Alpha decay

Introduction to Nuclear Science

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Outline

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1. The decay processes
2. The energetics
3. The observed decay rates

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2. The energetics
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Decay processes

- The classification of decay processes into the α, β and γ decay originates comes from the early XX-century studies of decay processes using nuclear emulsion.
- Nuclear emulsion is a photographic plate (as used in analog cameras for black and white photography) with a particularly thick emulsion layer and with a very uniform grain size.
- Nuclear emulsions exposed to radiation showed tracks which were classified into the three groups
  - class α: short and thick
  - class β: longer and thin
  - class γ: very long and very thin.
- Subsequent studies revealed the origin of the processes resulting in the observation of these different tracks, however, the names stick to the processes.

The α-decay process

- Ernest Rutherford pioneered application of α-decay process into nuclear science experimental studies.
- He correctly identified α particles as nuclei of helium atoms.
- This has been achieved using atomic spectroscopy. α particles from a source were collected in a discharge tube and then observed to show line spectra identical with these from helium reference gas.
- Rutherford experiments on scattering of α-particles led to the discovery of atomic nucleus in 1911 (100 years from now).
- Thus α decay process has been recognized as emission of energetic helium nucleus from a parent nucleus.

The α-decay process

- In the most general way the α-decay process can be written as

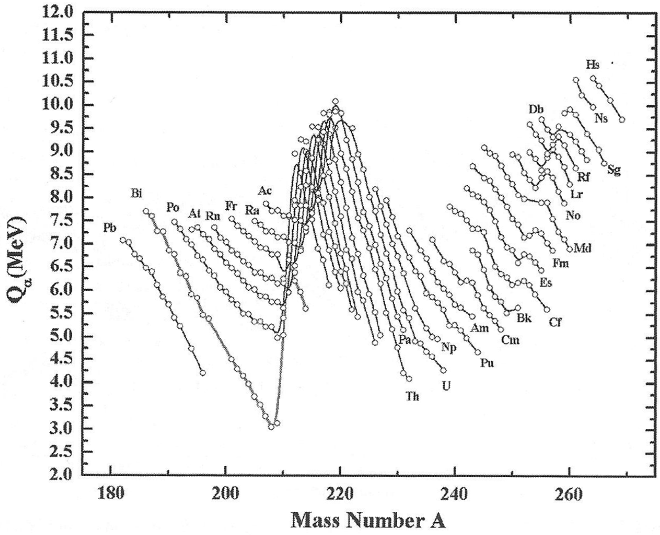
$$ {}^{A}_{Z}Y^{0}_{N} \quad \rightarrow \quad {}^{A-4}_{Z-2}X^{2-}_{N-2} + {}^{4}_{2}He^{2+}_{2} + Q_{\alpha} \tag{1} $$

- The Q_α value is

$$ Q_{\alpha} = (m_Y - m_X - m_{\alpha})c^2 \tag{2} $$

- The Q_α value has to be positive for nuclei to undergo a spontaneous (exothermic) decay.
- The Q_α values can be calculated from measured masses, or estimated using the Liquid Drop Model.
- In the Q_α calculations binding energies of electrons can be neglected as small and masses of neutral atoms can be used.
- For know α emitters the Q_α values vary between $\sim$3 and $\sim$ 11 MeV.

The Q_α values



The S_α values

- Note that Q_α values correspond to the negative separation energy S_α of an α particle from the parent nucleus.

$$ {}^{A}_{Z}Y_N + S_\alpha \quad \rightarrow \quad {}^{A-4}_{Z-2}X_{N-2} + {}^{4}_{2}He_2 \tag{3} $$

- Thus

$$ S_\alpha = (m_X + m_\alpha - m_Y)c^2 = -Q_\alpha \tag{4} $$

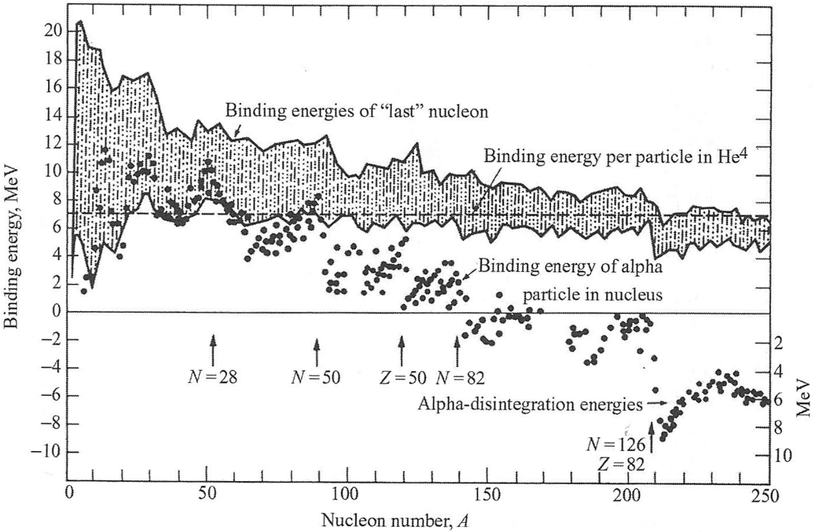
- α separation energies can be calculated using binding energies

