The nuclear force

Introduction to Nuclear Science

Simon Fraser University
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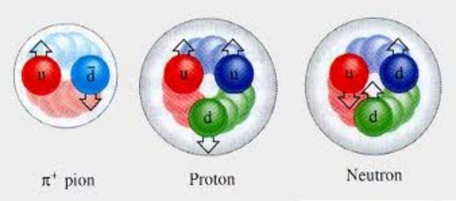
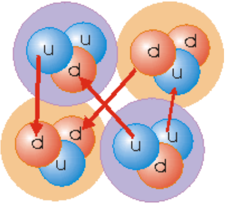
The strong vs. nuclear force

- Strong force acts between quarks in hadrons



- The nuclear force is the residual interaction between quarks localized in different hadrons.



Experimental studies of the nuclear force

- Our understanding of nuclear force is based on three types of experimental information:

  1. results of nucleon-nucleon (proton-proton, neutron-neutron, and proton-neutron) scattering experiments. Some of these experiments are conducted with spin-polarized projectiles/targets.
  2. Nuclear binding energies and masses, especially for light nuclei.
  3. Nuclear structure information, such as energies, spins, parities, magnetic and quadrupole moments, especially for light nuclei.

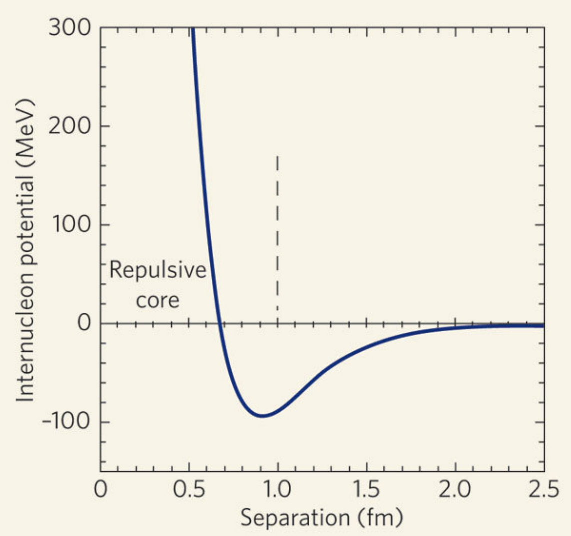
Experimental studies of the nuclear force

- Experimental results indicate that nuclear force depends on
  1. the distance between interacting nucleons (the radial part),
  2. spins and angular momenta of interacting nucleons (spin-orbit and tensor part).
- Experimental results indicate that nuclear force does not depend on the type of interacting nucleons.

The nuclear force: the radial part

- The nuclear force is short range, which implies it vanishes for distances longer than ~ 2 fm.
- The nuclear force is strongly attractive at distances of ~ 1 fm.
- The nuclear force is strongly repulsive on for distances shorter than ~ 0.5 fm (the hard core of nuclear potential).
- This is often represented in terms of the interaction potential shown on the next slide.

The radial potential for nucleon-nucleon interactions

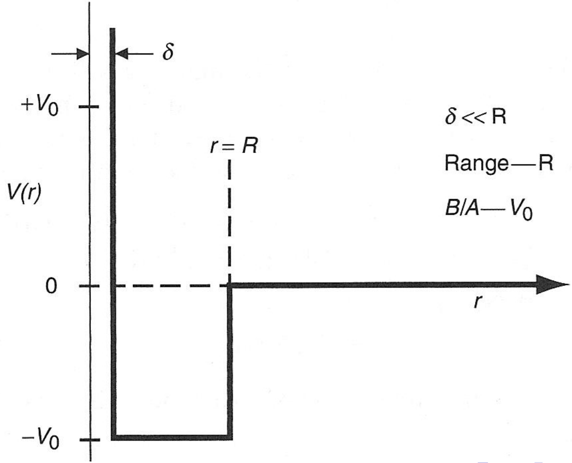


The nuclear force: the radial part

- The argument for the short range of nuclear forces are
  1. Binding energies per nucleon which are roughly constant indicating that nucleons in nuclei interact only with their immediate neighbours.
  2. Measurements of distances between nuclei at which nuclear reactions start to occur, these are $\sim$1-2 [fm] larger than corresponding nuclear radii.
  3. Nuclear densities which are only slightly smaller than nucleon density indicating very dense packing of nucleons in a nucleus.
- Arguments for the attractive nuclear force is the fact that nuclei are bound.
- Arguments for the repulsive nuclear force at short distances come from high-energy nucleon-nucleon scattering experiments.

