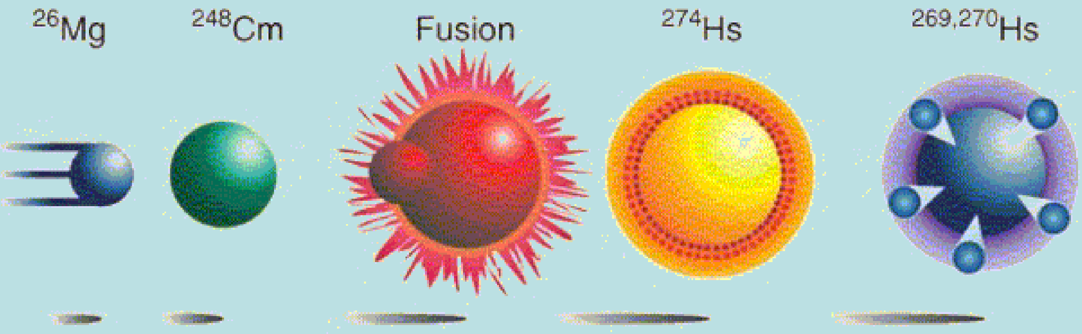
Fusion Reactions

Hans-Jürgen Wollersheim

“Hot Fusion”



$5 \cdot 10^{15} \longrightarrow 5 \cdot 10^{8} \longrightarrow$

projectiles on target compound nuclei 1 atom

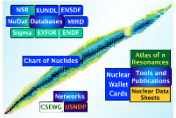
Hot fusion (~1952)

successful up to element 106 (Seaborgium)

- **Coulomb barrier V_C** between projectile and target nucleus has to be exceeded

$$V_C = \frac{Z_p \cdot Z_t \cdot e^2}{R_{int}} = 126.2\ MeV \quad \left({}^{26}Mg + {}^{248}Cm\right)$$

- reaction: a + A → C^* → B + b



https://www.nndc.bnl.gov/qcalc/

$$\Delta m = m_a + m_A - m_{CN}$$

$$\Delta m = (25.983 + 248.072 - 274.143) * 931.478\ MeV/c^2$$

$$= -82.153\ MeV/c^2$$

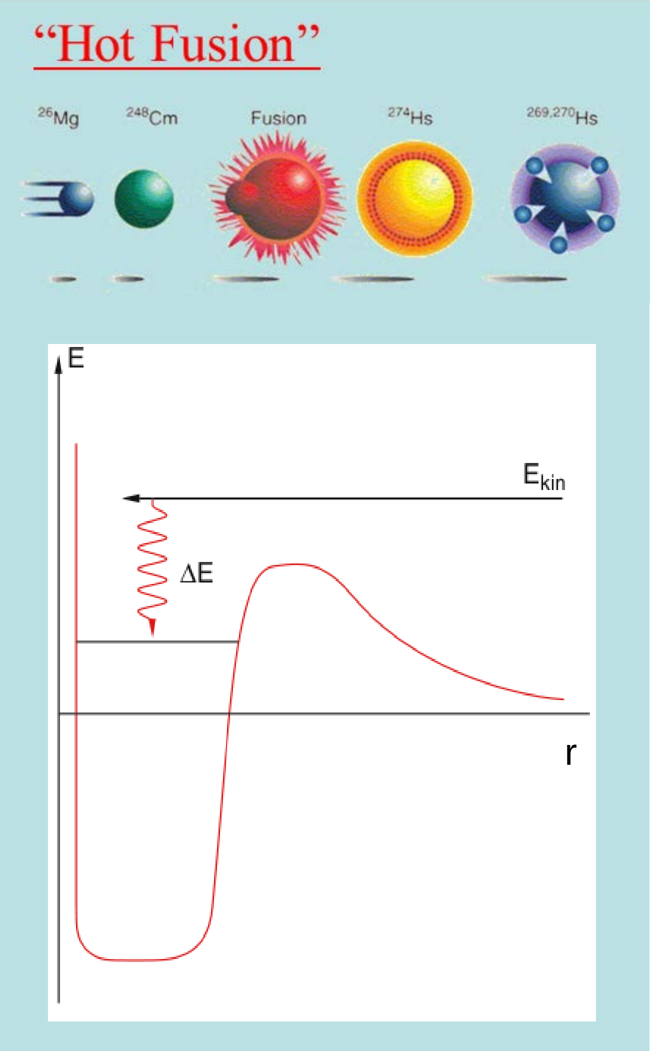
- excitation energy of compound nucleus

$$E^* = E_{kin} + \Delta m \cdot c^2$$

$$= 126.2\ MeV - 82.2\ MeV$$

$$= \mathbf{44.0\ MeV}$$

- approximate 4 neutrons will be evaporated to avoid fission

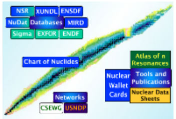


Cold fusion (1981-1996)

- **Coulomb barrier V_C** between projectile and target nucleus has to be exceeded

$$V_C = \frac{Z_p \cdot Z_t \cdot e^2}{R_{int}} = 223.3\ MeV \qquad \left({}^{58}Fe + {}^{208}Pb\right)$$