$$ \begin{aligned} S_\alpha &= B(A,Z) - B(A-2,Z-2) - B(4,2) \\ &= B(A,Z) - B(A-2,Z-2) - 28.3 \text{ MeV} \end{aligned} \tag{5} $$

taking into account the binding energy of 28.3 MeV for the α particle.

- Negative S_α (or positive Q_α) is indicative of nuclei which undergo spontaneous α decay.

The S_α values



The S_α values

- The known masses as well as the Liquid Drop model indicate that all nuclei with mass number $A > 150$ have negative S_α, positive Q_α and could undergo a spontaneous α decay.
- Experimentally, the α decay dominates in heavy nuclei with mass number $A > 210$
- For these heavy nuclei the emission of an α particle increases binding energy per nucleon for the system.
- This is a consequence of two factors:
  - α particle is very tightly bound with binding energy per nucleon of $B/A \sim 7.1$ MeV comparable to these of mid- and heavy-nuclei. Thus, there is only a little reduction of binding associated with emission of α.
  - The α decay process reduces the charge of the nucleus, thus increases the binding significantly due to reduction of Coulomb repulsion of protons inside a nucleus.

Kinetic energy of α particles

- The α decay process conserves momentum.

$$0 = \vec{p}_X + \vec{p}_\alpha \tag{6}$$

with zero being the initial momentum of the parent and $\vec{p}_X$ and $\vec{p}_\alpha$ being the final momenta of the daughter and the α particle.

- The zero initial momentum of the parent implicates that momenta of the daughter and the α particle have the opposite direction along the same line

$$\vec{p}_X = -\vec{p}_\alpha \tag{7}$$

- Conservation of energy implies that the sum of kinetic energy of the daughter and the α particle is equal to the Q_α value

$$K_X + K_\alpha = \frac{\vec{p}_X^2}{2m_X} + \frac{\vec{p}_\alpha^2}{2m_\alpha} = Q_\alpha \tag{8}$$

Kinetic energy of α particles

- Since, from the conservation of momentum

$$\vec{p}_X = -\vec{p}_\alpha \tag{9}$$

for the kinetic energy of α particle one can write

$$\begin{aligned} \frac{\vec{p}_X^2}{2m_X} + \frac{\vec{p}_\alpha^2}{2m_\alpha} &= Q_\alpha \\ \frac{\vec{p}_\alpha^2}{2m_X} + \frac{\vec{p}_\alpha^2}{2m_\alpha} &= Q_\alpha \\ \frac{\vec{p}_\alpha^2}{2m_\alpha}\left(\frac{m_\alpha}{m_X} + 1\right) &= K_\alpha\left(1 + \frac{m_\alpha}{m_X}\right) = Q_\alpha \end{aligned} \tag{10}$$

- This leads to

$$K_\alpha = Q_\alpha \frac{1}{1 + \frac{m_\alpha}{m_X}} = Q_\alpha \frac{m_X}{m_X + m_\alpha} \approx Q_\alpha \frac{m_X}{m_Y} \tag{11}$$

Kinetic energy of α particles

- The kinetic energy of the α particle is slightly smaller than the Q_α value

$$K_\alpha \approx Q_\alpha \frac{m_X}{m_Y} = Q_\alpha \frac{m_Y - m_\alpha}{m_Y} = Q_\alpha (1 - \frac{m_\alpha}{m_Y}) \approx Q_\alpha (1 - \frac{4}{A}) \quad (12)$$

with $A >> 4$ being the mass number of the parent.

- The recoil energy of the daughter is

$$K_X = Q_\alpha - K_\alpha = Q_\alpha \frac{m_\alpha}{m_Y} \approx Q_\alpha \frac{4}{A} \quad (13)$$

- For nuclei with mass number around $A = 200$ the kinetic energy of the α is ~98% of the Q_α value, while the kinetic energy of the daughter is $\sim$ 2% of the Q_α value.