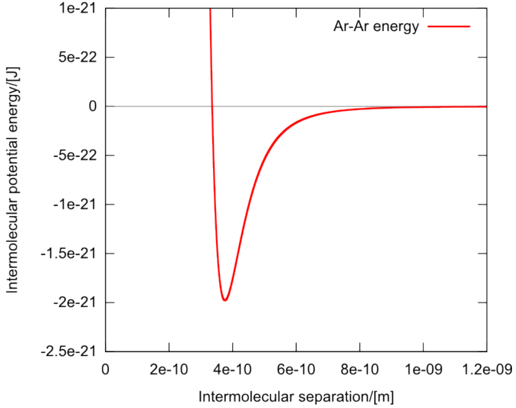
The nuclear force: the radial part

- Note the approximation shown for the nucleon-nucleon interaction potential in your textbook.



The nuclear force: the radial part

- Note also that the shape of the radial part of nucleon-nucleon potential is nearly identical to shape of intermolecular chemical interactions (except for the distance scale).



The two-nucleon system

- Let us examine nucleon-nucleon interactions in the simplest two-nucleon ($A = 2$) system.
- There are three two-nucleon systems which can be formed: the di-proton (pp), the di-neutron (nn) and the deuteron.
- Out of these three only the deuteron is bound.
- This observation alone and closer examination of the properties of the deuteron provide significant clues for the properties of the nuclear force.

The two nucleon system

- Note that nucleons are fermions with a spin 1/2.
- As such identical nucleons in the di-proton and the di-neutron have to obey the Pauli exclusion principle. They can not occupy the same quantum mechanical state.
- As a consequence the Pauli principle requires that nucleons in the di-proton and the di-neutron have the opposite spin.
- The di-proton and the di-neutron can only form a state of total spin of 0 (anti-parallel orientation of individual spins) since the spin 1 state (parallel orientation of individual spins) lead to violation of the Pauli principle as nucleons are in the same state.
- This is not true for the deuteron, however, the nucleons are different there and can form both, total spin 1 and total spin 0 state without violating the Pauli principle.

Properties of the deuteron

- The nucleons in the deuteron can form states of total spin 1 and 0.
- The ground state of the deuteron is measured to have spin 1.
- There are no indication of existence of any bound excited state in the deuteron. This implies that spin 0 state is not bound.
- Note, that spin 0 state in the deuteron has the same spin configuration as required for the di-proton and the di-neutron, which are also unbound.
- From this we conclude that the nuclear force between two nucleons is spin-dependent, and is attractive for parallel orientation of spins and repulsive for anti-parallel orientation of spins.

Properties of the deuteron: closer look

- The above argument looks all right, however, there is a bit of sweeping stuff under the carpet there since we only considered spin but neglected angular momentum.
- To examine angular momentum contribution we need to examine the Pauli principle.
- To examine the Pauli principle we need to introduce a concept of anti-symmetrization of fermionic wave functions.
- Pauli principles requires that for a system of two identical fermions the wave function has to change its sign when particles are interchanged.
- The di-proton and the di-neutron wave function has to have this property, but the deuteron wave function does not have to since the fermions are not identical.
- The antisymmetrization of the wave function implies that identical fermions will not be found in the same state.

Fermionic wave function and the Pauli principle

- Let us label protons positions as $\vec{r_1}$ and $\vec{r_2}$ and proton spins as $\vec{s_1}$ and $\vec{s_2}$. The wave function of the di-proton can be separated into three terms:
  1. Term Φ describing the motion of the centre between the two protons (centre of mass). This term depends on $\vec{r_1} + \vec{r_2}$, $\Phi = \Phi(\vec{r_1} + \vec{r_2})$.
  2. Term Y describing the orbital motion of the two protons around this centre. This term depends on $\vec{r_1}$ and $\vec{r_2}$, $\mathcal{Y} = \mathcal{Y}(\vec{r_1} - \vec{r_2})$
  3. Term χ describing spin orientation of the two protons. This term depends on $\vec{s_1}$ and $\vec{s_2}$, $\chi = \chi(\vec{s_1}, \vec{s_2})$.
- The full wave function is the product of these three partial wave functions

$$\Psi(\vec{r_1}, \vec{s_1}, \vec{r_2}, \vec{s_2}) = \Phi(\vec{r_1} + \vec{r_2}) \times \mathcal{Y}(\vec{r_1} - \vec{r_2}) \times \chi(\vec{s_1}, \vec{s_2}). \tag{1}$$