- reaction: a + A → C^* → B + b



https://www.nndc.bnl.gov/qcalc/

$$\Delta m = m_a + m_A - m_{CN}$$

$$\Delta m = (57.933 + 207.977 - 266.130) * 931.478\ MeV/c^2$$

$$= -205.092\ MeV/c^2$$

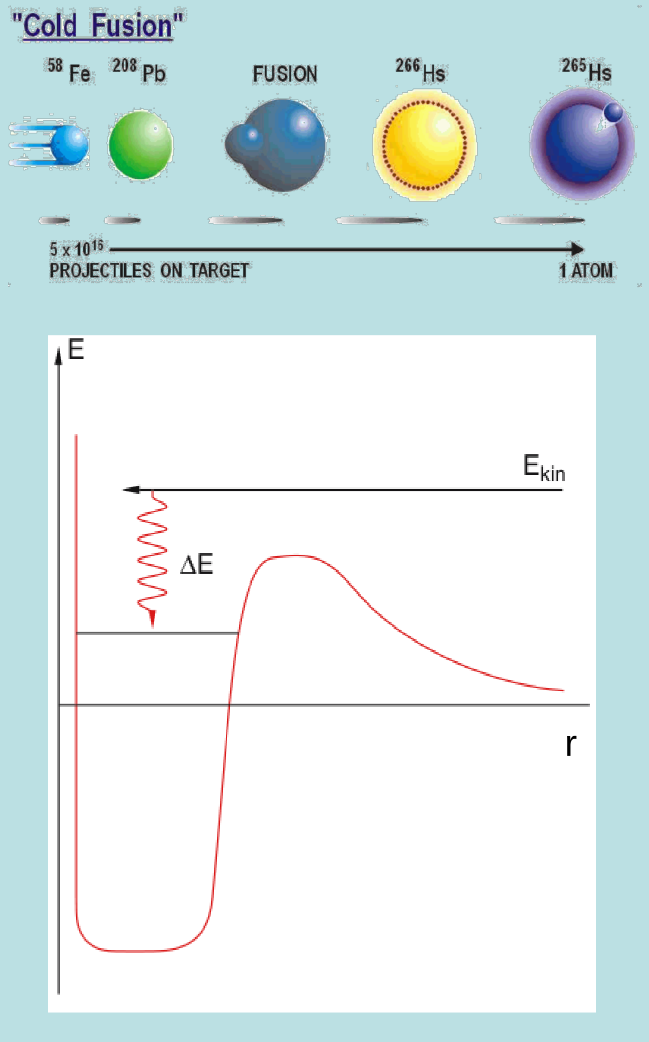
- excitation energy of compound nucleus

$$E^* = E_{kin} + \Delta m \cdot c^2$$

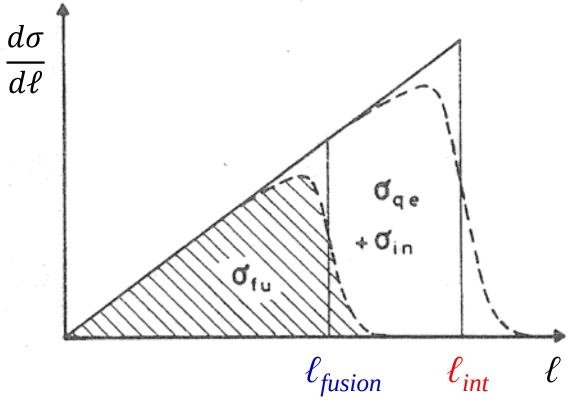
$$= 223.3\ MeV - 205.1\ MeV$$

$$\mathbf{= 18.2\ MeV}$$

- approximate 1-2 neutrons will be evaporated to avoid fission



Fusion cross section



Radius for fusion barrier:

$$R_{fusion} = R_{int} - \begin{Bmatrix} 0.3117 \cdot (Z_p \cdot Z_t)^{0.2122} & Z_p \cdot Z_t < 500 \\ 1.096 + 1.391 \cdot Z_p \cdot Z_t/1000 & Z_p \cdot Z_t \geq 500 \end{Bmatrix} \quad [fm]$$

	R_i [fm]	C_i [fm]	R_{int} [fm]	$V_C(R_{int})$ [MeV]	R_{fusion} [fm]	$V_C(R_{fusion})$ [MeV]
^{58}Fe	4.40	4.17	13.75	223.3	12.36	248.4
^{208}Pb	6.96	6.82				

Total cross section for fusion:

$$\sigma_{fusion} = \pi R_{fusion}^2 \cdot \left[1 - \frac{V_C(R_{fusion})}{E_{cm}}\right] \qquad with \quad E_{cm} = \frac{A_t}{A_t + A_p} \cdot E_{lab}$$

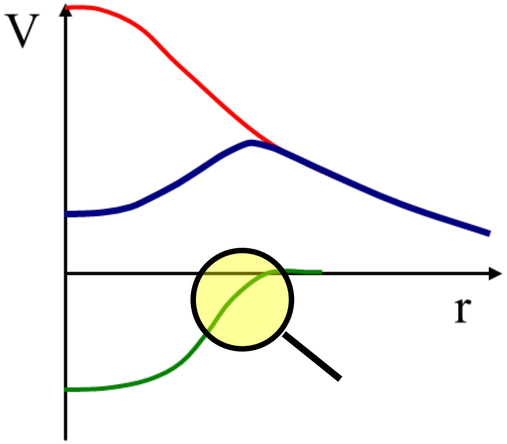
$$\sigma_{fusion} = \frac{\pi}{k_\infty^2} \cdot \ell_{fusion} \cdot (\ell_{fusion} + 1) \qquad with \quad k_\infty = 0.2187 \cdot \frac{A_t}{A_t + A_p} \cdot \sqrt{A_p \cdot E_{lab}} \quad [fm^{-1}]$$

Interaction potential

The potential between projectile and target nucleus is given by a function of the relative distance between them

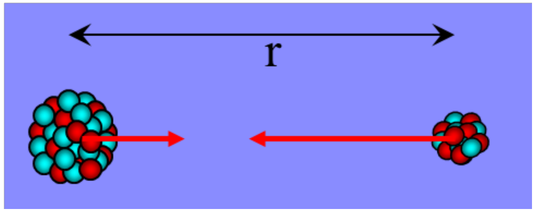
$$V(r) = V_N(r) + V_C(r)$$

nuclear potential + Coulomb potential



$$V_C(r) = \left\{ \begin{array}{ll} \dfrac{Z_1 Z_2 e^2}{2 \cdot R_C}\left(3 - \dfrac{r^2}{R_C^2}\right) & r < R_C \\ \dfrac{Z_1 Z_2 e^2}{r} & r \geq R_C \end{array} \right\}$$

$$V_N(r) = 4\pi \cdot \gamma \cdot \frac{C_p \cdot C_t}{C_p + C_t} \cdot b \cdot \Phi(\xi)$$