Comparison with the Coulomb barrier

- Let us compare the observed kinetic energies of α particles with the height of the Coulomb repulsion between the daughter and the ^4He.
- The energy of the Coulomb repulsion at a distance R is

$$V = \frac{e^2}{4\pi\epsilon_0 R}\frac{Z_\alpha(Z-2)}{R} = 1.44\ [\text{MeV fm}]\frac{Z_\alpha(Z-2)}{R} \tag{14}$$

- The height of the Coulomb barrier is the energy of the Coulomb repulsion at the distance corresponding to the sum of the radii for the daughter and the α-particle

$$V_C = 1.44\frac{Z_\alpha(Z-2)}{1.2(\sqrt[3]{A-4}+\sqrt[3]{4})} = 2.4\frac{Z-2}{\sqrt[3]{A-4}+1.59}\ [MeV] \tag{15}$$

with A and Z being mass and atomic number of the parent.

Comparison with the Coulomb barrier

- For ^{238}U $A = 238$, $Z = 92$ and

$$V_C = 2.4 \frac{Z-2}{\sqrt[3]{A-4} + 1.59} = 27.9 \; [MeV] \tag{16}$$

- The measured Q_α value is 4.3 MeV.
- Eq. 12 implies that the kinetic energy of α particles is 4.2 MeV.
- Thus the observed kinetic energy of α particles is by a significant factor (~ 7) smaller than the Coulomb barrier.
- One consequence of that fact is that α particles from naturally occurring radioactive sources are in general not energetic enough to induce nuclear reactions or nuclear transmutations.
- The exceptions are reactions on very light nuclei.

Pu-Be neutron source

- Let us calculate the Coulomb barrier for the reaction of α particles on beryllium

$$V_C = 1.44 \frac{2 \cdot 4}{1.2(\sqrt[3]{9} + \sqrt[3]{4})} = 2.6 \text{ [MeV]} \tag{17}$$

- This implies that for most of naturally occurring α emitters the energy of α particles is high enough to induce nuclear transmutation of beryllium.

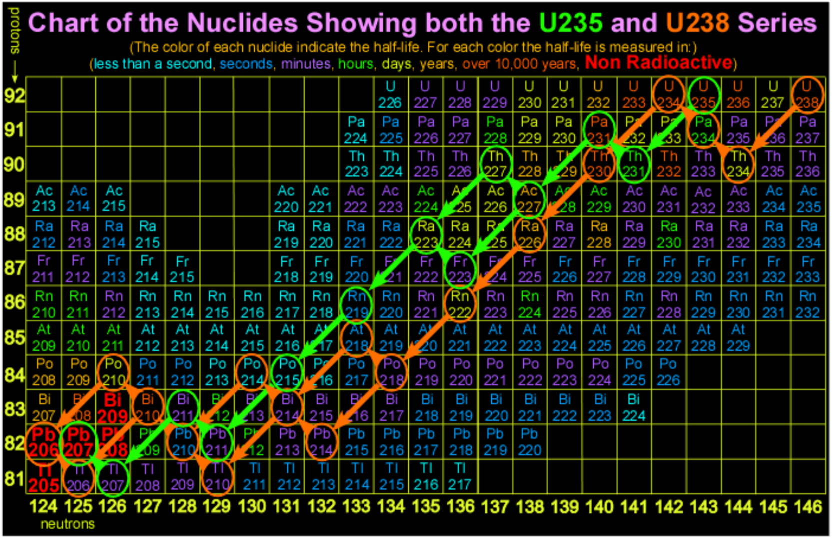
- Such mixtures are useful neutron sources since

$$^{9}Be + \alpha \rightarrow ^{13}C \rightarrow ^{12}C + n \tag{18}$$

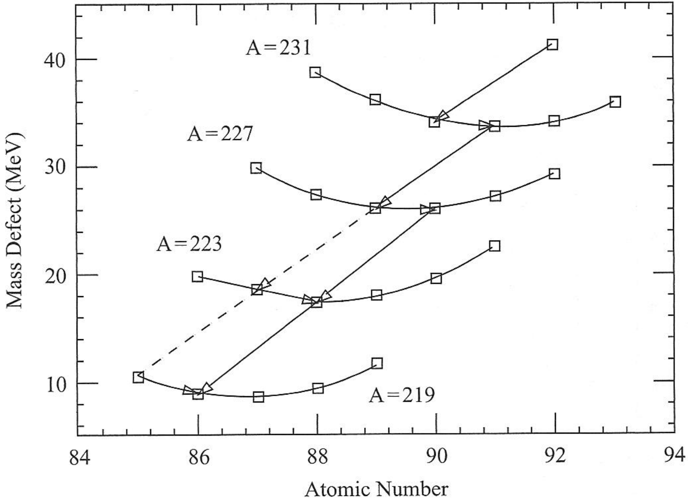
- A popular source is a mixture of ^{241}Pu with ^{9}Be.

