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Experimental test of special relativity by laser spectroscopy

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Abstract The Doppler-free laser-spectroscopic frequency measurement of Doppler-shifted optical lines in forward and backward direction of a fast ion beam permits a sensitive test of the relativistic Doppler-formula and, hence, the relativistic time dilation factor $\gamma_{\mathrm{SR}} = \left(1 - v^2/c^2\right)^{-1/2}$. An experiment on metastable $^7\mathrm{Li}^+$, stored at a velocity of v= 0.064c in the Heidelberg heavy-ion storage ring TSR, has confirmed time dilation with unprecedented accuracy. Latest tests at two different ion-velocities (v=0.03c and v=0.064c) will enhance these measurements. An improved version of this experiment will be carried out at the experimental storage ring (ESR) at the Gesellschaft für Schwerionenforschung (GSI) in Darmstadt. The ESR permits $^7\mathrm{Li}^+$ to be stored at v=0.33c which promises an improvement of the sensitivity to deviations from γ_{SR} by an order of magnitude. A first test at the ESR has shown the feasibility for this kind of experiment.

Key words precision spectroscopy · test of Lorentz invariance · storage ring · generation of metastabile lithium ions

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1 Lorentz invariance and time dilation

Lorentz invariance is a basic principle underlying all currently accepted theories describing the fundamental interactions of nature – the Quantum Field Theories of the strong and electroweak forces as well as General Relativity. This is a strong motivation to test the theory of Special Relativity (SR) experimentally with ever higher precision, as any deviation from its predictions would have profound consequences for our understanding of nature. In particular, many unifying theories, such as string theory, allow for violations of Lorentz invariance at some level [1]. While kinematic test theories describe possible violations of the observer Lorentz transformation (e.g., the rest frequency of an atomic transition is assumed to be the same in all inertial frames, while the transformation between the frames is tested), dynamical test theories like the Standard-Model Extension [1] deal with possible breakdowns of the particle Lorentz transformation. Time dilation experiments are sensitive to several parameters of kinematic as well as dynamic test theories [2, 3].

In order to quantify deviations from the relativity principle, Robertson developed a kinematical test theory [4] that was later modified by Mansouri and Sexl [5]. They consider generalized Lorentz transformations between a hypothetical preferred frame $\Sigma(T, \boldsymbol{X})$ and a frame $S(t, \mathbf{x})$ moving relative to Σ at a velocity V along the X axis, and the speed of light is assumed to be isotropic in Σ only. Using Einstein synchronization, these transformations read

$$T = \Gamma\left(\frac{t}{\widehat{a}} + \frac{Vx}{\widehat{b}c_0^2}\right); X = \Gamma\left(\frac{x}{\widehat{b}} + \frac{Vt}{\widehat{a}}\right); Y = \frac{y}{\widehat{d}}; Z = \frac{z}{\widehat{d}} \tag{1}$$

with $\Gamma = \left(1 - V^2/c_0^2\right)^{-\frac{1}{2}}$ and c_0 being the speed of light in Σ. Note that due to the abolition of the relativity principle these transformations are in general not valid between two arbitrary, constantly moving reference frames but only with respect to Σ. This model contains three velocity-dependent test functions $\widehat{a}(V^2)$, $\widehat{b}(V^2)$, and $\widehat{d}(V^2)$, which modify time dilation as well as Lorentz contraction in longitudinal and transverse direction. They reduce to $\widehat{a}(V^2) = \widehat{b}(V^2) = \widehat{d}(V^2) = 1$ in the case SR holds. In the low-velocity limit, these functions can be expanded in powers of V^2/c^2, i.e., $\widehat{a}(V^2) = \left[1 + \alpha V^2/c_0^2 + O\left(c_0^{-4}\right)\right]$, $\widehat{b}(V^2) = \left[1 + \beta^* V^2/c_0^2 + O\left(c_0^{-4}\right)\right]$ and $\widehat{d}(V^2) = \left[1 + \delta V^2/c_0^2 + O\left(c_0^{-4}\right)\right]$. In this regime one is therefore left with three test parameters α, β^* *and* δ.