$$\Phi(\xi) = \left\{ \begin{array}{ll} -0.5 \cdot (\xi - 2.54)^2 - 0.0852 \cdot (\xi - 2.54)^3 & \xi \leq 1.2511 \\ -3.437 \cdot exp(-\xi/0.75) & \xi \geq 1.2511 \end{array} \right\}$$

$$\xi = (r - C_p - C_t)/b$$

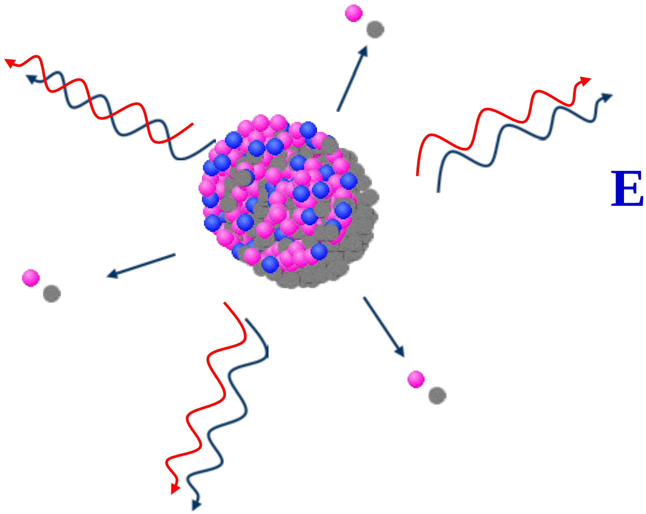
$$b = \frac{\pi}{\sqrt{3}} \cdot a \cong 1\ fm \quad with\ a = 0.55\ fm$$

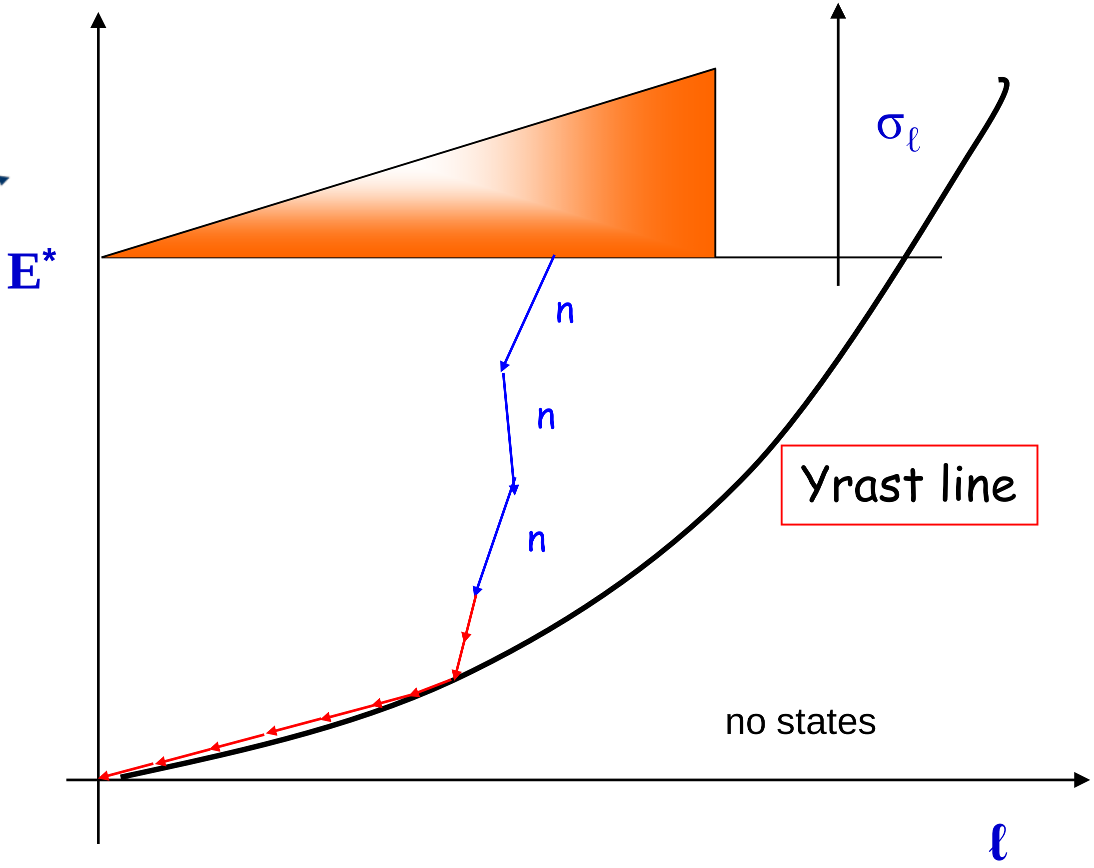
$$\gamma = 0.9517 \cdot \left\{1 - 1.7826 \cdot \left(\frac{N_c - Z_c}{A_c}\right)^2\right\} \quad \frac{MeV}{fm^2}$$

$$C_i = R_i \cdot \left(1 - R_i^{-2}\right) \quad [fm] \qquad R_i = 1.28 \cdot A_i^{1/3} - 0.76 + 0.8 \cdot A_i^{-1/3} \quad [fm]$$

