Light ion scattering

Introduction to Nuclear Science

Simon Fraser University
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Outline

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1. Overview
2. States in nuclei
3. Elastic scattering

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Overview

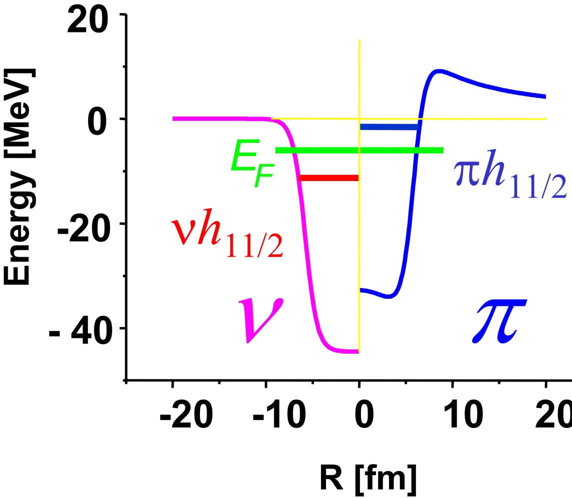
- So far we talked about Rutherford scattering, which is not truly a nuclear reaction since the only interaction involved is the electromagnetic force.
- From now we are going to talk about nuclear reactions, thus the reactions which involve the strong interactions.
- To understand the nuclear reactions we will start with classifying the states in nuclei which react.
- Next we will talk about different way the nucleons in nucleus can be re-arranged during a reaction.
- We will limit the discussion to the binary reactions, namely two nuclei in the entrance channel and two nuclei in the exit channel.
- Binary reactions obey the discussed binary kinematics resulting from the laws of conservation of energy, momentum, and angular momentum.

States in nuclei

- States in nuclei can be classified as bound and unbound.
- If a nucleon is in the bound state it means energy is required to remove this nucleon from a nucleus.
- Bound states are discrete, they result from a solution of many-body Schroedinger equation with nuclear Hamiltonian, for example the Shell Model Hamiltonian.
- As such, bound states are discrete.
- Binding energy of a nucleon can be calculated from nuclear models if the energy of the state is calculated.
- We also used the Liquid Drop Model to calculate binding and separation energies for a neutron, proton and α particle. These energies were average, and did not take into account quantization in nuclear potential.

Phenomenological shell-model potential

States with energy $E < 0$ are bound



Bound states

- The discussion of nuclear models lead to conclusion that bound states are characterized by
  - quantized negative energy
  - quantized angular momentum (integer for even-even and odd-odd nuclei, half integer for even-odd and odd-even nuclei)
  - positive or negative parity.
- The energy of the bound state has to be taken into account when calculating the Q-value of the reaction.
- The angular momentum and parity quantum number have an impact on the selection rules for the reaction which result from the laws of conservation of angular momentum and parity for strong and electromagnetic interactions.

Excited states

- State of the lowest energy for a nucleus is called the ground state.
- According to our definitions the ground state has negative energy of the largest absolute value.
- States above the ground state are called excited states.
- Most of the excited states are short lived and decay by γ-ray emission or electron conversion to the ground state on the time scale of a nanosecond.
- Some states are metastable and live for times longer than the nanosecond, these are called isomeric states.
- It is possible for the reaction products to emerge from the reaction in an isomeric state, in such a case the Q value is reduced by the excitation energy of the isomer with respect to the ground state.

Unbound states

- States which do not require energy to emit a nucleon or an α particle are called unbound.
- According to our definitions unbound states have positive energy.
- Quantization conditions for bound states were a consequence of the confinement of the wave corresponding to a nucleon within the potential well.
- Wave functions of positive energy states are not confined by the potential well.
- Unbound states are not quantized.
- The Schroedinger equation has a solution for any value of positive energy, therefore there is a continuum of unbound states.
- But not all unbound state are the same.

The natural line width

- You may recall the Heisenberg uncertainty principle

$$\Delta p_x \Delta x \geq \hbar \tag{1}$$

- It implicates that at a given time both momentum and position can not be defined exactly.
- Uncertainty principles of a similar kind hold between other, so called, conjugated variables. For example between the time and the energy

$$\Delta E \Delta t \geq \hbar \tag{2}$$

  Note that in the above equation ΔE represents the uncertainty in the energy, not the energy difference between the initial and the final state as in the previous slides.
- If we would have sufficient resolving power we would see that the state does not have a single energy, but a distribution of energies with the width defined by Γ.

