Introduction to nuclear reactions

Introduction to Nuclear Science

Simon Fraser University
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Outline

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Binary nuclear reactions

- The binary reaction between the projectile a and target nucleus A producing a nucleus B and outgoing particle b

$$a + A \rightarrow B + b \tag{1}$$

  is abbreviated as $A(a, b)B$.

- The entrance channel is $a + A$.

- The exit channel is $B + b$.

The Q-value

- The heat or Q-value for the reaction is the energy released in the reaction, which can be calculated from the mass difference between the entrance and exit channels:

$$Q_n = (M_{n,a} + M_{n,A} - M_{n,B} - M_{n,b})c^2. \tag{2}$$

- Exothermic reactions release heat and have $Q > 0$.
- Endothermic reactions have $Q < 0$.
- Endothermic reaction require heat above the threshold value of Q to proceed. This heat comes from the kinetic energy of the projectile.

The Q-value

- The Q value can be extracted from the atomic masses:

$$Q_a = Q_n + (Z_a + Z_A - Z_B - Z_b) m_e c^2 + B_e(Z_a) + B_e(Z_A) - B_e(Z_B) - B_e(Z_b). \tag{3}$$

- If all the exit channel charge is in the produced nuclei charge conservation requires $Z_a + Z_A - Z_B - Z_b = 0$.

- Therefore

$$Q_a = Q_n + \Delta B_e. \tag{4}$$

- The difference between electron binding in the entrance and exit channel is usually small compared to the nuclear Q_n value:

$$B_e = +B_e(Z_a) + B_e(Z_A) - B_e(Z_B) - B_e(Z_b) << Q_n. \tag{5}$$

- Therefore, usually $Q_a \approx Q_n$.

Atomic and nuclear Q-values

- For reactions involving charged lepton production the atomic Q value is differs from the nuclear Q value since the condition $Z_a + Z_A - Z_B - Z_b = 0$ is not fulfilled.
- Let us consider the Hydrogen burning reaction which proceeds via the weak interactions

$$p + p \rightarrow d + e^{+} + \bar{\nu}. \tag{6}$$

- The atomic Q value is

$$Q_a = (2 * M_{a,H} - M_{a,{}^2H})c^2 = 1.44 \text{ MeV}. \tag{7}$$

- The nuclear Q value is

$$Q_n = (2 * M_{n,p} - M_{n,d})c^2 = 0.93 \text{ MeV}. \tag{8}$$

- Atomic Q value includes the energy released in annihilation of the positron, while nuclear Q value does not include this energy.

The Q-value

- Atomic masses are usually tabulated in terms of atomic mass excess:

$$\Delta M_a = (M - AM_u)c^2 \tag{9}$$

with M_u being the atomic mass unit:

$$M_u c^2 = \frac{1}{12} M(^{12}C)c^2 = 931.50 \text{ MeV}. \tag{10}$$

- Note that $\Delta M_a \neq \Delta M_n$ but $\Delta M_a \simeq \Delta M_n$.

- The ratio $\frac{\Delta M_a}{M_a} \approx 0.1\% - 1\%$ represents the conversion factor from mass to energy in nuclear reactions.

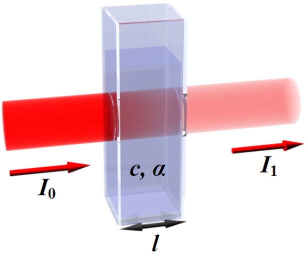
Q value

- Q value measures the energy liberated or consumed in a nuclear reaction.
- Q values are relatively easy to measure and are usually known with high precision from mass measurements.
- The Q value carry very limited information on reaction rates: for endothermic reactions the rate is zero if the kinetic energy of the projectile is not sufficient to overcome the Q value threshold.
- Other than that, the Q value alone does not allow to predict reaction rates.

Beer-Lambert law in optics

- Transmission intensity (number of particles per second) I_1 for a beam of particles traversing a thin target with thickness x depends on the target thickness and initial intensity:

$$I_1 = I_0 \exp\left(-\frac{x}{l}\right). \tag{11}$$



- This also applies to elementary particles passing through thin targets.