The Statistical Model

de-excitation of the hot compound system

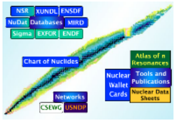




$$E^* = E_{kin} + \Delta m \cdot c^2$$

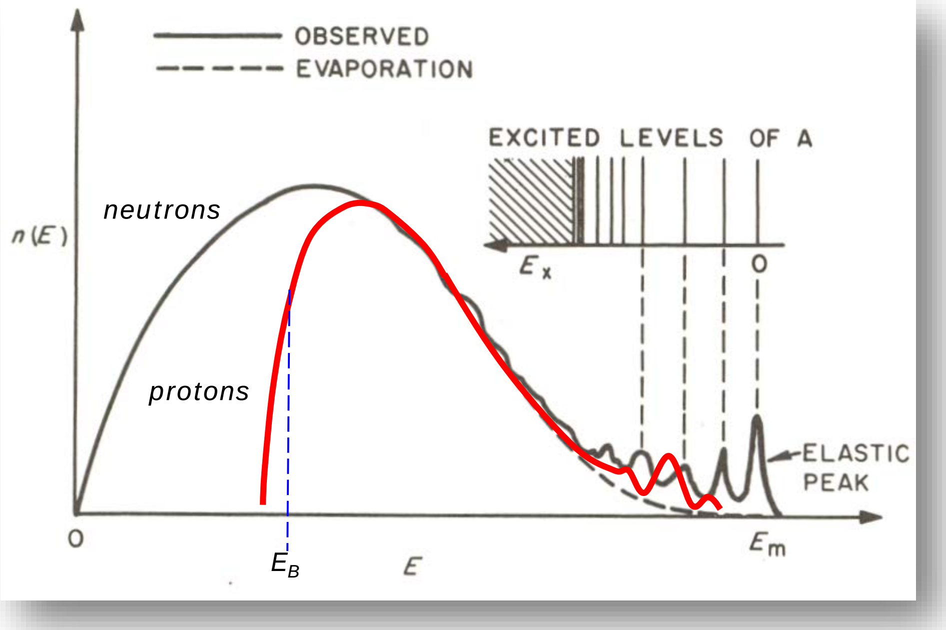
$$E_{kin} > V_C = \frac{Z_a \cdot Z_A \cdot e^2}{R_{int}}$$

$$\Delta m = m_a + m_A - m_{CN}$$



https://www.nndc.bnl.gov/qcalc/

Evaporation particles



compound nucleus reactions

direct reactions

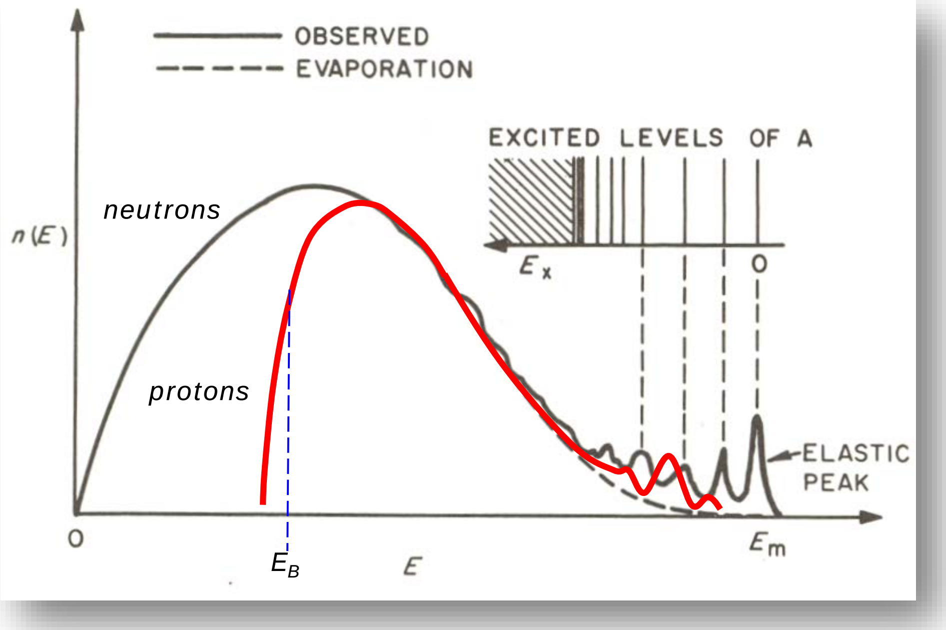
cm-spectra of particles statistically emitted from CN (evaporation) are of Maxwell Boltzmann type

$$\boxed{\frac{dN}{dE} \propto (E - E_B) \cdot e^{-E/T}}$$

E_B = Coulomb barrier
T = effective nuclear temperature

Typical energy spectrum of nucleons emitted at a fixed angle in inelastic nucleon-nucleon reactions.

Evaporation particles



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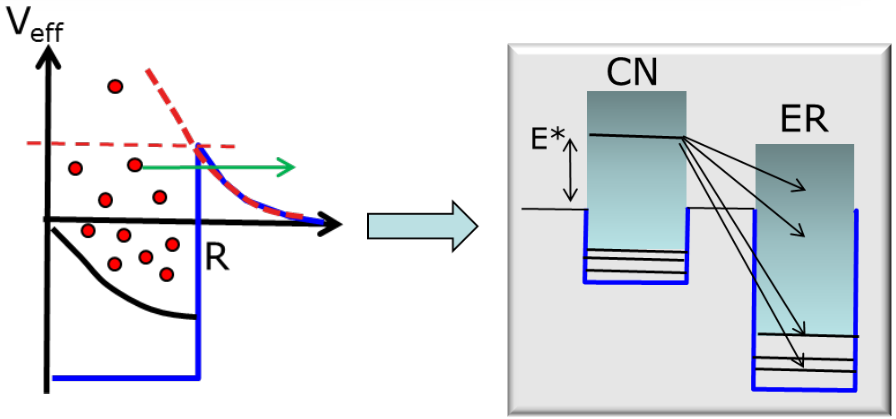
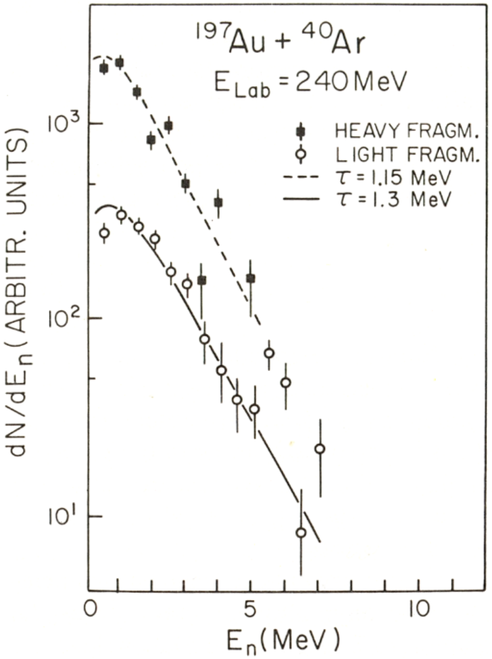


