Relativistic kinematics and phenomenology of particle production

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Phenomenology of Collisions at High Energy

from now on, use units of $\hbar = c = 1$

relativistic Kinematics

$E^2 = \vec{p}^{\,2} + m^2$, 4 vector $p = (E, \vec{p})$

$p^2 = m^2$ velocity $\vec{\beta} = \frac{\vec{p}}{E}$, $\gamma^2 = \frac{1}{1-\beta^2}$

viewed from a frame moving with velocity $\vec{\beta}_f$, we get

$$\begin{pmatrix} E^* \\ p_\parallel^* \end{pmatrix} = \begin{pmatrix} \gamma_f & -\gamma_f\beta_f \\ -\gamma_f\beta_f & \gamma_f \end{pmatrix} \begin{pmatrix} E \\ p_\parallel \end{pmatrix} \qquad p_\perp^* = p_\perp$$

$P_{\parallel(\perp)}$ is component of $\vec{P}$ parallel or perpendicular to $\vec{\beta}_f$

center of mass energy

$$E_{cm} = \sqrt{(P_1 + P_2)^2} = \sqrt{(E_1 + E_2)^2 - (\vec{P}_1 + \vec{P}_2)^2}$$

$$= \sqrt{m_1^2 + m_2^2 + 2E_1E_2(1 - \beta_1\beta_2\cos\theta_{12})}$$

in a frame with particle 2 at rest ($\vec{p}_2 = 0$,

$$E_{cm} = \sqrt{m_1^2 + m_2^2 + 2E_1^{lab} \cdot m_2}$$

velocity of center of mass in lab frame

$$\vec{\beta}_{cm} = \frac{\vec{P}_1^{lab}}{E_1^{lab} + m_2} \qquad \gamma_{cm} = \frac{E_1^{lab} + m_2}{E_{cm}}$$

For nucleus - nucleus collisions

$m_1 = A_1 \cdot m_u$ $\qquad m_u \cong 0.931\ \text{GeV}$ = atomic mass unit

$\Rightarrow$ E_1/A_1 : total energy / nucleon ...

example : in a frame with particle 2 at rest

E_{cm} for $A_1 + A_2$ collision :

$$E_{cm} = m_u \sqrt{A_1^2 + A_2^2 + 2 \frac{E_1^{lab}}{m_1} A_1 A_2}$$

$\Rightarrow$

$$\frac{E_{cm}}{m_u} = \sqrt{A_1^2 + A_2^2 + 2 \frac{E_1^{lab}}{m_u} \frac{A_1 A_2}{A_1}}$$

for $A_1 = A_2 = A$

$$\frac{E_{cm}}{A} = m_u \sqrt{2 + 2\gamma_1^{lab}}$$

Sometimes we need the energy available for particle production

$$E^* = E_{cm} - m_u (A_1 + A_2)$$

for $A_1 = A_2$

$$\frac{E^*}{A} = m_u \left[\sqrt{2 + 2\gamma_1^{lab}} - 2 \right]$$

the connection between β and γ

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$ suggest introducing

a new parameter y such that

$$\beta = \tanh y \quad \Rightarrow$$

$$\gamma = \cosh y \qquad \beta\gamma = \sinh y$$

and the Lorentz transformation reads

$$\begin{pmatrix} E^* \\ p_\parallel^* \end{pmatrix} = \begin{pmatrix} \cosh y_f & -\sinh y_f \\ -\sinh y_f & \cosh y_f \end{pmatrix} \begin{pmatrix} E \\ p_\parallel \end{pmatrix}$$

let us pick some direction, usually the beam direction

energy of a particle $E = \sqrt{p_{\parallel}^2 + p_{\perp}^2 + m^2}$

$m_{\perp} = \sqrt{m^2 + p_{\perp}^2}$: transverse mass

then the energy and momentum of the particle are obtained from $(m_{\perp}, 0)$ $(\beta = -\beta_f')$ as

$$\begin{pmatrix} E \\ p_{\parallel} \end{pmatrix} = \begin{pmatrix} \cosh y & \sinh y \\ \sinh y & \cosh y \end{pmatrix} \begin{pmatrix} m_{\perp} \\ 0 \end{pmatrix}$$

or $\quad E = m_\perp \cdot \cosh y \qquad p_{\parallel} = m_\perp \cdot \sinh y$

from this we get $\quad y = \tanh^{-1} \dfrac{p_{\parallel}}{E}$

$$= \ln \frac{E + p_{\parallel}}{m_\perp} = \frac{1}{2} \ln \frac{E + p_{\parallel}}{E - p_{\parallel}}$$

