  $b$  can be calculated via

$$N_{\text{part}_{pA}}(b) = \left\{ 1 - \left[ 1 - \frac{\sigma_{\text{NN}} T_A(b)}{A} \right]^A \right\} + T_A(b) \sigma_{\text{NN}} \quad (13)$$

$$N_{\text{part}_{AB}}(b) = \int d^2s T_A(\vec{s}) \left\{ 1 - \left[ 1 - \frac{\sigma_{\text{NN}} T_B(\vec{s} - \vec{b})}{B} \right]^B \right\} \quad (14)$$

$$+ \int d^2s T_B(\vec{s}) \left\{ 1 - \left[ 1 - \frac{\sigma_{\text{NN}} T_A(\vec{s} + \vec{b})}{A} \right]^A \right\}. \quad (15)$$

The two terms represents the contributions from the two colliding objects. The factor repeatedly present in these formulas represents the probability for a nucleon to pass through the nucleus without any collision:

$$p(ncol = 0) = \left[ 1 - \frac{\sigma_{\text{NN}} T_A(b)}{A} \right]^A \quad (16)$$

This probability is obtained using the binomial distribution of number of NN collisions. A numerically similar result can be obtained using the simpler Poissonian

$$p(ncol = 0) = e^{-\sigma_{NN} T_A(b)}. \quad (17)$$

The probability of becoming participant is  $p(ncol > 0) = 1 - p(0) = 1 - e^{-\sigma_{NN} T_A(b)}$ , and the number of participants is then

$$N_{part_{pA}}(b) = \left\{ 1 - e^{-\sigma_{NN} T_A(b)} \right\} + T_A(b) \sigma_{NN} \quad (18)$$

$$N_{part_{AB}}(b) = \int d^2s T_A(\vec{s}) \left\{ 1 - e^{-\sigma_{NN} T_B(\vec{s}-\vec{b})} \right\} + \int d^2s T_B(\vec{s}) \left\{ 1 - e^{-\sigma_{NN} T_B(\vec{s}+\vec{b})} \right\}. \quad (19)$$

### 3.5 Total geometric cross section

Total geometric cross section is the cross section for such A+B collisions in which at least one NN collision occurs:

$$\sigma_G = \int d^2b \left[ 1 - e^{-T_{AB}(b) \sigma_{NN}} \right]. \quad (20)$$

For the sharp sphere density profile and in the limit of an infinite nucleon-nucleon cross section  $\sigma_{NN}$  the total geometric cross section should approach

$$\sigma_G = \pi(R_A + R_B)^2 = \left( \frac{9\pi}{16n_0^2} \right)^{1/3} \left[ A^{1/3} + B^{1/3} \right]^2 \approx 3.94 \left[ A^{1/3} + B^{1/3} \right]^2. \quad (21)$$