Even for fixed E^* the particle spectrum is continuous (Maxwell Boltzmann), except for transitions to discrete spectrum at low E_{ER}^*

Nuclear temperatures and level densities

de-excitation of the hot compound system



$$\frac{dN}{dE_n} \propto E_n \cdot e^{-E_n/T}$$ spectrum of single neutron

$$\langle E_n \rangle = 2T \qquad \max \frac{dN}{dE_n} \ @ \ E_n = T$$

$$\frac{dN}{dE_n} \propto \sqrt{E_n} \cdot e^{-E_n/T_{eff}}$$ spectrum of cascade of neutrons

$$\langle E_n \rangle = 1.5T \qquad T_{eff} \approx 0.92 \cdot T \quad (1^{st}\ daughter)$$

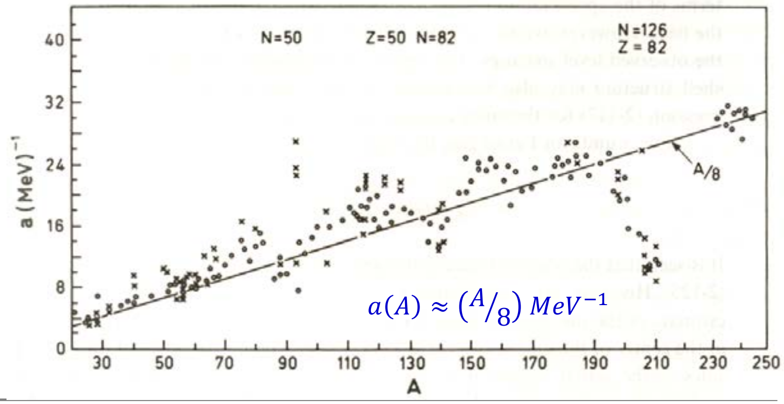
Fermi gas relations:

$$E^* = a \cdot T^2 \quad \text{"little} - \text{a"}$$

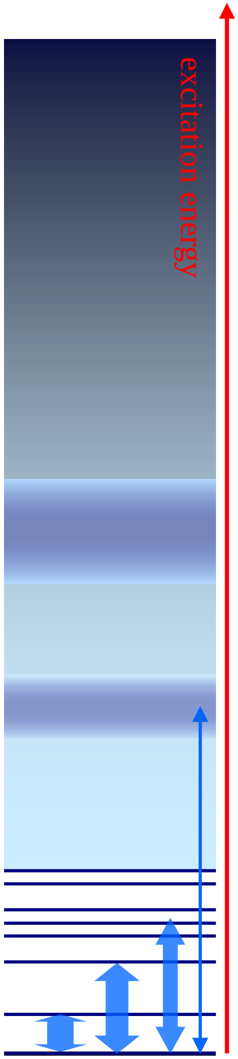
$$S = \int \frac{dE^*}{E^*} = 2\sqrt{a \cdot E^*}$$

$$\rho\left(E^*\right) = \rho_0 \cdot e^{2\sqrt{a \cdot E^*}}$$

deviations at shell closures



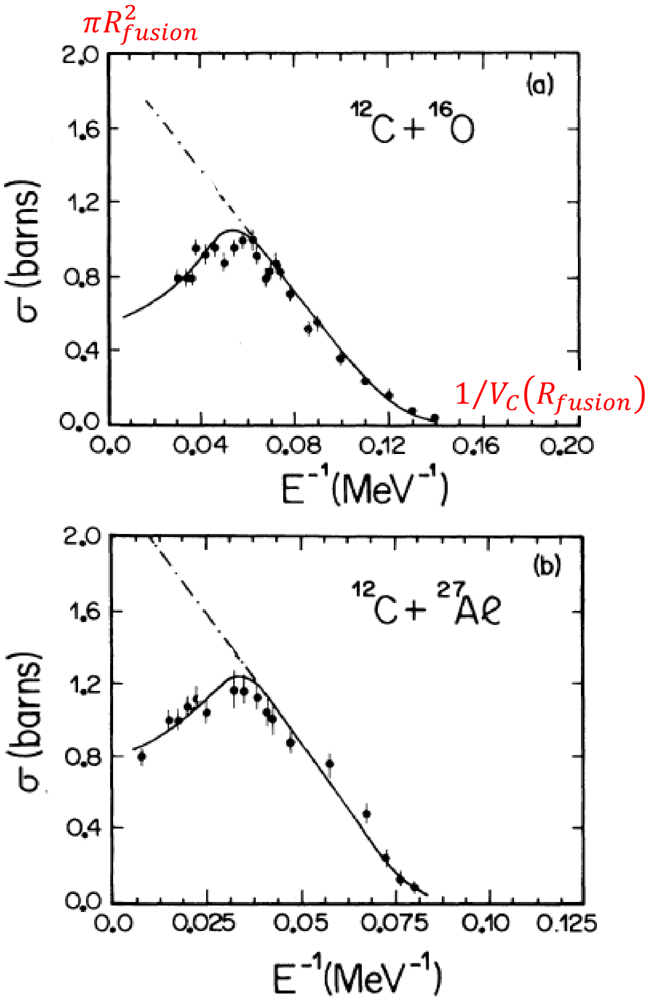
$a(A) \approx \left({}^{A}/_{8}\right) MeV^{-1}$



