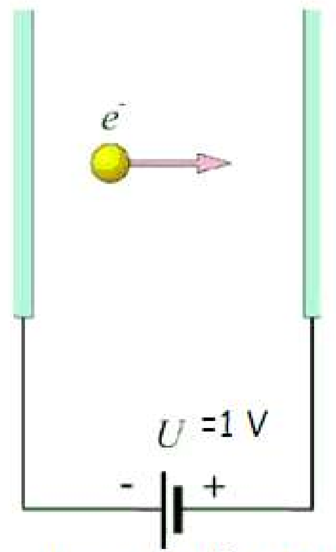
Accelerators

- Survey of various accelerator types
    - electrostatic generators
    - linacs
    - cyclotrons
    - synchrotrons
- Beam optics
    - betatron oscillations
    - weak focusing
    - strong focusing

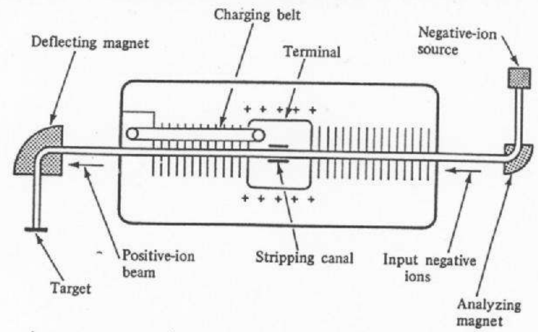


eU=1.602 10^{-19} J=1 eV

1 keV= 10^3 eV
1 MeV= 10^6 eV
1 GeV= 10^9 eV
1 TeV= 10^{12} eV

	Energies in eV
Expected mass of the Higgs boson	> 100 GeV
W, Z exchange bosons of the weak force	82 bzw. 93 Ge
Rest mass of the anti proton: $E=mc^2$	0.938 GeV
Excitation energies in nuclei	MeV
Ionisation energies in heavy atoms	keV
Ionisation energy of the hydrogen atom	13 eV

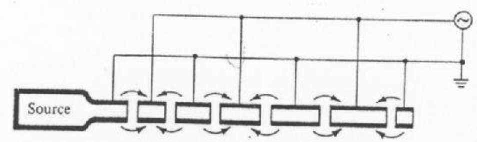
bis 20 MV (q+1)



Elektro-stat. Beschleunigr

Tandem Van de Graaff. Negative ions are first accelerated to the central terminal. There they are stripped of their electrons and accelerated as positive ions to the target.

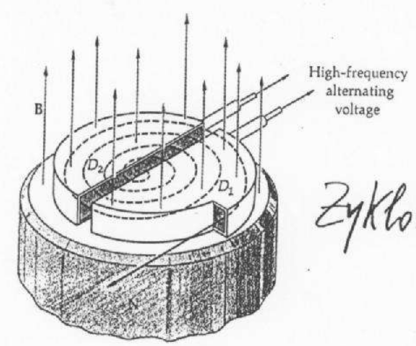
~50 GeV



Linac (Linearbeschl.)

Drift tube linac. The arrows at the gaps indicate the direction of the electric field at a given time.

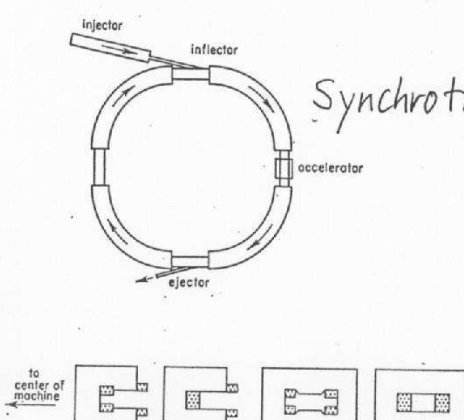
~500 MeV



Zyklotron

Schematic drawing of a cyclotron. The upper (south) pole face of the magnet has been omitted.

~ 10 TeV · $\frac{A}{q}$



Synchrotron

Proton synchrotron of four sectors, and schematic enlarged cross sections of "C" and "picture-frame" magnets, with and without poles that protrude beyond the exciting windings.

Equation of motion:

$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

Relativistic parameters:

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \frac{v}{c}$$

Change of particle energy:

$$\frac{d}{dt}(\gamma mc^2) = q\vec{E}\vec{v} = q\vec{E}\frac{d\vec{s}}{dt}$$

$$\frac{d}{dt}(\gamma mc^2 + U) = 0 \quad \text{with} \quad U = q\phi = -q\int \vec{E}d\vec{s}$$

1. Electrostatic acceleration:

$$-\varepsilon_0 \Delta\phi = \rho_b$$ (ρ_b beam charge density)

2. 'Radiofrequency' (RF) acceleration:

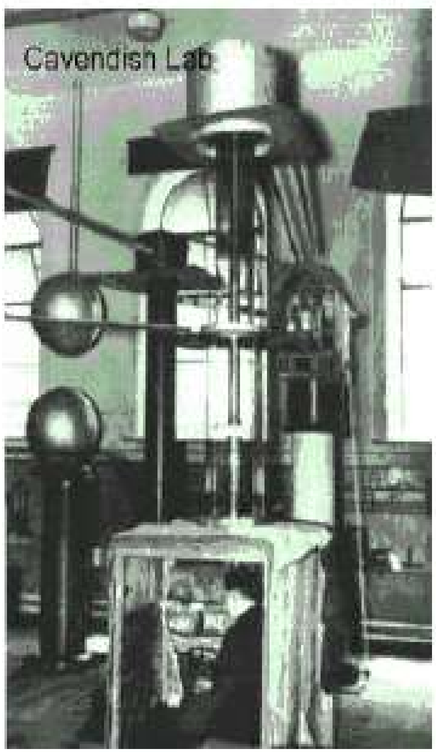
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

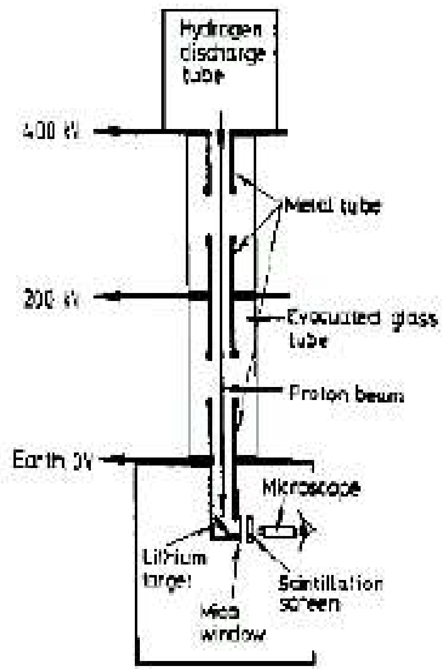
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

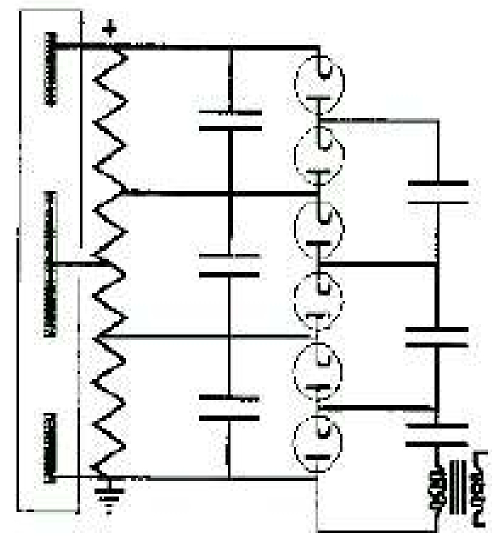
Electrostatic accelerators

1928 Cockcroft & Walton start designing a 800 kV generator encouraged by Rutherford

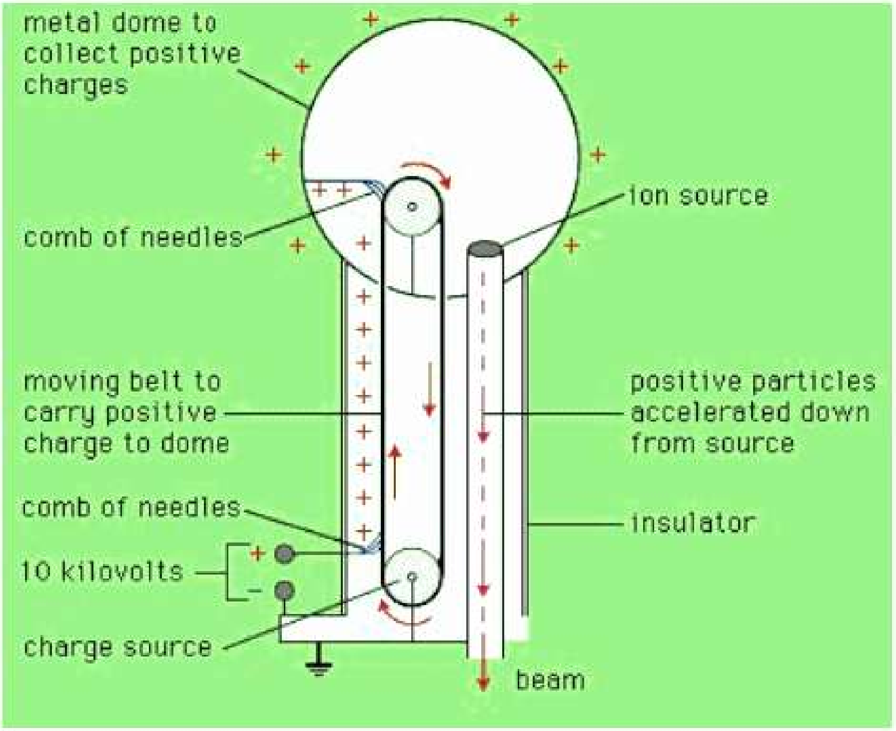
1932 Generator reaches 700 kV and C&W split lithium with 400 kV protons.



