Gamma-ray decay

Introduction to Nuclear Science

Simon Fraser University
Spring 2011

NUCS 342 — March 7, 2011

Outline

Outline

Outline

Outline

Outline

Recoil in the absorption and emission

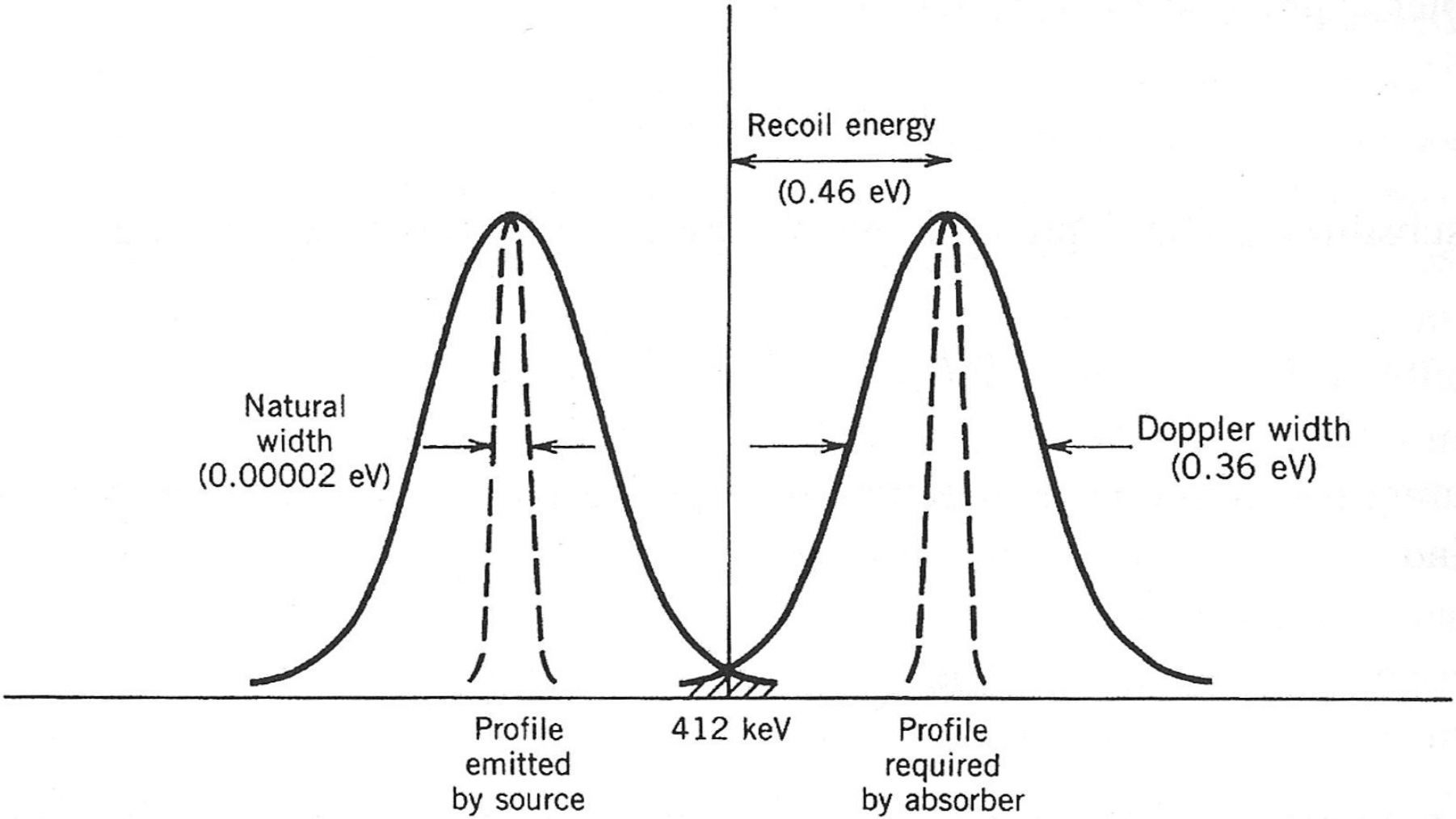
- Energy conservation in γ-ray decay leads to the following result for the energy of the γ ray in the absorption or emission process

$$E_\gamma = \Delta E \pm \delta$$
$$\delta = \frac{1}{2}\frac{(\Delta E)^2}{M_d c^2} \quad (1)$$

  with ΔE being the absolute value of the energy difference between the initial and the final state, δ being the recoil energy correction, M_d being the mass of the daughter and plus/minus sign referring to absorption/emission.

- δ is large comparing to the natural line width and also to the Doppler-broaden line width in room temperature.

The recoil on emission and absorption



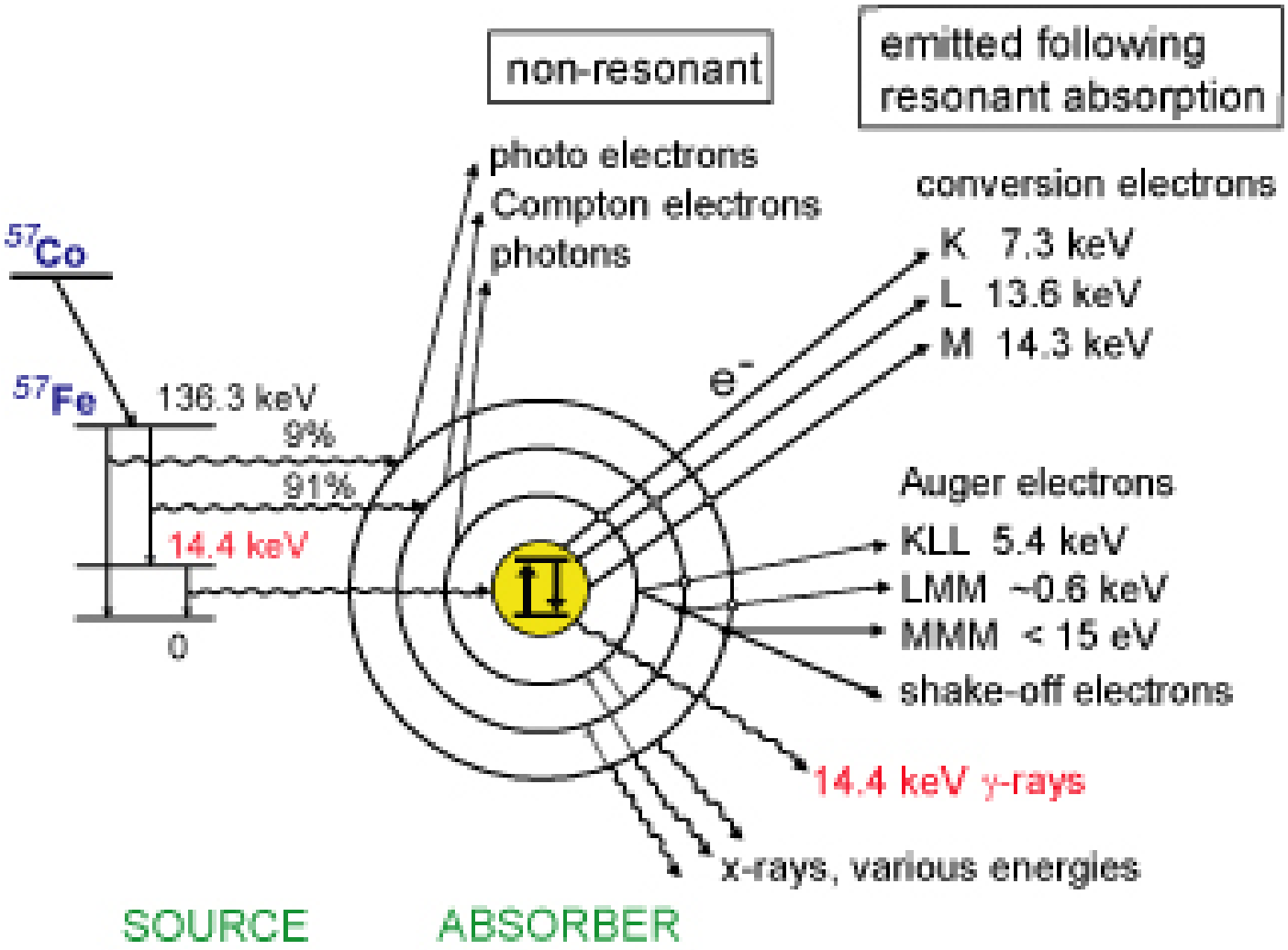
Recoil less absorption and emission

- Because of the recoil terms a γ-ray emitted by an isotope can not be absorbed by the same isotope.
- In 1957 R. Mössbauer postulated in his PhD thesis the effect of recoil-less absorption and emission for sources and absorbers embedded into crystal latices.
- In such a case the mass of the recoiling object is not the mass of the single daughter nucleus M_d in the denominator in the expression for δ but rather the mass of the whole crystal which is on the order of $N_A M_d$.

$$E_\gamma = \Delta E \pm \delta \qquad \delta = \frac{1}{2}\frac{(\Delta E)^2}{N_A M_d c^2} \approx 0 \tag{2}$$

- Since this mass is large the recoil correction δ becomes nearly zero and a gamma-ray emitted by a source embedded in a crystal lattice can be absorbed by a nucleus of the same isotope embedded into a crystal lattice.