(remember $\quad \sinh^{-1} x = \ln\left[x + \sqrt{x^2+1}\right]$

$\cosh^{-1} x = \ln\left[x + \sqrt{x^2-1}\right] \qquad x \geq 1$

$\tanh^{-1} x = \frac{1}{2} \ln \dfrac{1+x}{1-x} \qquad 0 \leq x^2 < 1$)

under boost in z-direction with velocity β

$$y_2 = y_1 + \tanh^{-1}\beta$$

because

$$\tanh^{-1} z_1 + \tanh^{-1} z_2 = \tanh^{-1}\frac{z_1+z_2}{1+z_1 z_2}$$

and

$$\beta_2 = \frac{\beta_1+\beta}{1+\beta_1\beta}$$ is the general

addition theorem for relativistic velocities

Since $\tan\theta = \frac{p_\perp}{p_\parallel}$ $\Rightarrow$

$$y = \frac{1}{2} \ln \frac{\cos^2\theta/_2 + m^2/_{4p^2} + \cdots}{\sin^2\theta/_2 + m^2/_{4p^2} + \cdots} \cong -\ln \tan \theta/_2$$

$$= \frac{1}{2} \ln \frac{p + p_\parallel}{p - p_\parallel} \equiv \eta$$ with

η : Pseudorapidity

η only depends on angle (can be easily measured)

Simple identities:

$$\sinh\eta = \cot\theta$$
$$\cosh\eta = 1/\sin\theta$$
$$\tanh\eta = \cos\theta$$
$$\frac{d\eta}{d\theta} = \frac{1}{\sin\theta}$$

Cross sections

define invariant cross section

$E \cdot \frac{d^3\sigma}{dp^3}$ since $p = \sqrt{p_{\parallel}^2 + p_{\perp}^2}$

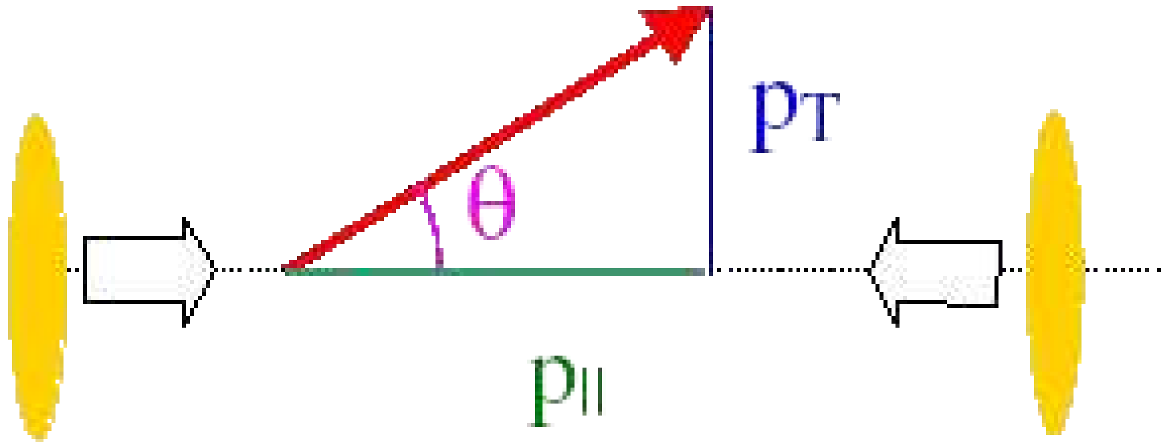
$\Rightarrow dp^3 = \pi \, dp_{\perp}^2 \, dp_{\parallel} = 2\pi \, p_{\perp} \, dp_{\perp} \, dp_{\parallel}$