- It produces 30 neutrons per million of emitted α particles.

^{235}U and ^{238}U decay chains



^{235}U decay chain sequence



Correlation between Q_α and α decay half-life

- Investigation of α decay by Rutherford's students and collaborators Hans Geiger and John M. Nuttall lead to the observation of correlation between the Q_α value and α decay half-life

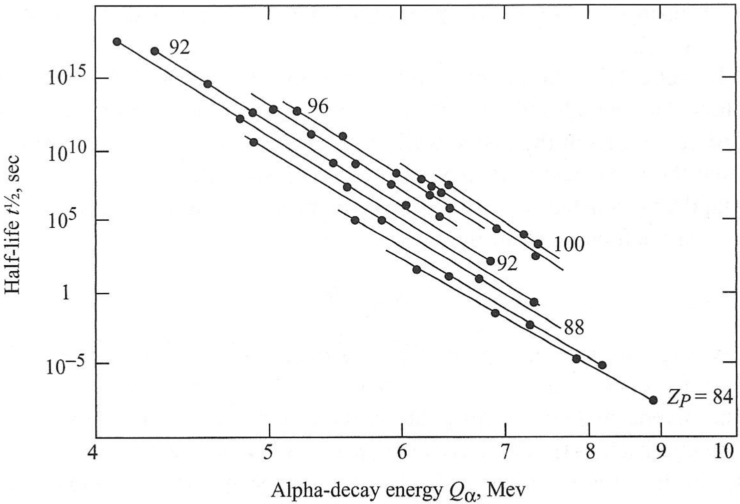
$$\ln(\tau_\alpha) = a - \frac{b}{\sqrt{Q_\alpha}} \tag{19}$$

- Initially, this correlation was observed as a correlation between the half-life and the track length (range) of α particles emitted from various natural sources of α radioactivity.

- The track length (range) is proportional to the initial momentum which is proportional to the square root of the initial kinetic energy, and thus proportional to the square root of the Q_α value.

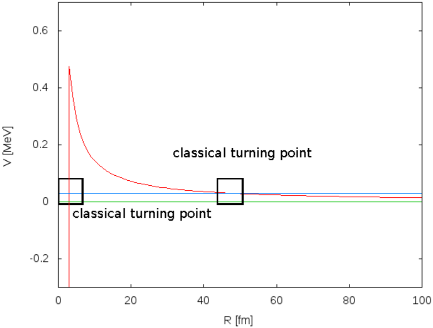
- The constants A and B depend on the Z of the parent.

The Geiger-Nuttall plot



Classical turning point

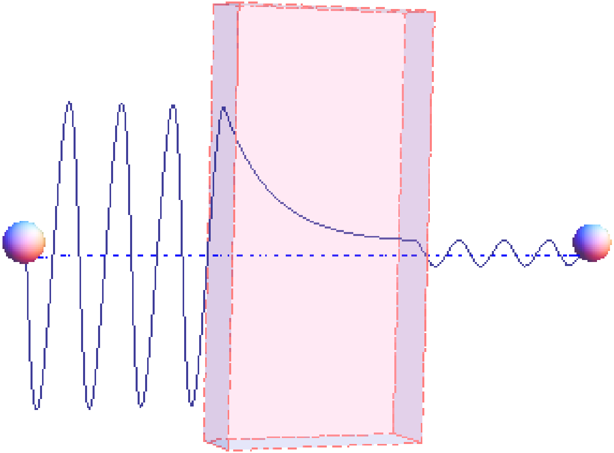
- Particles at energy lower than the Coulomb barrier should start moving away from the interaction centre at the classical turning point.



- On the graph the red line is the Coulomb energy as the function of radius for the $p+p$ reaction, green line represents zero energy, while the blue line represents centre of mass energy of 30 keV. The classical turning points for these conditions are at $R \sim 5$ and $R \sim 47$ [fm].

Tunnelling

- Quantum mechanics allow particles to tunnel through the barrier.



Tunnelling

- The probability of finding a particle across the barrier is given by the transmission coefficient defined as the ratio of amplitude squared of the incoming and transmitted wave functions

$$T = \frac{\mid \Psi(R_N) \mid^2}{\mid \Psi(R_C) \mid^2}, \tag{20}$$

with R_N and R_C being the nuclear radius and the distance to the classical turning point, respectively.