The recoil less emission and absorption



Mössbauer spectroscopy

- The effect of recoil less emission and absorption is called the Mössbauer effect and lead to the Nobel Prize in 1961.
- The Mössbauer effect become very swiftly a very important analytical tool and prompted development of the field of Mössbauer spectroscopy.
- The key to understand the Mössbauer spectroscopy is to realize our ability to change the γ ray energy of the emitter utilizing the Doppler effect.
- The next key factor to recognize is that the absorption of the γ rays is resonant, means only gamma rays with energies matching the absorption energy within the line width are absorbed.

Mössbauer spectroscopy experiment

transmissions method

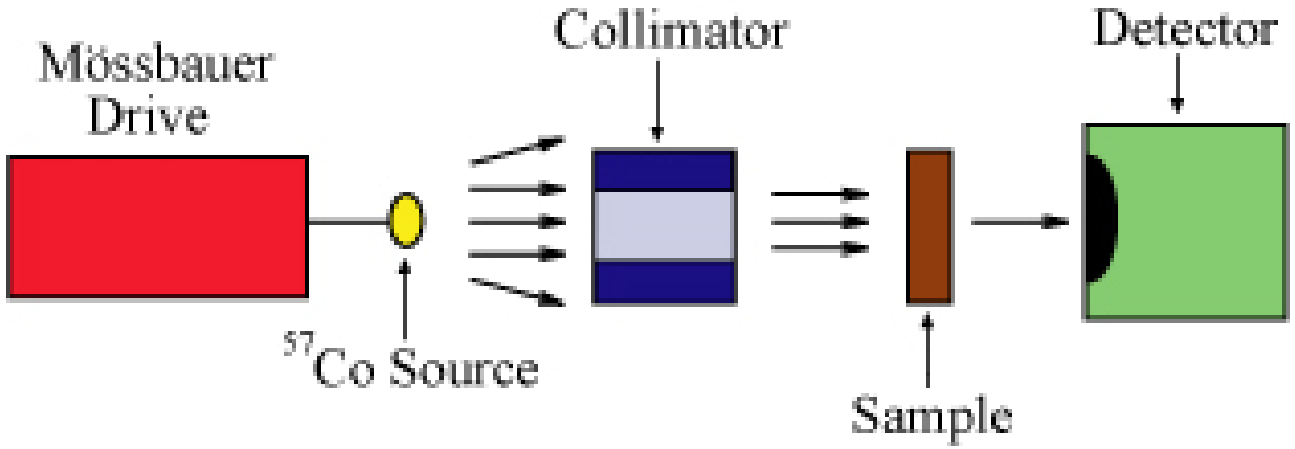
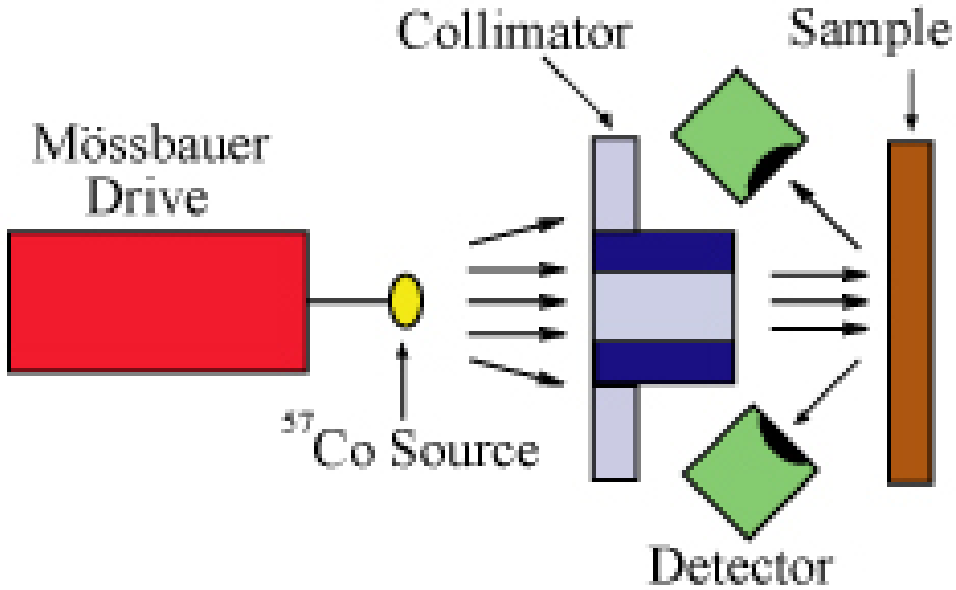


photo activation method



The speed of the drive

- The Doppler effect is used to change the energy of the γ rays.
- In a Mössbauer spectroscopy experiment the energies are scanned over an energy interval in search for the resonant absorption.
- The energy interval is defined by the line width of the absorber. The scan should encompass several line widths around the expected resonant energy.
- The question arises what should be the speed (and mechanical construction) of the drive to realize this energy interval.
- This question can be addressed by calculating the speed of the drive which corresponds to the Doppler shift equal to the line width of interest.
- Let us do this calculations for the 14.4 keV γ-ray emitted as the $3/2^- \rightarrow 1/2^-$ transition in ^{57}Fe which is a common Mössbauer source.

The speed of the drive

- The $3/2^-$ state in ^{57}Fe has a half-life of $t_{1/2}$=98 ns.
- The natural line width of the state is

$$\Gamma = \frac{\hbar}{\tau} = \frac{\hbar ln(2)}{t_{1/2}} = 4.65 \times 10^{-9} \text{ eV} \tag{3}$$

- The Doppler shift is

$$E = E_0 \frac{\sqrt{1-\beta^2}}{1-\beta\cos\theta} \tag{4}$$

- Let us assume the drive approaching the sample at $\theta = 0$. This implies

$$E = E_0 + \Gamma = E_0 \frac{\sqrt{1-\beta^2}}{1-\beta} = E_0 \sqrt{\frac{1+\beta}{1-\beta}} \approx E_0(1+\beta) \tag{5}$$

The speed of the drive

- Simplifying

$$E_0 + \Gamma \approx E_0(1+\beta) \implies \Gamma \approx E_0\beta$$

$$\beta = \frac{\Gamma}{E_0} = \frac{4.65 \times 10^{-9}}{14.4 \times 10^3} = 0.32 \times 10^{-12} \tag{6}$$

$$v = \beta c = 0.32 \times 10^{-12} * 3 \times 10^8 = 10^{-4}\ [\mathrm{m/s}] = 100\ [\mu\mathrm{m/s}]$$

- Scanning requires an interval of speeds.
- Relatively small drive speeds on the order of mm or cm per second can scan the energy interval of several line widths.
- A way to handle scanning is to mount the source on a rotating turntable. The change of the radial speed with respect to the sample is realized then by rotation of the turntable.
- The energy of the γ-ray is derived from the measured speed of the drive at the time of absorption.