In this framework, the speed of light $c(\theta, V)$ in the moving frame S,

$$\frac{c(\theta, V)}{c_0} = 1 + (\beta^* - \delta)\frac{V^2}{c_0}\sin^2(\theta) + (\alpha - \beta^*)\frac{V^2}{c_0} \tag{2}$$

is in general not constant, but is dependent on the angle θ between the direction of $c(\theta, V)$ and the motion of the moving frame S as well as on the velocity V between Σ and S. Michelson–Morley type experiments are sensitive to an anisotropy of the speed of light thus determining the parameter combination $|\beta^* - \delta|$ and so called Kennedy–Thorndike tests the velocity-dependence of c described by $|\alpha - \beta^*|$. Ives–Stilwell type experiments independently measure the parameter α that describes time dilation.

No deviation from SR has been found to date in any experiment. The most sensitive interferometric experiments have yielded limits of $|\beta^* - \delta| < (0.5 \pm 3 \pm 0.7) \times 10^{-10}$ [6] and $< (-0.9 \pm 2.0) \times 10^{-9}$ [7], respectively, for an anisotropy and of $|\alpha - \beta^*| < (1.6 \pm 3.0) \times 10^{-7}$ [8] for a velocity-dependence of c. The best limit on deviations from time dilation prior to the storage ring experiment was $|\alpha| < 1.4 \times 10^{-6}$ [9].

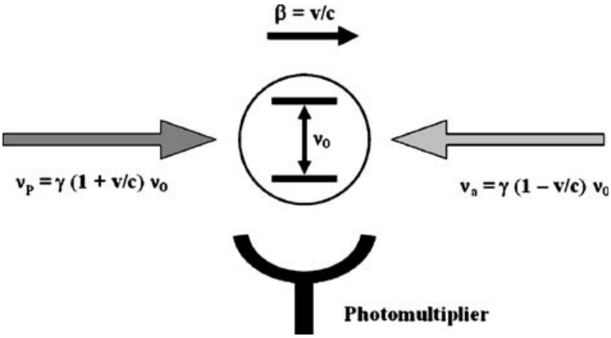


Fig. 1 Principle of Ives–Stilwell experiments with spectroscopy of fast ions

2 Ives and Stilwell experiments

Time dilation can be measured using the optical Doppler Effect, for example via parallel and anti-parallel excitation of a clock moving at a velocity $v=\beta c$, see Fig. 1.

In SR the Doppler-shifted frequencies are given by the relativistic Doppler-formula $\nu_{p,a} = \gamma_{\mathrm{SR}}(1 \pm \beta)\nu_0$, where the Lorentz factor γ_{SR} appears as a direct consequence of time dilation. Multiplication of these equations yields the β-independent relation $\nu_a \nu_p = \nu_0^2$ if SR holds. The Mansouri–Sexl test theory parameterizes possible deviations of time dilation from SR with the test parameter α by $\gamma = (1-\beta^2)^{-1/2+\alpha} = \gamma_{\mathrm{SR}}(1+\alpha\beta^2+\ldots)$ [5]. A non-vanishing test parameter α would indicate a violation of SR and modify the outcome of the Ives–Stilwell experiment as

$$\frac{\nu_p \cdot \nu_a}{\nu_0^2} = \left(1-\beta^2\right)^{-2\alpha} \approx 1 + 2\alpha \cdot \beta^2 \tag{3}$$

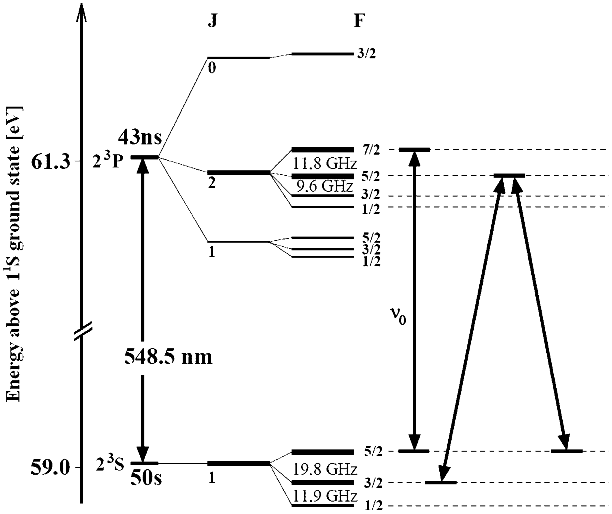
The first experiment of this kind was performed by Ives and Stilwell (IS), who measured the Doppler-shifted frequencies ν_p and ν_a of the H_β line (ν_0) of a hydrogen beam in parallel and anti-parallel direction. The result of the original IS experiment set an upper bound on α of $|\alpha| < 1 \times 10^{-2}$ [10].