from $p_{\parallel} = m_{\perp} \cdot \sinh y \Rightarrow$

$dp_{\parallel} = m_{\perp} \cdot \cosh y \, dy = E \, dy$

$\Rightarrow E \frac{d^3\sigma}{dp^3} = \frac{1}{2\pi p_{\perp}} \frac{d^2\sigma}{dp_{\perp} dy}$

Kinematics for colliders



Rapidity: $y = \frac{1}{2} \ln\left(\frac{E + p_{||}}{E - p_{||}}\right)$

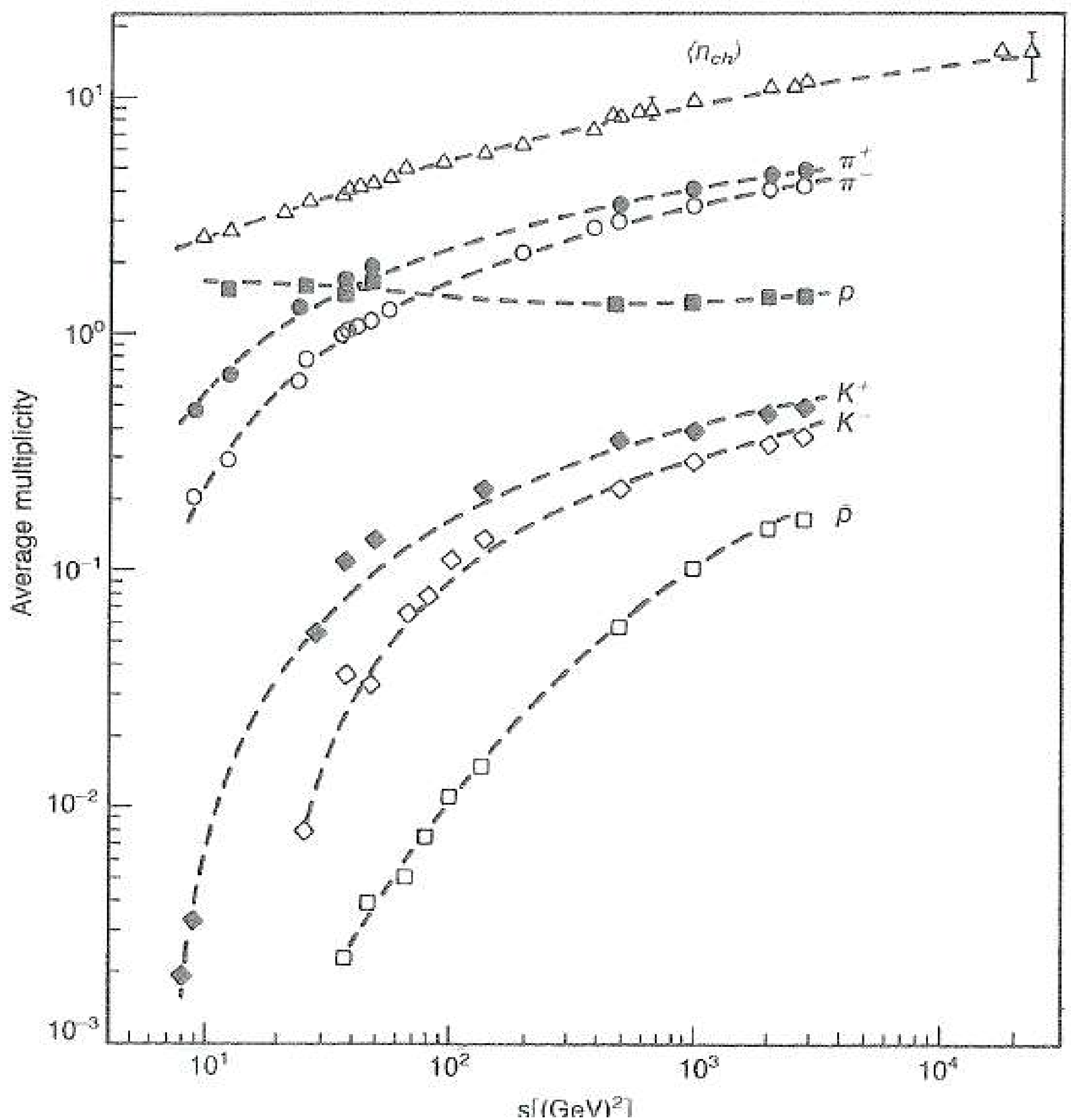
Invariant cross section: $E\frac{d^3\sigma}{d^3p} = \frac{d^2\sigma}{2\pi p_T\, dy\, dp_T}$