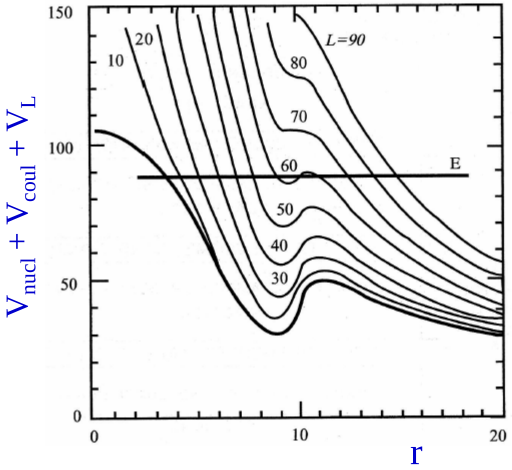
collective states

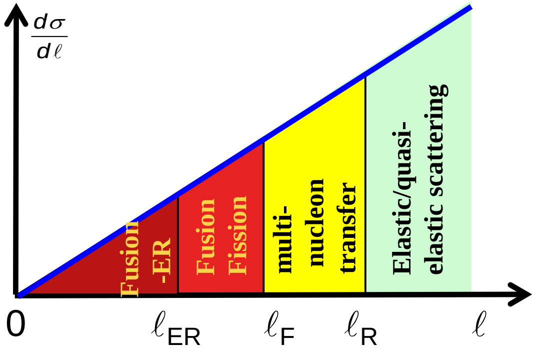
ground state

Fusion excitation functions



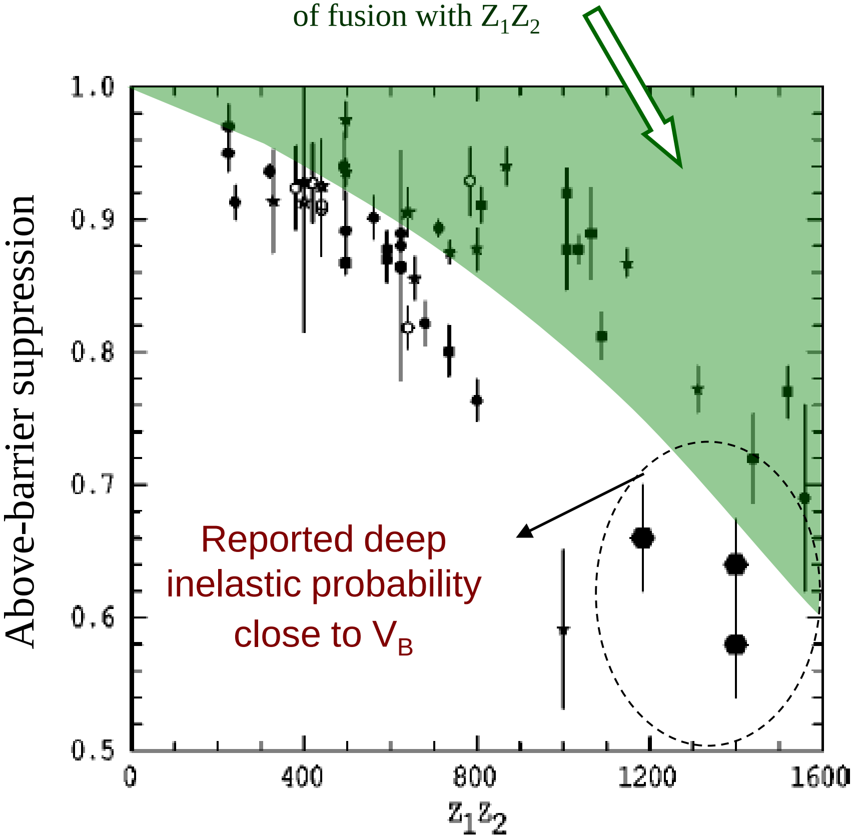
maximum ℓ_{fusion} due to nuclear centrifugal stability





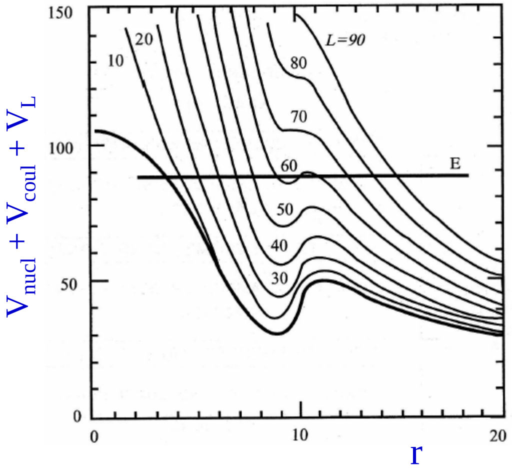
P. Sperr et al., PRL37, 321(1976)

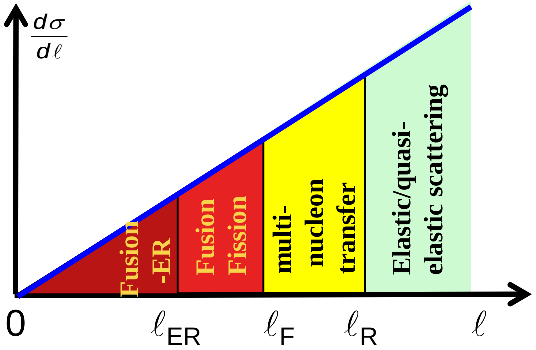
Reduction in fusion at above barrier energies



increased reduction of fusion with Z_1Z_2

Reported deep inelastic probability close to V_B





P. Sperr et al., PRL37, 321(1976)

A limiting nuclear angular momentum

rotating charged liquid drop

surface energy:

