Control of the fission chain reaction

Introduction to Nuclear Science

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Outline

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2. Energy in nuclear fission

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3. Self sustained chain reaction

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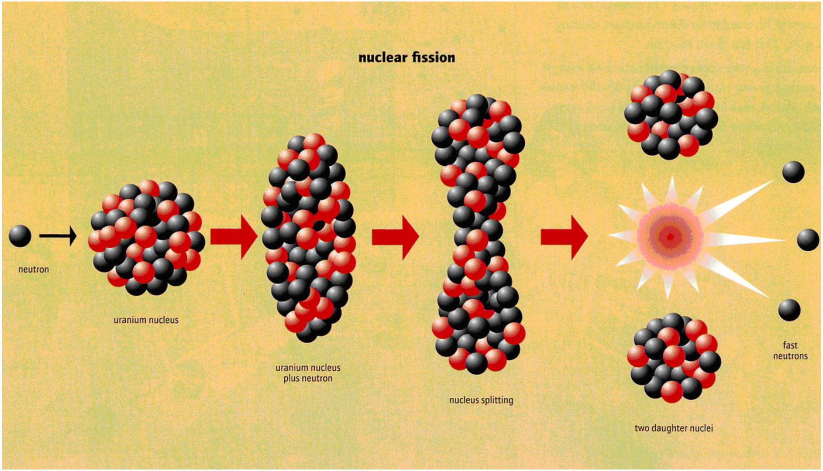
4. Moderation and thermalization of neutrons
5. Steady state in nuclear reactors

Significance of fission chain reaction

- Fission chain reaction is the key to:
  - conversion of nuclear energy to electrical energy,
  - harvesting isotopes for medical and industrial applications,
  - production of neutron beams for basic and applied research,
  - production of neutrino beams for basic research,
  - nuclear weapon applications.
- The control of the parameters of the fission reaction is the key to above applications.
- It is possible, however, not necessarily easy, to control the fission chain reaction.

Nuclear fission



Energy balance in ^{235}U fission

- Let us investigate the energy balance in a single fission of ^{235}U
    - average kinetic energy of fission fragments is 168 MeV
    - there are 2.5 neutrons emitted on average with average energy of 2 MeV energy per neutron
    - thus the average energy of neutrons is 5 MeV
    - average energy of prompt γ-rays after fission is 8 MeV
    - average energy of β^- particles from fission fragment decay is 8 MeV
    - average energy of γ rays following β^- decay of fission fragments is 7 MeV
    - average energy of electron antineutrinos following fission fragment β^- decay is 12 MeV.
- The total average energy is 208 MeV
- Since antineutrinos escape 196 MeV per fission is recoverable.

The energy perspective

- Let us calculate the energy release from fissioning of 1 g of ^{235}U
- There number of ^{235}U atoms in 1 g is

$$n = \frac{m}{\mu} N_A = \frac{1}{235} 6.02 \times 10^{23} = 2.56 \times 10^{21} \quad (1)$$

- The conversion of MeV to Jules is

$$196 \times 10^{6} \text{ [eV]} \times 1.6 \times 10^{-19} \text{ [J/ev]} = 3.1 \times 10^{-11} \text{ [J]} \quad (2)$$

- The energy content in 1 g of ^{235}U is

$$2.56 \times 10^{21} \times 3.1 \times 10^{-11} \text{ [J]} \approx 8 \times 10^{10} \text{ [J]} = 80 \text{ [GJ]} \quad (3)$$

- An energy content of 1 g of coal is at most 35 kJ, 2.3 million less.

The energy perspective

- So we have noted that energy content of 1 g of ^{235}U is equivalent to the energy content of 2.3 tons of coal.
- What about radiation? Burning coal produces quite a lot, since long-lived radioactive isotopes are dug out from the crest of the Earth and released in burning.
- What about solar power?
- Average solar energy delivered to 1 m^2 on the surface of the earth is 1360 W.
- Let us assume that we have access to solar panels which convert solar light into electric energy 12 h per day.
- Let us calculate surface area of panels delivering energy equivalent to 1 g of ^{235}U.