Pseudo-rapidity: $\boxed{y \rightarrow \eta = -\ln\left[\tan\left(\theta/2\right)\right]}$ for m/p << 1

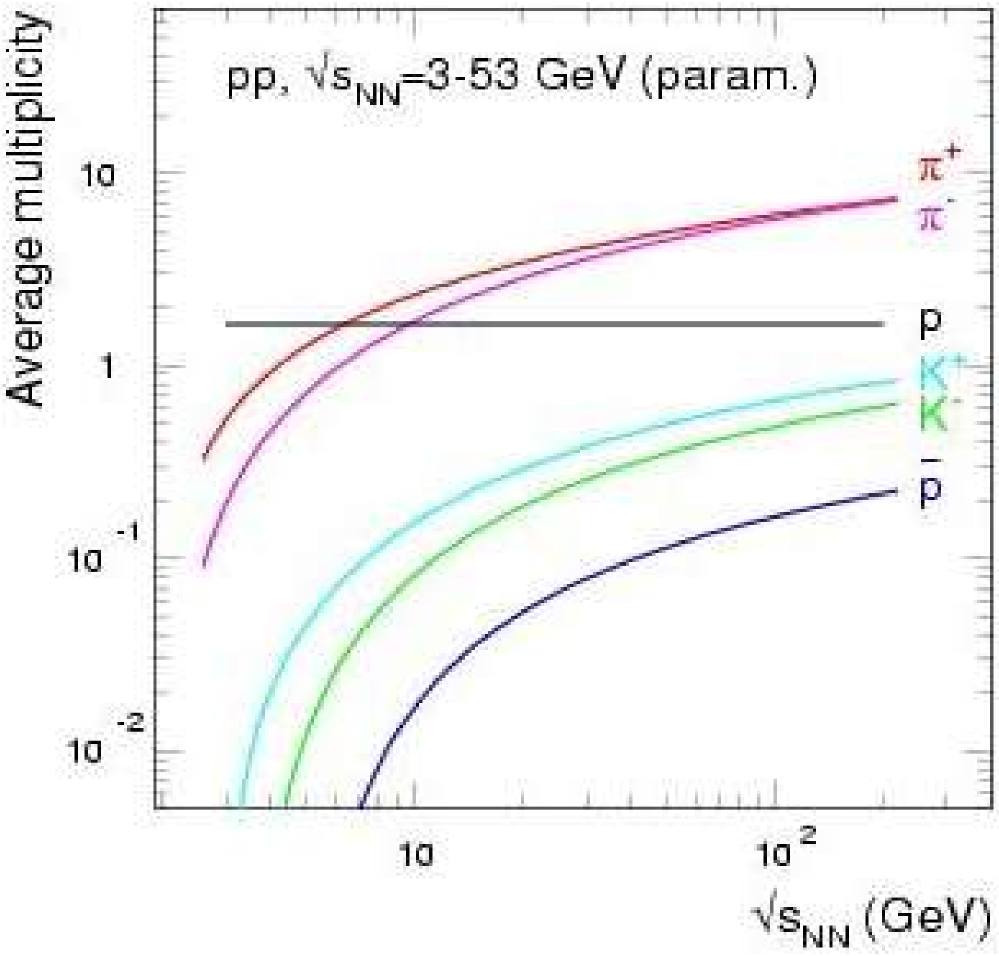
Transverse momentum (p_T) and **(pseudo-)rapidity (*y* or η)** provide a convenient description