Mössbauer spectroscopy experiment

transmissions method

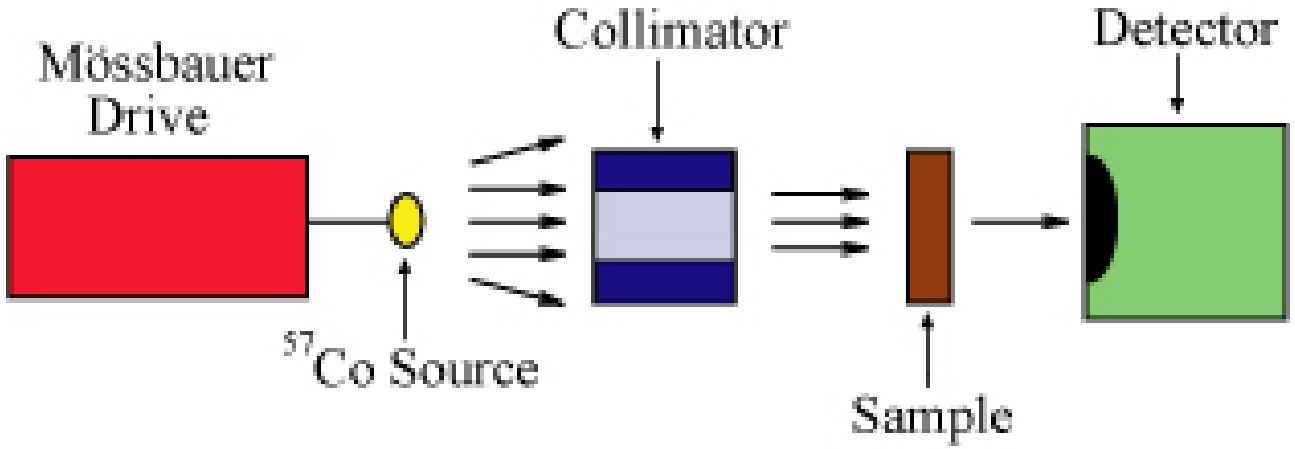
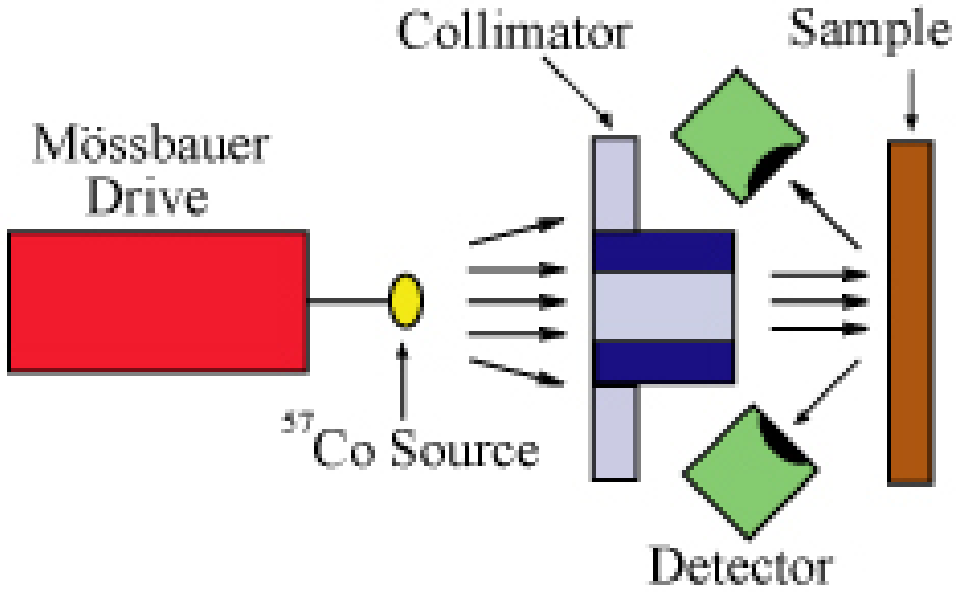


photo activation method



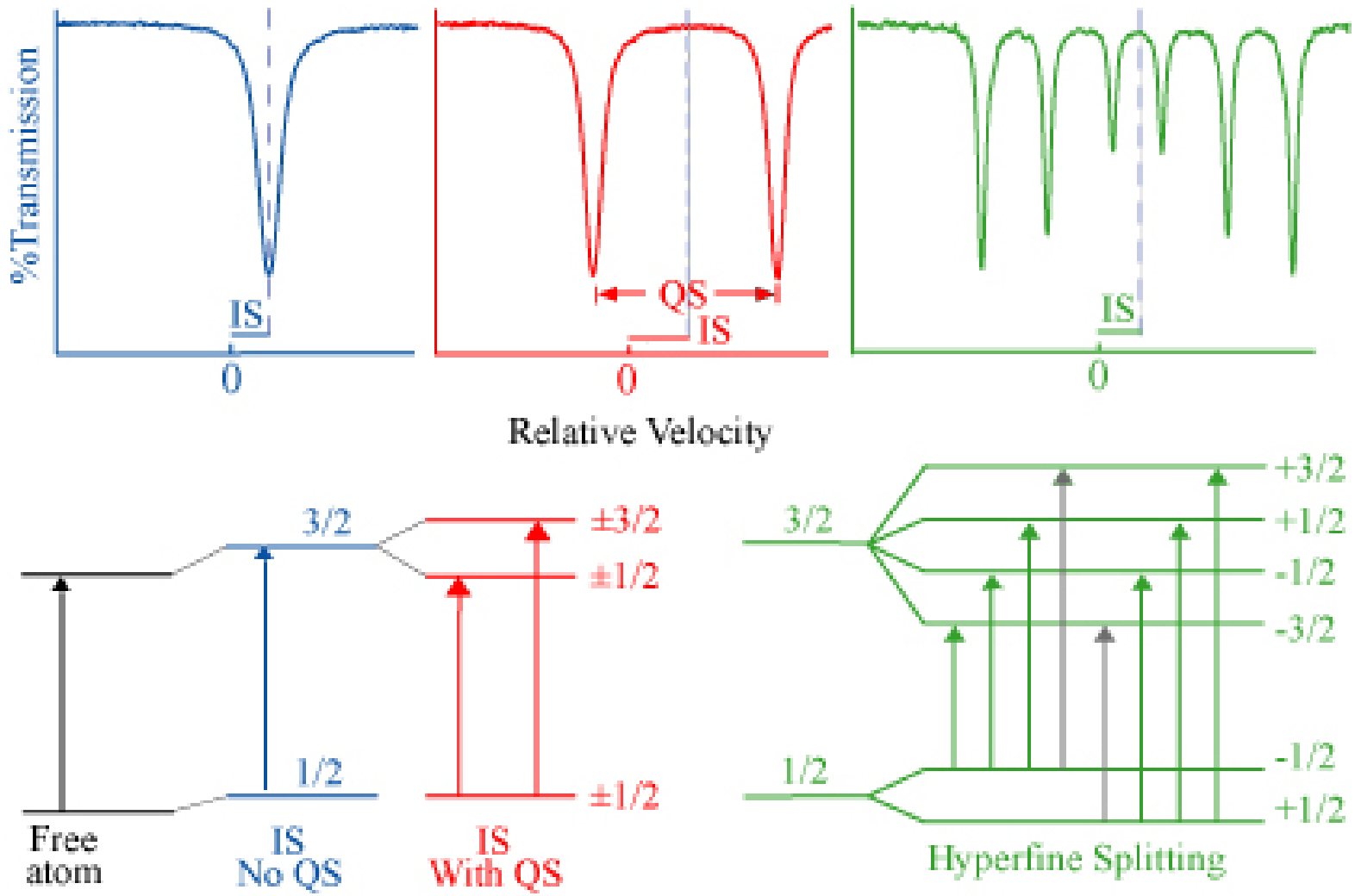
Why is Mössbauer spectroscopy useful

- Mössbauer spectroscopy can be used to detect the impact of the environment on the nuclear transitions.
- This is because of the fact that that the Mössbauer spectroscopy is capable of picking up the energy shifts for the transition of interest which are comparable to the line width or small fraction of eV.
- On that level of sensitivity atomic effects have an impact on the nuclear transitions through the so called hyperfine interactions.
- The atomic effects are defined by atomic structure, which is of interest to Chemistry.
- Thus the Mössbauer spectroscopy probes the chemical environment of a nucleus and provides information on properties like, for example, the number of electrons on the nucleus of interest.