## 4 Output

### 4.1 Output format

The code prints the values of the arguments, then the calculated total cross section. The rest of output consists of several columns of numbers. The first column is the abscissa and represents the radius  $r$  or the impact parameter  $b$  in fm. Following are the densities of the colliding nuclei  $n_A(r)$  and  $n_B(r)$ , their thickness functions  $T_A(b)$  and  $T_B(b)$ , the overlap function  $T_{AB}(b)$ , then again the overlap function but calculated using a different method (a disagreement between the two is a sign that the used statistics was too low), numbers of participants from A and from B ( $A_{part}, B_{part}$ ), the contribution of the given impact parameter slice to the total cross section ( $sigma$ ), and finally the centrality, i.e. the cross section for collisions with an impact parameter between 0 and  $b$  divided by the total cross section. An example of the output is shown below.

```
*****
Nuclear overlap calculation
Enter A,B
Select density profile: 1 for sharp sphere or 2 for Woods-Saxon
Enter sigma_NN in mb
Enter number of shots per integral (e.g. 10000)
*****
A ..... 208.
B ..... 197.
sigma_NN ..... 38. mb
statistics ..... 30000
density profile ... Woods-Saxon
*****
Calculating TA for A and B
Calculating TAB for A+B collision
Calculating Npart and TAB for A+B collision
Calculating cross section for A+B collision
cross section ..... 7.092 barn
```

```

*****
b ..... impact parameter or radius (fm)
nA ..... density of nucleus A (1/fm^3)
nB ..... density of nucleus B (1/fm^3)
TA ..... thickness function of A (1/fm^2)
TB ..... thickness function of B (1/fm^2)
TAB ..... overlap function of A+B (1/fm^2)
TABcheck..... TAB calculated via TA and TB
Apart ..... number of participants from A
Bpart ..... number of participants from B
sigma ..... cross section ds/db*0.1 fm (fm^2)
cent ..... fraction of total cross section
*****
  b      nA      nB      TA      TB      TAB      TABcheck      Apart      Bpart      sigma      cent
0.05  0.170  0.170  2.215  2.174  300.524  306.128  198.495  190.450  0.031  0.000
0.15  0.170  0.170  2.202  2.153  304.809  305.482  198.380  190.315  0.094  0.000
0.25  0.170  0.170  2.212  2.167  299.831  302.380  196.504  188.693  0.157  0.000
0.35  0.170  0.170  2.215  2.160  305.086  309.382  200.418  192.350  0.220  0.001
0.45  0.170  0.170  2.189  2.162  301.327  300.823  196.282  188.342  0.283  0.001
0.55  0.170  0.170  2.191  2.147  288.262  311.906  201.823  193.817  0.346  0.002
0.65  0.170  0.170  2.194  2.169  300.122  303.332  197.622  189.716  0.408  0.002
0.75  0.170  0.170  2.185  2.144  300.392  302.418  196.487  188.337  0.471  0.003
0.85  0.170  0.170  2.187  2.155  304.341  299.117  195.480  187.891  0.534  0.004
0.95  0.170  0.170  2.189  2.138  292.538  297.281  194.179  186.329  0.597  0.004
1.05  0.170  0.170  2.171  2.143  289.995  297.381  193.628  186.793  0.660  0.005
1.15  0.170  0.170  2.170  2.121  301.532  292.714  192.400  185.654  0.723  0.006
1.25  0.170  0.170  2.169  2.137  291.566  294.301  192.815  185.939  0.785  0.007
1.35  0.170  0.170  2.150  2.129  300.017  289.067  190.803  183.807  0.848  0.009
1.45  0.170  0.170  2.157  2.102  294.802  294.474  192.743  186.157  0.911  0.010
1.55  0.170  0.170  2.132  2.096  292.712  280.834  185.871  178.748  0.974  0.011
1.65  0.170  0.170  2.122  2.098  288.827  284.375  187.610  181.018  1.037  0.013
1.75  0.170  0.170  2.126  2.069  280.473  283.571  187.595  180.799  1.100  0.014
1.85  0.170  0.170  2.119  2.076  280.946  283.873  187.305  181.206  1.162  0.016
1.95  0.170  0.170  2.107  2.059  277.115  280.157  187.221  178.951  1.225  0.018
2.05  0.170  0.170  2.089  2.058  281.614  276.426  183.524  177.734  1.288  0.020