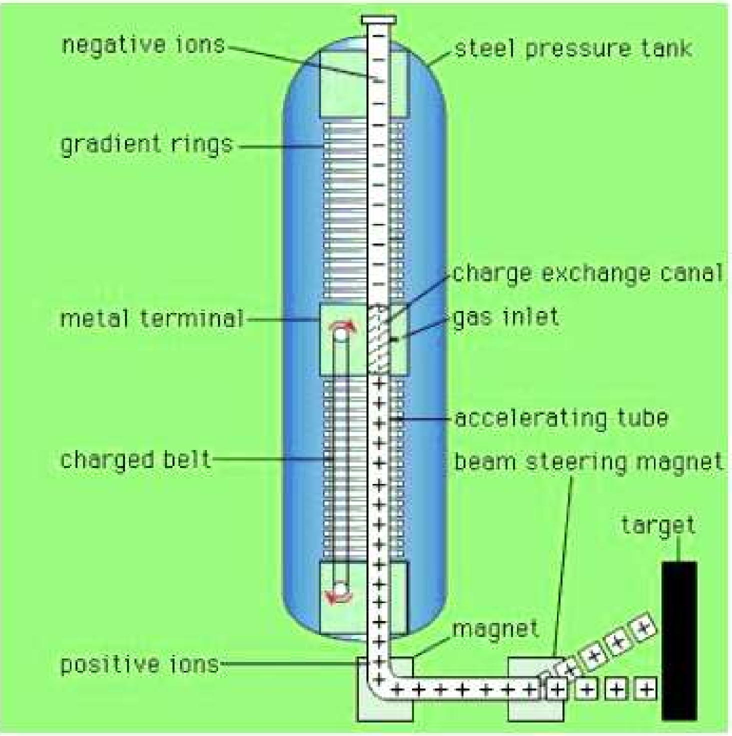




1931 Van de Graaff generator (basic design)



Two-stage tandem accelerator



10 MV Van de Graff accelerator (HMI)

Tandem accelerator at Brookhaven Nat'l Labs

Linear accelerators (I) the Wideroe

1924 Ising proposed time-varying fields across drift tubes: 'resonant acceleration'.

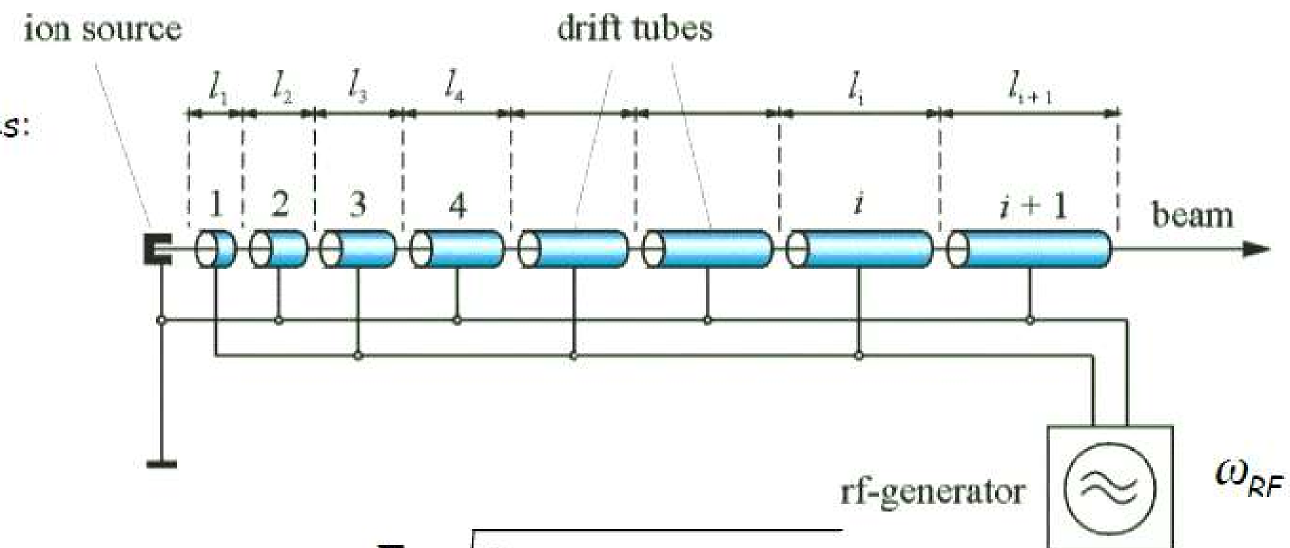
1928 Wideröe (in Aachen) demonstrates Ising's principle with a 1 MHz, 25 kV oscillator.

Length of the drift tubes:

$$l_i = \frac{1}{2} v_i T_{RF}$$

Particle energy:

$$(E_s)_i = \frac{1}{2} m v_i^2$$

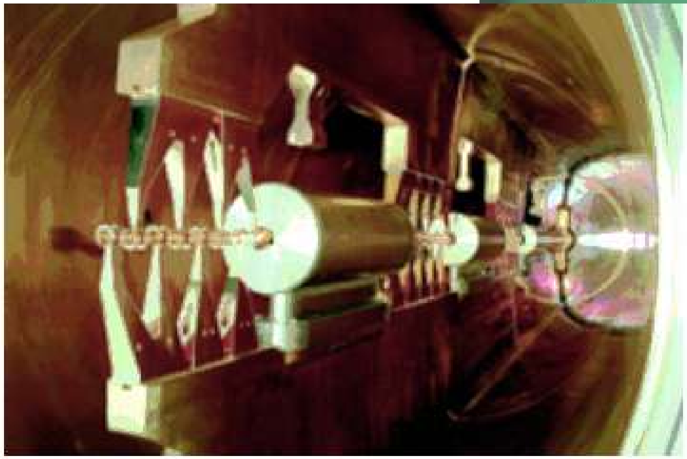


$$\omega_{RF} = \frac{2\pi}{T_{RF}}$$

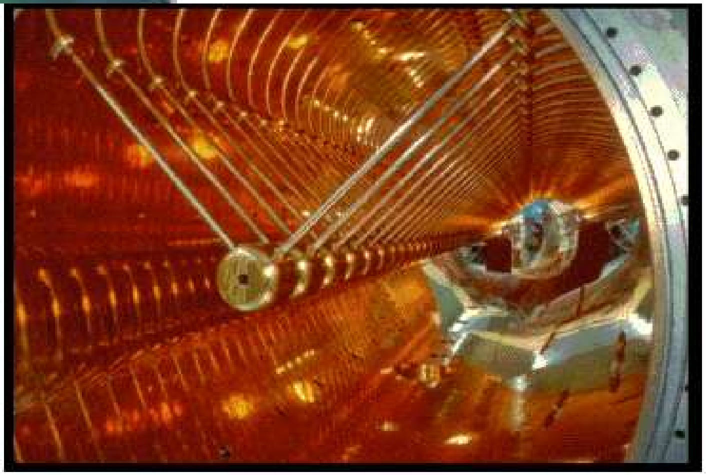
$$(E_s)_{i+1} = (E_s)_i + qV_0 \sin\phi_s \quad \Rightarrow l_i = \frac{T_{RF}}{2} \sqrt{\frac{2}{m}(E_{s0} + iqV_0 \sin\phi_s)}$$

For 10 (100) MHz and 2 MeV protons we get a maximum drift tube length of 1 (0.1) m !

Modern structures in use at GSI



f_{HF}=36 MHz (‚IH')



f_{HF}=108 MHz (‚Alvarez')

Principle of RF accelerators

ΔE: Energy kick

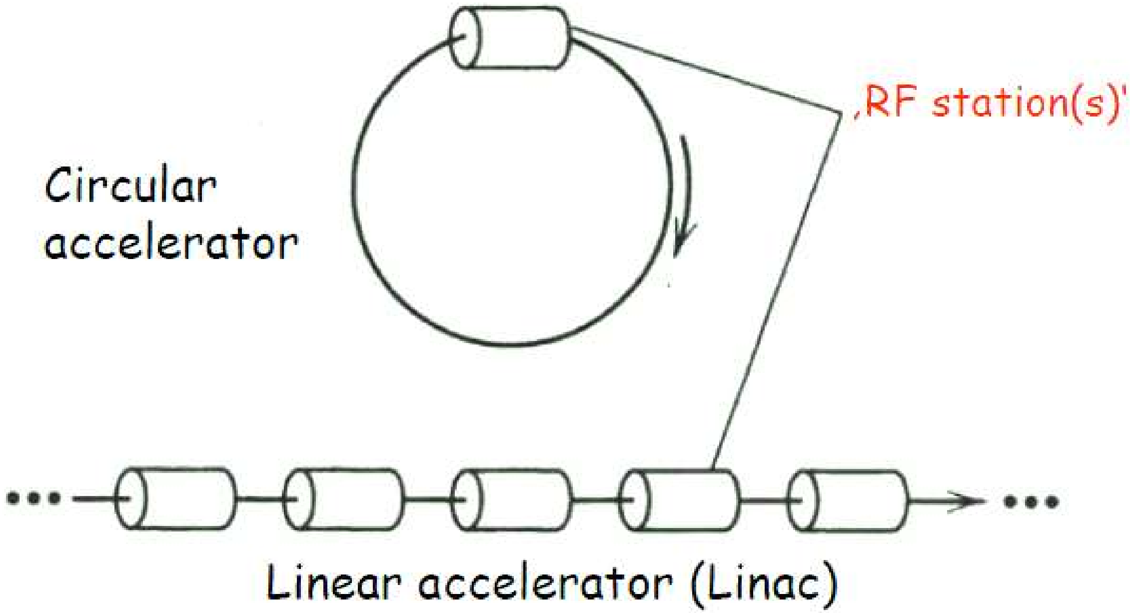
t: time

q: Particle charge

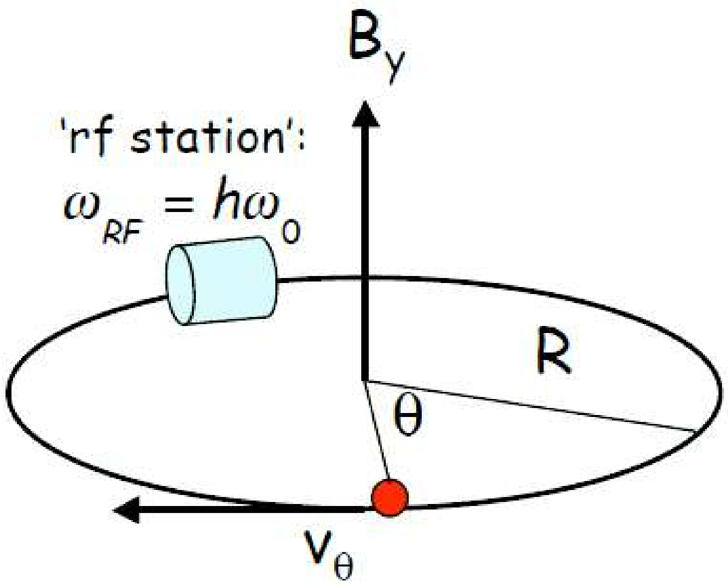
V_0: Gap voltage

f_{RF}: RF frequency

$$\Delta E = qV_0^{RF} \sin(2\pi f_{RF} t)$$



Circular accelerators: principle



Homogenous B-field in y-direction:

$$\dot{\theta} = \omega_0 = \frac{v_\theta}{R} = \frac{qB_y}{\gamma m}$$

('cyclotron frequency')