Mean free path

- Parameter l referred to as the mean free path is the mean distance for particles to be removed from the beam.
- Intensity decreases due to the beam interactions with atoms or nuclei in the target.
- Mean free path has to be related to the number of atoms per unit volume, referred to as the number density N.
- The relation is the inverse proportionality: larger N smaller l.
- The proportionality constant multiplying N is called the cross section σ. With this definition the relation between l, n and σ is

$$l = \frac{1}{N\sigma} \tag{12}$$

The reaction rate

- Inserting the cross section to the Beer-Lambert law results in:

$$I_1 = I_0 \exp\left(-N\sigma x\right). \tag{13}$$

- Number of particles removed from the beam is equal to the number of reactions per second or the reaction rate:

$$R = I_0 - I = I_0 \left(1 - \exp\left(-N\sigma x\right)\right). \tag{14}$$

- A thin target is a target for which

$$\frac{x}{l} = N\sigma x << 1. \tag{15}$$

- In this case

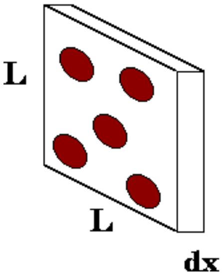
$$\exp\left(-N\sigma x\right) \approx 1 - N\sigma x \quad \text{and} \quad R = I_0 N\sigma x. \tag{16}$$

The cross section

- The dimension of a cross section is that of an area:

$$\sigma = \frac{1}{Nl}, \quad [\sigma] = \left[\frac{1}{\frac{1}{m^3} m} \right] = [m^2]. \tag{17}$$

- The cross section represents an effective area for interaction of a single target atom.
- For a infinitesimally thin target Ndx is the number of atoms per target surface area. This arises from a fact that probability of an atom to be screened from the interaction with the beam by another atom is very small.
- In such a case $N\sigma dx$ is the fraction of atoms interacting with the beam.



The unit of a cross section

- The unit of a cross section is a barn 1 [b]= 10^{-28} [m^2].
- From the nuclear interaction points of view the target is mostly empty. Indeed a separation between atoms is $\sim 10^{-10}$ m while a nuclear sizes are on the order of $\sim 10^{-15}$ m.
- Consequently nuclear cross sections are small.
- If a reaction has 1 barn cross section it implies that it is as easy to hit the target nucleus as to hit a barn.

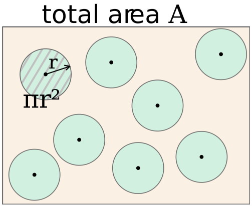
Geometric approximation

- If we knew the radii of the target R_t and the projectile R_p the geometric approximation for the cross section would be an area of a disc with the radius of $R_S = R_t + R_p$

$$\sigma = \pi R_s^2 = \pi(R_t + R_p)^2. \tag{18}$$

- With the nuclear radius definition $R_N = R_0 A^{\frac{1}{3}}$ and $R_0 = 1.3$ fm the geometric cross sections are:

  - $\sigma = 0.2$ [b] for ^{1}H+^{1}H,
  - $\sigma = 2.8$ [b] for ^{1}H+^{238}U,
  - $\sigma = 4.8$ [b] for ^{238}U+^{238}U,