- The Pauli principle requires that the full wave function is anti-symmetric with respect to interchange of the particles

$$\Psi(\vec{r_1}, \vec{s_1}, \vec{r_2}, \vec{s_2}) = -\Psi(\vec{r_2}, \vec{s_2}, \vec{r_1}, \vec{s_1}) \tag{2}$$

Fermionic wave function and the Pauli principle

- Note that the part which describes the centre of mass motion is always symmetric when particles are interchanged since

$$\vec{r_1} + \vec{r_2} = \vec{r_2} + \vec{r_1} \implies \Phi(\vec{r_1} + \vec{r_2}) = \Phi(\vec{r_2} + \vec{r_1}) \tag{3}$$

- Thus for $\Psi(\vec{r_1}, \vec{s_1}, \vec{r_2}, \vec{s_2})$ to be antisymmetric under particle exchange there are two possibilities:
  1. Y is symmetric and χ is antisymmetric

$$\mathcal{Y}(\vec{r_1} - \vec{r_2}) = \mathcal{Y}(\vec{r_2} - \vec{r_1}) \text{ and } \chi(\vec{s_1}, \vec{s_2}) = -\chi(\vec{s_2}, \vec{s_1}) \tag{4}$$

  2. Y is antisymmetric and χ is symmetric

$$\mathcal{Y}(\vec{r_1} - \vec{r_2}) = -\mathcal{Y}(\vec{r_2} - \vec{r_1}) \text{ and } \chi(\vec{s_1}, \vec{s_2}) = \chi(\vec{s_2}, \vec{s_1}) \tag{5}$$

- The case of symmetric Y and symmetric χ as well as antisymmetric Y and antisymmetric χ leads to symmetric Ψ and is forbidden by the Pauli principle.

The symmetric spin functions

- Let us first examine symmetric spin functions for a two-nucleon system.
- There are two orientation of spins for each nucleon, for simplicity, let us use arrows up and arrows down to denote spin orientations.
- Symmetric spin function for a two nucleon system are

$$\begin{aligned} |\uparrow\rangle_1 \, |\uparrow\rangle_2 &= |\uparrow\rangle_2 \, |\uparrow\rangle_1 \\ |\downarrow\rangle_1 \, |\downarrow\rangle_2 &= |\downarrow\rangle_2 \, |\downarrow\rangle_1 \\ \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 \, |\downarrow\rangle_2 + |\downarrow\rangle_1 \, |\uparrow\rangle_2\right) &= \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_2 \, |\downarrow\rangle_1 + |\downarrow\rangle_2 \, |\uparrow\rangle_1\right) \end{aligned} \quad (6)$$

- These three wave functions correspond to three magnetic substates, $M = 1, -1, 0$, respectively, of the spin $S = 1$ state resulting from the coupling of two spin $1/2$ particles.
- Spin $S = 1$ coupling is often referred to as the triplet.

The antisymmetric spin functions

- Let us next examine the antisymmetric spin function for a two-nucleon system.
- We use the same notation as before.
- There is only one antisymmetric spin function for a two nucleon system:

$$\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 \, |\downarrow\rangle_2 - \, |\downarrow\rangle_1 \, |\uparrow\rangle_2\right) \quad = \quad -\frac{1}{\sqrt{2}}\left(|\uparrow\rangle_2 \, |\downarrow\rangle_1 - \, |\downarrow\rangle_2 \, |\uparrow\rangle_1\right) \qquad (7)$$

- This wave function correspond to the only magnetic substate, $M = 0$, respectively, of the spin $S = 0$ state resulting from the coupling of two spin $1/2$ particles.
- Spin $S = 0$ coupling is often referred to as the singlet.
- Note that two $1/2$ particles can couple to the total spin 0 or 1 and that we constructed the wave function for all magnetic substates of the singlet and triplet couplings.