Later, significant improvements have been achieved using laser techniques instead of conventional spectrometers; two-photon spectroscopy on a β=0.0036 neon atomic beam has set an absolute bound of $|\alpha| < 2.3 \times 10^{-6}$ [11] and, considering the cosmic background frame as the preferred one (β_{lab}≈350 km/s), even $|\alpha| < 1.4 \times 10^{-6}$ [9] from limits on sidereal variations.

3 A modern Ives–Stilwell experiment at the TSR

Today, heavy ion storage rings equipped with electron coolers like the TSR in Heidelberg or the ESR in Darmstadt provide low-divergence ion beams at high velocities. The combination of these fast ion beam techniques with high resolution laser spectroscopy allows for a significant improvement of the time dilation measurement. The latest experiment, at the TSR, uses metastable $^7\mathrm{Li}^+$ ions in a storage ring at a velocity of β= 0.064 and β=0.03, respectively. The triplet spectrum of the helium-like $^7\mathrm{Li}^+$ has a strong optical transition $2s\,^3S_1 \rightarrow 2p\,^3P_2$ at 548.5 nm with a well-resolved and precisely known hyperfine structure multiplet. A first version of this experiment that employed collinear optical-optical double-resonance spectroscopy on a Λ-type three-level system formed by the

Fig. 2 Level-scheme of ortho-helium-like triplet system of $^7\mathrm{Li}^+$



$2^3S_1(F = 3/2)$, $2^3S_1(F = 5/2)$, and $2^3P_2(F = 5/2)$ states, compare Fig. 2, has set the thitherto best absolute bound of $|\alpha| < 8 \times 10^{-7}$ [12]. The limiting factor was the large observed linewidth of the Λ-resonance of almost 60 MHz, compared to a natural width of 3.8 MHz. This broadening was caused by velocity changes between subsequent excitations of the two transitions of the Λ-system. Changing the experimental scheme to collinear saturation spectroscopy on the $2^3S_1(F = 5/2) \rightarrow 2^3P_2(F = 7/2)$ two-level transition avoids this problem.

The preparation process for the production of metastable $^7\mathrm{Li}^+$ ions starts with negative charged Li ions, which are accelerated and stripped in a Tandem Van de Graaff accelerator. A fraction of about 10% emerges in the metastable $^3S_1$ state from the stripping process and typically an amount of 10^8 ions can be injected into the TSR. There the ions are subjected to electron cooling to narrow the velocity distribution. While the natural lifetime of the metastable ions is 50 s, collisions with the residual gas reduce the beam lifetime to about 13 s. At equilibrium (after a cooling time of about 5–10 s) a longitudinal momentum spread of $\Delta p/p = 3.5 \times 10^{-5}$ leads to a Doppler-width of the transition of about 2.5 GHz (FWHM). A more detailed description of the storage ring can be found in [13] and [14].

For each experiment (viz for each ion-velocity) a frequency-stabilized laser system for the parallel and the anti-parallel excitation of the metastable $^7\mathrm{Li}^+$ ions has been established. All lasers that are used to perform one of the spectroscopy experiments in the storage ring are listed in Table 1. The laser setup is exemplarily described for the case of the faster ion-velocity (β=0.064) below.

