Alpha decay

Introduction to Nuclear Science

Simon Fraser University
Spring 2011

NUCS 342 — February 21, 2011

Outline

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2. The hindrance factor
3. The barrier radius

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Correlation between Q_α and α decay half-life

- Investigation of α decay by Rutherford's students and collaborators Hans Geiger and John M. Nuttall lead to the observation of correlation between the Q_α value and α decay half-life

$$\ln(\tau_\alpha) = A + \frac{B}{\sqrt{Q_\alpha}} \tag{1}$$

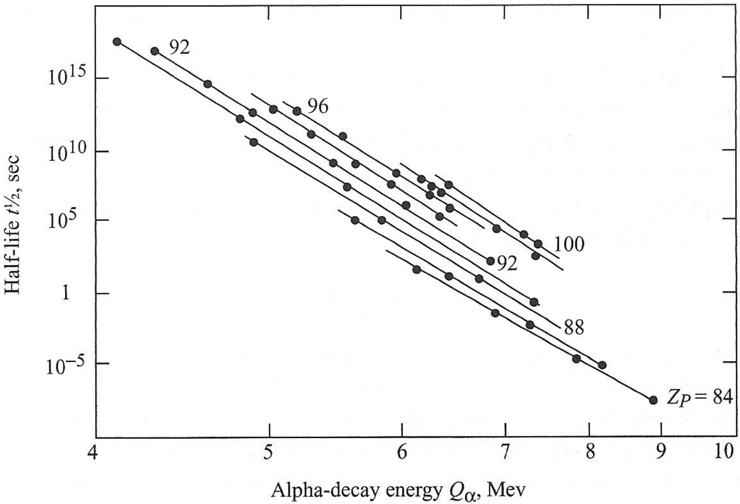
- As realized by Gorge Gamow calculations based on the tunnelling result in a nearly similar correlation

$$\ln(\tau_\alpha) = a + \frac{b}{\sqrt{Q_\alpha}} \tag{2}$$

with the parameters defined by the Z and A of the parent:

$$b = \frac{\sqrt{2\mu}}{4\epsilon_0}\frac{2(Z-2)e^2}{\hbar} \quad a = -\ln\left(\sqrt{\frac{2(Q_\alpha + V_0)}{\mu}}\frac{1}{2R}\right) \quad \mu = \frac{4(A-4)}{A}u \tag{3}$$

The Geiger-Nuttall plot



The hindrance factor

- With the derivation of the correlation between the lifetime and Q_α it is now possible to calculate the lifetimes for the decays with the Q_α measured.
- This calculations yield lifetimes by a factor of 10-10^6 times shorter than observed experimentally.
- In a sense this may indicate the failure of the tunnelling model in predicting α-decay rates.
- Remember, however, that for a given element, the tunnelling model predicted proper dependence on the Q_α.
- This indicate rather that we have partial, but incomplete understanding and it is worth while to revise our basic assumptions.

The hindrance factor

- So far we assumed that the rate for the α decay is

$$\lambda_{\alpha}^{calc.} = \frac{1}{\tau_{\alpha}^{calc.}} = fT \tag{4}$$

- But we measure

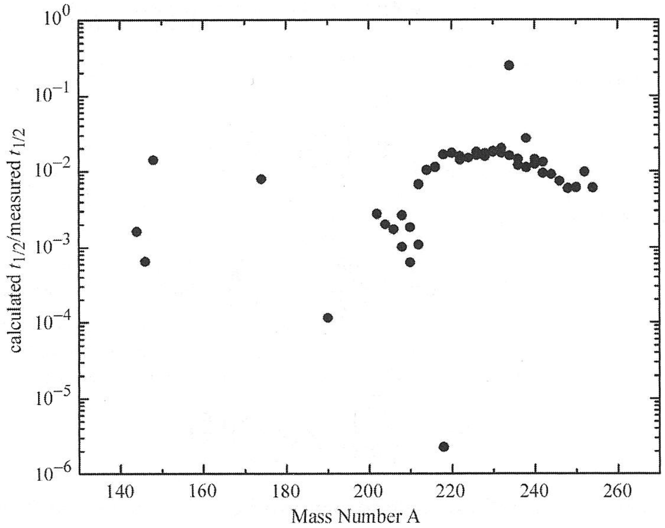
$$\lambda_{\alpha}^{exp.} = \frac{1}{\tau_{\alpha}^{exp.}} \tag{5}$$