The energy perspective

- Let us calculate area of panels delivering energy of 80 GJ in 12 h.
- The power is energy over time which results in

$$P = \frac{E}{t} = \frac{80 \times 10^9}{12 * 60 * 60} = 1.85 \times 10^6 \text{ [W]} = 1.85 \text{ [MW]} \tag{4}$$

- The solar power is 1360 W per meter square.
- Thus to get the equivalent energy one needs

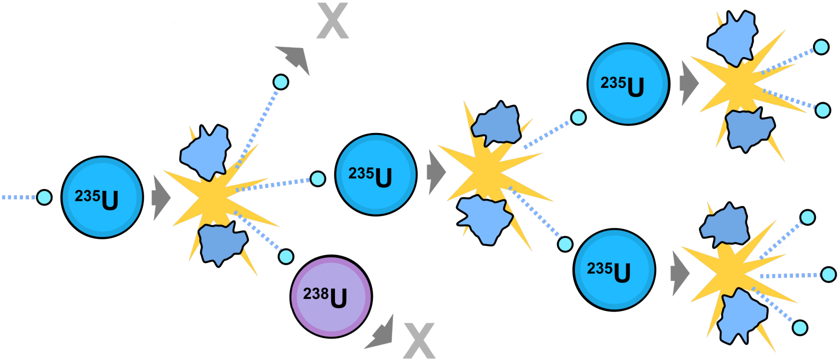
$$A = \frac{1.85 \times 10^6}{1360} = 1.36 \times 10^3 \text{ m}^2 \tag{5}$$

- This corresponds to a square of 37 m side.
- Current maximum efficiency of panels is 40%, typical will be smaller.
- Efficiency of 10% gives a square of a 117 m side.

Self sustained chain reaction

- So in principle, nuclear chain reaction should be self sustained.
- There are 2.5 neutrons per fission, 5 neutrons per 2 fission, so naively every 2 fissions should produce 5 fissions with 196 MeV energy release.
- But life is not so simple, predominantly for three reasons
  - Fission cross sections are energy-dependent and rather small for 2 MeV neutrons.
  - Neutrons can be lost in reactions which do not produce fission.
  - Neutrons can leak out from the volume of the reactor without inducing fission.
- For the above reasons, building and controlling nuclear reactor is an art, even more so for nuclear weapon.

Nuclear chain reaction

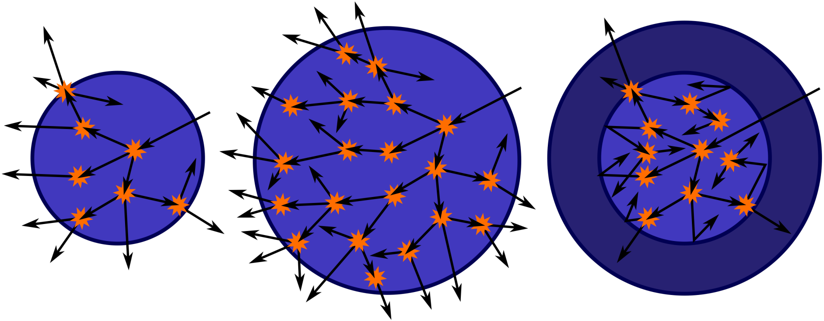


The critical mass

- The critical mass is the smallest amount of fissile material needed for a sustained nuclear chain reaction.
- The critical mass of a fissionable material depends on
    - nuclear cross section thus the type of material
    - density of the material
    - shape of the material
    - enrichment (or content of non-fissile impurities)
    - the temperature of the material
    - surroundings of the material.
- For a self-sustaining fission reaction in a critical mass of a material here is no increase or decrease in power, temperature or neutron population.