Particle production in proton-proton collisions

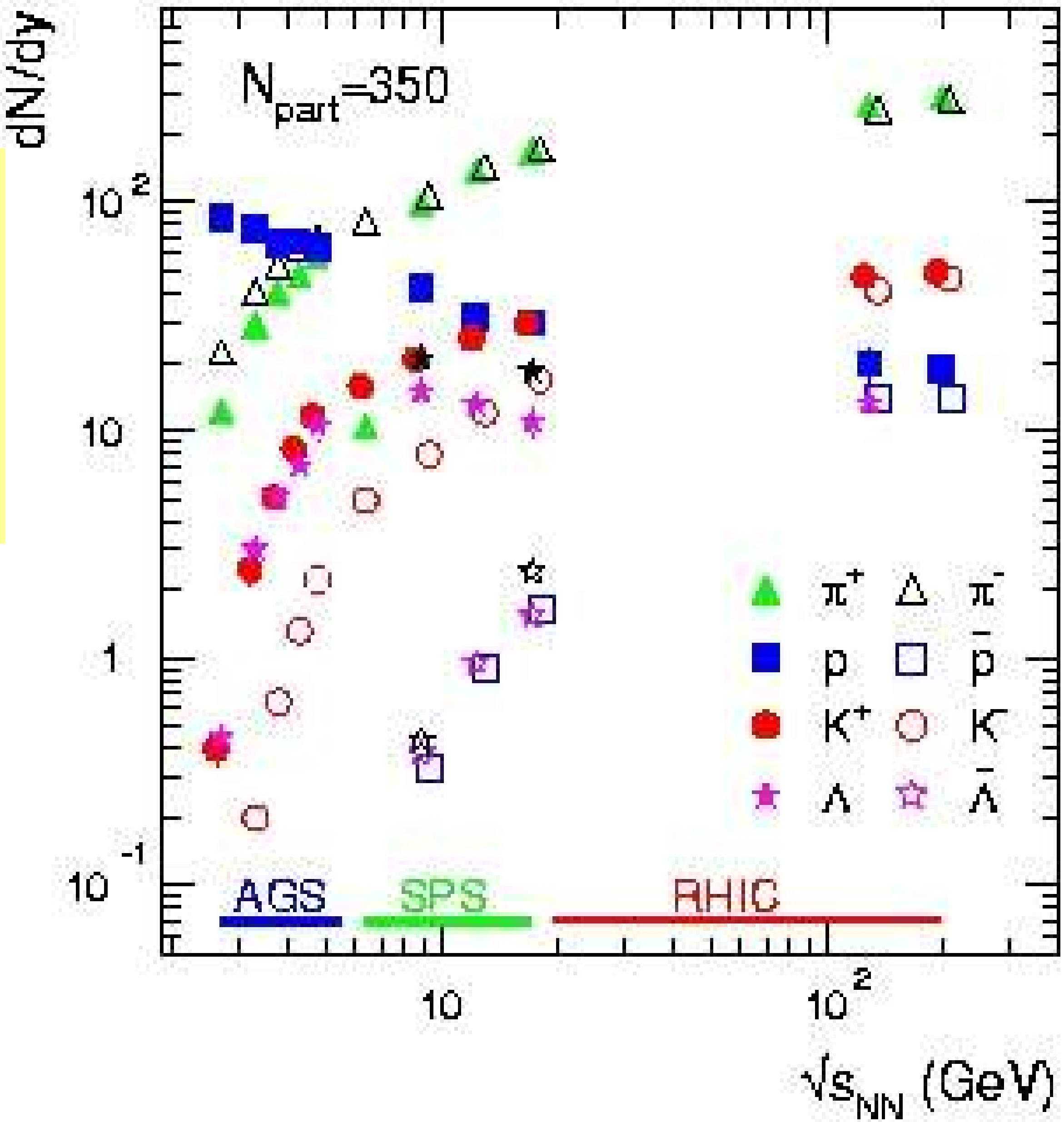
Particle multiplicity in proton-proton collisions