'Rigidity':

$$p = qBR \qquad E = \gamma mc^2 \approx pc$$

Example: 1 TeV protons and B=2 T results in R=1.6 km (L=10 km)

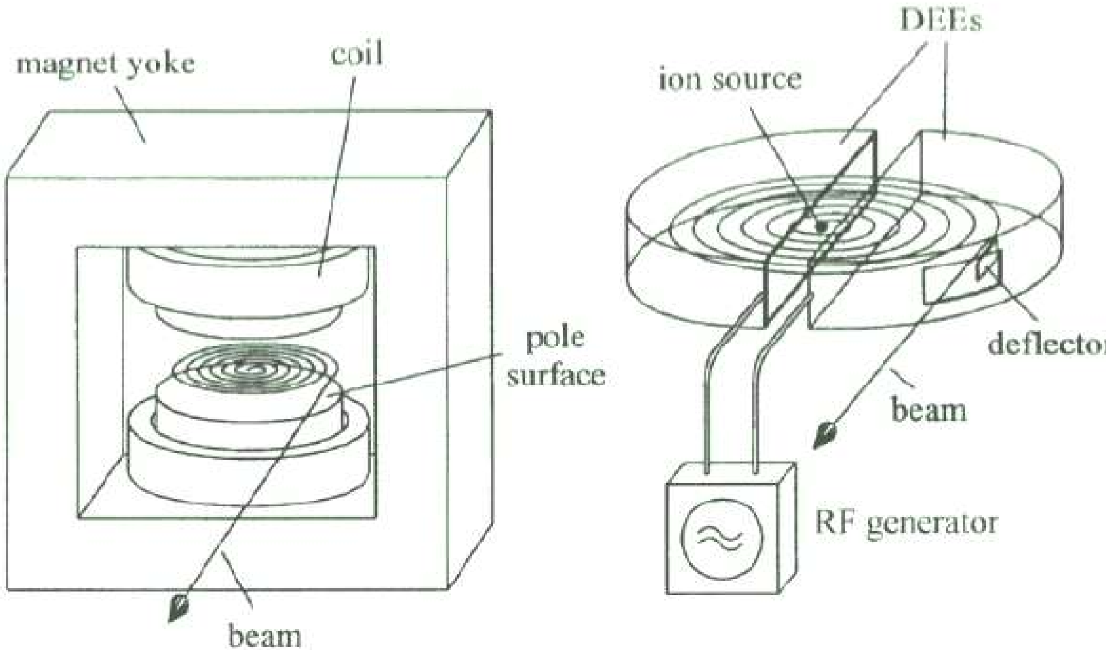
The cyclotron

Constant (magnetic) bending field ⇒ increasing radius

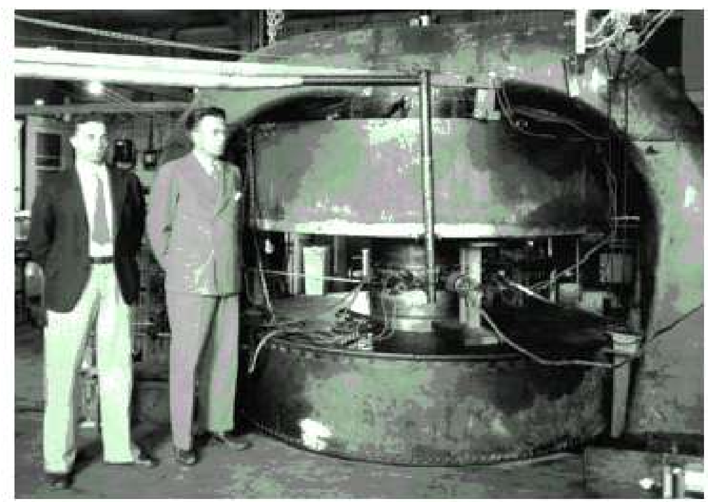
= 1

$$\omega_0 = \frac{v_\theta}{R} = \frac{qB_y}{\gamma m} = const.$$

(Cyclotron frequency)



$$\omega_0 = \omega_{HF} = const.$$



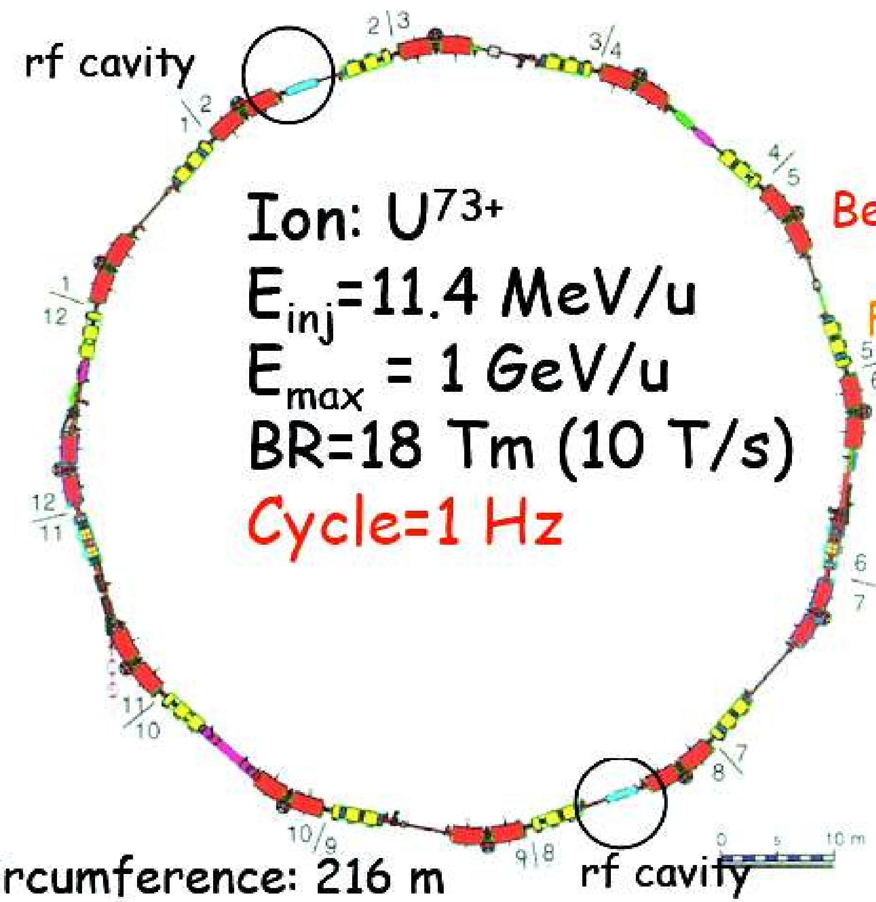
Lawrence and Livingston, Berkeley, 1932

Synchrotron: the principle

'Synchronous' rf frequency: $\omega_{HF} = h\omega_0 = h\frac{qB_\gamma}{\gamma m}$

Constant radius ⇒ variable B Feld

$$R = \frac{p}{qB} = const.$$



Ion: U^{73+}

E_{inj}=11.4 MeV/u

E_{max} = 1 GeV/u

BR=18 Tm (10 T/s)