- We can define the hindrance factor as the ratio of the measured to calculated rate equal to the ration of the calculated to measured lifetime

$$h = \frac{\lambda_{\alpha}^{exp.}}{\lambda_{\alpha}^{calc.}} = \frac{\tau_{\alpha}^{calc.}}{\tau_{\alpha}^{exp.}} = \frac{1}{fT\tau_{\alpha}^{exp.}} \tag{6}$$

- The hindrance factors vary between 0.1 to 10^{-6}

The hindrance factor plot



The hindrance factor

- Knowing the hindrance factor h the rate for α decay is

$$\lambda_\alpha = hfT \tag{7}$$

  with f being the frequency at which α particle impinges on the barrier and T being the transmission factor.

- The above assumption separates the tunnelling through the barrier, defined by the fT factor, from other effects which may impact the decay.
- The impact of the other effects on the rate is manifested by the magnitude of the hindrance factor h.
- These other effects are most likely related to the structure of the decaying system.
- With the above assumption the effort in calculations is shifted towards predicting the hindrance factors.

Impact of structure effects

- We will now examine an impact of several structure effects on the rate of α decay.
- These are
  - the barrier radius,
  - the overlap between the initial and final states,
  - the pre-formation factor,
  - the conservation laws,
  - the centrifugal barrier.

The barrier radius

- For calculations of the transmission factor we used the approximation

$$T = \exp^{-2G} = \exp\left(-\frac{b}{Q_\alpha}\right) \quad b = \frac{\sqrt{2\mu}}{4\epsilon_0}\frac{2(Z-2)e^2}{\hbar} \tag{8}$$

  which is valid for α-decay Q-values much smaller than the Coulomb barrier $Q_\alpha << V_C$.

- This is not a good approximation since the Coulomb barrier is larger than the Q_α value by a factor smaller than an order of magnitude.

- The full expression for the transmission factor

$$T = \exp\left(-2KR_C\left[\frac{\arctan\sqrt{\frac{R_C}{R_B}-1}}{\sqrt{\frac{R_C}{R_B}-1}} - \frac{R_B}{R_C}\right]\right) \quad K = \sqrt{\frac{2\mu}{\hbar^2}(V_C - Q_\alpha)}$$

  with R_B and R_C being the barrier radius and the distance to the classical turning point is a sensitive function of the barrier radius R_B.

The pre-formation factor

- So far we assumed that the rate for the α decay is product of the frequency at which α particle impinges on the barrier and the tunnelling transmission factor.
- This relationship, however, assumes that a cluster of nucleons resembling an α-particle exists inside the parent nucleus, or in another words, that the parent can be seen as a daughter- α system.
- This picture is rather inconsistent with our assumption about the spherical shell model in which nucleons are moving on independent orbitals.
- Pairing actually helps since it clusters pairs of like nucleons.
- However, we have not yet accounted for a probability to cluster two pairs into an α particle.

The pre-formation factor

- We can define the pre-formation factor as a probability for the parent nucleus to exist in a state which can be represented as a state in the daughter nucleus and an α particle at the radius corresponding to the radius of the barrier.
- According to the quantum mechanics this probability is given by the squared matrix element of the overlap between the parent and daughter/α wave functions

$$p = \langle \Psi(A, Z) \mid [\Psi(A-4, Z-2) \otimes \Psi(4, 2)]_{r=R_B} \rangle^2 \tag{9}$$

- The pre-formation factor can in principle be calculated from nuclear models. I practise, complex cluster models are needed.
- The hindrance factor is proportional to the pre-formation factor

$$h \sim p \tag{10}$$

but can be smaller than p if other factors reduce the rate.

Impact of conservation laws

- In addition to energy conservation the decays are subject to angular momentum and parity conservation laws.