To control the frequency of the laser for the parallel excitation, this has been locked directly to a molecular iodine reference line. The dye laser for the anti-parallel excitation has been controlled via a frequency-offset-locking technique relative to a second dye laser that is, again, directly locked to an iodine reference, see Fig. 3. This method allows tuning the first dye laser in the MHz range, while its frequency is, due to the beating with the second dye laser, kept under control on the kHz level. Therefore an accuracy in the frequency determination of $\Delta\nu/\nu = 2 \times 10^{-10}$ has been achieved. The used iodine references were calibrated at the Max Planck Institute for Quantum Optics in Garching in

Table 1 Lasers used in the latest experimental series for testing time dilation at the TSR

β [%c]	Excitation direction	Denotation	Laser	Wavelength [nm]	Iodine-Frequency [kHz]
6.4	parallel	$\nu_{p1}=\nu_p$	Ar-Ion	514	582 490 603 430±3
3	parallel	ν_{p2}	Nd:YAG	532	563 209 278 600±7
3	anti-parallel	ν_{a2}	Dye	565	530 222 434 291±73
6.4	anti-parallel	$\nu_{a1}=\nu_a$	Dye	585	512 671 028 075±73

Iodine frequencies are taken from [14].

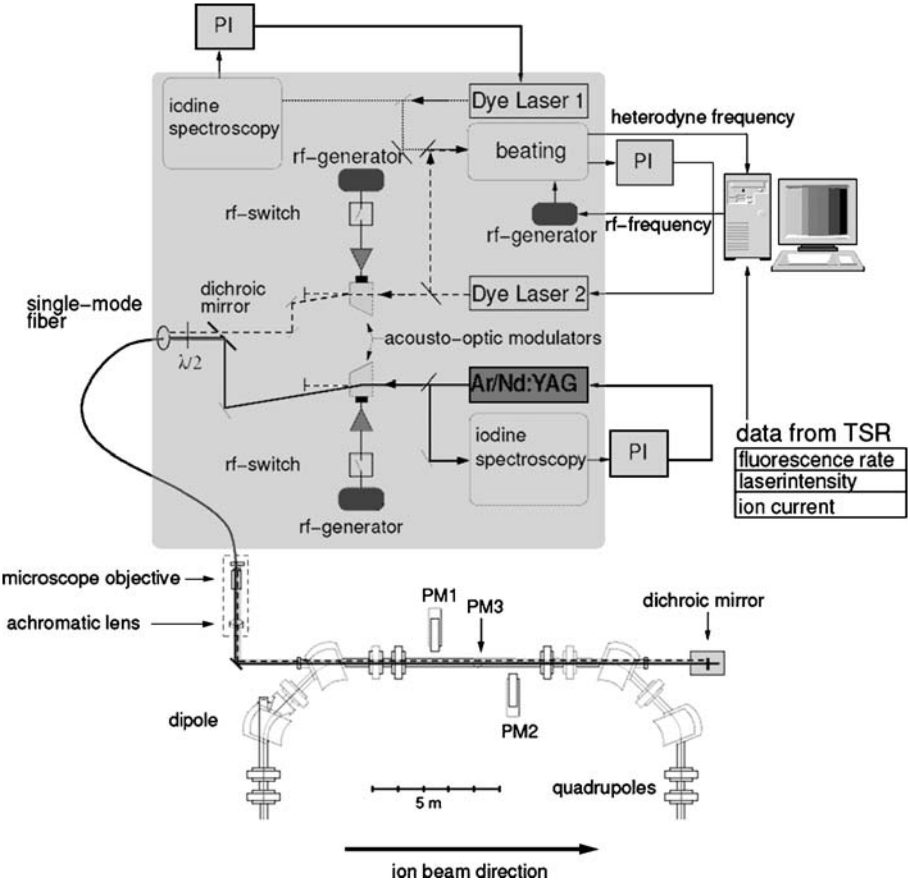


Fig. 3 Setup of the experiment at the ion storage ring TSR. The dye lasers have been used in both experiments (viz for both ion-velocities). Depending on the ion-velocity, the Ar ion laser or the Nd:YAG laser is used

cooperation with the Group of T.W. Hänsch with a frequency comb [15]. Both laser beams going into the TSR are passed through acousto-optic frequency shifters in order to switch the light on and off using fast radiofrequency switches. The beams are then merged with a dichroic mirror and guided to the TSR by a single-mode polarization-maintaining fiber. At the TSR, the laser beams are linearly polarized in the same direction and their intensities are