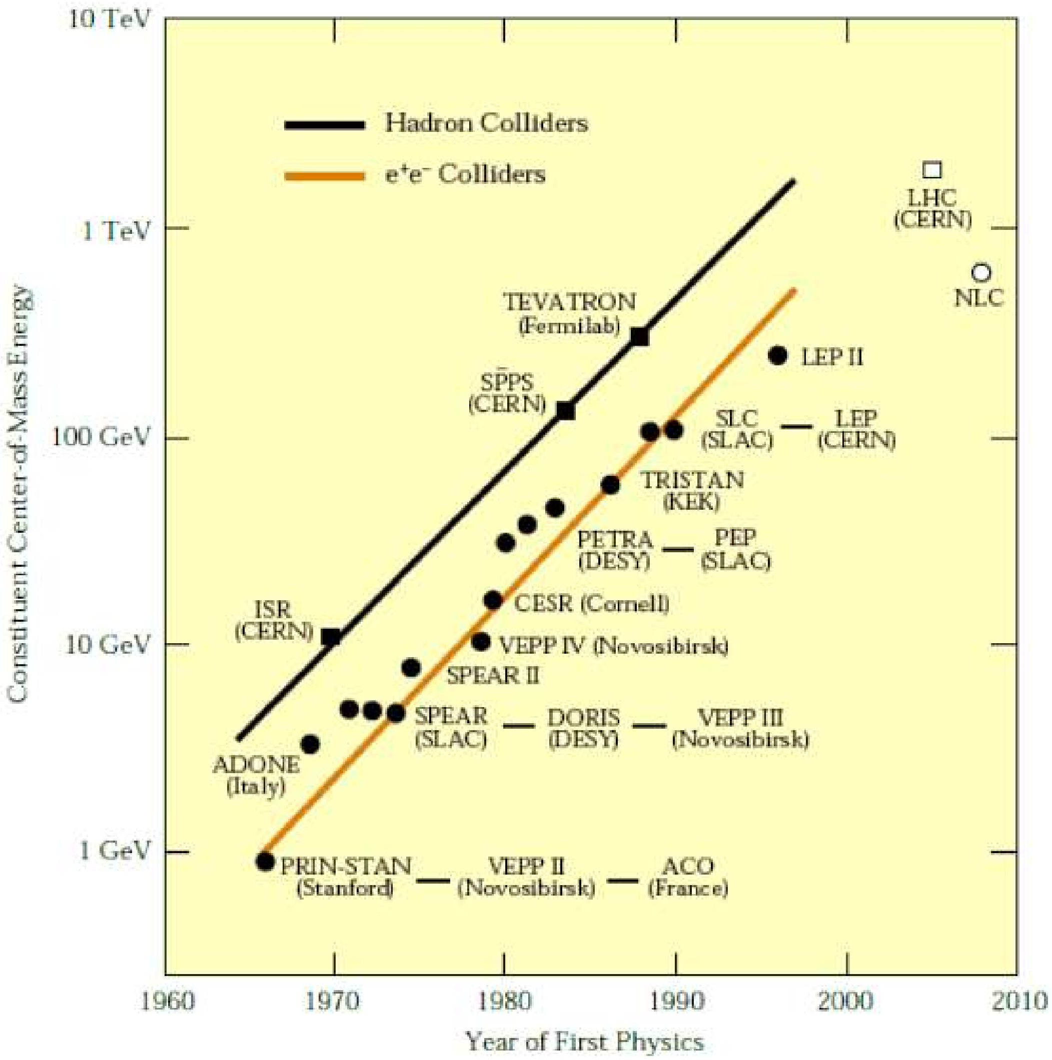
Cycle=1 Hz

Circumference: 216 m

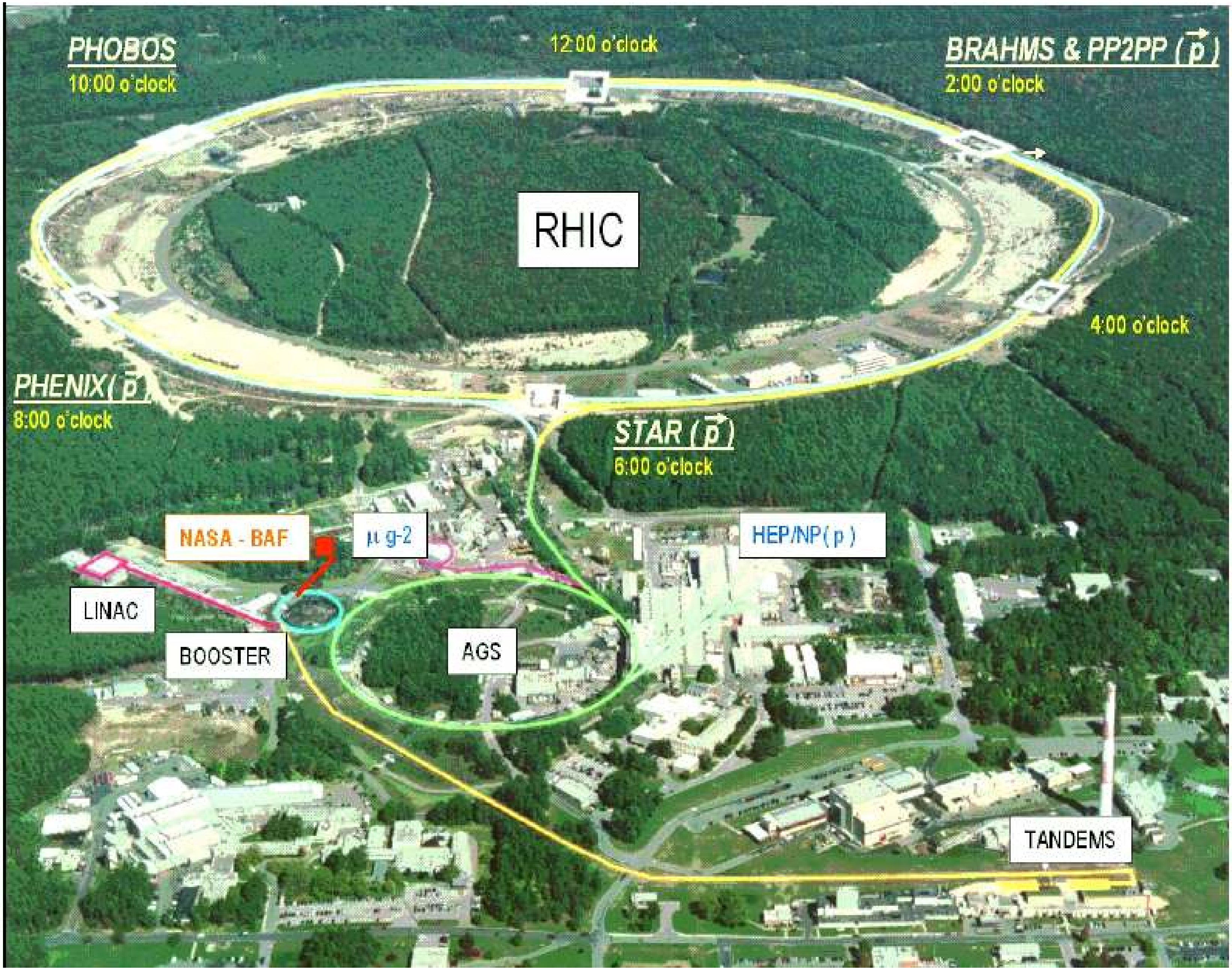
Synchrotron principle:
E.M. McMillan,
Uni. of California, 1945

Storage ring: no acceleration !

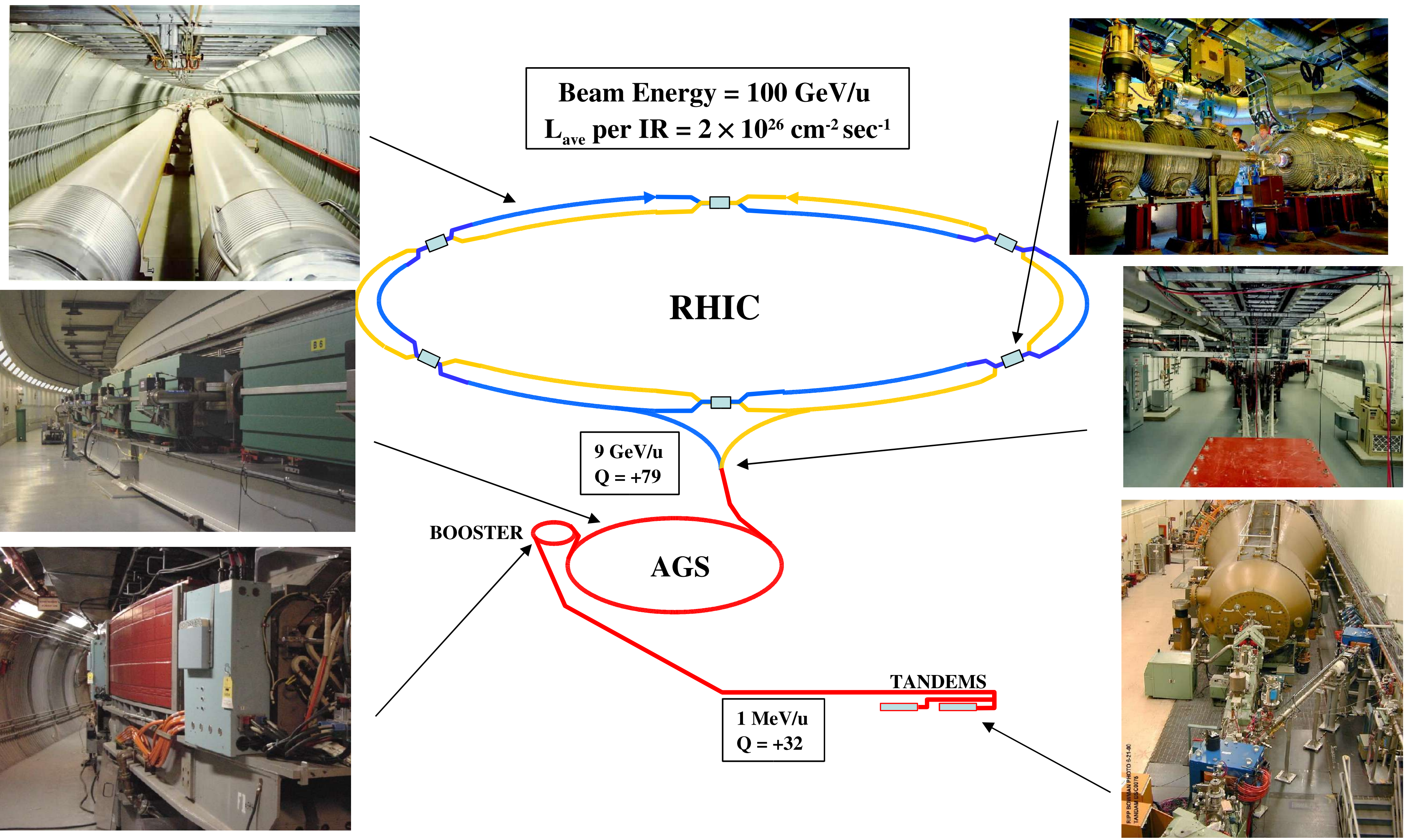
The „Livingston“ diagram



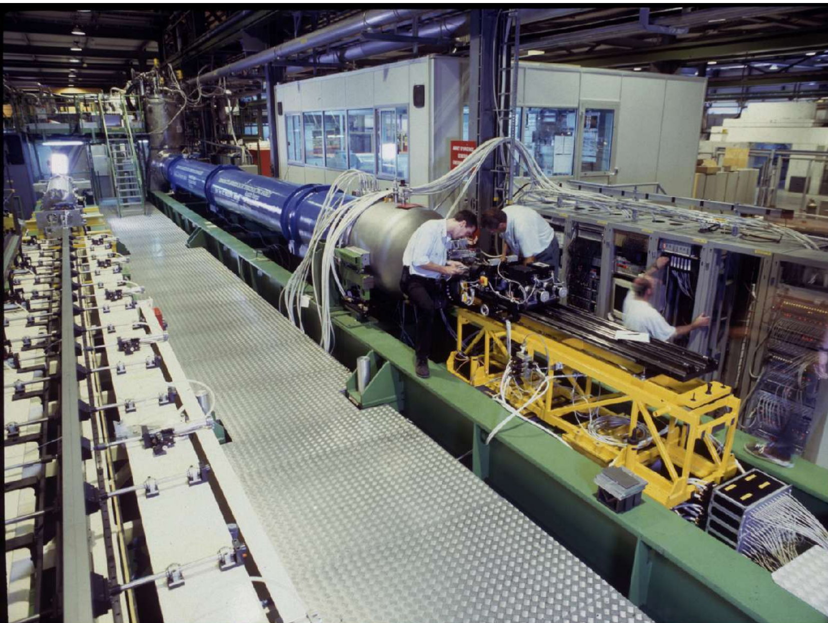
The RHIC accelerator facility and experiments