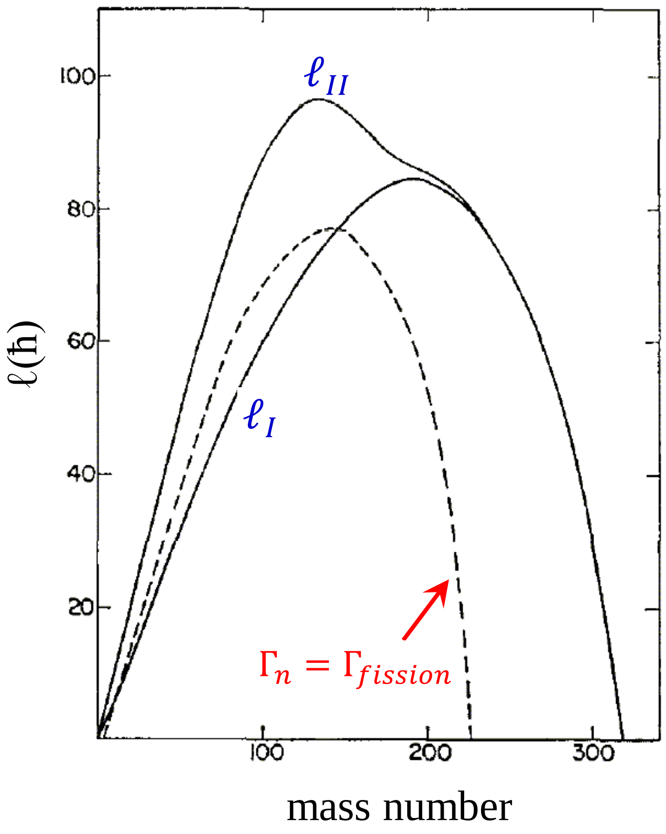
$$E_S^{(0)} = 17.9439 \cdot \left[1 - 1.7826 \cdot \left(\frac{N-Z}{A}\right)^2\right] \cdot A^{2/3} \quad [MeV]$$

Coulomb energy:

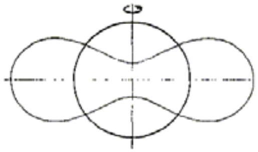
$$E_{Coul}^{(0)} = 0.7053 \cdot \left(Z^2/A^{1/3}\right) \quad [MeV]$$

rotational energy:

$$E_{Rot}^{(0)} = \frac{1}{2}\frac{\hbar^2 \cdot \ell^2}{(2/5) \cdot A \cdot m \cdot R^2} = 34.54 \cdot \frac{\ell^2}{A^{5/3}} \quad [MeV]$$



change of the nuclear shape



$$\frac{E_{Rot}^{(0)}}{E_S^{(0)}} = \begin{cases} 0.2829 - 0.3475 \cdot X - 0.0016 \cdot X^2 + 0.0501 \cdot X^3 & 0 \leq X \leq 0.75 \\ (7/5) \cdot (1-X)^2 - 4.5660 \cdot (1-X)^3 + 6.7443 \cdot (1-X)^4 & 0.75 \leq X \leq 1.0 \end{cases}$$

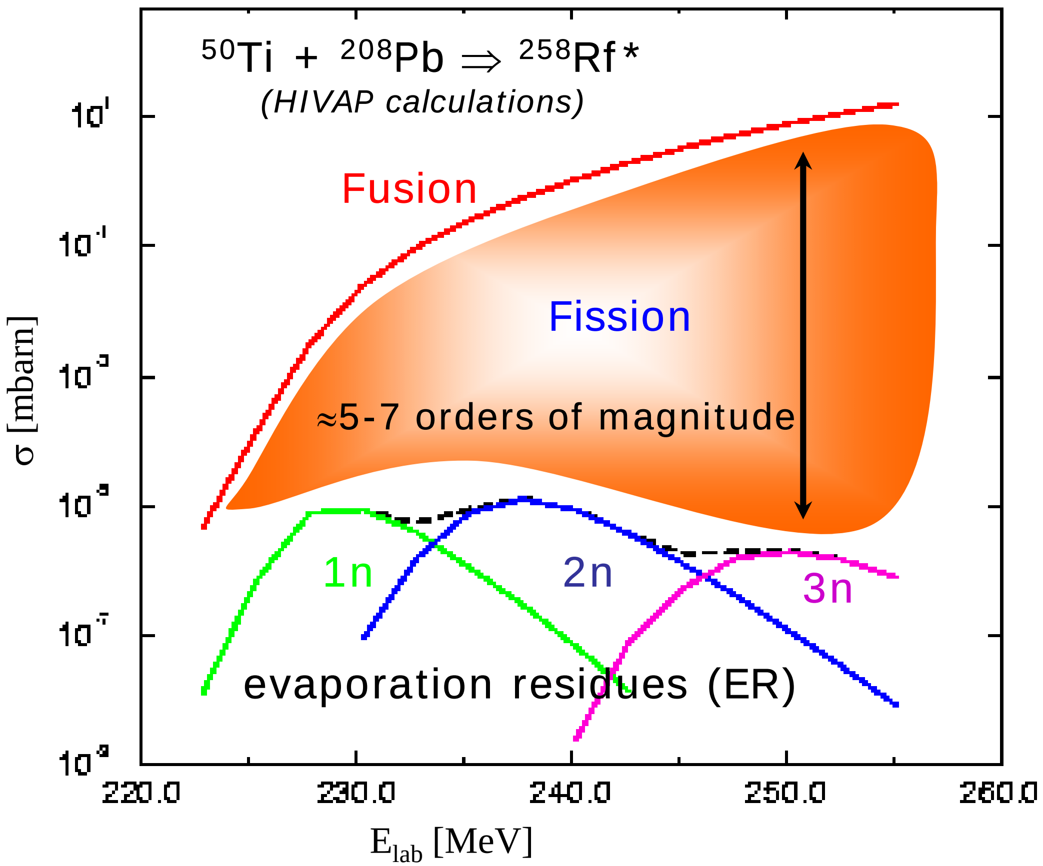
with $X = \dfrac{E_{Coul}^{(0)}}{2 \cdot E_S^{(0)}}$ "fissility parameter"