The critical mass

Left: subcritical Middle: critical Right: subcritical with a reflector



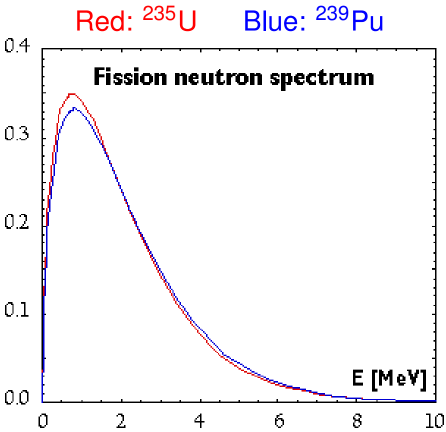
The critical mass

- The critical mass can be changed by
  - varying the amount of fuel
  - varying the shape of the fuel
  - varying the temperature (hot fuel is less reactive)
  - varying the density of the fuel
  - application of neutron moderators
  - application of neutron reflectors
- Critical mass for a bare sphere of ^{239}Pu is 10 kg (radius of 10 cm).
- Critical mass for a bare sphere of ^{235}U is 52 kg (radius of 17 cm).
- Critical mass of 20% enriched ^{235}U is 400 kg.

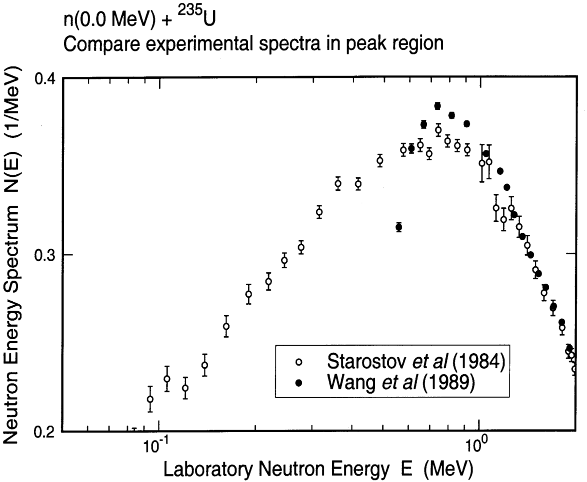
Nuclear reactors

- Nuclear reactor is a device to initiate and control a self-sustained fission chain reaction.
- Nuclear reactors can operate with the fuel mass significantly below the critical mass of a bare sphere.
- This is achieved by changing the property of the neutron flux.
- A moderator in a reactor shifts the intensity of the neutron flux to the thermal region where cross sections for fission are high.
- Moderators allow low enriched uranium which is a mixture of 3% ^{235}U and 97% of ^{238}U to be used as a reactor fuel.
- Note, that natural uranium contains 0.7% of ^{235}U and 99.3% of ^{238}U.

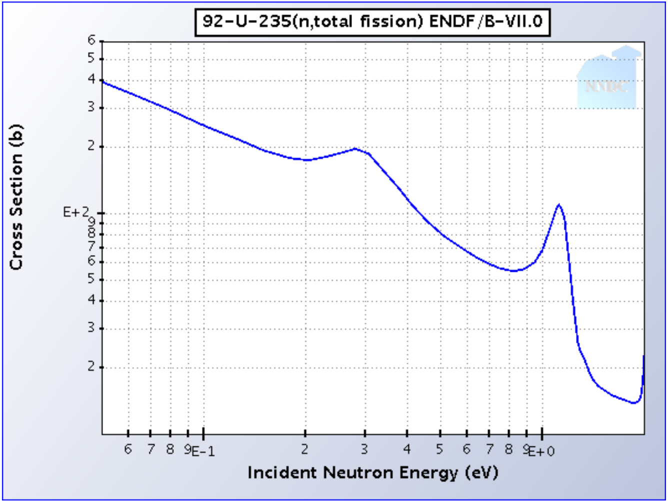
Fission neutron spectra



^{235}U fission neutron spectra



Neutron-induced fission cross section for ^{235}U



The need for moderation

- A comparison between the neutron energy spectra from fission of ^{235}U and the cross section for ^{235}U fission indicates the need for moderation of neutrons.
- Indeed, the most probable energy for neutrons is 2 MeV.
- The fission cross section at this energy is 1.3 b.
- If neutrons are moderated and brought to thermal equilibrium with the surroundings they will have temperature of 20 deg. C or 293 deg. K, the speed of 2200 m/s and the energy of 0,025 eV.
- The cross section at this energy is 584 b.
- For ^{23}U moderation increases the cross section by a factor of 450.
- Some neutrons will be lost during the moderation.