The RHIC accelerator



CERN Large Hadron Collider (LHC)
(under construction)



Protons anf heavy ions (Pb)

Energy: > 1 TeV

Protons in the ring: 3×10^{14}

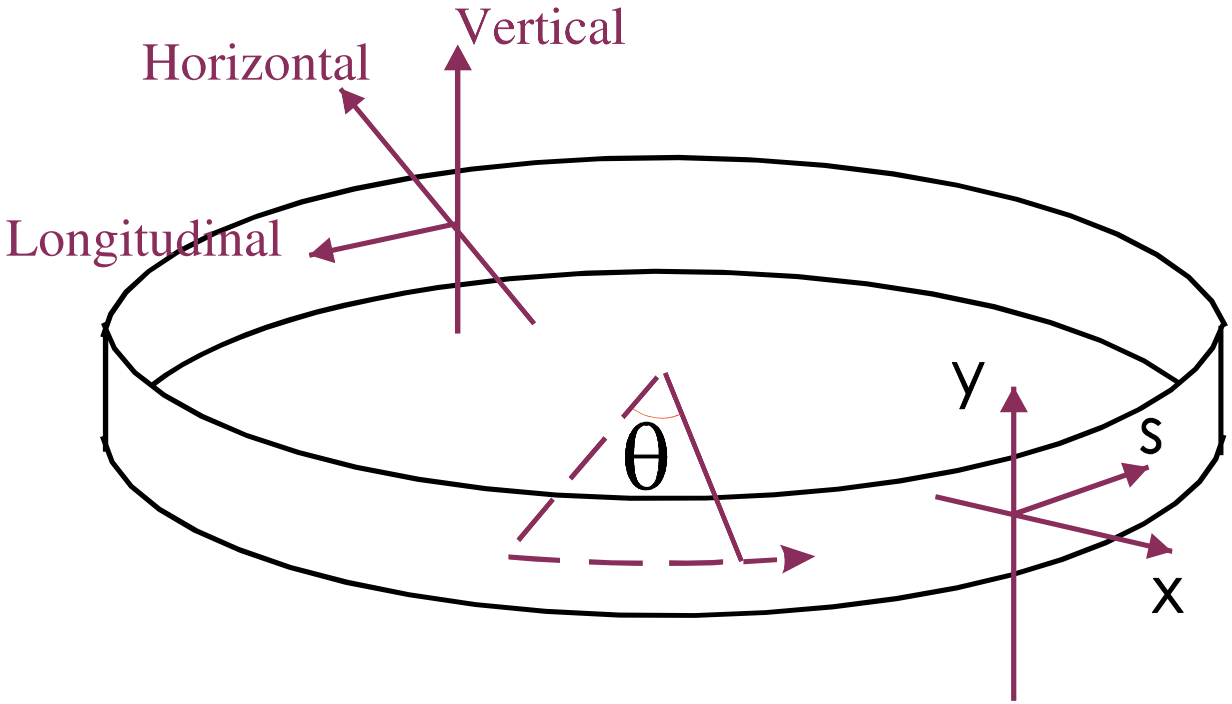
Current: 0.5 A

Total beam energy: 3 MJ

Magnetic bending field: 8 T

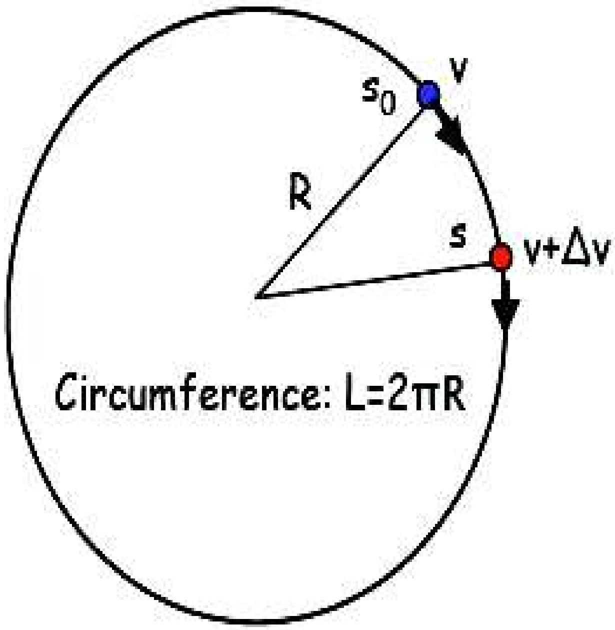
Circumference: 27 km !

coordinate system for accelerators



Synchrotron oscillations and phase stability

Circular accelerator



Revolution time (synchronous particle):

$$T = \frac{L}{v}$$

Revolution time (non-synchronous particle):

$$\frac{\Delta T}{T} = \frac{\Delta L}{L} - \frac{\Delta v}{v}$$

$$\frac{\Delta v}{v} = \frac{1}{\gamma^2}\frac{\Delta p}{p}$$

$$\Delta p = \gamma m \Delta v + m v \gamma' \Delta v$$
$$\gamma' = \frac{v}{c^2}\gamma^3$$
$$\beta^2 = 1 - \frac{1}{\gamma^2}$$

$$\frac{\Delta L}{L} = \frac{1}{\gamma_t^2}\frac{\Delta p}{p}$$ ('gamma transition')

$$\frac{\Delta T}{T} = \eta \frac{\Delta p}{p} \quad \text{with} \quad \eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

(frequency slip factor)

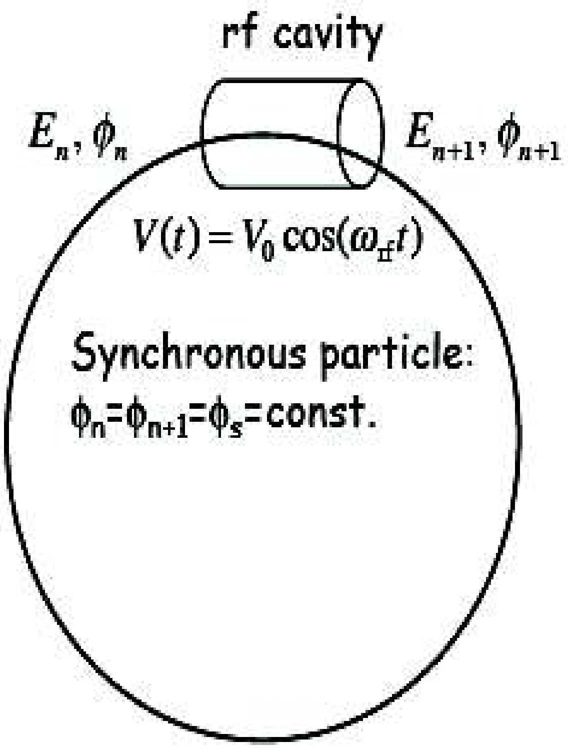
Linac: $R \to \infty, \quad \gamma_t \to \infty, \quad \eta \to -\frac{1}{\gamma^2}$

Phase dynamics

Non-synchronous particle:

$$\phi_{n+1} = \phi_n + \omega_{rf}\Delta T_{n+1} = \phi_n + \omega_{rf}T_{n+1}\left(\frac{\Delta T}{T}\right)_{n+1} = \phi_n + \eta\omega_{rf}T_{n+1}\left(\frac{\Delta p}{p}\right)_{n+1}.$$

$$\omega_{\rm rf}T_{n+1} \approx \omega_{\rm rf}T_0 = \text{const.} = 2\pi h$$ (harmonic number)



Substituting $\frac{\Delta p}{p} = \frac{c^2}{v^2}\frac{\Delta E}{E_s}$

$$\phi_{n+1} = \phi_n + \frac{2\pi h\eta c^2}{v^2}\left(\frac{\Delta E_{n+1}}{E_s}\right)$$

Energy change (synchronous particle):