- Conservation of angular momentum requires that the vector sum of the spin of the daughter J_D, spin of the α-particle J_α and the orbital angular momentum in the centre of mass L is equal to the spin of the parent J_P:
$$\vec{J_D} + \vec{J_\alpha} + \vec{L} = \vec{J_P} \tag{11}$$

- Conservation of parity requires that the product of the parity for the daughter $\pi(J_D)$, the α-particle $\pi(J_\alpha)$ and the orbital angular momentum in the centre of mass $(-1)^l$ is equal to the parity of the parent $\pi(J_P)$:
$$\pi(J_D)\pi(J_\alpha)(-1)^l = \pi(J_D) \tag{12}$$

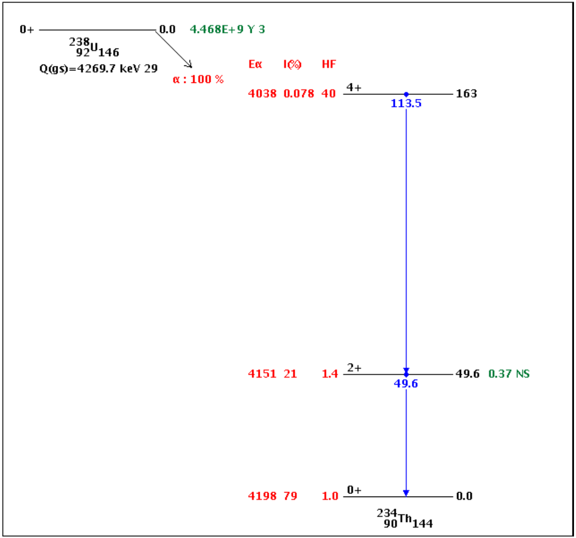
Impact of conservation laws

- The impact of the selection rules is very dramatic for the emission of the spin-less α particle from spin 0^+ ground state in even-even nuclei.
- In this case $\vec{J}_P = \vec{J}_\alpha = 0$ and $\pi(J_P) = \pi(J_\alpha) = 1$.
- The selection rules are $\vec{L} = -\vec{J}_D$ and $\pi(J_D) = (-1)^l = (-1)^{J_D}$.
- This implies direct correspondence between the spin/parity of the state populated by the decay in the daughter nucleus and the angular momentum channel of the decay.
- In consequence the only states which can be populated by α decay from ground states of even-even nuclei are these of even spin and positive parity or odd spin and negative parity.
- Most even-even nuclei have no low-lying negative parity states, therefore, predominantly even spin parity states are populated in the α decay of ground states of even-even nuclei.

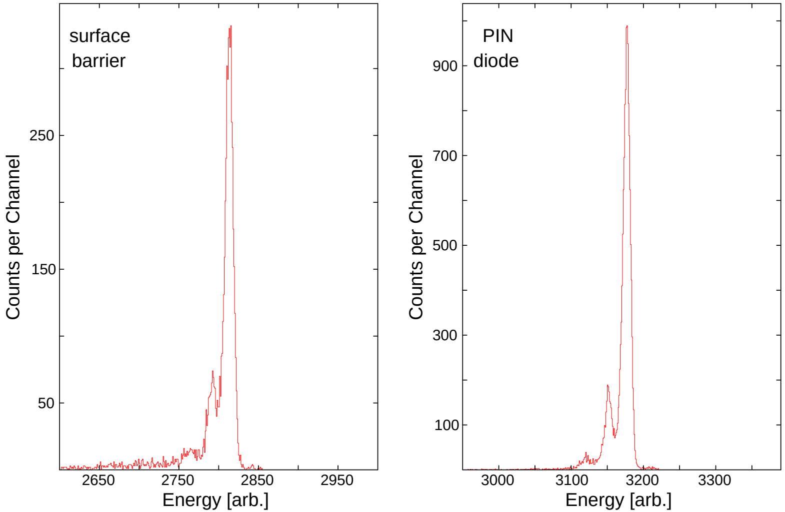
The fine structure in α decay

- So far we were discussing α decay from a ground state of an even-event nucleus, however, without specifying the final state in the daughter.
- In this case for most of the time the decay populates the ground state in the daughter.
- This is not true always though, in fact we have seen from the conservation laws that excited states of natural parity can be populated in the daughter nuclei.
- Population of multiple states in the daughter give rise to the fine structure in the α decay, the name indicates splitting of the line in the α spectrum into several peaks with comparable energy.

