Masses and binding energies

Introduction to Nuclear Science

Simon Fraser University
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Notation

- For element X: $${}^{A}_{Z}X_{N}$$ denotes
  - $A = Z + N$-number of nucleons,
  - Z-number of protons,
  - N-number of neutrons.
- Isotopes: constant Z for varying A and N,
- Isotones: constant N for varying A and Z,
- Isobars: constant A for varying N and Z,

Mass-energy equivalence

- In 1905 Albert Einstein following his derivation of the Special Theory of Relativity identifies relation between mass and energy of an object at rest:

$$E = mc^2 \quad . \tag{1}$$

- The corresponding relation for moving object is

$$E = \frac{1}{\sqrt{1+\left(\frac{v}{c}\right)^2}} mc^2. \tag{2}$$

- This discovery explains the energy powering nuclear decay. The question of energy release in nuclear decay was a major scientific puzzle from the time of the discovery of natural radioactivity by Henri Becquerel (1896) until Einstein's postulate of mass-energy equivalence.

- Subsequent experiments confirm conversion between mass and energy in atomic and nuclear processes.

Why is life complicated?

- Mass-energy conversion during the decay or reactions happens in nuclei.
- But we measure masses of atoms or ions.
- Atoms comprise of nuclei and electrons.
- Electrons are much lighter then nuclei. Mass of an electron is 0.511 MeV/c^2 to be compared with a mass of a proton of 938.27 MeV/c^2 or a neutron 939.56 MeV/c^2.
- For the energy-mass equivalence to work in an atom binding energy of electrons and nucleons have to be taken into account.

Why is life complicated (furthermore)?

- To measure mass we need a mass unit.
- Since the Avogadro number is so large $N_A = 6.02 \times 10^{23}$ macroscopic units like kg or g are very inconvenient (see your sheet of constants).
- But we could measure mass of nuclei in the unit of proton mass!
- However, if we use protons, why not the mass of hydrogen? That would take care of the electron mass and some binding energy.
- Or another thought, why not use the neutron mass as a unit?
- Or there may be so many other great ideas. Fortunately, scientists agreed to define the atomic mass unit as 1/12 of the mass of ^{12}C.
- Atomic mass unit u=931.49 MeV/c^2 is smaller than the mass of a proton of 938.27 MeV/c^2 or a neutron 939.56 MeV/c^2 since it includes nuclear and atomic binding energy of ^{12}C atom constituents.

Mass excess

- Mass excess (aka mass defect) is equal to the difference between atomic mass and the atomic number times the atomic mass unit.

$$\Delta M(A,Z) = M(A,Z) - A \times u$$
$$M(A,Z) = A \times u + \Delta M(A,Z) \tag{3}$$

- Since

$$u = \frac{1}{12} M(12,6) \tag{4}$$

mass excess for ^{12}C is

$$\Delta M(12,6) = M(12,6) - 12u = M(12,6) - M(12,6) = 0 \tag{5}$$

- Mass excess is convenient to use in calculations of nuclear decay and reactions energetics. It is frequently used to tabulate atomic masses. It does not provide the best insight into the nuclear binding.

Atomic mass vs. nuclear mass

- Most of experiments yield atomic masses M_{atom} or masses of ions.

- Nuclear mass m_{nuc} can be obtained by accounting for masses m_e and binding energies B_e of electrons . For an atomic mass:

$$M_{atom} = m_{nuc} + Z \times m_e - \frac{B_e(Z)}{c^2}. \tag{6}$$

- The electron binding energy can be estimated using atomic models. In particular the Thomas-Fermi model yields

$$B_e(Z) = 15.73 Z^{\frac{7}{3}} \text{ [eV]}. \tag{7}$$

- Binding energy of electrons is small compared to mass of a nucleus (several tens of GeV), and even mass of electrons (several tens of MeV). As such B_e it is often neglected in calculations.

Why are masses important?

- Masses are important since they provide information on energy balance in nuclear processes.
- An important consequence of the mass-energy equivalence and the universal conservation of energy is that we can calculate the energy released in a process just by taking a mass difference between initial and final states. Without any detailed knowledge of the process!
- One straightforward consequence of the above and energy release in nuclear decay is the fact that mass of the parent (the initial nucleus) has to be larger than the mass of the daughter (the final nucleus). The decay can not happen starting from a lighter mass nucleus ending in a heavier mass nucleus.
- Decays which are allowed by the energetics do not always happen. For example, mass-energy equivalence allows for emission of heavy fragments like ^{12}C from heavy nuclei like ^{238}U. Such decays are not observed.