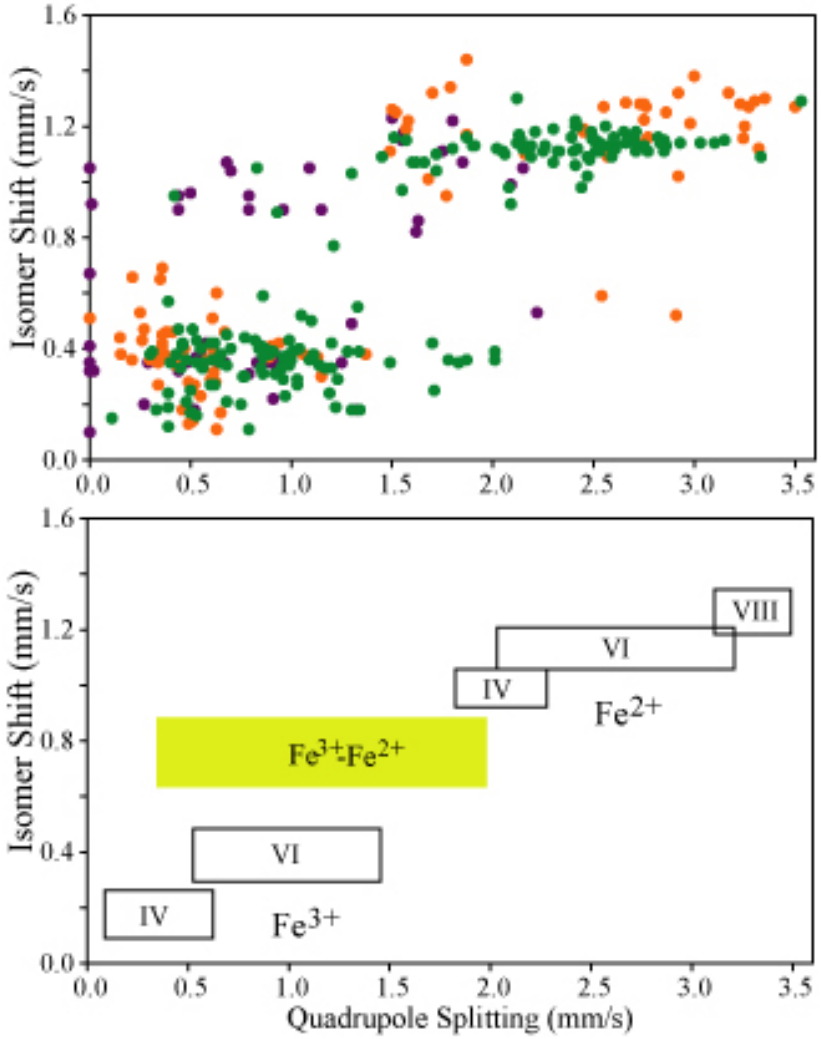
Hyperfine interactions

- The hyperfine interactions are the interaction of the electric and magnetic fields outside of a nucleus with the nuclear electric quadrupole and magnetic monopole moments.
- The nuclear electric quadrupole moment interacts with the electric field gradient at the location of the nucleus which leads to the quadrupole splitting of the magnetic substates.
- The nuclear magnetic dipole moment interacts with the magnetic field at the location of the nucleus which leads to the Zeeman splitting of the magnetic substates.
- The hyperfine interactions can be used as a probe of the atomic fields at the location of the nucleus.
- In molecules or solids hyperfine interactions provide a sensitive probe for access to the information on chemical environment the nucleus is embedded into.

Hyperfine interactions from Mössbauer spectroscopy



Charge of the molecular Fe from Mössbauer spectroscopy



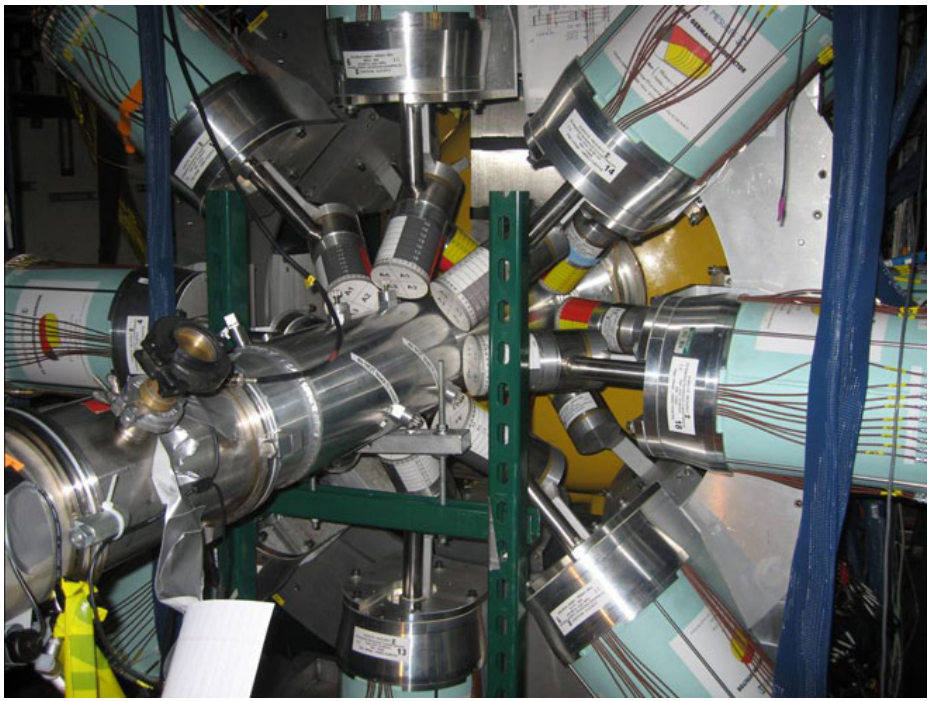
Gamma-ray spectroscopy

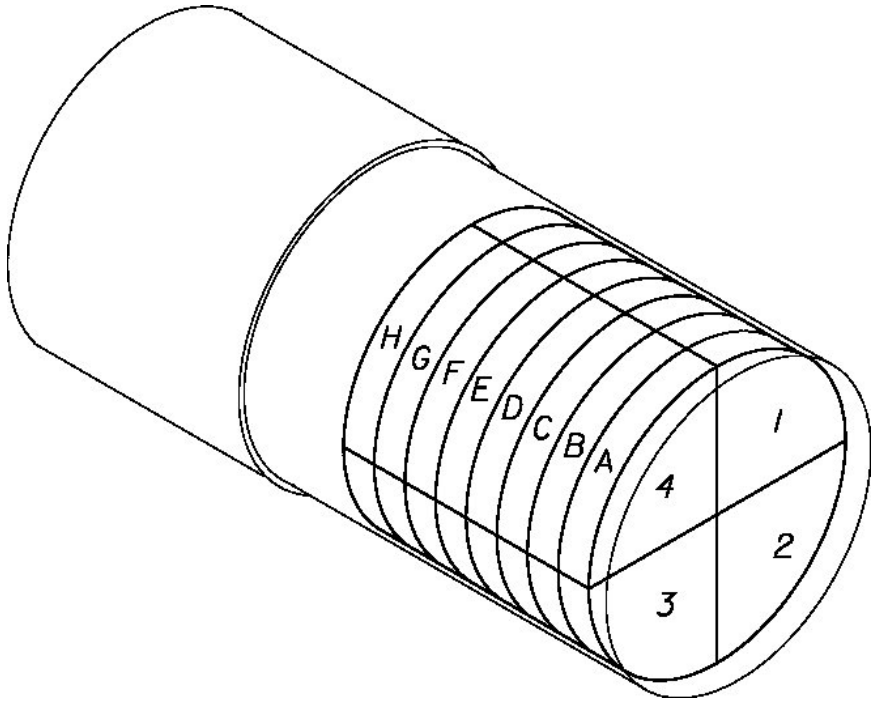
- While the Mössbauer spectroscopy is predominantly the tool to study the atomic/chemical effects and structure the γ-ray spectroscopy provides the tool to study the nuclear effects and structure.
- While the tiny shift between the initial and final state energy difference ΔE is crucial for the Mössbauer spectroscopy it is insignificant in the γ-ray spectroscopy which leads to the assumption

$$\delta << \Delta E \implies \delta \approx 0 \implies \Delta E = E_\gamma. \tag{7}$$