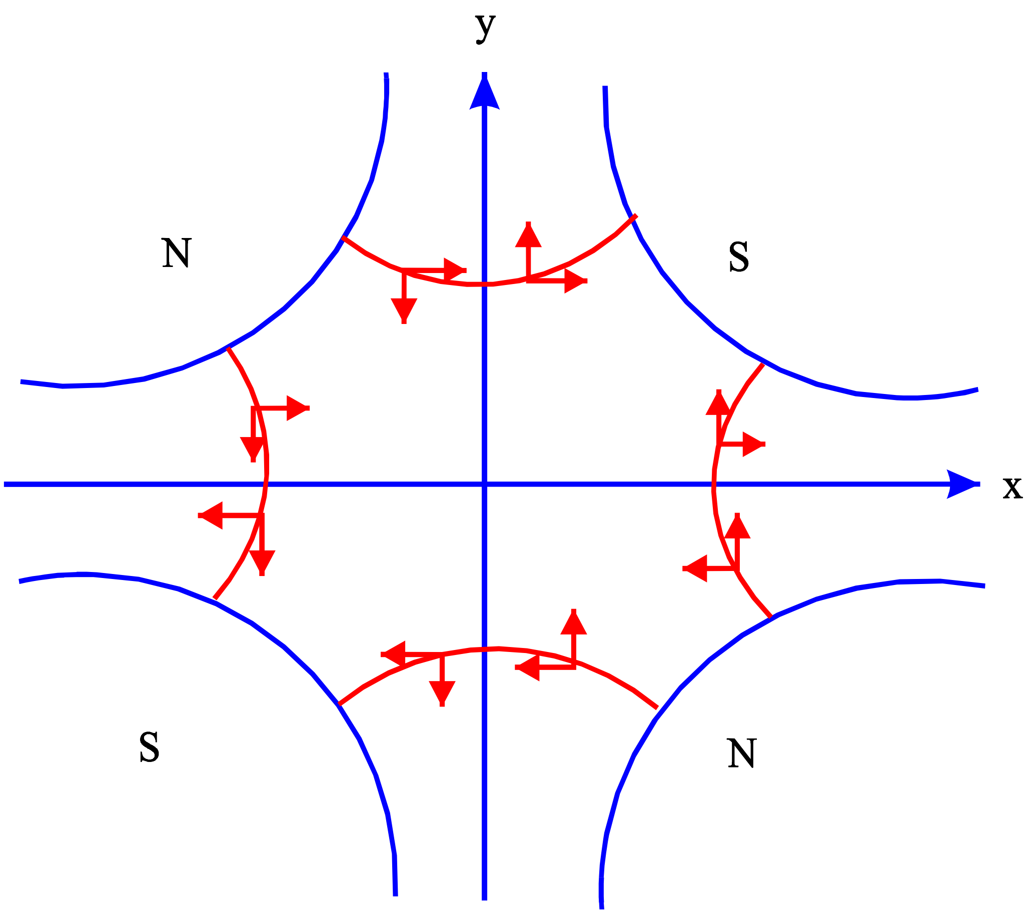
$$(E_s)_{n+1} = (E_s)_n + qV\sin\phi_s$$

Non-synchronous particle:

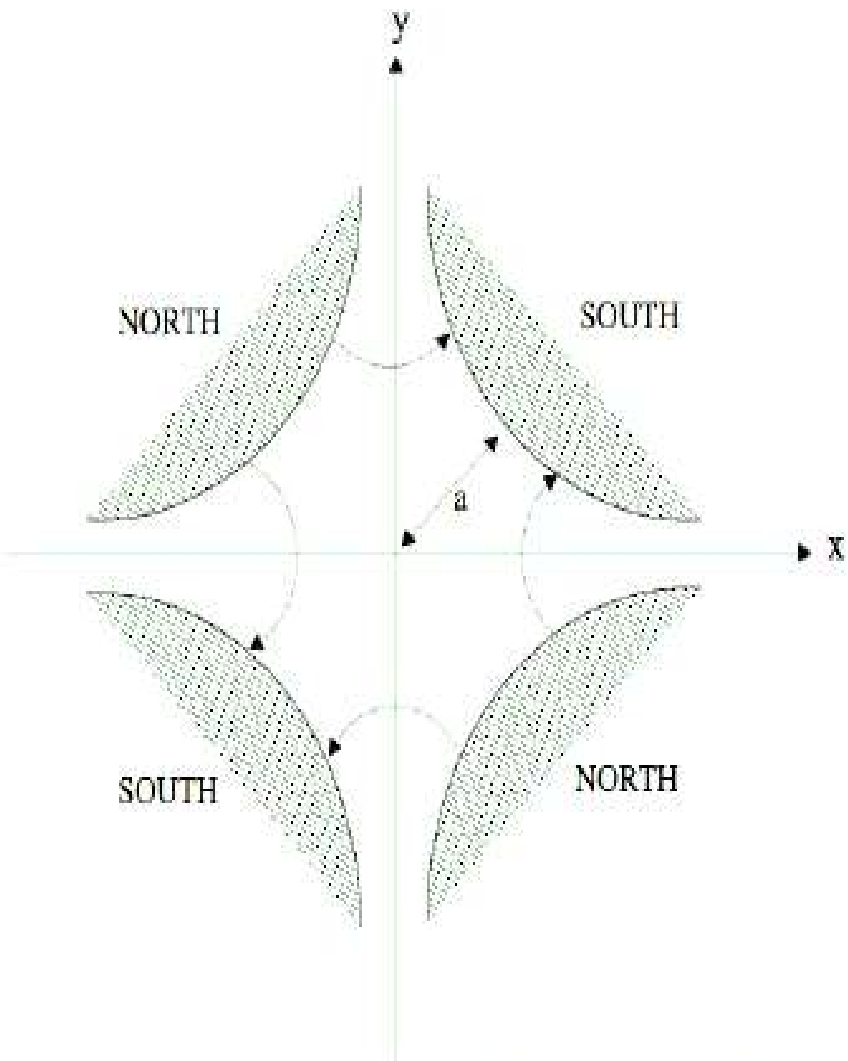
$$E_{n+1} = E_n + qV\sin\phi_n$$

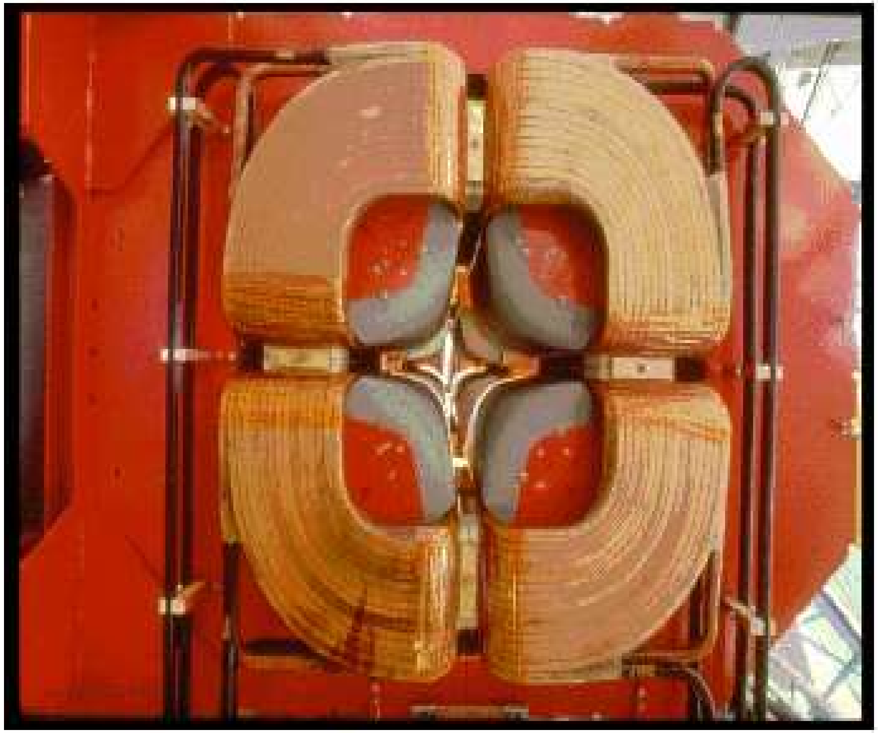
$$\Delta E_{n+1} = \Delta E_n + qV(\sin\phi_n - \sin\phi_s)$$

Quadrupole magnet



Transverse focusing with quadrupole magnets



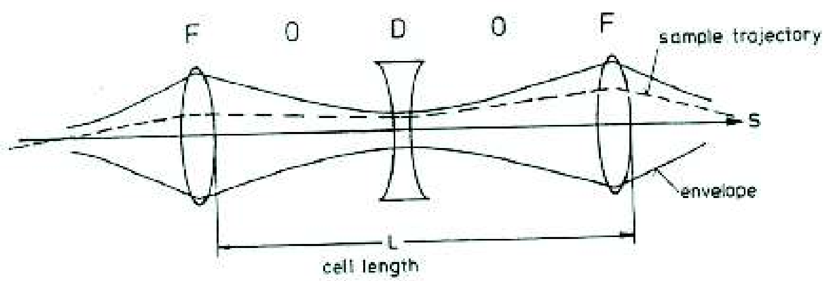


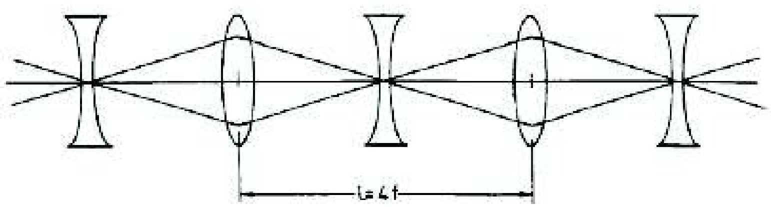
Quadrupole magnet in the ESR

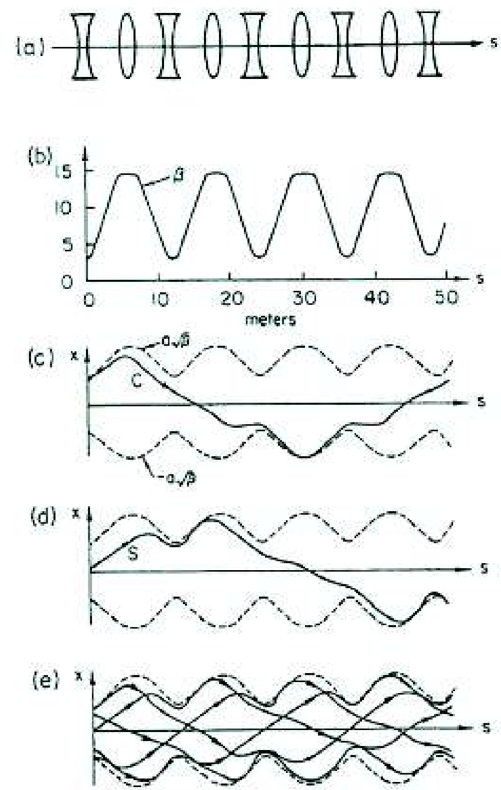
Magnetic field: $B_y = B_0 \dfrac{x}{a}, \quad B_x = B_0 \dfrac{y}{a}$