Nuclear energetics

- Note that measured and tabulated masses are for atoms not nuclei! This fact has to be accounted for in nuclear energy balance calculations.
- Let us calculate the difference between nuclear masses for a nucleus with Z_i, A_i with a nuclear mass m_i and a nucleus with Z_f, A_f with nuclear mass m_f using the atomic mass M_i for the initial and M_f for the final atom:

$$\begin{aligned} m_i - m_f &= M_i - Z_i m_e + \frac{B_e(Z_i)}{c^2} - M_f + Z_f m_e - \frac{B_e(Z_f)}{c^2} \\ &= (M_i - M_f) + (Z_f - Z_i)m_e + \frac{1}{c^2}(B_e(Z_i) - B_e(Z_f)) \qquad (8) \end{aligned}$$

Nuclear energetics

- Using the mass excess the first term can be evaluated as

$$\begin{aligned} M_i &= A_i u + \Delta M_i(Z_i, A_i) \\ M_f &= A_f u + \Delta M_f(Z_f, A_f) \\ (M_i - M_f) &= (A_i - A_f)u + (\Delta M_i(Z_i, A_i) - \Delta M_f(Z_f, A_f)) \end{aligned} \tag{9}$$

- The third term is very small and can be dropped from the equation

$$\frac{1}{c^2}(B_e(Z_i) - B_e(Z_f)) \approx 0 \tag{10}$$

Nuclear energetics

- The difference between nuclear masses becomes then

$$
\begin{aligned}
& m_i - m_f = \qquad (11)\\
& = (\Delta M_i(Z_i, A_i) - \Delta M_f(Z_f, M_f)) + (A_i - A_f)u + (Z_f - Z_i)m_e
\end{aligned}
$$

- The advantage in using the mass excess is in the fact that in many cases $A_i = A_f$ and the second term is zero.
- This is, for example, the case for β-decay.

β^- decay of a neutron

- β^- decay is a process which converts a neutron into a proton with emission of an electron e^- and electron anti-neutrino.

$$n \to p + e^- + \bar{\nu}_e \tag{12}$$

- More detail will be given later, today we only talk about energetics.
- Neutron decay can take place since neutron is heavier than a proton so there is energy available to drive the decay, however, this energy is partially converted into the mass of the electron and anti-neutrino.
- Let us examine carefully the energy balance.

β^- decay of a neutron

- The energy Q turned into the kinetic energy of a proton, electron, and anti-neutrino in the final state is the difference between the mass of the neutron and the mass of the proton, electron and anti-neutrino (multiplied by c^2).

$$Q = (m_n - m_p - m_e - m_{\bar{\nu}_e})c^2 \tag{13}$$

- Mass of the anti-neutrino is very small (a few eV) and can be dropped from the equation without any significant loss of accuracy.

- The nuclear mass difference between neutron and proton can be calculated using Eq. 11 realizing that $A_i = A_f = 1$, $Z_i = 0$, $Z_f = 1$. Thus

$$\begin{aligned} Q &= (\Delta M(0,1) - \Delta M(1,1) + (1-0)m_e - m_e)c^2 = \\ &= (\Delta M(0,1) - \Delta M(1,1))c^2 = 8.071 - 7.289 = 0.782 \text{ [MeV]} \end{aligned} \tag{14}$$

β^- decay of a nucleus

- β^- decay can convert a neutron in a nucleus into a proton.
- The process proceeds with emission of an electron e^- and electron anti-neutrino in a similar way as the β^- decay of a neutron. For example

$$^{14}C \rightarrow ^{14}N + e^- + \bar{\nu_e} \quad \tau = 5730 \text{ [y]} \tag{15}$$

- Nuclear β^- decay can take place if the energy balance allows it, meaning, that the sum of the masses in the final state is smaller than the mass in the initial state.
- Let us examine carefully the energy balance for the example case.

β^- decay energetics for ^{14}C decay

- The energy Q turned into the kinetic energy of ^{14}N, electron, and anti-neutrino in the final state is the difference between the mass of ^{14}C and the mass of ^{14}N, electron and anti-neutrino (times c^2).

$$Q = (m_{^{14}C} - m_{^{14}N} - m_e - m_{\bar{\nu_e}})c^2 \tag{16}$$

- We drop the mass of anit-neutrino again.
- The nuclear mass difference between ^{14}C and ^{14}N can be calculated using Eq. 11 realizing that $A_i = A_f = 14$, $Z_i = 6$, $Z_f = 7$. Thus

$$Q = (\Delta M(6,14) - \Delta M(7,14) + (7-6)m_e - m_e)c^2 = \tag{17}$$
$$= (\Delta M(6,14) - \Delta M(7,14))c^2 = 3.020 - 2.863 = 0.157 \text{ [MeV]}$$

Forbidden β^+ decay of a proton

- β^+ decay turns a proton into a neutron in a nucleus.

$$p \rightarrow n + e^+ + \nu_e \tag{18}$$

- Note that compared to a neutron decay the positron e^+ is an anti-particle of an electron, and the neutrino ν_e is an anti-particle of an anti-neutrino $\bar{\nu}_e$. Consequently the mass of a positron is the same as for an electron, and the mass of an anti-neutrino is the same as for a neutrino.
- This process of a conversion of a free proton into a free neutron by β^+ decay is forbidden by energetics. Let us examine the energy balance.