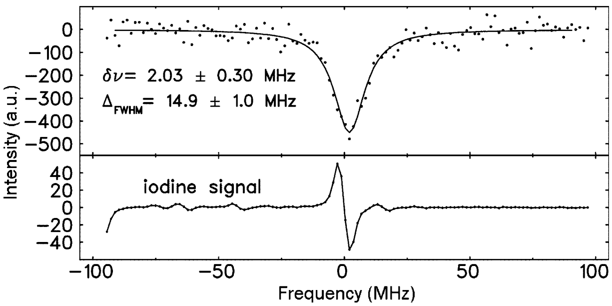


Fig. 4 Upper part: fluorescence signal for a multiple scan of the Lamb-dip at a velocity of β=0.064 (the Doppler-background is subtracted). Lower part: corresponding iodine signal of the tuned dye laser. The zero of the frequency scale corresponds to the position of the iodine reference line

kept equal in order to balance the laser forces on the ions. The bichromatic beam coming out of the fiber is directed via an achromatic telescope through the experimental section of the TSR and retro-reflected by a flat mirror. The laser beams are accurately superimposed with the ion beam with computer-controlled motorized translation/rotation stages. The fluorescence signal is recorded by three photomultipliers that are placed in the ion-laser-beam interaction zone along the beam pipe. By simultaneously maximizing the fluorescence yield, both laser beams can be aligned with respect to the ion beam to better than 70 µrad.

A typical spectrum of the lithium spectroscopy in the storage ring is shown in Fig. 4. The laser for the parallel excitation has been frequency-fixed while the dye laser is tuned by 200 MHz across the resonance.

Applying this technique at an ion-velocity of β=0.064 the Doppler-shifted frequencies ν_p and ν_a could be measured to an accuracy of $\Delta\nu/\nu = 1 \times 10^{-9}$. Considering (3) the latest upper limit of $|\alpha| < 2.2 \times 10^{-7}$ [16] has been achieved. A tribute to the quality of the storage ring spectroscopy is the fact that the limitation at this stage comes from the insufficient knowledge of the rest frame frequency $\Delta\nu_0$=400 kHz [17], which enters (3) in quadrature.

To overcome the limitation due to the uncertainty of the rest frequency, a second measurement at a lower velocity of β=0.03 was carried out. Multiplication of the expressions for Doppler-shifted frequencies according (3) for both ion-velocities leads to

$$\frac{\nu_{p2}\nu_{a2}}{\nu_{p1}\nu_{a1}} = \frac{1 + 2\alpha \cdot \beta_2^2}{1 + 2\alpha \cdot \beta_1^2} \approx 1 + 2\alpha\left(\beta_2^2 - \beta_1^2\right) \tag{4}$$

From the measurements at two different ion velocities it is possible to determine the rest frequency and α independently. Additionally the measurement at high velocity of β=0.064 were repeated with an improved stabilization/frequency scanning scheme for the lasers.

The comparison of the different excitation frequencies at the different velocities cancels out the influence of the rest frame frequency, compare (4), and will lead to an increase of the upper limit for α (the data is currently analyzed and the final value will be presented elsewhere soon).

4 Feasibility-test at the ESR

The next generation of Ives–Stilwell type experiments will be carried out at GSI in the experimental storage ring ESR. Here storage velocities up to β=0.4 are possible and due to the fact that α scales with the square of the ion-velocity, an increase of the upper limit of α