Parity

- Before we examine the orbital wave functions let us investigate the concept of parity.
- A function F of $\vec{r}$ is said to have a positive parity $\pi = +$ if

$$F(\vec{r}) = F(-\vec{r}) \tag{8}$$

- In one dimension the cosine function has positive parity since

$$\cos(x) = \cos(-x) \tag{9}$$

- A function F of $\vec{r}$ is said to have a negative parity $\pi = -$ if

$$F(\vec{r}) = -F(-\vec{r}) \tag{10}$$

- In one dimension the sine function has positive parity since

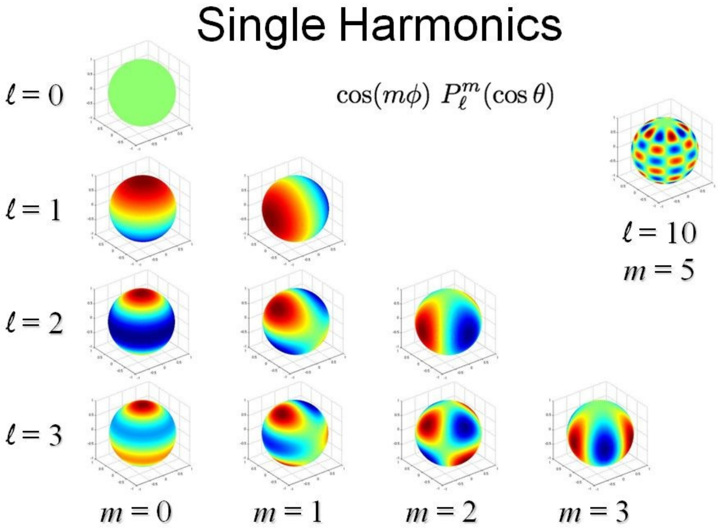
$$\sin(x) = \sin(-x) \tag{11}$$

Orbital wave function

- Orbital wave functions for the mass $A = 2$ system are the same as the orbital wave functions for an electron in the hydrogen atom, except, of course, for the different argument, which for the mass A=2 system is $\vec{r_1} - \vec{r_2}$
- These wave functions are assigned the angular momentum quantum number as well as parity.
- Energy of states of increasing angular moment increases.
- Thus the lowest energy states are:

$$\text{the } \mathcal{S}\text{-state with } \; l = 0, \; \pi = +$$
$$\text{the } \mathcal{P}\text{-state with } \; l = 1, \; \pi = -$$
$$\text{the } \mathcal{D}\text{-state with } \; l = 2, \; \pi = +$$

Orbital wave function



Orbital wave function

- Note that the positive parity for the S and the D states implies the symmetry under exchange of particles 1 and 2

$$\mathcal{S}_{l=0}^{\pi=+}(\vec{r_1}-\vec{r_2}) = \mathcal{S}_{l=0}^{\pi=+}(-(\vec{r_1}-\vec{r_2})) = \mathcal{S}_{l=0}^{\pi=+}(\vec{r_2}-\vec{r_1}) \qquad (12)$$

$$\mathcal{D}_{l=2}^{\pi=+}(\vec{r_1}-\vec{r_2}) = \mathcal{D}_{l=2}^{\pi=+}(-(\vec{r_1}-\vec{r_2})) = \mathcal{D}_{l=2}^{\pi=+}(\vec{r_2}-\vec{r_1}) \qquad (13)$$

- By the same token the negative parity for the P state implies the antisymmetry under exchange of particles 1 and 2

$$\mathcal{P}_{l=1}^{\pi=-}(\vec{r_1}-\vec{r_2}) = -\mathcal{P}_{l=1}^{\pi=-}(-(\vec{r_1}-\vec{r_2})) = -\mathcal{P}_{l=1}^{\pi=-}(\vec{r_2}-\vec{r_1}) \qquad (14)$$

Pauli principle again

- For the di-proton or the di-neutron ground state the lowest-energy, symmetric, orbital $\mathcal{S}$ state require anti-symmetric singlet spin function. If bound the di-proton and di-neutron ground state would have spin zero and positive parity.
- For the deuteron, the the lowest-energy, symmetric, orbital $\mathcal{S}$ state can couple to the anti-symmetric singlet as well as symmetric triplet spin function. The coupling with the triplet spin $S = 1$ is lower in energy than the coupling with the singlet spin $S = 0$. Moreover, coupling with the singlet is unbound as for the di-proton and di-neutron.
- We come to the same conclusions on spin dependence of nuclear force, this time with better understanding of angular momentum contributions.