Force: $F_x = -qv_0 B_y = -qv_0 B_0 \dfrac{x}{a}, \quad F_y = qv_0 B_x = qv_0 B_0 \dfrac{y}{a}$

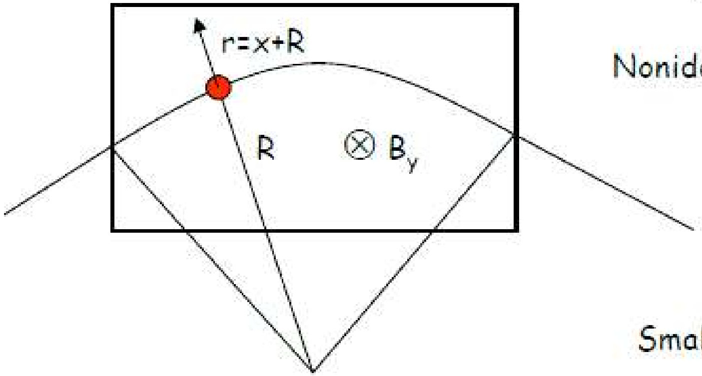
Net action of quadrupole doublet: focussing







Deflection (dipole) magnet



Ideal particle: $R = \frac{p_0}{qB_y}$

Nonideal particle: $\gamma m \ddot{r} = -q v_s B_y$

$$\ddot{r} \approx \ddot{x} - \frac{v^2}{r}$$

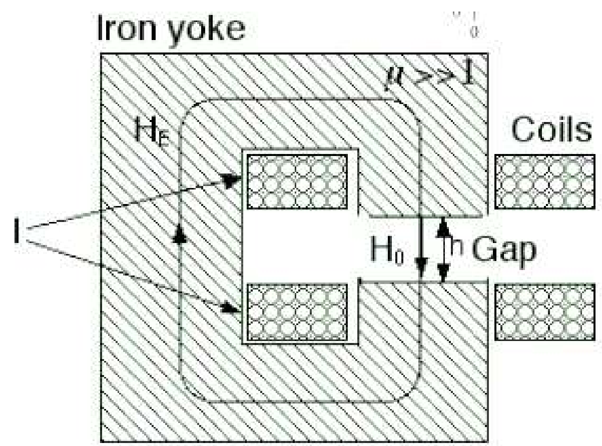
$$\Rightarrow x'' = \frac{1}{r} - \frac{qB_y}{p}$$

Small deviations from the ideal orbit:

$$\frac{1}{r} \approx \frac{1}{R}\left(1 - \frac{x}{R}\right) \qquad \frac{1}{p} \approx \frac{1}{p_0}\left(1 - \frac{\Delta p}{p_0}\right)$$

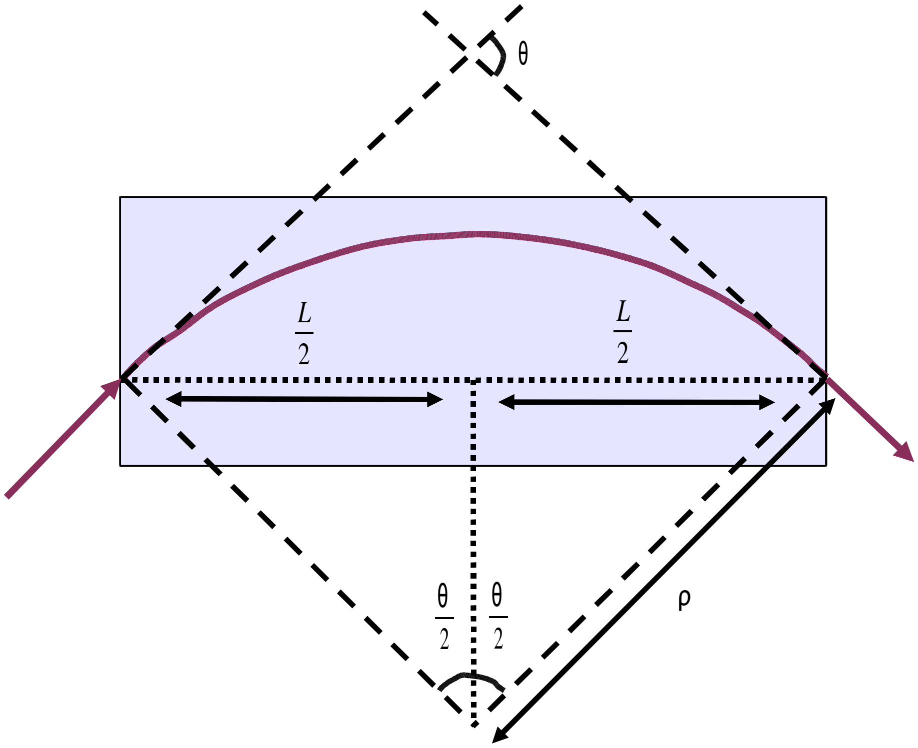
$$x'' + \frac{1}{R^2}x = \frac{1}{R}\frac{\Delta p}{p_0}$$

'weak' focusing — inhomogeneous part



Deflection in a dipole magnet with uniform field B

$$\sin\left(\frac{\theta}{2}\right)=\frac{L}{2\rho}=\frac{1}{2}\frac{LB}{(B\rho)}$$