The way to moderate

- The moderation of neutrons should proceed predominantly through the elastic scattering process since other processes, like reactions, will reduce neutron flux.
- Elastic collision conserves momentum and energy.
- Let us consider a central elastic collision between a neutron and a nucleus of mass A. Conservation of energy and momentum yields:

$$\begin{aligned} \vec{p}_i &= \vec{p}_f + \vec{p}_A \\ \frac{p_i^2}{2m} &= \frac{p_f^2}{2m} + \frac{p_A^2}{2Am} \end{aligned} \tag{6}$$

- A bit of algebra leads to

$$A(p_i^2 - p_f^2) = (p_i - p_f)^2 = p_A^2 \tag{7}$$

The way to moderate

- The solution to the equation

$$A(p_i^2 - p_f^2) = (p_i - p_f)^2 \tag{8}$$

is provided by

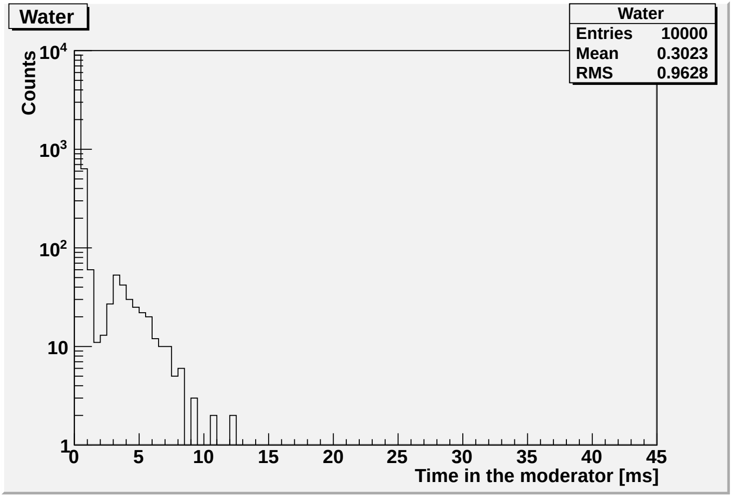
$$\begin{aligned} A(p_i^2 - p_f^2) &= A(p_i + p_f)(p_i - p_f) = (p_i - p_f)^2 \\ A(p_i + p_f) &= p_i - p_f \\ Ap_i + Ap_f &= p_i - p_f \\ p_f &= -p_i \frac{A-1}{A+1}, \quad E_f = E_i \left(\frac{A-1}{A+1}\right)^2 \end{aligned} \tag{9}$$

- Note that for $A = 1$ the momentum after collision is $p_f = 0$
- For $A >> 1$ the momentum after collision is $p_f = -p_i$.

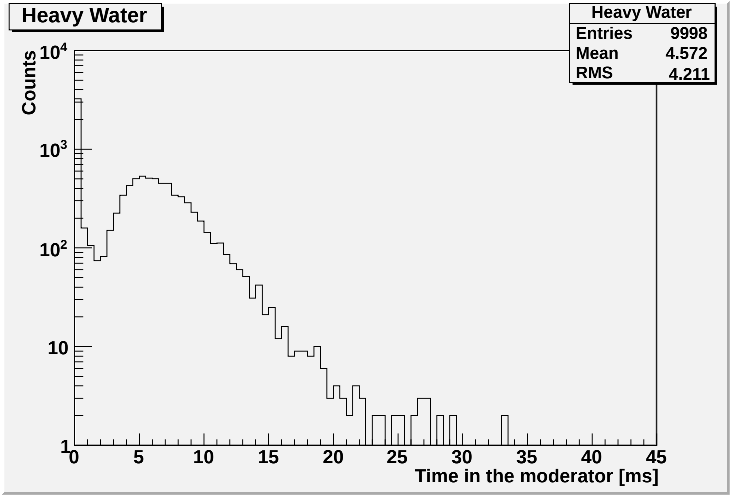
The way to moderate

- We have observed that the moderation via elastic scattering works best for moderators of small A.
- From the collision point of view hydrogen at $A = 1$ seems best.
- Water, containing significant quantities of hydrogen is used as a moderator in Light Water Reactors.
- A good moderator will reduce energy of neutrons without reducing the neutron flux by absorption.
- Hydrogen captures neutrons and forms deuterium.
- Deuterium at $A = 2$ is a reasonably good moderator with low cross section for a neutron capture.
- Heavy water containing deuterium instead of hydrogen is used as a moderator in Heavy Water Reactors.