The fine structure in α-decay of ^{238}U



The fine structure in α-decay of ^{241}Am



The centrifugal barrier

- You might have noticed that the probability of α decay decreases as a function of decreasing energy of the decay.
- This is always true for α decay of an even-even parent.
- Note that α decays at the highest energy populates the ground state in the daughter with spin parity 0^+. Lower energy α decays populate states of higher spin and energy in the daughter.
- Conservation laws imply that higher spin of the daughter J_D corresponds to the higher angular momentum channel L for the decay.
- We observe that higher angular momentum channels are suppressed more in the α-decay process.
- The reason is that α particles truly has to tunnel through the Coulomb and the centrifugal barrier and that the height and the width of the centrifugal barrier depends on the angular momentum of the decay channel.

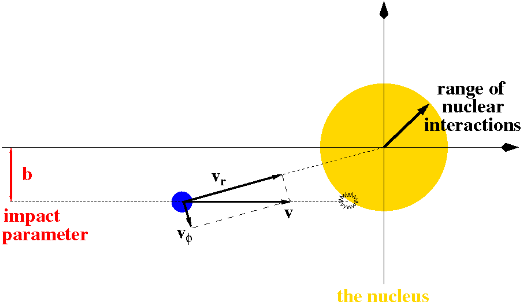
Centrifugal barrier

- The energy of the α in the centre of mass is

$$E = K + V = \frac{mv^2}{2} + V(r). \tag{13}$$

- Using the radial coordinates

$$v^2 = v_r^2(r) + v_\phi^2(r) = v_r^2(r) + r^2\omega^2(r). \tag{14}$$



Centrifugal barrier

- The angular speed ω can be expressed using angular momentum L

$$L = \mu r^2 \omega, \quad \text{or} \quad \omega = \frac{L}{\mu r^2}. \tag{15}$$

- Combining the above gives the following for the energy

$$E = \frac{\mu v_r^2}{2} + \frac{L^2}{2\mu r^2} + V(r). \tag{16}$$

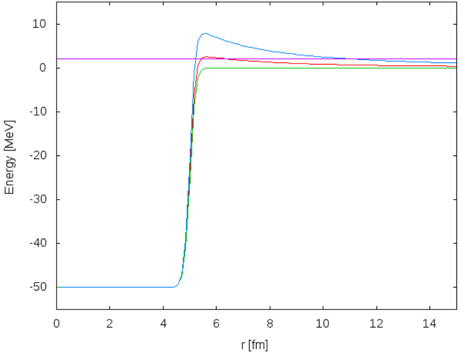
- Angular momentum for the decay is conserved and quantized

$$L^2 = l(l+1)\hbar^2. \tag{17}$$

- The energy can be split into the part which depends on radial speed and one which depends on the relative distance r.

Centrifugal barrier

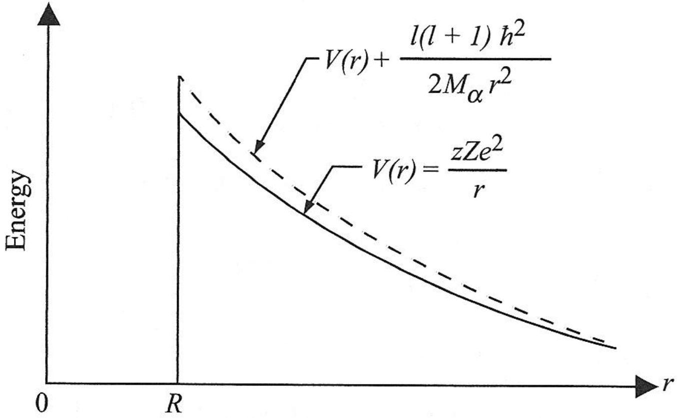
- The part of energy which depends on the relative distance consist of the potential energy $V(r)$ and the centrifugal barrier $\frac{L^2}{2\mu r^2}$.
- The diagram of r-dependent terms of energy as a function of the relative distance is a convenient way to represent decay energetics.