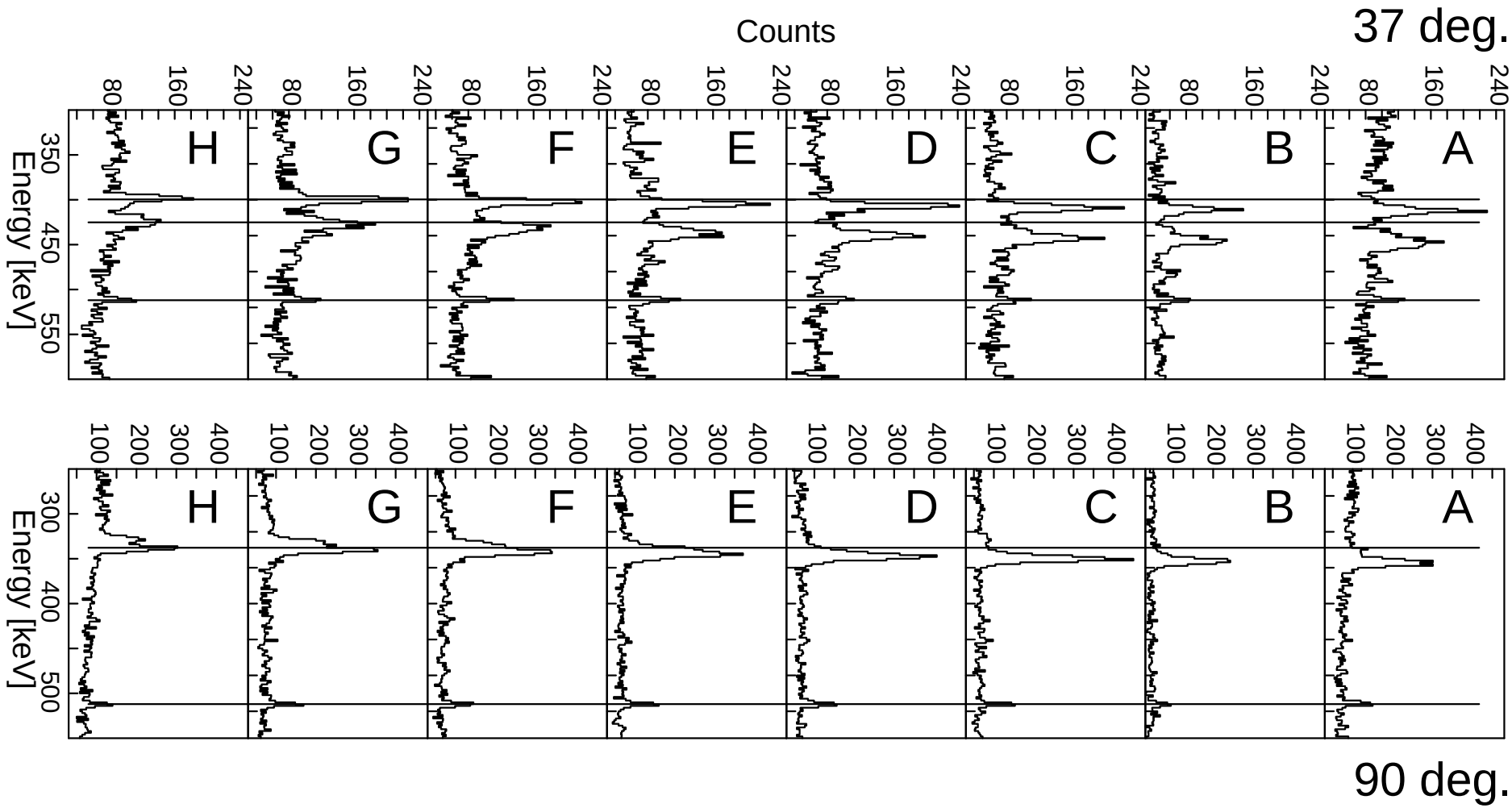
- Reactions and decays of various types can be used to produce excited nuclei of interest which then de-excite by γ-ray emission.
- Since the accuracy of γ-ray energy measurement can be better than part per thousand, γ-ray spectroscopy it the key tool to provide information on the level structure in these nuclei.