Magnetic moment of the deuteron

- Parallel spin orientation and lack of orbital angular momentum (l=0 for the $\mathcal{S}$ state) implies that the magnetic moment of the deuteron should be the sum of magnetic moment of the proton and neutron.

$$\begin{aligned} \mu_{th} &= \mu_p + \mu_n = g_p s + g_n s = \frac{1}{2}(5.585 - 3.826)\ [\mu_{\rm B}] \\ &= 0.879\ [\mu_{\rm B}] \end{aligned} \tag{15}$$

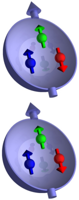
- The measured value is pretty close, but different

$$\mu_{exp} = 0.857\ [\mu_{\rm B}] \tag{16}$$

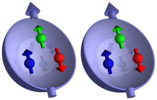
- The difference is explained by a small ($\sim$ few percent) admixture of the orbital $\mathcal{D}$ state to the $\mathcal{S}$ state in the ground state.
- The $\mathcal{P}$ state does not admix since parity is a conserved quantity which implies no mixing between positive and negative parity states.

Quadrupole moment of the deuteron

- Deuteron has a positive quadrupole moment of $Q = 0.2860(15)$ [fm^2] measured with respect to the spin axis.
- Positive quadrupole moment indicates prolate deformation with the spin axis being the symmetry axis. Or it indicates that the deuteron looks like the picture on the left then on the right.



prolate



oblate

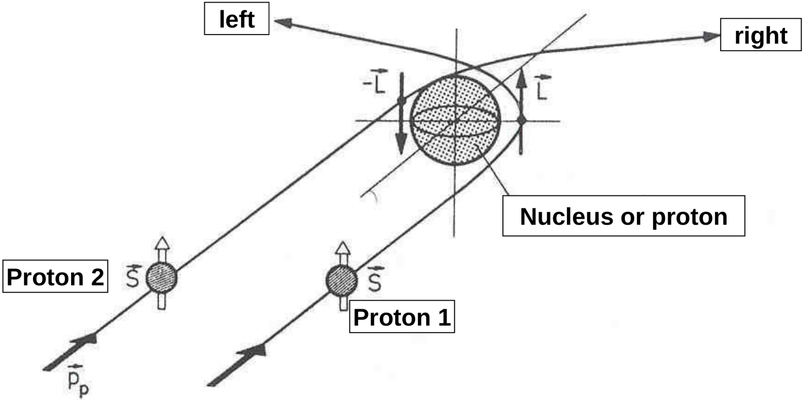
Quadrupole moment of the deuteron

- Positive quadrupole moment of the deuteron indicates that there is a part of the nuclear force which depends on the spin and spatial position of nucleons at the same time.
- This part of the nuclear force is called the tensor force.
- An analogues classical force occurs between two bar magnets.

Attractive Repulsive

The spin-orbit force

- Scattering of spin-polarized nucleons indicates that nuclear force has a component which depends on the spin and angular momentum of the interacting nucleon



Charge independence of nuclear force

- Nucleon-nucleon scattering experiments indicate that the nuclear force is independent of electric charge.
- If you think about it, why should it? Nuclear force comes from the residual strong force, while the charge plays a role in electromagnetic interactions.
- Charge-independence of the nuclear force implies that if electromagnetic effects are eliminated the scattering of protons on protons, protons on neutrons and neutrons on neutrons yields (nearly) the same results.
- Electromagnetic effects to be eliminated need to be calculated and subtracted from the data (there is no other way to "turn off" the electromagnetic force).
- In fine details the charge independence is broken, for example by the small mass difference between the proton and neutron mass.