The natural line width

- In nuclear processes the statistical nature of radioactive decay implicates an uncertainty in the existence of a quantum state which is on the order of the lifetime. This results in uncertainty of the energy, or the natural width of the state on the order of

$$\Gamma \approx \Delta E = \frac{\hbar}{\Delta t} = \frac{\hbar}{\tau} \tag{3}$$

- Typical lifetimes of bound states are on the order of 10^{-12}s.
- Thus typical natural width for such states is

$$\Gamma \approx \frac{\hbar}{\tau} = \frac{6.58 \times 10^{-22}}{10^{-12}} \text{ [MeV]} = 6.58 \times 10^{-10} \text{ [MeV]} = 0.7 \text{ [meV]}$$

- Typical lifetimes of an unbound state is on the order of 10^{-20}s.
- Thus typical natural width for an unbound states is

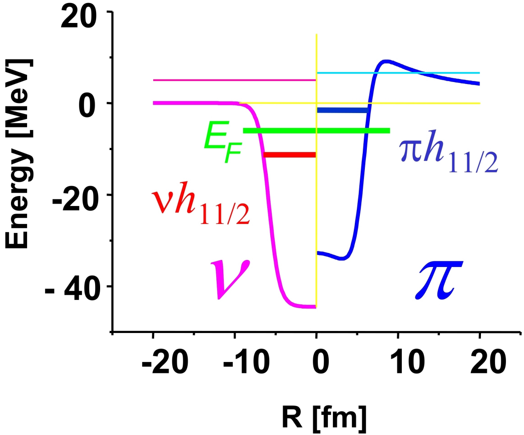
$$\Gamma \approx \frac{\hbar}{\tau} = \frac{6.58 \times 10^{-22}}{10^{-20}} \text{ [MeV]} = 6.58 \times 10^{-2} \text{ [MeV]} = 70 \text{ [keV]}$$

Resonant states

- The continuum of the positive energy states can be thought of as a superposition of a large number of resonances.
- Some of these resonances are very short lived.
- The width then is so broad that they overlap with other broad resonances and the identity of states is lost.
- However, some of the resonances are narrow.
- Narrow resonances correspond to long-lived states in continuum.
- Narrow resonances have a relatively well defined energy and can be assigned spin and parity quantum numbers.

Phenomenological shell-model potential

States with energy $E > 0$ are unbound. The $E > 0$ neutron state in magenta is a broad short-lived resonance while the $E > 0$ proton state in light blue is a narrow long-lived resonance.

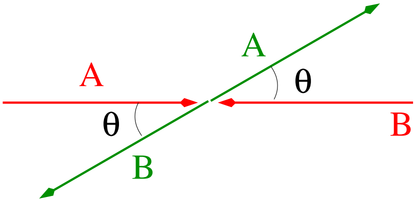


Elastic scattering

- Elastic scattering is the reaction in which the exit channel is the same as the entrance channel $B(A, A)B$.
- Rutherford scattering is an example of the elastic scattering via the electromagnetic (Coulomb) force.
- Elastic scattering does also occur via the strong interactions.
- The analysis of the elastic scattering is particularly easy in the centre of mass reference frame.
- In the centre of mass reference frame both particles have momenta of equal magnitude but opposite direction, both in the entrance and the exit channel.
- The result of the scattering is the change of the angle between the line defined by the initial and final momenta without any change to the kinetic energy in the centre of mass.

Elastic scattering

- Kinematics of elastic scattering in the centre of mass:

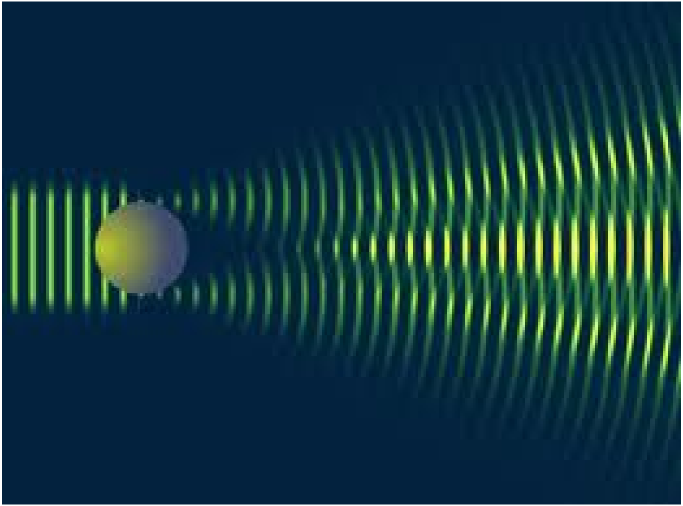


- From the centre of mass transformation the centre of mass angle can be converted into the laboratory scattering angle.
- Elastic scattering of nucleons is one of the means to study the nuclear potential.

Scattering of laser light

- Elastic scattering via the strong interactions can be treated in quantum mechanics as a scattering of the wave function representing the particles in the initial state on a nuclear potential well.
- As such, the nuclear elastic scattering resembles optical scattering of coherent (laser) light from a semi-transparent glass sphere.
- Scattering of the laser light shows angular distribution of intensity which is characteristic to the size of the sphere.
- If the sphere is completely transparent there is no loss of light.
- For a semi-transparent sphere part of the light is absorbed thus lost from the beam.