Segmented Germanium Array (SeGA) at the NSCL

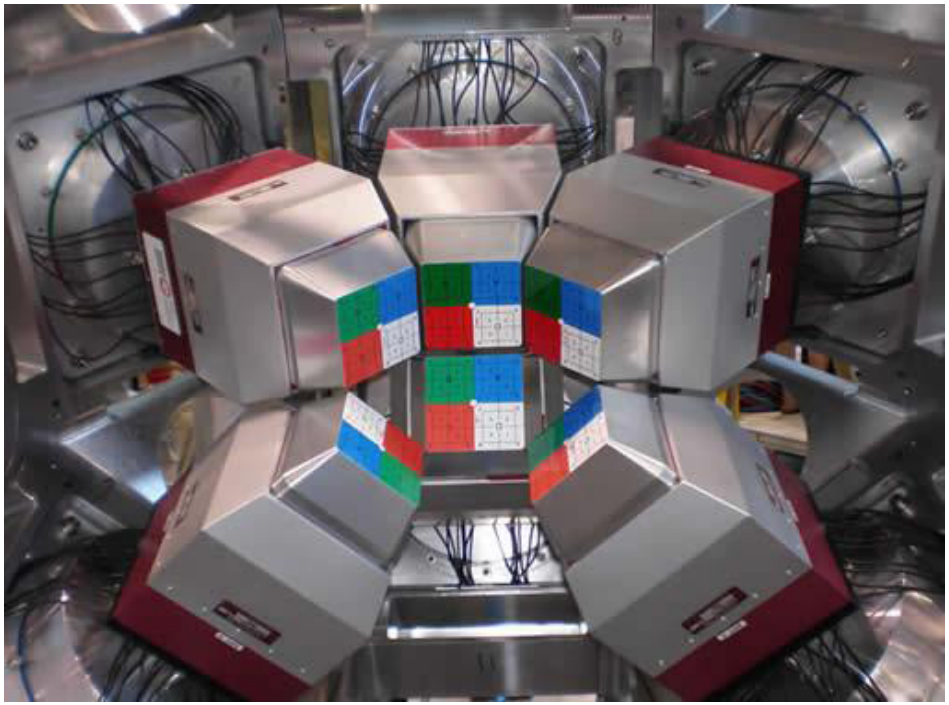


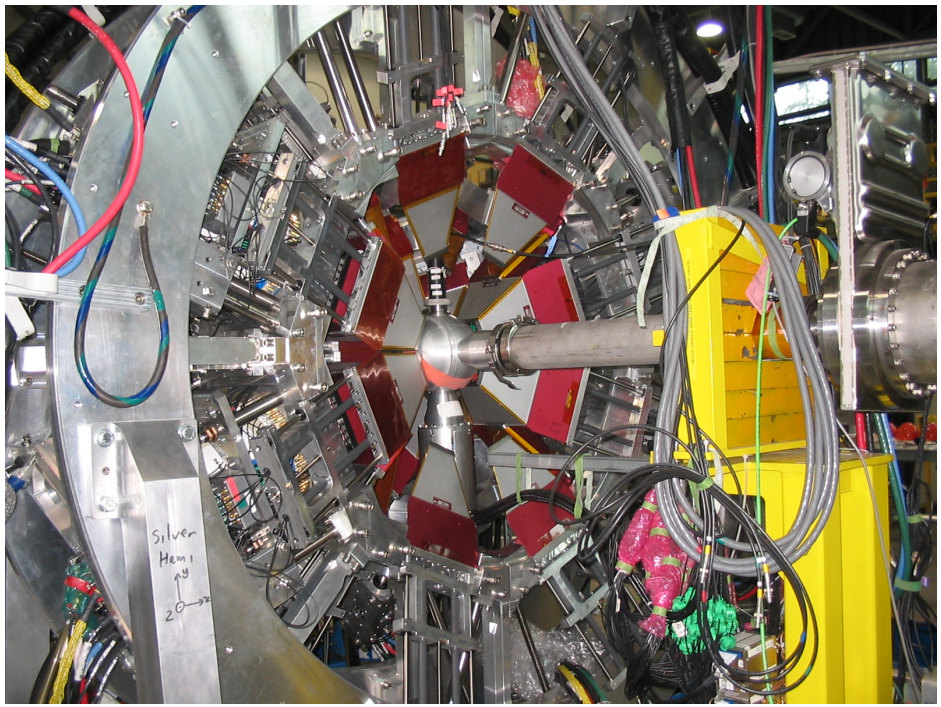


v/c sensitivity

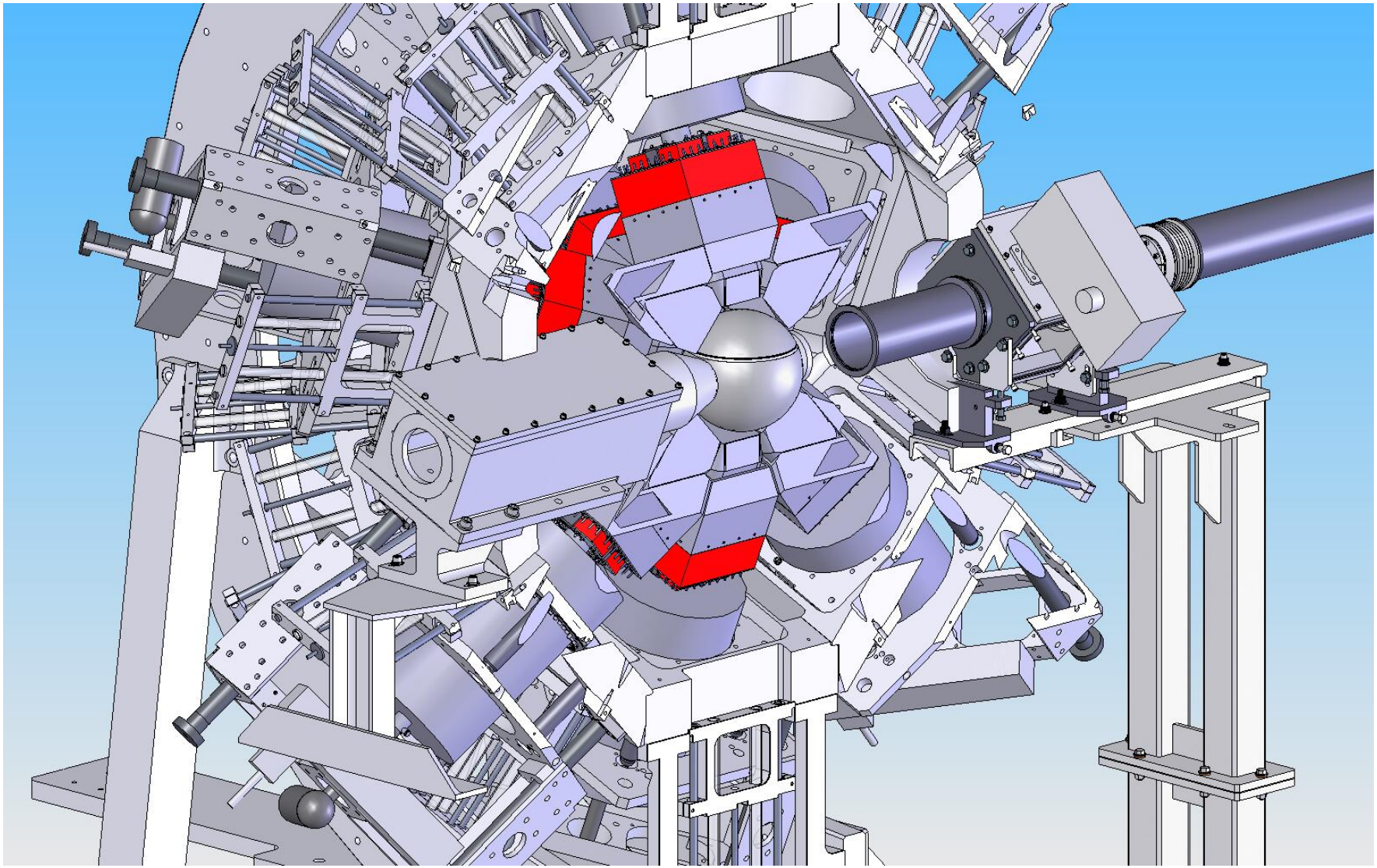


The TIGRESS array at TRIUMF

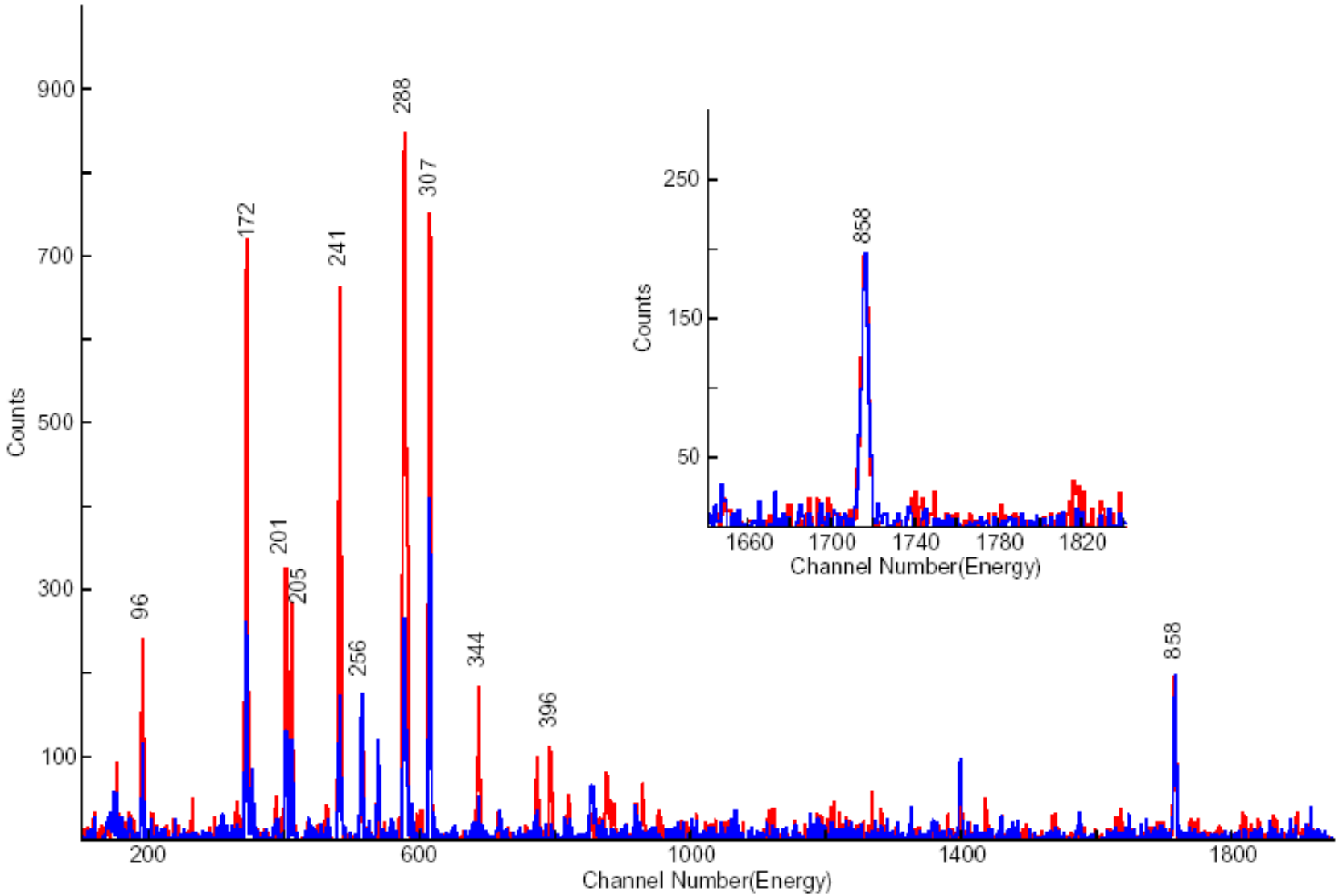




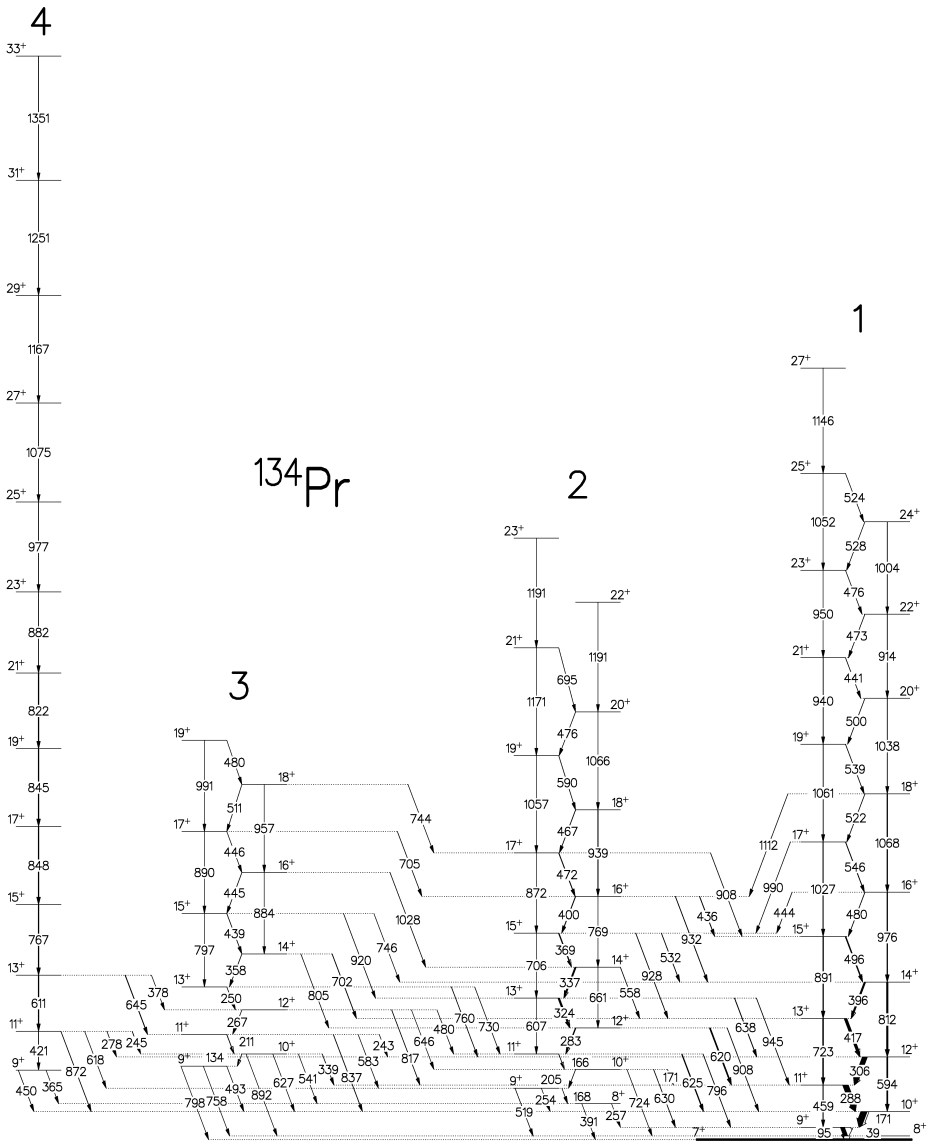
TIGRESS Integrated Plunger (TIP) device



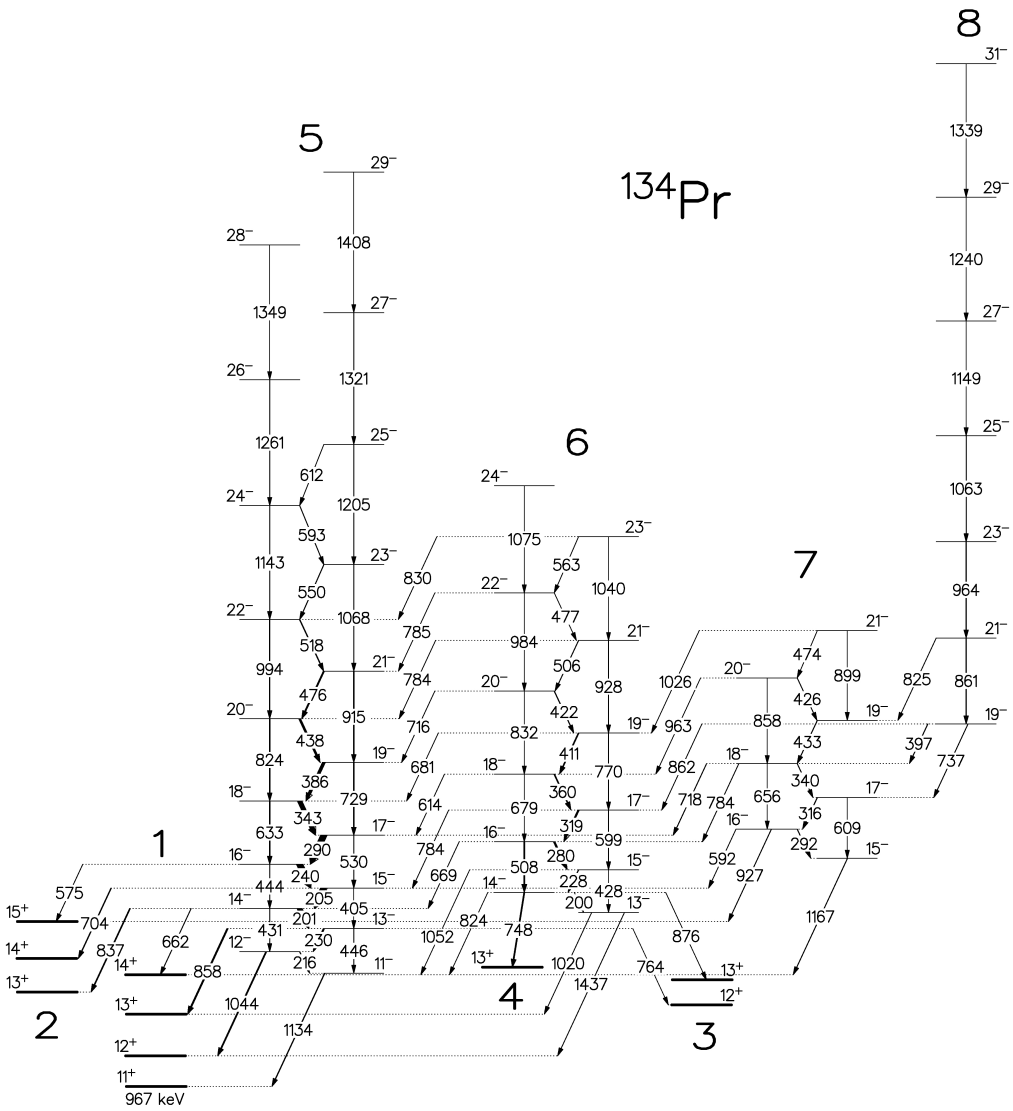
Gamma-ray spectra: ^{134}Pr



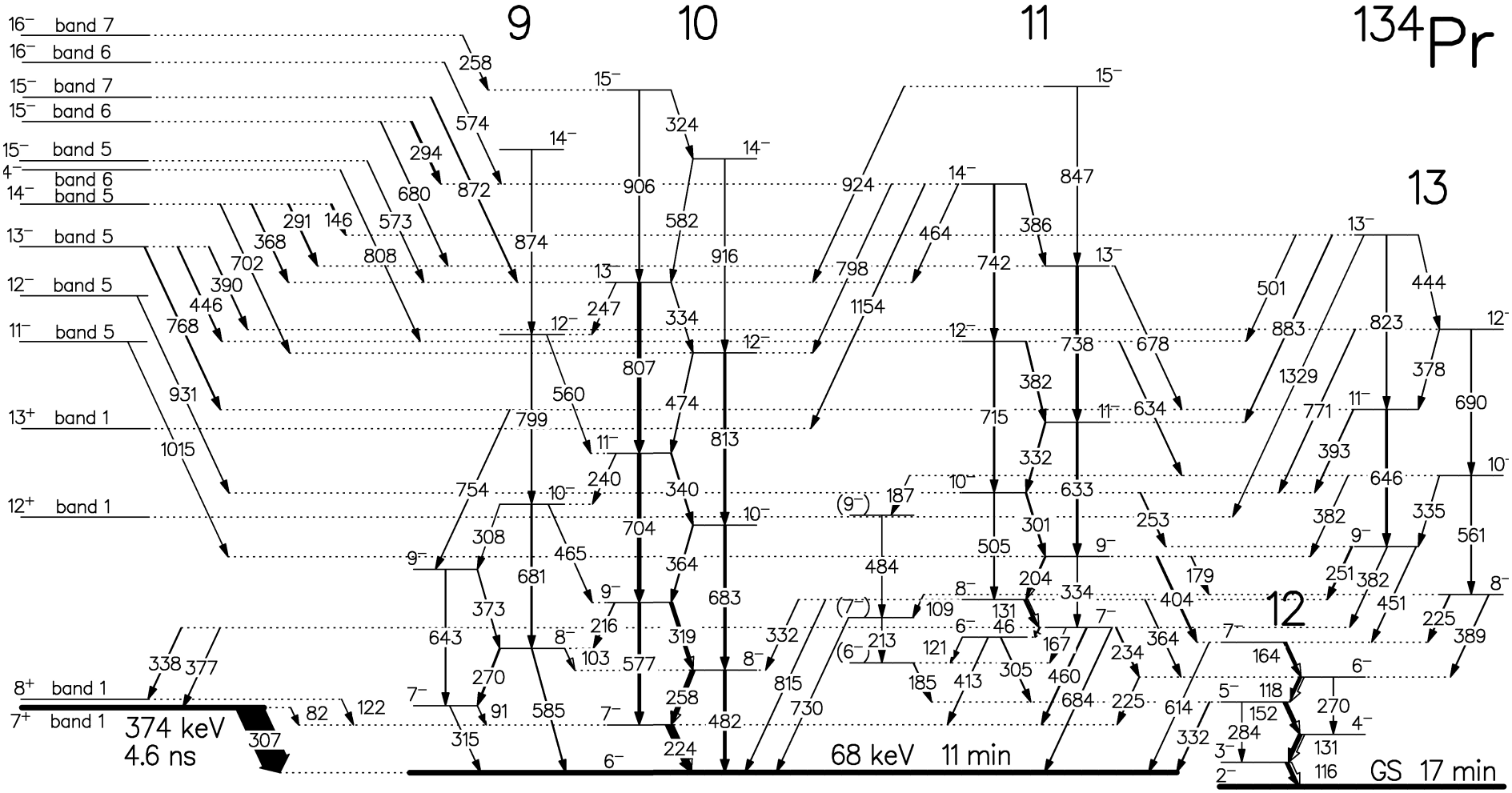
Band structure: ^{134}Pr



Band structure: ^{134}Pr



Band structure: ^{134}Pr



Selection rules in γ-decay

- Selection rules in γ-ray decay play a very important role in γ-ray spectroscopy as they can be used for measurements of spins and parties.
- The selection rules for γ-ray decay result from the laws of conservation of angular momentum and parity.
- A photon from the decay carries out an angular momentum of $\vec{L}$ and parity π.
- The conservation laws require that

$$\begin{aligned} \vec{I_i} &= \vec{I_f} + \vec{L} \\ \pi_i &= \pi_f \cdot \pi \end{aligned} \tag{8}$$

with I_i, π_i and I_f, π_f being the spin/parities for the initial and the final states, respectively.

Multipolarity of the decay

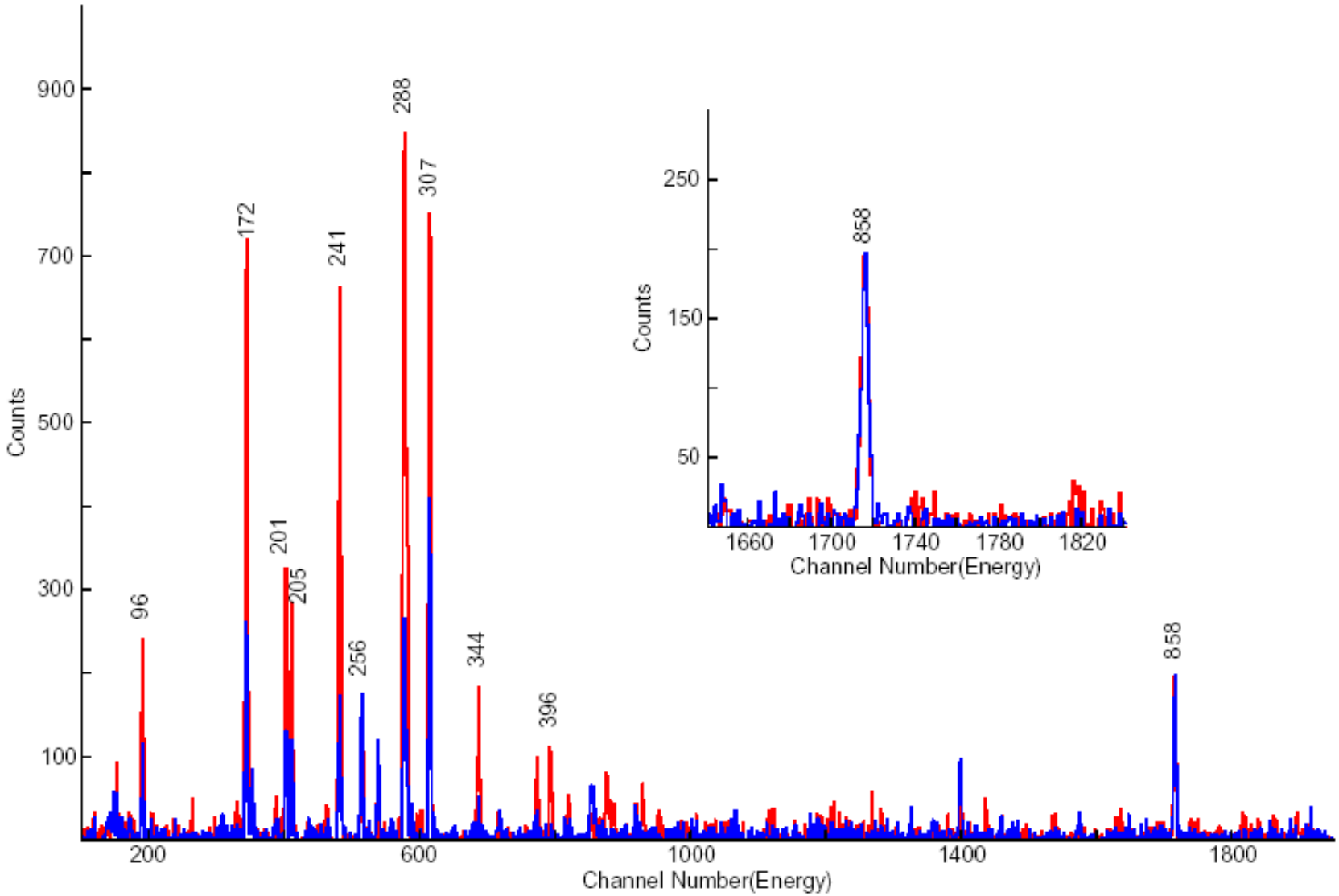
- Based on the angular momentum and parity change between the initial and the final state the γ-ray decays are classified according to their multipolarity L and the character π