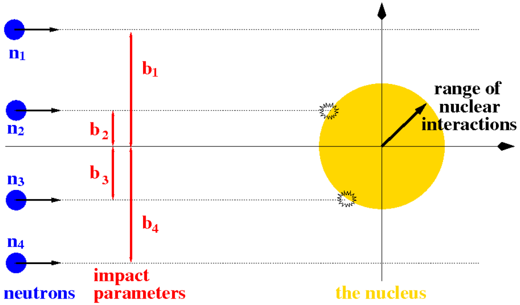


The proper treatment

- The proper derivation of a reaction cross section includes quantum mechanical treatment of interaction between the target and projectile.
- There are significant discrepancies between the realistic cross sections and geometrical estimates.
- For example, real cross sections are energy dependent, while the geometric approximation is not.
- Various fundamental interactions give very different scales for typical cross sections. Assuming 2 MeV for the projectile energy
  - for the weak force and $p(p, e^+\nu)d$ reaction $\sigma \approx 10^{-20}$ [b],
  - for the E-M force and $^3\text{He}(\alpha,\gamma)^7\text{Be}$ reaction $\sigma \approx 10^{-6}$ [b],
  - for the strong force and $^{15}\text{N}(p,\alpha)^{12}\text{C}$ reaction $\sigma \approx 0.5$ [b],
- If possible, cross sections are measured rather than derived.

Neutron-induced reactions

- Neutrons do not have charge, there is no long-range electromagnetic interactions with the nucleus, neutron trajectories are straight lines.
- The only interactions are the short-range ∼fm nuclear interactions.
- A good approximation is provided by the "black disc" model, all neutrons with impact parameter b smaller than the range of nuclear interactions R will react.



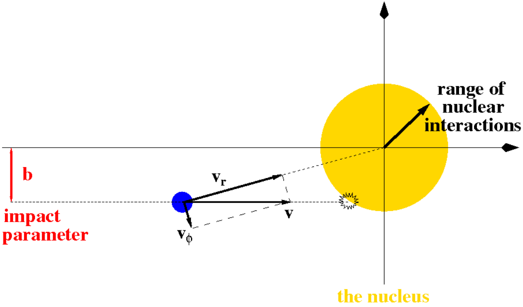
Centrifugal barrier

- The energy of the neutron in the center of mass is

$$E = K + V = \frac{mv^2}{2} + V(r). \tag{19}$$

- Using the radial coordinates

$$v^2 = v_r^2(r) + v_\phi^2(r) = v_r^2(r) + r^2\omega^2(r). \tag{20}$$



Centrifugal barrier

- The angular speed ω can be expressed using angular momentum L

$$L = \mu r^2 \omega, \quad \text{or} \quad \omega = \frac{L}{\mu r^2}. \tag{21}$$

- Combining the above gives the following for the energy

$$E = \frac{\mu v_r^2}{2} + \frac{L^2}{2\mu r^2} + V(r). \tag{22}$$

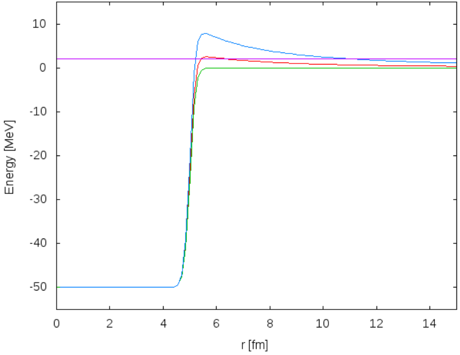
- Angular momentum for a collision is conserved and quantized

$$L^2 = l(l+1)\hbar^2. \tag{23}$$

- The energy can be splitted into the part which depends on radial speed and one which depends on the relative distance r.

Centrifugal barrier

- The part of energy which depends on the relative distance consist of the potential energy $V(r)$ and the centrifugal barrier $\frac{L^2}{2\mu r^2}$.
- The diagram of r-dependent terms of energy as a function of the relative distance is a conveninet way to represent collision energetics.



- Potential for nuclear interactions assumed to have -50 MeV depth and radius of 5 fm. Graphs are:
  - green for l=0 $\hbar$
  - red for l=1 $\hbar$
  - blue for l=2 $\hbar$
  - purple for $E = 2$ MeV.