2.15  0.170  0.170  2.079  2.037  273.103  267.322  180.559  173.641  1.351  0.021
2.25  0.170  0.170  2.061  2.016  266.790  270.640  180.946  176.012  1.414  0.023
2.35  0.170  0.170  2.056  2.021  272.931  261.966  176.429  171.052  1.477  0.026
2.45  0.170  0.170  2.027  1.980  265.807  273.173  182.624  177.808  1.539  0.028
2.55  0.170  0.170  2.017  1.981  256.821  269.147  180.497  175.334  1.602  0.030
2.65  0.170  0.170  2.011  1.956  256.049  258.155  174.493  169.258  1.665  0.032
2.75  0.170  0.170  1.988  1.950  250.209  253.325  170.716  166.772  1.728  0.035
2.85  0.170  0.170  1.972  1.940  250.668  251.617  172.166  166.258  1.791  0.037
2.95  0.170  0.170  1.948  1.907  244.754  250.879  171.457  166.992  1.854  0.040
3.05  0.170  0.170  1.943  1.893  252.517  245.776  168.228  163.369  1.916  0.043
3.15  0.170  0.170  1.905  1.876  245.027  242.584  166.632  162.837  1.979  0.045
3.25  0.170  0.169  1.893  1.866  236.961  242.516  166.361  163.004  2.042  0.048
3.35  0.169  0.169  1.874  1.830  237.807  234.087  162.111  157.320  2.105  0.051
3.45  0.169  0.169  1.856  1.816  238.464  236.683  163.793  159.612  2.168  0.054
3.55  0.169  0.169  1.823  1.781  230.838  234.208  162.615  158.587  2.231  0.057
3.65  0.169  0.169  1.810  1.766  231.862  223.043  156.247  152.728  2.293  0.061
3.75  0.169  0.169  1.786  1.734  227.528  225.927  157.622  154.567  2.356  0.064
3.85  0.169  0.168  1.747  1.696  210.380  217.560  153.424  150.190  2.419  0.067
3.95  0.168  0.168  1.717  1.682  223.550  214.991  151.775  148.671  2.482  0.071
4.05  0.168  0.168  1.702  1.650  209.885  217.344  152.942  149.982  2.545  0.074
4.15  0.168  0.167  1.676  1.611  201.937  203.763  145.635  142.618  2.608  0.078
4.25  0.167  0.167  1.632  1.573  206.610  203.748  146.803  143.782  2.670  0.082
4.35  0.167  0.166  1.615  1.541  202.463  200.713  144.125  141.442  2.733  0.086
4.45  0.166  0.165  1.568  1.504  197.503  197.960  144.203  140.753  2.796  0.090
4.55  0.165  0.164  1.535  1.474  194.053  188.864  137.449  135.095  2.859  0.094
4.65  0.165  0.163  1.491  1.429  195.888  190.419  137.499  135.400  2.922  0.098
4.75  0.163  0.162  1.446  1.380  185.580  183.938  134.705  131.160  2.985  0.102
4.85  0.162  0.160  1.414  1.342  181.589  181.672  134.770  131.709  3.047  0.106
4.95  0.161  0.159  1.366  1.284  178.346  175.311  130.461  126.446  3.110  0.111
5.05  0.159  0.156  1.318  1.246  173.757  177.727  130.948  128.992  3.173  0.115
5.15  0.157  0.154  1.282  1.190  170.356  163.767  124.367  121.069  3.236  0.120
5.25  0.154  0.151  1.225  1.146  164.250  163.723  123.847  121.984  3.299  0.124
5.35  0.152  0.148  1.171  1.102  158.491  163.844  123.802  120.046  3.362  0.129
5.45  0.148  0.144  1.122  1.044  160.115  162.438  122.609  120.586  3.424  0.134
5.55  0.145  0.139  1.059  0.968  149.566  162.292  123.342  120.132  3.487  0.139
5.65  0.140  0.134  1.006  0.918  156.551  154.345  118.167  116.525  3.550  0.144
5.75  0.136  0.129  0.959  0.873  147.020  146.136  113.127  111.721  3.613  0.149
5.85  0.130  0.123  0.886  0.797  145.015  144.633  113.254  110.865  3.676  0.154
5.95  0.124  0.116  0.822  0.747  144.700  138.271  109.195  107.907  3.739  0.159
6.05  0.118  0.109  0.765  0.677  134.790  139.807  110.156  108.725  3.801  0.165
6.15  0.111  0.102  0.705  0.620  133.980  134.977  106.914  106.877  3.864  0.170
6.25  0.104  0.094  0.644  0.568  133.752  131.886  106.737  103.210  3.927  0.176
6.35  0.096  0.087  0.579  0.503  134.261  126.877  102.774  100.185  3.990  0.181
6.45  0.088  0.079  0.521  0.454  126.816  118.877  97.455  96.310  4.053  0.187
6.55  0.080  0.071  0.471  0.407  119.377  117.256  96.236  94.573  4.115  0.193
6.65  0.073  0.063  0.420  0.356  115.103  118.588  96.476  95.884  4.178  0.199