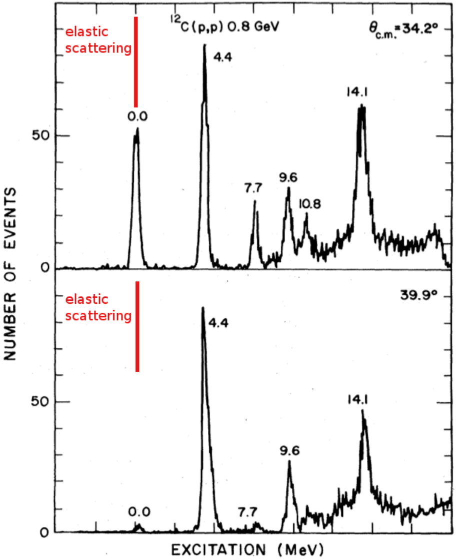
Laser light diffraction



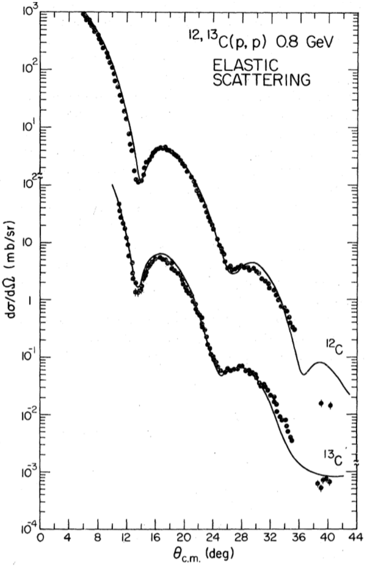
The optical model

- The optical model is a nuclear model developed to describe elastic scattering of nucleons by nuclear potential.
- In this model
  - the wave function in the entrance channel represents the laser light
  - the real (Wood-Saxon Shell-Model) potential well represents the transparent scattering sphere
  - the imaginary (Wood-Saxon Shell-Model) potential well represents a light-absorbing sphere
  - mixture of real and imaginary potential represents a semi-transparent scattering/absorbing sphere
- The optical model predicts diffractive pattern of angular distribution for scattering probability as a function of the centre of mass scattering angle.
- The diffractive pattern carries information on nuclear potential.

Angular distribution in $^{12}C(p,p)^{12}C$ elastic scattering



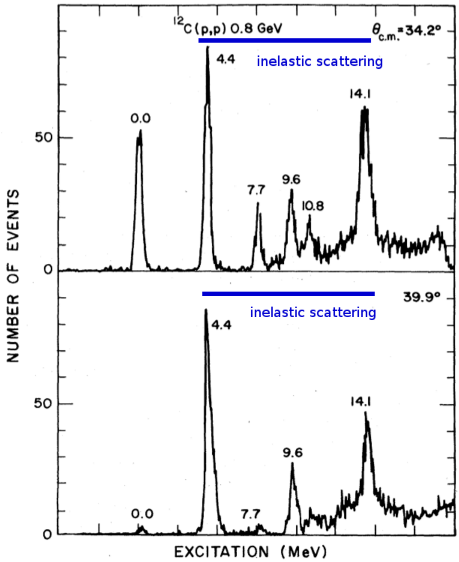
Angular distribution in $^{12}C(p,p)^{12}C$ elastic scattering



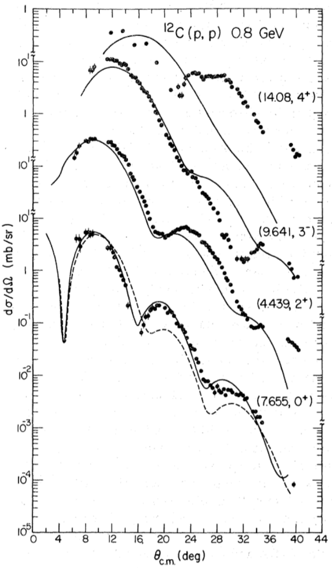
Inelastic scattering

- Inelastic scattering is the reaction in which the exit channel is the same as the entrance channel, except one of the nuclei in the exit channel emerges in an excited state $B(A, A)B^*$.
- Coulomb scattering is an example of the inelastic scattering via the electromagnetic (Coulomb) force.
- Inelastic scattering does also occur via the strong interactions.
- The optical model can be used to analyze the inelastic scattering, however, the model space has to include the ground state and excited states for nuclei in the exit channel.
- The model space for the exit channel has to include also the couplings between the ground state and excited states.
- The inelastic scattering provides thus the information on the potential well and the couplings between excited states in interacting nuclei.

$^{12}C(p,p)^{12}C$ inelastic scattering



Angular distribution in $^{12}C(p,p)^{12}C$ inelastic scattering



Transfer reactions

- Transfer reaction occure when one of the nuclei in the entrance channel has a structure of a core coupled to a nucleon or light cluster (the α particele, and when, as the result of the reaction, this nucleon or the cluster is transfered to the other nucleus in the entrance channel.

$$\begin{aligned} A + B &\rightarrow C + D \\ (A = C + n) + B &\rightarrow C + (D = B + n) \end{aligned} \tag{4}$$

- Nucleon or cluster transfer reactions are the tools to study nucleon or cluster separation energies since for the Q-value

$$S_D = S_A + Q \implies Q = S_D - S_A \tag{5}$$

thus if the nucleon/cluster separation energy is known in A the S_D can be deduced from Q-value measurement.

- The (d, p), (d, n) are examples of transfer reactions.