Light Water moderator of SIMON



Heavy Water moderator of SIMON



Neutron reflectors

- Analysis of elastic collision with a nucleus of large mass $A >> 1$ implies that the momentum of the incoming neutron is reversed with nearly unchanged magnitude.
- This implies that materials of $A >> 1$ and low neutron absorption cross section can be used to reflect neutrons.
- Materials used for reflectors are graphite, beryllium, lead, steel, tungsten, carbide and other.
- A reflector can make nuclear chain reaction which is not self-sustaining to become self-sustaining.
- In 1946 a 6.2 kg core of ^{239}Pu become critical (developed self-sustained chain reaction) after a brick made of neutron reflecting material accidentally fell on it. Brick was removed promptly but neutron irradiation resulted in death of a worker 25 days later.

Neutron multiplication factor

- The critical parameter for operating nuclear reactor is the neutron multiplication factor denoted by k which is defined as the average number of neutrons from one fission which cause another fission.
- The neutrons which do not cause fission are either absorbed or escape from the reactor.
- If the the time scale for neutron multiplication by a factor of k is denoted by τ the change of the neutron flux N as a function of time is defined by

$$\frac{dN}{dt} = \frac{kN - N}{\tau} = \frac{k-1}{\tau} N \tag{10}$$

- The solution of this equation is define the operating condition of a reactor.

Reactor in the critical mode

- The neutron multiplication factor $k = 1$ is the special case corresponding to reactor operating in the critical mode.
- The equation for the flux becomes in this case

$$\frac{dN}{dt} = \frac{k-1}{\tau} N = 0 \tag{11}$$

- The solution of this equation gives a constant flux

$$N(t) = N(0) \tag{12}$$

- This is the solution for a steady state of the reactor without any growth or reduction in the number of neutrons.
- The energy or heat generation in the steady state is the flux N times the recoverable energy per fission.

Reactor in the subcritical mode

- The neutron multiplication factor $k < 1$ corresponds to reactor operating in the subcritical mode.
- The equation for the flux becomes in this case

$$\frac{dN}{dt} = \frac{kN - N}{\tau} = \frac{k-1}{\tau}N = -\frac{1-k}{\tau}N \tag{13}$$

- The solution of this equation is

$$N(t) = N(0)\exp\left(-(1-k)\frac{t}{\tau}\right) \tag{14}$$

- In the subcritical mode the flux decreases which leads to a shutdown of the reactor within the timescale defined by the effective lifetime of

$$\tau/(1-k). \tag{15}$$

Reactor in the supercritical mode

- The neutron multiplication factor $k > 1$ corresponds to reactor operating in the supercritical mode.
- The equation for the flux becomes in this case

$$\frac{dN}{dt} = \frac{kN - N}{\tau} = \frac{k-1}{\tau}N \tag{16}$$

- The solution of this equation is

$$N(t) = N(0)\exp\left((k-1)\frac{t}{\tau}\right) \tag{17}$$

- In the supercritical mode the flux increases within the timescale defined by the effective lifetime of

$$\tau/(k-1). \tag{18}$$

The control of a reactor

- The goal of the control of a reactor is to maintain a steady state at the criticality with the multiplication factor $k = 1$.
- This is not an easy task since it is practically impossible to maintain a constant k without even slight deviations.
- What helps to control the reactor is the fact that increased temperatures reduce reactivity of the neutrons as cross sections decrease with increased energy.
- This feature provides a negative feedback needed for control: supercriticality leads to decreased reactivity which lowers the temperature giving subcriticality which increases reactivity etc.
- As a consequence the multiplication factor k oscillates around the critical value $k = 1$.