```

6.75	0.065	0.056	0.367	0.313	110.514	114.716	95.800	93.229	4.241	0.205
6.85	0.058	0.049	0.319	0.271	108.120	111.950	92.493	92.660	4.304	0.211
6.95	0.051	0.043	0.283	0.234	103.300	103.864	87.453	86.736	4.367	0.217
7.05	0.045	0.038	0.247	0.206	101.995	102.951	88.986	85.378	4.430	0.223
7.15	0.039	0.032	0.212	0.174	99.222	100.927	86.590	85.215	4.492	0.230
7.25	0.033	0.028	0.181	0.149	97.376	95.427	83.418	80.672	4.555	0.236
7.35	0.029	0.024	0.158	0.128	92.586	93.376	81.550	79.385	4.618	0.243
7.45	0.025	0.020	0.133	0.107	87.821	90.211	78.730	78.633	4.681	0.249
7.55	0.021	0.017	0.114	0.092	86.584	86.684	76.759	76.658	4.744	0.256
7.65	0.018	0.015	0.097	0.078	86.116	84.392	75.303	74.954	4.807	0.263
7.75	0.015	0.012	0.082	0.065	80.289	81.637	73.533	72.348	4.869	0.269
7.85	0.013	0.010	0.069	0.055	78.867	77.348	71.054	70.602	4.932	0.276
7.95	0.011	0.009	0.058	0.047	77.872	74.950	68.849	68.639	4.995	0.283
8.05	0.009	0.007	0.049	0.039	73.964	71.445	66.565	66.340	5.058	0.291
8.15	0.008	0.006	0.041	0.033	74.792	69.825	65.473	64.956	5.121	0.298
8.25	0.006	0.005	0.034	0.028	68.475	66.144	63.270	62.186	5.184	0.305
8.35	0.005	0.004	0.029	0.023	67.783	64.049	61.589	61.201	5.246	0.313
8.45	0.004	0.004	0.024	0.020	59.608	61.346	59.548	59.409	5.309	0.320
8.55	0.004	0.003	0.020	0.016	59.443	59.525	58.754	57.368	5.372	0.328
8.65	0.003	0.002	0.017	0.014	56.385	56.964	56.867	55.962	5.435	0.335
8.75	0.003	0.002	0.014	0.011	56.644	56.001	55.929	54.912	5.498	0.343
8.85	0.002	0.002	0.012	0.010	50.866	52.092	53.357	52.169	5.561	0.351
8.95	0.002	0.001	0.010	0.008	49.712	48.628	50.306	49.657	5.623	0.359
9.05	0.001	0.001	0.008	0.007	46.466	47.980	49.994	49.368	5.686	0.367
9.15	0.001	0.001	0.007	0.006	47.500	45.004	47.884	47.096	5.749	0.375
9.25	0.001	0.001	0.006	0.005	44.230	43.505	47.087	45.985	5.812	0.383
9.35	0.001	0.001	0.005	0.004	40.375	40.607	44.537	43.753	5.875	0.391
9.45	0.001	0.001	0.004	0.003	39.624	37.742	41.843	41.417	5.938	0.400
9.55	0.001	0.000	0.003	0.003	37.242	37.117	41.074	40.752	6.000	0.408
9.65	0.000	0.000	0.003	0.002	32.536	35.251	40.270	39.836	6.063	0.417
9.75	0.000	0.000	0.002	0.002	34.081	33.107	38.013	37.575	6.126	0.425
9.85	0.000	0.000	0.002	0.002	30.728	31.131	36.022	35.861	6.189	0.434
9.95	0.000	0.000	0.002	0.001	29.514	27.940	33.093	33.559	6.252	0.443
10.05	0.000	0.000	0.001	0.001	27.184	27.773	33.722	32.964	6.315	0.452
10.15	0.000	0.000	0.001	0.001	25.260	27.471	33.210	33.191	6.377	0.461
10.25	0.000	0.000	0.001	0.001	23.936	24.835	30.750	30.422	6.440	0.470
10.35	0.000	0.000	0.001	0.001	22.186	22.725	29.079	28.242	6.503	0.479
10.45	0.000	0.000	0.001	0.001	21.891	21.744	28.073	27.969	6.566	0.488
10.55	0.000	0.000	0.001	0.000	19.365	20.731	26.774	26.820	6.629	0.498
10.65	0.000	0.000	0.000	0.000	16.989	18.089	24.326	24.108	6.692	0.507