Character/multipolarity	Symbol	L	$\mid I_i - I_f \mid$	$\pi = \pi_i \pi_f$
Electric dipole	$E1$	1	≤ 1	-1
Magnetic dipole	$M1$	1	≤ 1	+1
Electric quadrupole	$E2$	2	≤ 2	+1
Magnetic quadrupole	$M2$	2	≤ 2	-1
Electric octupole	$E3$	3	≤ 3	-1
Magnetic octuploe	$M3$	3	≤ 3	+1

Multipolarity of the decay

- In general transitions at increasing multipolarites have decreasing transition rates and increasing lifetimes.
- Transitions which differ by the multipolarity L have different angular distributions.
- Transitions of different character (magnetic/electric) differ by the linear polarization of the photon.
- Since the multipolarities of the decay can be distinguished the are the workhorse of the γ-ray spectroscopy for spins and parity measurements.

Angular distributions of the decay intensity



Single-particle γ-ray decay rates

- Crude estimates for γ-ray decay rates were calculated by Weiskopff assuming a single proton transition within a nucleus, nuclear radii proportional to $A^{\frac{1}{3}}$ and uniform, structure-less wave function. For γ-ray energies given in MeV the rates are:

$$\begin{aligned}
&\text{Electric dipole:} && \Gamma(E_\gamma) = \hbar\lambda = \hbar/\tau = 6.8 \times 10^{-2} A^{\frac{2}{3}} E_\gamma^3 \\