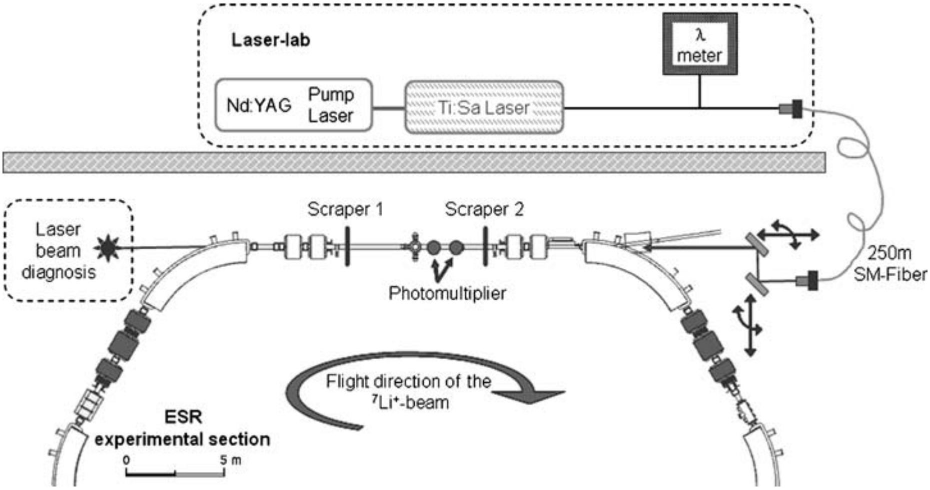


Fig. 5 Setup of the test experiment at the ion storage ring ESR

by more than one order of magnitude is possible. This high velocity leads, of course, to a larger Doppler-shift of the rest frequency and requires new references on the corresponding frequency ranges. With the development of the frequency comb, such a reference with the needed accuracy has become available.

A first test beam-time at the ESR has been performed in October 2005 to show the feasibility of a test of SR at GSI. Before this beam-time only one experiment at GSI dealt with lithium-ions and it had been located just behind the Unilac accelerator (Tinschert, personal communication.), hence, no experience with lithium ions in the subsequent heavy ion synchrotron (SIS) and in the ESR had so far been made. Furthermore at GSI no ion source is available that works with negatively charged ions. Thus, a new production method for metastable $^7\mathrm{Li}^+$ ions, different from the one used at the TSR, has to be established. In principle there are two processes to gain metastable ions from an ion source that produces positively charged ions: electron capture of $^7\mathrm{Li}^{2+}$ ions or excitation of $^7\mathrm{Li}^+$ from the ground state. Indeed a common process for the metastable production is electron capture at energies in the range of 50 keV [18]. Unfortunately no foil nor gas stripper is available at those ion energies at GSI. Hence, the production has to be performed through excitation processes directly in the source. For energy reasons the electron cyclotron resonance (ECR) ion source was chosen. There the raw material (lithium fluoride) was evaporated and ionized using radio frequency radiation. In this process an equilibrium of all possible ion-charges ($^7\mathrm{Li}^+$, $^7\mathrm{Li}^{2+}$, $^7\mathrm{Li}^{3+}$ and even the metastable $^7\mathrm{Li}^{+*}$) will appear. The ion-cloud has been extracted from the source, pre-accelerated to 15 keV and separated by a mass to charge analyzer to the desired ratio of seven to one. Those ions were accelerated to an energy of 58.8 MeV/u by the Unilac and the SIS. About 10^8 ions, corresponding to a current signal of 20–30 μA, were injected into the ESR. There the ions were subjected to electron cooling to narrow the velocity distribution.

The laser system applied for this first test consisted only of one laser for the excitation from the anti-parallel direction. In contrast to the TSR experiment described above, this test gains no Doppler-free signal but shows the velocity distribution of the metastable ions in the storage ring and gives information about the lifetime and the ion beam behaviour. Due to the high velocity of β=0.33 the Doppler-shifted wavelength is λ_α=780.2 nm. This laser

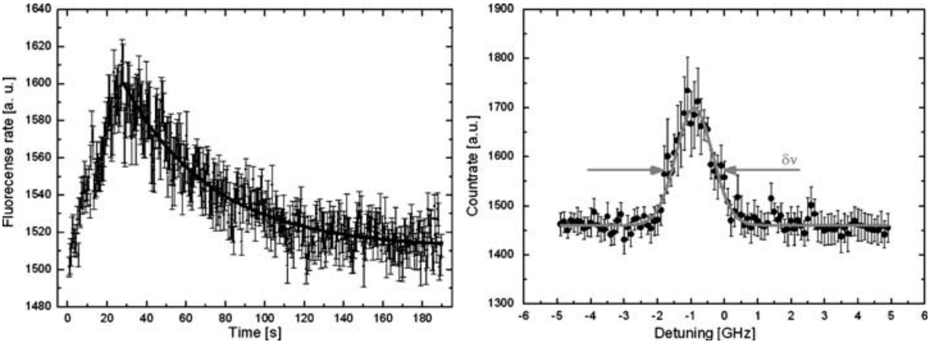


Fig. 6 Left panel: measurement of the lifetime of the metastable ^{7}Li$^+$ ions in the ESR. Right panel: velocity distribution of the ions in terms of frequency in the ESR