10.75	0.000	0.000	0.000	0.000	17.007	17.499	23.211	23.792	6.754	0.517
10.85	0.000	0.000	0.000	0.000	16.930	16.248	22.510	22.344	6.817	0.526
10.95	0.000	0.000	0.000	0.000	17.012	15.406	21.600	21.534	6.880	0.536
11.05	0.000	0.000	0.000	0.000	14.102	14.787	21.128	20.943	6.943	0.546
11.15	0.000	0.000	0.000	0.000	12.022	12.562	18.447	18.437	7.006	0.556
11.25	0.000	0.000	0.000	0.000	11.616	12.392	18.466	18.144	7.069	0.566
11.35	0.000	0.000	0.000	0.000	12.367	11.386	17.507	17.017	7.131	0.576
11.45	0.000	0.000	0.000	0.000	10.363	10.594	16.276	16.433	7.194	0.586
11.55	0.000	0.000	0.000	0.000	9.684	9.522	15.205	14.927	7.257	0.596
11.65	0.000	0.000	0.000	0.000	8.406	8.459	13.844	13.391	7.320	0.606
11.75	0.000	0.000	0.000	0.000	8.091	8.114	13.237	13.321	7.383	0.617
11.85	0.000	0.000	0.000	0.000	7.169	7.315	12.359	12.205	7.446	0.627
11.95	0.000	0.000	0.000	0.000	7.137	6.484	11.047	11.238	7.508	0.638
12.05	0.000	0.000	0.000	0.000	6.115	5.970	10.460	10.393	7.571	0.649
12.15	0.000	0.000	0.000	0.000	5.642	5.370	9.627	9.649	7.634	0.659
12.25	0.000	0.000	0.000	0.000	4.871	5.084	9.409	9.136	7.697	0.670
12.35	0.000	0.000	0.000	0.000	4.350	4.455	8.354	8.320	7.760	0.681
12.45	0.000	0.000	0.000	0.000	4.152	4.043	7.709	7.762	7.823	0.692
12.55	0.000	0.000	0.000	0.000	3.681	3.534	6.811	6.938	7.885	0.703
12.65	0.000	0.000	0.000	0.000	3.138	3.193	6.461	6.278	7.948	0.714
12.75	0.000	0.000	0.000	0.000	2.829	2.836	5.770	5.732	8.011	0.726
12.85	0.000	0.000	0.000	0.000	2.427	2.620	5.412	5.455	8.073	0.737
12.95	0.000	0.000	0.000	0.000	2.299	2.385	5.030	5.051	8.135	0.749
13.05	0.000	0.000	0.000	0.000	2.231	2.068	4.493	4.399	8.198	0.760
13.15	0.000	0.000	0.000	0.000	1.942	1.885	4.165	4.072	8.257	0.772
13.25	0.000	0.000	0.000	0.000	1.497	1.623	3.644	3.596	8.297	0.783
13.35	0.000	0.000	0.000	0.000	1.552	1.476	3.409	3.296	8.365	0.795
13.45	0.000	0.000	0.000	0.000	1.289	1.294	3.000	2.984	8.388	0.807
13.55	0.000	0.000	0.000	0.000	1.164	1.130	2.724	2.610	8.411	0.819
13.65	0.000	0.000	0.000	0.000	0.967	1.040	2.506	2.462	8.359	0.831
13.75	0.000	0.000	0.000	0.000	0.863	0.892	2.194	2.150	8.314	0.842
13.85	0.000	0.000	0.000	0.000	0.803	0.778	1.958	1.894	8.290	0.854
13.95	0.000	0.000	0.000	0.000	0.658	0.677	1.733	1.673	8.045	0.865
14.05	0.000	0.000	0.000	0.000	0.580	0.606	1.565	1.531	7.855	0.877
14.15	0.000	0.000	0.000	0.000	0.490	0.517	1.368	1.312	7.510	0.887
14.25	0.000	0.000	0.000	0.000	0.461	0.464	1.247	1.185	7.402	0.898
14.35	0.000	0.000	0.000	0.000	0.444	0.418	1.143	1.075	7.349	0.908
14.45	0.000	0.000	0.000	0.000	0.330	0.348	0.952	0.921	6.486	0.917
14.55	0.000	0.000	0.000	0.000	0.276	0.298	0.828	0.793	5.936	0.925
14.65	0.000	0.000	0.000	0.000	0.254	0.263	0.736	0.718	5.701	0.934
14.75	0.000	0.000	0.000	0.000	0.214	0.215	0.615	0.585	5.156	0.941