&\text{Magnetic dipole:} && \Gamma(E_\gamma) = \hbar\lambda = \hbar/\tau = 2.1 \times 10^{-2} E_\gamma^3 \\
&\text{Electric quadrupole:} && \Gamma(E_\gamma) = \hbar\lambda = \hbar/\tau = 4.9 \times 10^{-6} A^{\frac{4}{3}} E_\gamma^5 \\
&\text{Electric quadrupole:} && \Gamma(E_\gamma) = \hbar\lambda = \hbar/\tau = 1.5 \times 10^{-8} A^{\frac{2}{3}} E_\gamma^5
\end{aligned}$$

- Nuclear structure effects are know to modify these estimates by enhancement or hindrance factors up to three orders of magnitude.
- Measurements or state-of-the-art calculations have to be employed for realistic estimates of the rates.

Multipole effects for γ-ray decay

- Note, that γ-ray decay carries away at least one unit of angular momentum, therefore, a single-photon γ-ray decay between the initial state with spin zero and the final state with spin zero is forbidden by the conservation of angular momentum.

- The above decay can proceed through a double-photon γ-ray decay or electron-positron decay with rates much different then the Weiskopff estimate.

- Also, multipoles with the same parity and comparable decay widths can mix which has an impact on the total Γ-ray decay width. For example, mixing of magnetic dipole with electric quadrupole happens frequently and require measurement of the mixing ratio to disentangle the individual partial widths.