Thermal neutrons

- The energy of the neutrons is defined by the temperature.
- The temperature range of 10^6 up to 10^9 deg. K corresponds to the energies of 0.13 keV up to 130 keV.
- Neutrons in this energy range are called thermal.
- The energy of a neutron required to cross the centrifugal barrier at $l = 1$ and $R = 5$ fm in a collision with a heavy nucleus $\mu \sim m_n$

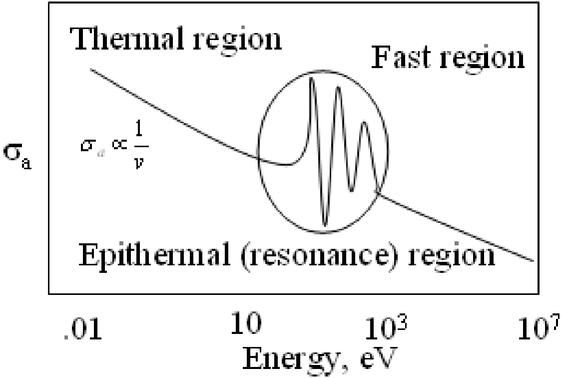
$$E > \frac{l(l+1)\hbar^2}{2\mu R^2} = \frac{2 * 197^2}{2 * 940 * 5^2} \sim 1.65 \quad [\text{MeV}]. \tag{24}$$

- Therefore thermal neutron induced reactions have $l = 0$.

Thermal neutron cross section

- For thermal neutron the cross section for capture is

$$\sigma \propto \frac{1}{v}. \tag{25}$$



Energy for charged-particle interactions

- The total energy is

$$E = K + V = \frac{mv^2}{2} + V_C(r) + V_N(r), \quad (26)$$

with $V_C(r)$ and $V_N(r)$ representing the electromagnetic (Coulomb) and nuclear interactions.

- Centrifugal and Coulomb barrier can be separated which leads to:

$$E = \frac{\mu v_r^2}{2} + \frac{L^2}{2\mu r^2} + \frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{r} + V_N(r) \quad (27)$$

Energy for charged-particle interactions

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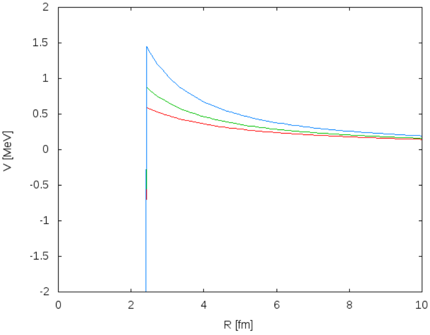
- For heavier nuclei at distances larger than the range of nuclear interactions the Coulomb barrier dominates the centrifugal barrier.

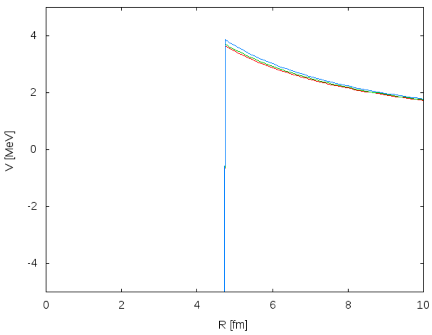
Energy for charged-particle interactions

- The graphs show

$$\frac{L^2}{2\mu r^2} + \frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{r} + V_N(r) \tag{30}$$

for the $p + p$ on the left and the $^{12}C + \alpha$ reaction on the right. The red, green, and blue are for $L = 0$, 1, and 2 $[\hbar]$, respectively.



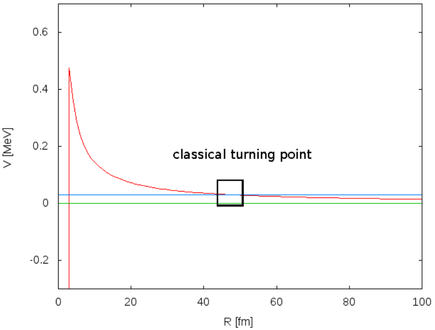


Classical turning point

- Coulomb barrier for the hydrogen burning is $E_C \sim 550$ keV.
- This corresponds to the temperature of $T = 6.4 \times 10^9$ K much higher then $T_\odot = 0.01 \times 10^9$ observed for the Sun.
- Average energy at $T_\odot = 0.01 \times 10^9$ is 0.86 keV.
- At this temperature the classical mechanics forbids protons to interact.
- The fraction of particles with energy above 550 keV in the Maxwell-Boltzmann distribution with mean energy of 0.86 keV is very small.
- According to classical mechanics stars do not shine.