14.85	0.000	0.000	0.000	0.000	0.207	0.196	0.571	0.535	5.085	0.948
14.95	0.000	0.000	0.000	0.000	0.166	0.171	0.497	0.477	4.393	0.954
15.05	0.000	0.000	0.000	0.000	0.141	0.139	0.411	0.393	3.932	0.960
15.15	0.000	0.000	0.000	0.000	0.120	0.123	0.368	0.349	3.482	0.965
15.25	0.000	0.000	0.000	0.000	0.109	0.108	0.329	0.312	3.258	0.969
15.35	0.000	0.000	0.000	0.000	0.087	0.086	0.260	0.249	2.724	0.973
15.45	0.000	0.000	0.000	0.000	0.087	0.077	0.239	0.224	2.736	0.977
15.55	0.000	0.000	0.000	0.000	0.069	0.066	0.209	0.193	2.267	0.980
15.65	0.000	0.000	0.000	0.000	0.060	0.057	0.181	0.168	1.998	0.983
15.75	0.000	0.000	0.000	0.000	0.049	0.049	0.159	0.145	1.679	0.985
15.85	0.000	0.000	0.000	0.000	0.043	0.040	0.131	0.118	1.512	0.987
15.95	0.000	0.000	0.000	0.000	0.037	0.036	0.119	0.107	1.304	0.989
16.05	0.000	0.000	0.000	0.000	0.028	0.030	0.098	0.088	1.011	0.991
16.15	0.000	0.000	0.000	0.000	0.026	0.026	0.088	0.078	0.947	0.992
16.25	0.000	0.000	0.000	0.000	0.021	0.021	0.072	0.064	0.794	0.993
16.35	0.000	0.000	0.000	0.000	0.020	0.018	0.064	0.055	0.762	0.994
16.45	0.000	0.000	0.000	0.000	0.017	0.016	0.055	0.047	0.637	0.995
16.55	0.000	0.000	0.000	0.000	0.013	0.013	0.045	0.039	0.514	0.996
16.65	0.000	0.000	0.000	0.000	0.012	0.011	0.039	0.033	0.476	0.996
16.75	0.000	0.000	0.000	0.000	0.010	0.010	0.034	0.029	0.404	0.997
16.85	0.000	0.000	0.000	0.000	0.008	0.008	0.028	0.024	0.322	0.998
16.95	0.000	0.000	0.000	0.000	0.008	0.006	0.023	0.020	0.300	0.998
17.05	0.000	0.000	0.000	0.000	0.005	0.006	0.020	0.017	0.210	0.998
17.15	0.000	0.000	0.000	0.000	0.005	0.004	0.016	0.014	0.188	0.998
17.25	0.000	0.000	0.000	0.000	0.004	0.004	0.014	0.012	0.170	0.999
17.35	0.000	0.000	0.000	0.000	0.004	0.003	0.012	0.010	0.147	0.999
17.45	0.000	0.000	0.000	0.000	0.003	0.003	0.010	0.009	0.126	0.999
17.55	0.000	0.000	0.000	0.000	0.002	0.002	0.008	0.007	0.103	0.999
17.65	0.000	0.000	0.000	0.000	0.002	0.002	0.007	0.006	0.086	0.999
17.75	0.000	0.000	0.000	0.000	0.002	0.002	0.006	0.005	0.064	0.999
17.85	0.000	0.000	0.000	0.000	0.001	0.001	0.005	0.004	0.063	1.000
17.95	0.000	0.000	0.000	0.000	0.001	0.001	0.004	0.004	0.046	1.000
18.05	0.000	0.000	0.000	0.000	0.001	0.001	0.004	0.003	0.045	1.000
18.15	0.000	0.000	0.000	0.000	0.001	0.001	0.003	0.003	0.036	1.000
18.25	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.002	0.030	1.000
18.35	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.002	0.026	1.000
18.45	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.001	0.020	1.000
18.55	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.001	0.018	1.000
18.65	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.014	1.000
18.75	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.014	1.000
18.85	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.010	1.000
18.95	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.009	1.000
19.05	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.008	1.000
19.15	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.006	1.000
19.25	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	1.000
19.35	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	1.000
19.45	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.004	1.000
19.55	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.003	1.000
19.65	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	1.000
19.75	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	1.000
19.85	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	1.000
19.95	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	1.000