Particle multiplicity in proton-proton collisions



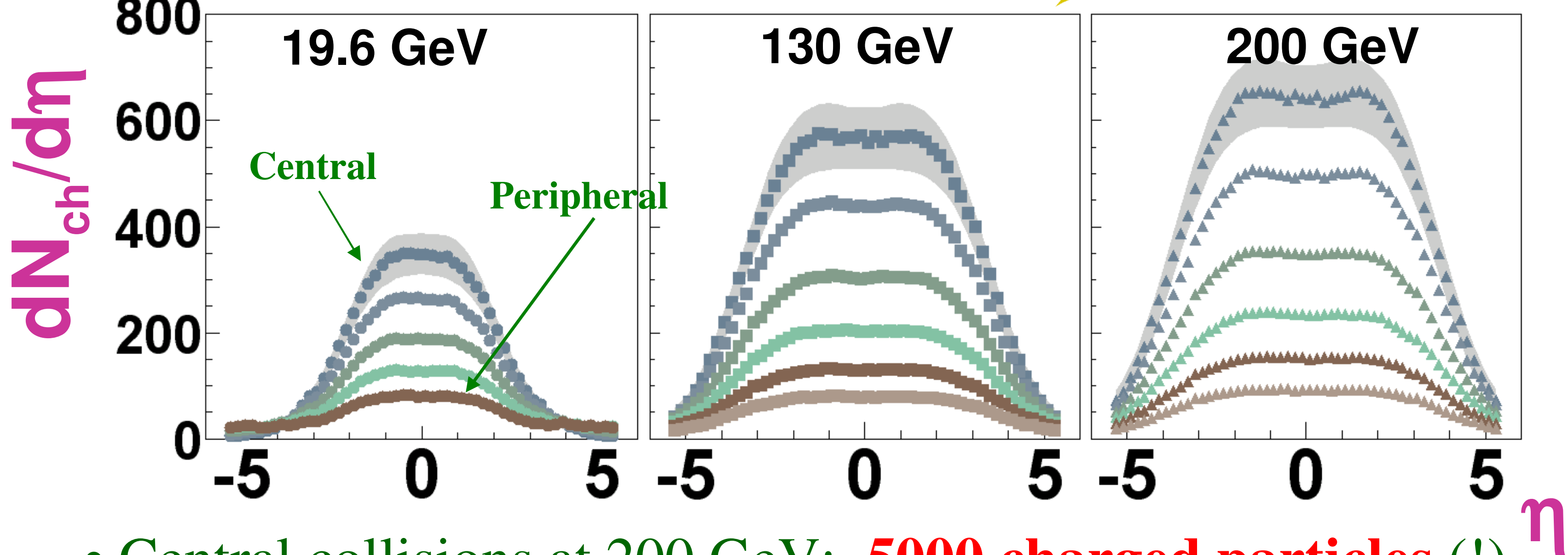
Particle production in central nucleus-nucleus collisions



rapidity density
dN/dy

Bulk particle production in Au+Au

PRL 91, 052303



- Central collisions at 200 GeV: **5000 charged particles** (!)
- mid-rapidity: ~ boost invariance
- Energy density: boost invariant hydrodynamics (Bjorken)