Energetics of a forbidden β^+ decay of a proton

- The energy Q is the difference between the mass of a proton and the mass of a neutron , positron and neutrino (multiplied by c^2). We use the fact that the mass of a positron is the same as electron.

$$Q = (m_p - m_n - m_e - m_{\nu_e})c^2 \tag{19}$$

- Mass of the neutrino is very small and can be dropped from the equation without any significant loss of accuracy.
- The nuclear mass difference between a proton and a neutron can be calculated using Eq. 11 realizing that $A_i = A_f = 1$, $Z_i = 1$, $Z_f = 0$:

$$\begin{aligned} Q &= (\Delta M(1,1) - \Delta M(0,1) + (0-1)m_e - m_e)c^2 = \\ &= (\Delta M(1,1) - \Delta M(0,1) - 2m_e)c^2 = \\ &= 7.289 - 8.071 - 2 * 0.511 = -1.804 \text{ [MeV]} \end{aligned} \tag{20}$$

- Negative Q indicates that process does not release energy but rather requires energy to proceed.

β^+ decay of a nucleus

- β^+ decay can convert a neutron in a nucleus into a proton if the energy balance allows it. This is the case for a large number of nuclei which have excess of protons as compared to neutrons.
- The process proceeds with emission of a positron e^+ and electron neutrino. For example

$$^{64}Cu \rightarrow ^{64}Ni + e^+ + \nu_e \quad \tau = 12.7 \text{ [h]} \tag{21}$$

- The decay of ^{64}Cu is only partially (39% cases) through the nuclear β^+ decay, the remaining (61%) is through the electron capture (to be discussed later).
- Let us examine carefully the energy balance for the example case of β^+ decay.

β^+ decay energetics for ^{64}Cu decay

- The energy Q turned into the kinetic energy of ^{64}Ni, positron, and neutrino in the final state is the difference between the mass of ^{64}Cu and the mass of ^{64}Ni, positron and neutrino (times c^2). Mass of the positron is the same as electron.

$$Q = (m_{^{64}Cu} - m_{^{64}Ni} - m_e - m_{\nu_e})c^2 \tag{22}$$

- We drop the mass of neutrino again.
- The nuclear mass difference between ^{64}Cu and ^{64}Ni can be calculated using Eq. 11 realizing that $A_i = A_f = 64$, $Z_i = 29$, $Z_f = 28$. Thus

$$\begin{aligned} Q &= (\Delta M(29,64) - \Delta M(28,64) + (28-29)m_e - m_e)c^2 = \\ &= (\Delta M(29,64) - \Delta M(28,64) - 2m_e)c^2 = \\ &-65.421 - (-67.096) - 2*0.511 = 0.653 \text{ [MeV]} \end{aligned} \tag{23}$$

- Note, that in general the atomic mass difference has to be at least $2*m_ec^2$=1.022 MeV for the β^+ decay to proceed.

Forbidden electron capture in a hydrogen atom

- Electron capture decay turns a proton into a neutron in a nucleus via capture of an electron from an atomic orbit.

$$p + e^- \rightarrow n + \nu_e \tag{24}$$

- Note that compared to a neutron decay we deal with an electron again and that the neutrino ν_e is an anti-particle of an anti-neutrino $\bar{\nu}_e$.
- This process of a conversion of a proton in a hydrogen atom into a neutron by electron capture decay is forbidden by energetics. Let us examine the energy balance.

Forbidden electron capture in hydrogen

- The energy Q is the difference between the mass of a proton plus mass of the electron and the mass of a neutron plus mass of the neutrino (multiplied by c^2). Note that the mass of a proton plus mass of an electron is effectively mass of hydrogen, thus the electron capture would be allowed if hydrogen atom is heavier than neutron.

$$Q = (m_p + m_e - m_n - m_{\nu_e})c^2 \tag{25}$$

- We drop the neutrino mass.
- The nuclear mass difference between a proton and a neutron can be calculated using Eq. 11 realizing that $A_i = A_f = 1$, $Z_i = 1$, $Z_f = 0$:

$$\begin{aligned} Q &= (\Delta M(1,1) - \Delta M(0,1) + (0-1)m_e + m_e)c^2 = \\ &= (\Delta M(1,1) - \Delta M(0,1))c^2 = \\ &= 7.289 - 8.071 = -0.782 \text{ [MeV]} < 0 \end{aligned} \tag{26}$$

Electron capture energetics for ^{64}Cu decay

- The energy Q turned into the kinetic energy of ^{64}Ni and neutrino in the final state is the difference between the mass of ^{64}Cu plus an electron and the mass of ^{64}Ni and neutrino (times c^2).

$$Q = (m_{^{64}Cu} + m_e - m_{^{64}Ni} - m_{\nu_e})c^2 \tag{27}$$

- We drop the mass of neutrino again.
- The nuclear mass difference between ^{64}Cu and ^{64}Ni can be calculated using Eq. 11 realizing that $A_i = A_f = 64$, $Z_i = 29$, $Z_f = 28$. Thus

$$\begin{aligned} Q &= (\Delta M(29,64) - \Delta M(28,64) + (28-29)m_e + m_e)c^2 = \\ &= (\Delta M(29,64) - \Delta M(28,64) - m_e + m_e)c^2 = \\ &-65.421 - (-67.096) = 1.675 \text{ [MeV]} \end{aligned} \tag{28}$$

- Electron capture is allowed if β^+ decay is allowed and results in larger energy release (since it does not need to create a positron).