## 4.2 Do the obtained numbers make sense?

The calculated numbers can be compared to the popular expectations. The agreement should be exact for hard sphere and reasonable for Woods-Saxon.

1. Compare the radius at which the density drops to 50% with  $1.2 \text{ fm } A^{1/3}$ :

$$R_A \stackrel{?}{=} 1.2 \text{ fm } A^{1/3}, \quad n(R_A) = n_0/2. \quad (22)$$

2. Compare the peak density function with the product of nuclear density and nuclear diameter:

$$T_A(0) \stackrel{?}{=} 2 R n_0 \approx 0.38 A^{1/3}. \quad (23)$$

3. Compare the integral of the thickness function with the number of nucleons:

$$A \stackrel{?}{=} \int d^2b T_A(b) \approx \sum_{i=1}^{200} 2\pi b \cdot 0.1 \text{ fm} \cdot T_A(b). \quad (24)$$

4. Compare the peak overlap function with the analytic formula:

$$T_{AB}(0) \stackrel{?}{=} 2\pi R^4 n_0^2 \approx 0.29 A^{4/3}. \quad (25)$$

5. Compare the integral of the overlap function with  $AB$ :

$$AB \stackrel{?}{=} \int d^2b T_{AB}(b) \approx \sum_{i=1}^{200} 2\pi b \cdot 0.1 \text{ fm} \cdot T_{AB}(b). \quad (26)$$

6. Compare the total cross section with  $\pi b_{\max}^2$ :

$$\sigma_G \stackrel{?}{=} \pi(R_A + R_B)^2 \approx 3.94 \text{ fm}^2 \left[ A^{1/3} + B^{1/3} \right]^2. \quad (27)$$

7. Compare the mean number of collisions in minimum bias the value expected from the normalization conditions of  $T_A$  and  $T_{AB}$  (Eqs. 7,11):

$$\langle N_{\text{coll}} \rangle_{\text{minb}} \stackrel{?}{=} AB \frac{\sigma_{\text{NN}}}{\sigma_G} \quad (28)$$