- Potential for nuclear interactions assumed to have -50 MeV depth and radius of 5 fm. Graphs are:
  - green for l=0 $\hbar$
  - red for l=1 $\hbar$
  - blue for l=2 $\hbar$
  - purple for $E = 2$ MeV.

The tunnelling barrier

The height of the barrier for α tunnelling increases with increasing angular momentum channel for the decay.



Branching ratios in the fine structure

- To understand relative intensities of α branches to excited states in the daughter from the ground state of an even-even parent one needs to recognize that there are two factors which impact the rate

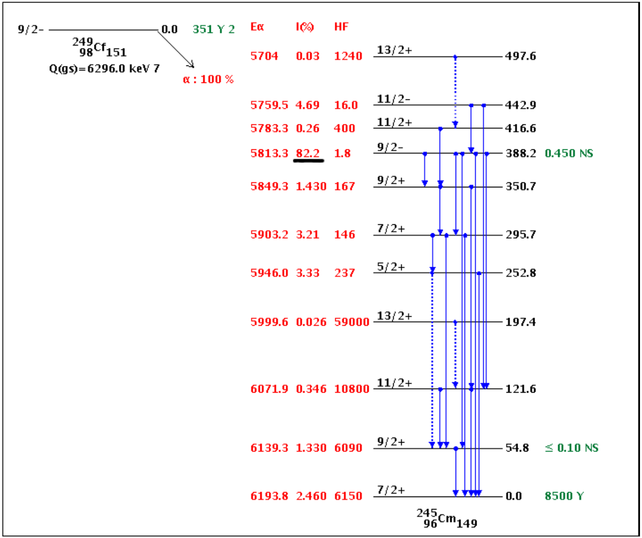
  1. The branches to the excited states have lower Q_α values.
  2. The branches to the excited states are suppressed by the centrifugal barrier.

- Both of these factors reduce the rate for α decay.
- The lower Q_α reduces the rate more than the increased angular momentum. This is a consequence of the fact that the centrifugal barrier is significantly smaller than the Coulomb barrier.

Decay of odd mass parents

- The nuclear structure effects are manifested very strongly in α decay of odd-mass parents.
- In such decays, the ground state of the parent has a structure defined by the valence nucleon.
- Typically, the daughter in the ground state has a structure defined by a different valence nucleon. Therefore there is a mismatch in structure between the parent and the daughter ground state.
- However, there are excited states in the daughter/α system which have a structure matching the ground state parent.
- For that reason the decay of odd-mass parents proceed to excited states in daughter nuclei.

The fine structure in α-decay of ^{249}Cf



Heavy cluster emission

- The mechanism of tunnelling through the barrier which we discussed for α-decay can give rise to emission of other heavy, strongly bound clusters, like ^{12}C or ^{16}O.
- The energetics of such emission can be worked out from measured masses aided by the Liquid Drop Model.
- The decay rates can be estimated by a model analogues to the Gamow model.
- The estimates of the rates yield branching ratios which should allow for observation of heavy cluster emission.
- The searches are undergoing, the emission of ^{14}C from ^{223}Ra has been observed at the level of 10^{-9} of the α emission.

Proton radioactivity

- The proton drip line is define by nuclei for which the separation energy of the valence proton is zero or negative.
- The protons in nuclei at and beyond the proton drip line are not ejected instantaneously, they must tunnel through the Coulomb barrier to get free!
- Since the tunnelling is slow, the proton-rich nuclei beyond the drip line can have lifetimes which are relatively long, on the millisecond (10^{-6}) scale.
- This has to be contrasted with the nuclei on the neutron drip line for which there is no Coulomb barrier for a neutron decay. These nuclei decay on the time-scale of 10^{-20} s.