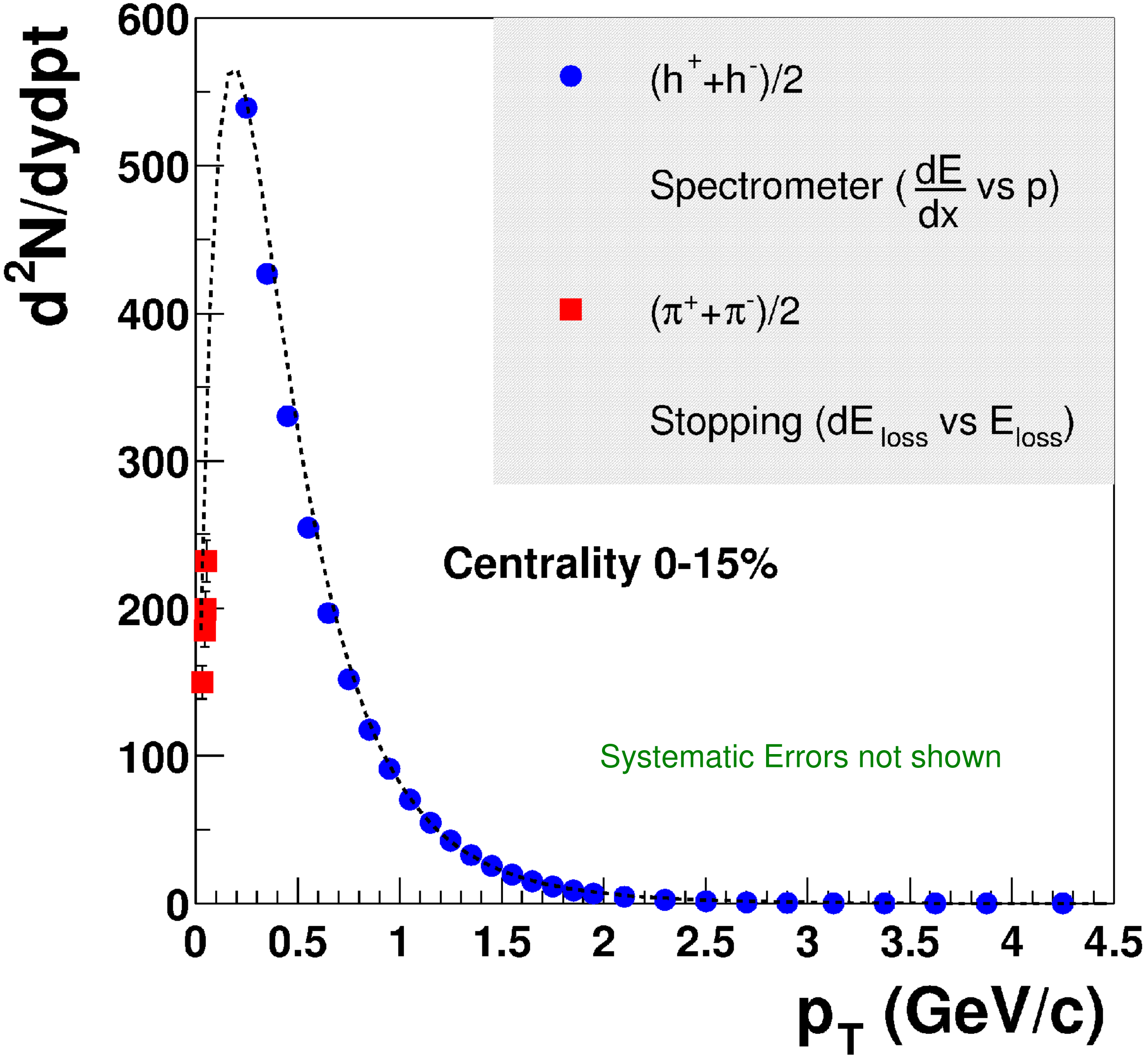
$$\epsilon = \frac{1}{\pi R^2 \tau}\frac{dE_T}{dy} \approx \frac{1}{\pi R^2 \tau}\langle p_T \rangle \frac{3}{2}\frac{dN_{ch}}{d\eta} \qquad (R \sim A^{1/3},\ \tau = 1\ \text{fm/c})$$

Central Au+Au at 130 GeV: **$\varepsilon = 4.6$ GeV/fm³**

(PHENIX, PRL 87, 052301)

p_T distribution of charged particles

nucl-ex/0401006



Experimental determination of centrality