Mirror nuclei

- Charge independence of nuclear force is manifested in properties of mirror nuclei.
- Mirror nuclei are these which have numbers of protons and neutrons exchanged one with respect to the other like

$$ {}^{A}_{Z}X_{N} \text{ is a mirror nucleus of } {}^{A}_{N}Y_{Z} \tag{17} $$

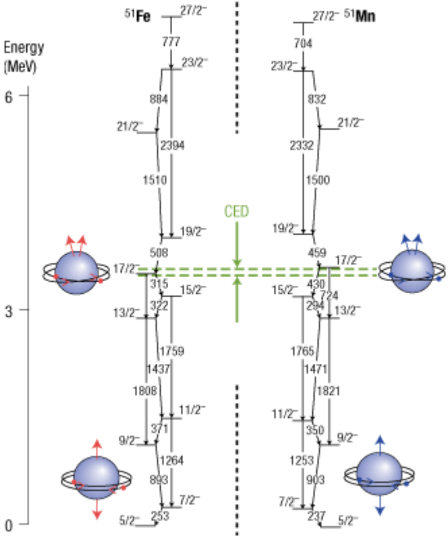
- Examples of mirror nuclei are ^{3}He and ^{3}H, ^{7}Li and ^{7}Be, etc.
- Due to the charge independence of the nuclear force the masses, binding energies and excitation spectra, when corrected for the electromagnetic (Coulomb) terms are very similar in light mirror nuclei.
- In heavy mirror nuclei the effect breaking charge independence of nuclear force are strong, and the similarity does not hold.

Binding energies in mirror nuclei

TABLE 5.1 Properties of Light Nuclei

A	Nucleus	Total Binding Energy (MeV)	Coulomb Energy (MeV)	Net Nuclear Binding Energy (MeV)
3	^{3}H	−8.486	0	−8.486
	^{3}He	−7.723	0.829	−8.552
13	^{13}C	−97.10	7.631	−104.734
	^{13}N	−94.10	10.683	−104.770
23	^{23}Na	−186.54	23.13	−209.67
	^{23}Na	−181.67	27.75	−209.42
41	^{41}Ca	−350.53	65.91	−416.44
	^{41}Sc	−343.79	72.84	−416.63

Excitation energies in mirror nuclei



The isospin

- The isospin is a concept introduced in $\sim$1930 by W. Heisenberg to account for charge independence of nuclear force and existence of two types of nucleons/
- In this formalism we assign a new quantum vector property called isospin to a nucleon. The isospin of a nucleon is $T = 1/2$.
- In analogy to spin, the isospin can have two projections, $T_z = +1/2$ and $T_z = -1/2$. The positive projection identifies a proton, the negative projection the neutron.
- The convenience in using the isospin formalism is in the fact that the nuclear force does not depend on the isospin projection T_z.
- It does, however, depend on the isospin T.
- For example, mirror pairs are nuclei which have the same isospin T but have the opposite isospin projection $T_Z = 1/2(Z - N)$

Isospin and spin in the mass A=2 system

- Let us consider the isospin in the simplest two-nucleon $A = 2$ system.
- The isospin couples like spin, thus for a two nucleon system we have a symmetric isospin triplet coupling and antisymmetric isospin singlet coupling.
- The wave function now has the orbital part, the spin part and the isospin part.
- The advantage is in the fact that this wave function describes the di-proton, the di-neutron and the deuteron simultaneously through the projection of the isospin while previously we had to describe each system separately.

Spin, isospin and the Pauli principle

- The Pauli principle requires that the whole wave function is anti symmetric.
- The lowest-energy orbital wave function, the $\mathcal{S}$ state is symmetric.
- Therefore the symmetric isospin triplet have to couple with the antisymmetric isospin singlet or the antisymmetric isospin singlet has to couple with the symmetric spin triplet.
- Spin singlet is $S = 0$, spin triplet is $S = 1$.
- From the properties of the deuteron we know that spin triplet is bound.
- Thus we conclude that the nuclear force is attractive in the isospin singlet coupling.

Spin, isospin and the Pauli principle

- The di-proton and the di-neutron are the opposite projections of the isospin triplet ($T_z = +1$ and $T_z = -1$, respectively), and therefore are spin singlet or have spin 0.
- There is also the $T_z = 0$ projection of the isospin triplet. It corresponds to the deuteron ($N = Z \implies T_z = 0$) and is spin singlet. That implies an existence in the deuteron of a spin 0 excited state.
- All members of the isospin triplet, the di-proton, the di-neutron, and the excited spin 0 state in the deuteron are unbound.
- Thus we conclude that the nuclear force is not attractive in the isospin triplet coupling.