Classical turning point

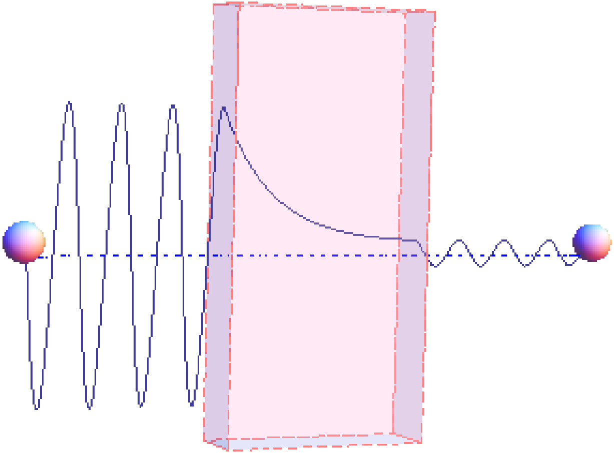
- This is because of the fact that the particles at energy lower than the Coulomb barrier should start moving away from the interaction center at the classical turning point.



- On the graph the red line is the Coulomb energy as the function of radius for the hydrogen burning, green line represents zero energy, while the blue line represents center of mass energy of 30 keV. The classical turning point for these conditions is at $R \sim 47$ [fm].

Tunneling

- Quantum mechanics allow particles to tunnel through the barrier.



Tunneling

- The probability of finding a particle across the barrier is given by the amplitude squared of the wave function.

$$P = \frac{\mid \Psi(R_N) \mid^2}{\mid \Psi(R_C) \mid^2}, \tag{31}$$

  with R_N and R_C being the nuclear radius and the distance to the classical turning point, respectively.

- The barrier suppresses the amplitude of the wave function, but not completely.

- The degree of suppression is calculable for the Coulomb potential and is given by

$$P = \exp\left(-2KR_C\left[\frac{\arctan\sqrt{\frac{R_C}{R_N}-1}}{\sqrt{\frac{R_C}{R_N}-1}} - \frac{R_N}{R_C}\right]\right), \quad K = \sqrt{\frac{2\mu}{\hbar^2}(E_C - E)}$$

The approximation for P

- If the distance to the classical turning point is much larger than the nuclear radius $R_C >> R_N$ which is equivalent to $E << E_C$ the tunneling probability is well approximated by

$$P = \exp\left(-b\frac{1}{\sqrt{E}}\right), \tag{32}$$

with

$$b = \frac{\sqrt{2\mu}}{4\epsilon_0}\frac{Z_1 Z_2 e^2}{\hbar} = 31.29 Z_1 Z_2 \sqrt{\frac{A_1 A_2}{A_1 + A2}} \quad \sqrt{\text{keV}}. \tag{33}$$

- P is a very rapidly changing function of the energy.

Excitation function

- Reaction cross section depends on the energy available in the entrance channel.
- The change of cross section as a function of energy in the entrance channel is referred to as the excitation function.
- The shape of the excitation function is very different for the neutron-induced reaction as compared to the charged-particle induced reaction.
- The excitation function increases with decreasing energy for neutrons, but decreases very fast for charged particles.
- This is directly related to the suppression of the reaction rate by the Coulomb barrier for charged particles which is not present in neutron-induced reactions.
- The sub-barrier charged particle reaction are strongly suppressed.

Excitation function

- For both, neutron and charge-particle induced reactions the excitation functions comprises of a superposition of a slow varying part, which is called the non-resonant part, and a fast changing part, which is called the resonant part.
- The resonant part depends very strongly on nuclear structure, in particular on the location of energy states in the compound system form in the fusion of the projectile and the target.
- The non-resonant part depends on the global properties of the projectile/target systems such as the height of the centrifugal/Coulomb barrier for the neutral/charged particles.
- At large energies the cross section approach the geometric cross section from the black disk model.

Total reaction cross section