light was provided by a cw Ti:Sa laser that was situated in a laboratory ~250 m away from the ESR guided to the storage ring through a single mode fiber. The frequency of this laser was maintained by a λ-meter and tuned in the range of 1 to 10 GHz across the excitation wavelength, to cover the whole velocity distribution of the ions. For the fluorescence detection two photomultipliers where placed at a distance of about 30 cm apart from each other in the experimental section of the ESR, see Fig. 5. To control the overlap of the ion- and the laser-beam, two scrapers were used. These are placed in the middle of the analysis section about 6.5 m apart. At the ESR, the laser beam is guided using fully motorized linear and rotation stages to provide excellent remote control of the laser beam, resulting in an uncertainty of the ion-laser-beam alignment of ±77 μrad.

In contrast to the TSR, where electron cooling takes 5–10 s and the metastable ^{7}Li$^+$ ions have a lifetime of 13 s [13], it turned out that the lifetime at the ESR is close to the natural one of 50 s, see Fig. 6, left panel. The time for the electron cooling in the ESR is in the range of about 30 s. Nevertheless, after reaching equilibrium, the width of the velocity distribution is $\delta\nu$=1.09 GHz±0.08 GHz in the laboratory system (see Fig. 6, right panel), compared to 2.5 GHz at the TSR [16]. Together with the measured centre frequency of $\nu_a = 383\ 621.0\ \text{GHz} \pm 0.4\ \text{GHz}$ for the anti-parallel excitation, a velocity ratio of $\Delta\beta/\beta = 3.4 \times 10^{-6}$ is found. This value provides a verification of the velocity of the electrons from the cooler, with an order-of-magnitude better accuracy compared to the electronic measurement setup routinely used at the ESR (Steck, personal communication).

Furthermore it turned out that the amount of stored ions in the metastable state is at most 0.1% of the whole beam (so in the range of <10^5 ions). Comparing this with the efficiency at TSR (~10% of 10^8 ions) it is obvious that the production process has to be significantly improved to increase the fluorescence signal.

5 Recent development

The first test at the ESR has given a positive feedback for the feasibility of the time dilation experiments. Nevertheless some further steps have to be done to perform a new test of special relativity. First of all the production process for the metastable ions has to be optimized (e.g., using a different source material) to a fraction of 1–10% of the whole ion-beam (at a typical ion number of 10^8 stored in the ESR). Tests at an ECR source addressing this point are in progress at the University of Giessen by the group of A. Müller.