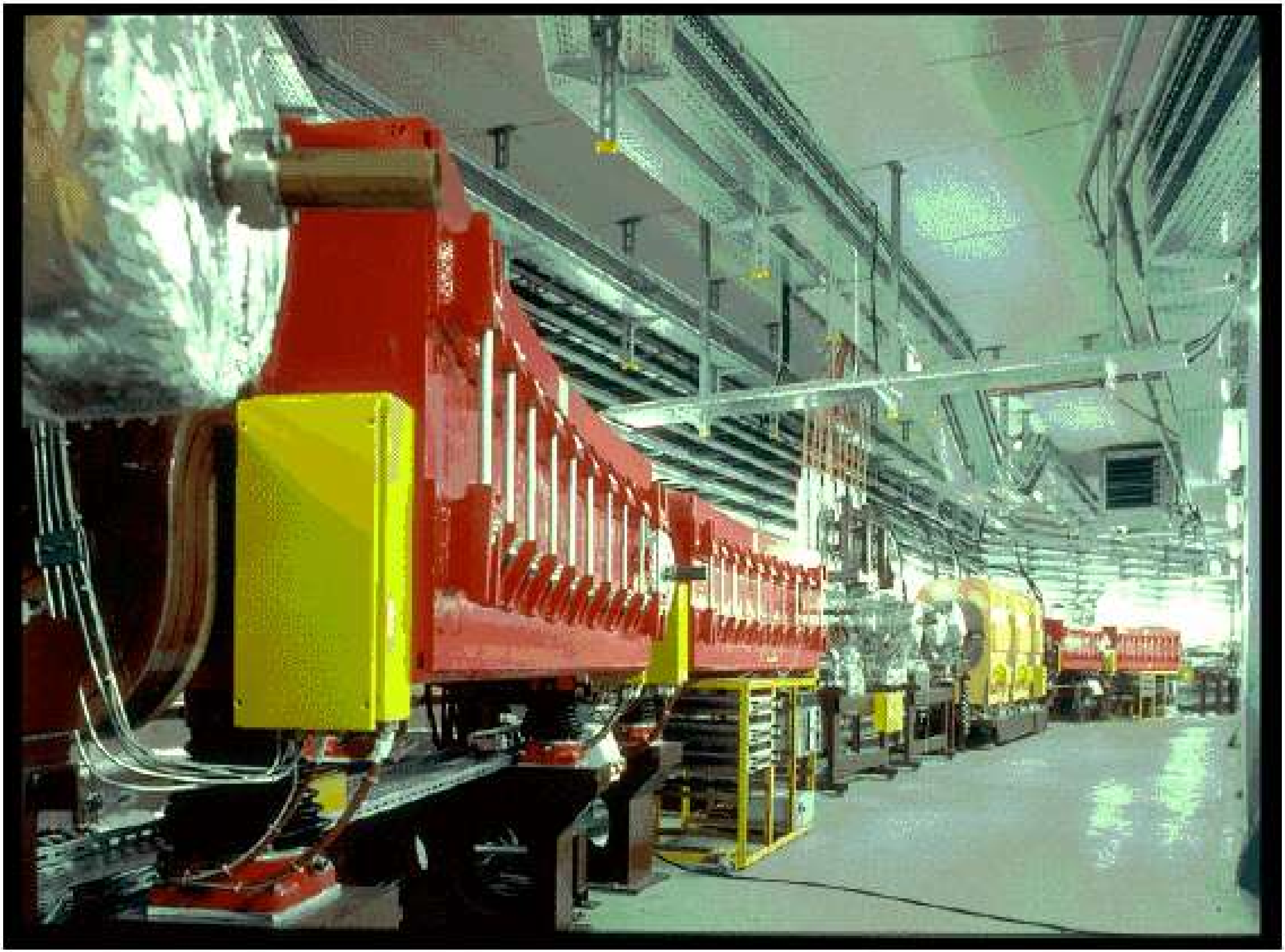
Magnetic rigidity

$$F = evB = \frac{mv^2}{\rho}$$

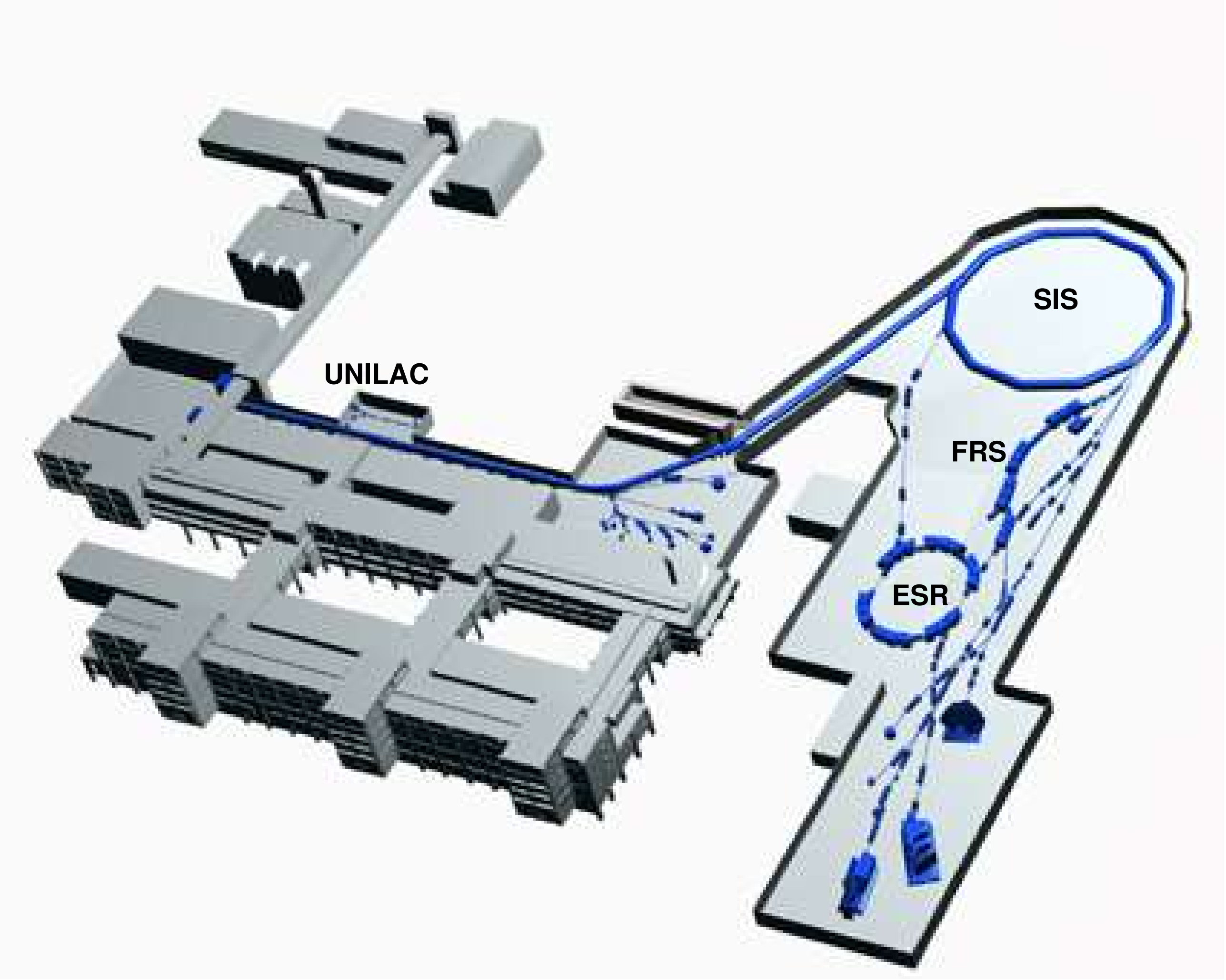
$$B\rho = \frac{mv}{e} = \frac{p}{e}$$

B = 33.356·p [KG·m] = 3.3356·p [T·m] (if p is in [GeV])

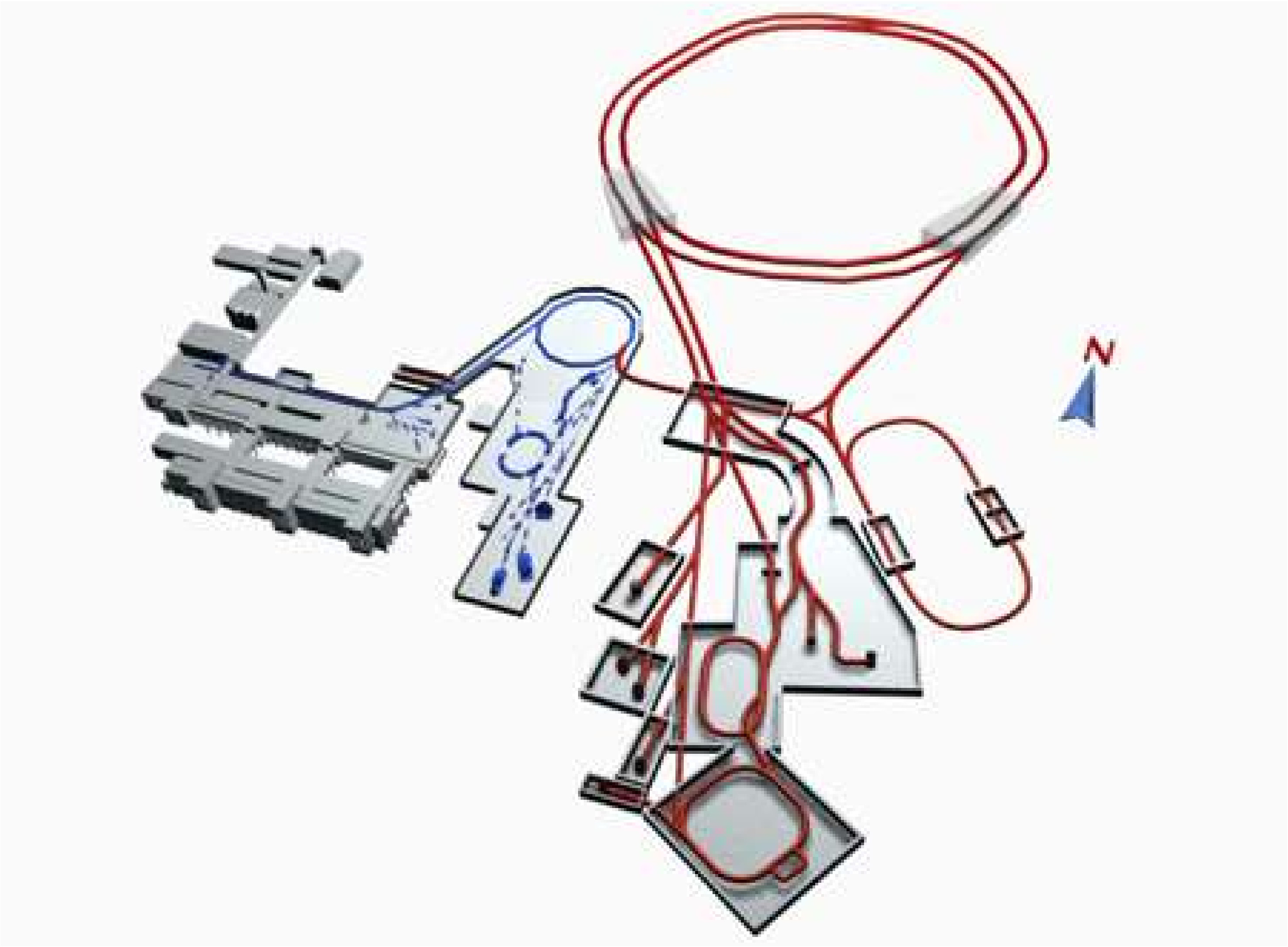
View into SIS18 at GSI



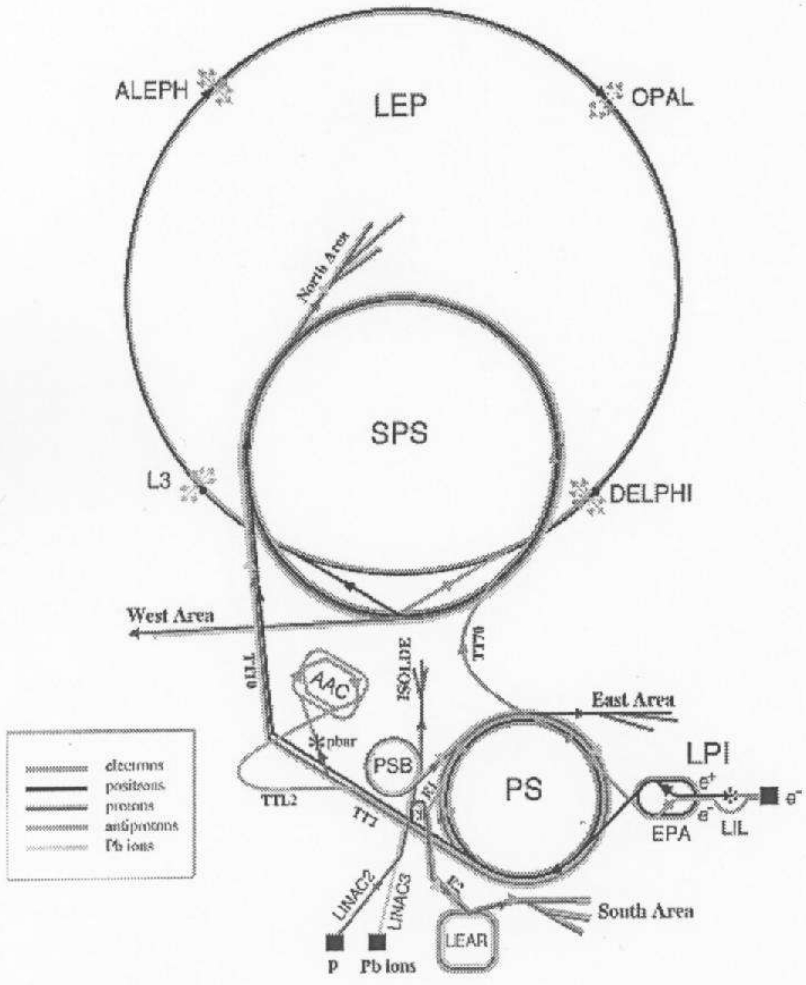
GSI now



GSI future   - the FAIR facility



CERN Accelerators



LEP: Large Electron Positron collider
SPS: Super Proton Synchrotron
AAC: Antiproton Accumulator Complex
ISOLDE: Isotope Separator OnLine DEvice

LPI: Lep Pre-Injector
EPA: Electron Positron Accumulator
LIL: Lep Injector Linac
LINAC2: LINear ACcelerator 2