- The barrier suppresses the amplitude of the wave function, but not completely.
- The degree of suppression is calculable for the Coulomb potential and is given by

$$T = \exp\left(-2KR_C\left[\frac{\arctan\sqrt{\frac{R_C}{R_N}-1}}{\sqrt{\frac{R_C}{R_N}-1}} - \frac{R_N}{R_C}\right]\right), \quad K = \sqrt{\frac{2\mu}{\hbar^2}(E_C - E)}$$

The approximation for P

- If the distance to the classical turning point is much larger than the nuclear radius $R_C >> R_N$ which is equivalent to $E << E_C$ the tunnelling probability is well approximated by

$$T = \exp\left(-b\frac{1}{\sqrt{E}}\right), \tag{21}$$

with

$$b = \frac{\sqrt{2\mu}}{4\epsilon_0}\frac{Z_1 Z_2 e^2}{\hbar} \tag{22}$$

- Symbol μ represents the reduced mass of the system

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \approx \frac{A_1 A_2}{A_1 + A_2} \tag{23}$$

- P is a very rapidly changing function of the energy.

The Gamow factor

- The transmission coefficient for α tunnelling with the energy equal to the Q_α value is expressed in terms of the Gamow factor

$$T = \exp^{-2G} \implies 2G = b\frac{1}{\sqrt{Q_\alpha}} = 2\sqrt{\frac{2\mu}{\hbar^2 Q_\alpha}} Z_\alpha (Z-2)\frac{e^2}{4\pi\epsilon_0}\frac{\pi}{2} \quad (24)$$

- Above $Z_\alpha = 2$ and Z and $Z-2$ are the atomic numbers of the α, the parent, and the daughter nucleus.
- Note that the above equation is equivalent to Eq. 7.17 in the textbook except for the difference in the units of charge.
- The textbook uses for the Coulomb energy

$$V_C = \frac{Z_1 Z_2 e^2}{r} \quad (25)$$

while in the SI units

$$V_C = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r} \quad (26)$$

The α-decay rate

- The decay rate, given by the inverse lifetime,

$$\lambda_\alpha = \frac{1}{\tau_\alpha} \tag{27}$$

  represents average number of α decays per second.

- Let us assume that the decay rate is completely determined by the quantum mechanical tunnelling process. This idea was originated by Gorge Gamow in late $\sim$ 1920.

- Gamow postulated that the decay rate is a product of the frequency f at which the α particle impinges on the barrier and the transmission coefficient T through the barrier

$$\lambda_\alpha = fT = f \exp^{-2G} \tag{28}$$

The frequency factor

- The average time at which the α particles trapped inside the parent potential well impinges on the barrier is given by the diameter of the well $D = 2R$ divided by the velocity of the α-particle

$$t = \frac{2R}{v} \tag{29}$$

- The frequency is the inverse of that time

$$f = \frac{v}{2R} \tag{30}$$

- The velocity can be calculated from the kinetic energy of the α particle in the well

$$\frac{\mu v^2}{2} = V_0 + Q_\alpha \tag{31}$$

with V_0 representing the depth of the well.

The decay rate

- With the frequency factor

$$f = \frac{v}{2R} = \sqrt{\frac{2(Q_\alpha + V_0)}{\mu}}\frac{1}{2R} \tag{32}$$

and the transmission factor

$$T = \exp^{-2G} = \exp\left(-\frac{b}{Q_\alpha}\right) \tag{33}$$

the rate is

$$\lambda_\alpha = fT = \sqrt{\frac{2(Q_\alpha + V_0)}{\mu}}\frac{1}{2R}\exp\left(-\frac{b}{Q_\alpha}\right) \tag{34}$$

with Z_D being the atomic number of the daughter.

The Geiger-Nuttall law

- The above result from the tunnelling model yields

$$\ln(\lambda_\alpha) = \ln(\frac{1}{\tau_\alpha}) = -\ln(\tau_\alpha) =$$

$$\ln\left(\sqrt{\frac{2(Q_\alpha + V_0)}{\mu}}\frac{1}{2R}\exp\left(-\frac{b}{\sqrt{Q_\alpha}}\right)\right)$$

$$= -b\frac{1}{\sqrt{Q_\alpha}} + \ln\left(\sqrt{\frac{2(Q_\alpha + V_0)}{\mu}}\frac{1}{2R}\right) \quad (35)$$

- Since usually $Q_\alpha < V_0$ the last term in the above can be approximated as a constant

$$-a = \ln\left(\sqrt{\frac{2(Q_\alpha + V_0)}{\mu}}\frac{1}{2R}\right) \quad (36)$$

The Geiger-Nuttall law

- With the above substitution we derive the Geiger-Nuttall law

$$\ln(\tau_\alpha) = \frac{b}{\sqrt{Q_\alpha}} + a \quad (37)$$