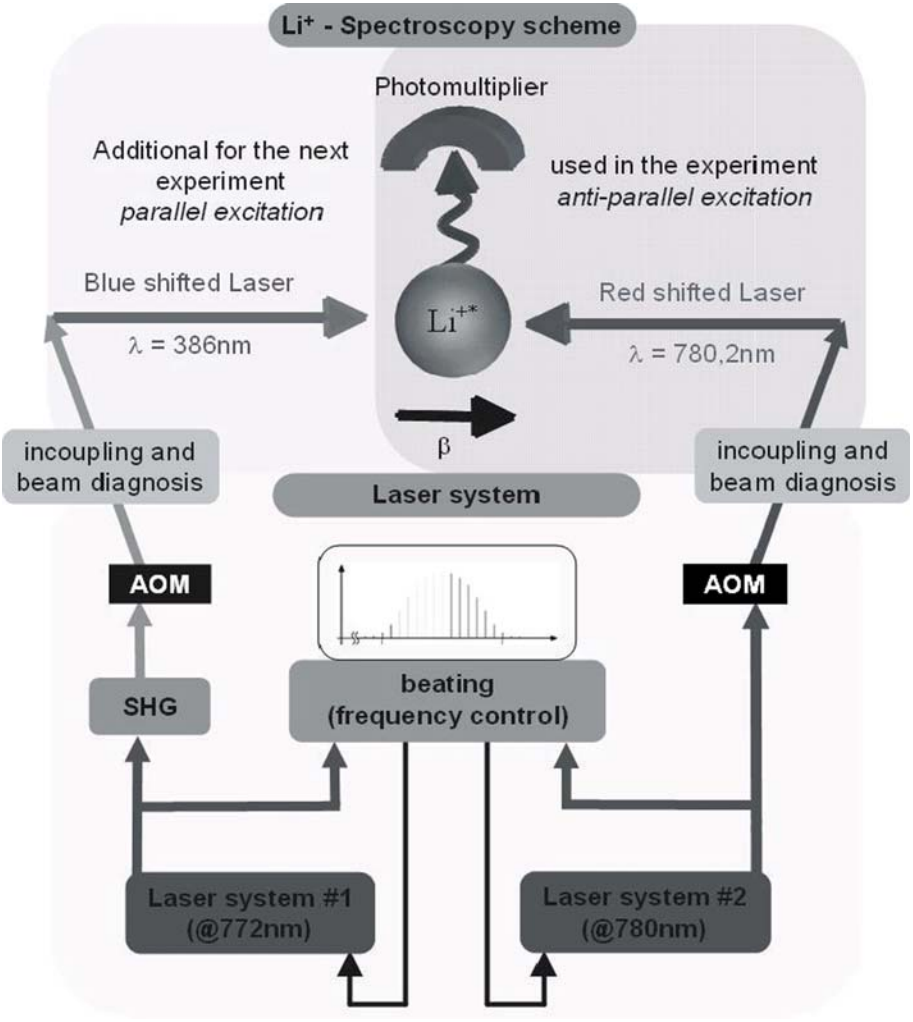


Fig. 7 Scheme of the time dilation experiment at the ESR

Furthermore the laser system for the parallel excitation of the ^{7}Li$^+$ ions has been established. The high ion-velocity induces a Doppler-shift in the parallel excitation scheme to λ_p=386 nm. To reach this wavelength, a laser system has been developed, that consists of a frequency-doubled cw Ti:Sa laser (the fundamental wave is λ_f=771.5 nm) that is pumped by a Nd:YAG laser. The efficiency of the frequency doubler is about 10%. To guide the 386 nm light a photonic crystal fiber, with an efficiency of about 10% and a length of 20 m, is used. To switch the laser light on and off, an acousto-optic modulator is placed right before the fiber, see Fig. 7. With an output power of about 1 W from the Ti:Sa laser, the frequency-doubled light power in the ESR will be in the mW range.

For the anti-parallel excitation, a laser system consisting of a second cw Ti:Sa laser (also pumped with another Nd:YAG laser) at a wavelength of λ_a=780.2 nm is build up. The laser light is guided to the ESR incoupling window via a 50 m fiber. Again, an acousto-optic modulator is applied for the on/off switching of the light in the ring.

The frequency determination of both laser systems is realised with a frequency comb synthesizer. This frequency standard has a comb structure of some hundred thousand modes in the VIS-NIR range with a spacing of 100 MHz. The idea for the time dilation experiment is to simultaneously stabilize both lasers (λ_f=771.5 nm and λ_a=780.2 nm) to different modes of the comb. This provides, together with a Rb-clock reference, the laser frequencies to an accuracy of 200 kHz. Due to the narrow mode spacing of 100 MHz a coarse, online frequency control is necessary to maintain the mode number. This has been realised with an atomic and a molecular frequency standard. λ_a is referenced to the atomic Rubidium line (384 227 995.7 MHz) and λ_f to the molecular iodine line (388 605 093.6 MHz), respectively, using saturation spectroscopy. (The spectroscopy at the molecular iodine was performed at a temperature of 550°C.) With this method the frequency can be maintained in the MHz regime.

For the control of the laser-laser-ion-beam overlap in the ESR, another observation window at a distance of 6 m from the other windows has been installed. Furthermore this window gives another possibility for the fluorescence detection at a different position in the ESR and therefore a chance to gain information about systematic effects in the ring.

6 Conclusions

An upper limit for possible deviations of the time dilation factor from Special Relativity $|\alpha| < 2.2 \times 10^{-7}$ has been found at the TSR in Heidelberg. After finishing the data analysis of the latest experiments at two different velocities at TSR, this upper limit is expected to be further improved.

Furthermore the feasibility of performing time dilation tests at the ESR has been shown in October 2005. The development of the laser excitation systems for the time dilation experiment at an ion-velocity of β=0.33 and their online frequency determination is completed. Experiments to increase the production rate of metastable ^{7}Li$^+$ ions from an ECR ion source are in progress.

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