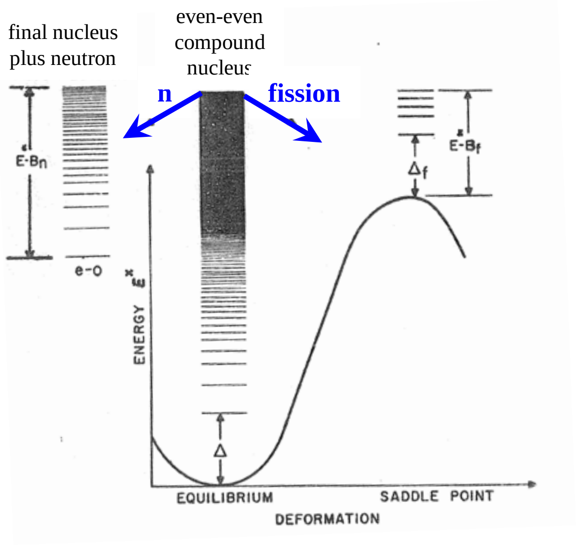
example: ${}^{127}_{57}La$ $E_S^{(0)} = 444.9\ [MeV]$ $E_{Coul}^{(0)} = 455.9\ [MeV]$ $X = 0.512$ $E_{Rot}^{(0)}/E_S^{(0)} = 0.1112$ $E_{Rot}^{(0)} = 49.48\ [MeV]$ $\ell_I = 67.8\ [\hbar]$

Cohen, Plasil, Swiatecki; Ann. Phys. 82, 557 (1974)

Fusion and evaporation

cold fusion





Both decay processes are determined by the level density, either from the residual nucleus or at the saddle point.

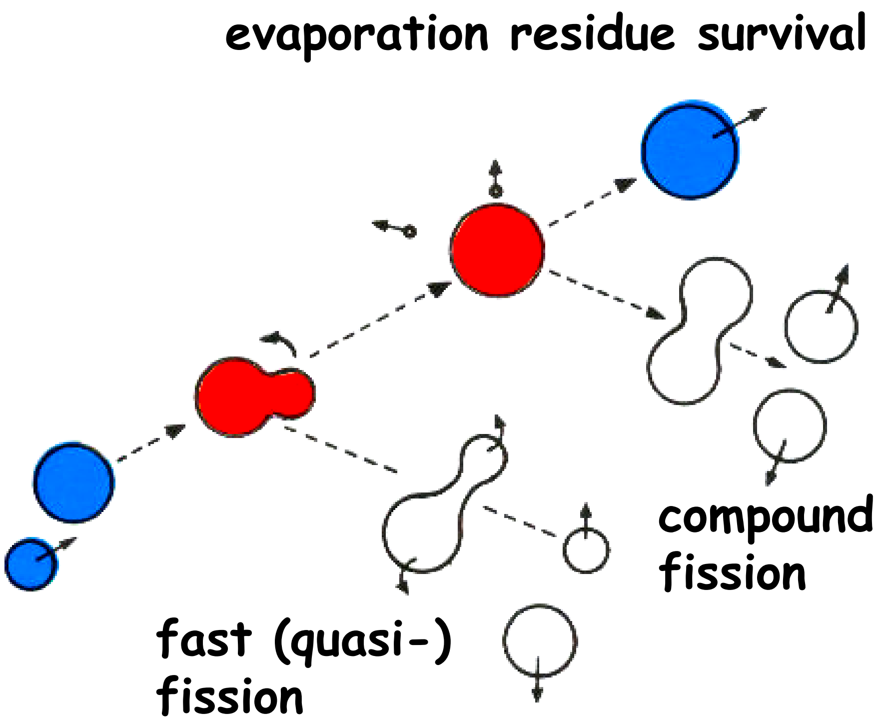
level density: $\rho(E^*) = const \cdot \exp(E^* / T)$

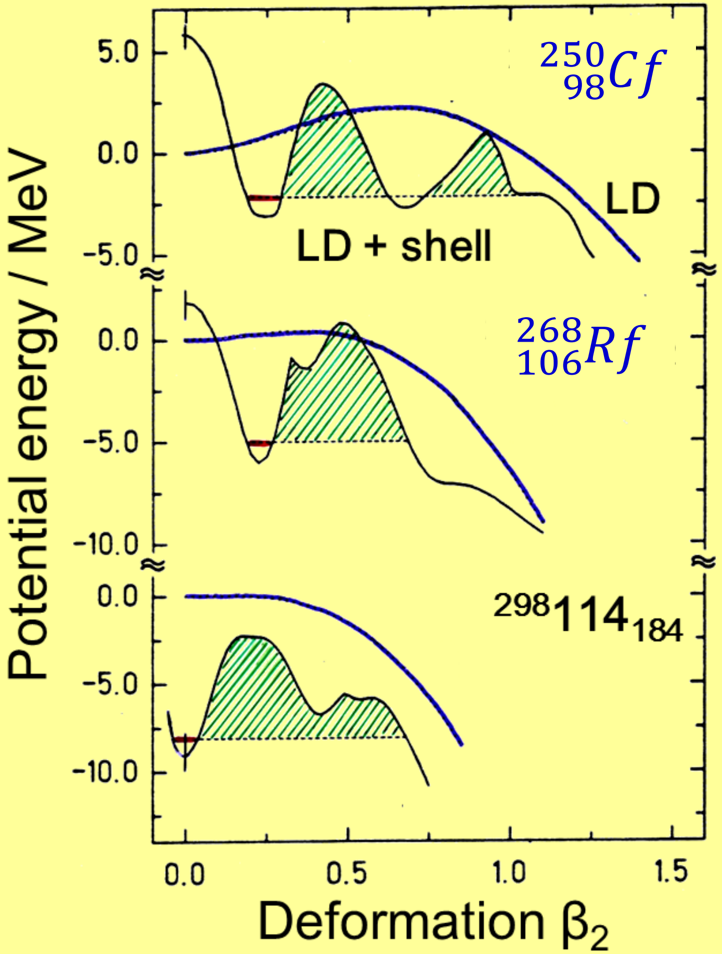
$$\frac{\Gamma_n}{\Gamma_f} = \frac{2 \cdot T \cdot A_{CN}^{2/3}}{K_0} \cdot \exp\left[\left(B_f - B_n\right)/T\right]$$

$K_0 = \hbar^2 / 2 \cdot m \cdot r_0^2 \approx 11.4 MeV$ $\quad T = \sqrt{8 \cdot E^* / A_{CN}}$

Fusion/Fission competition for SHE

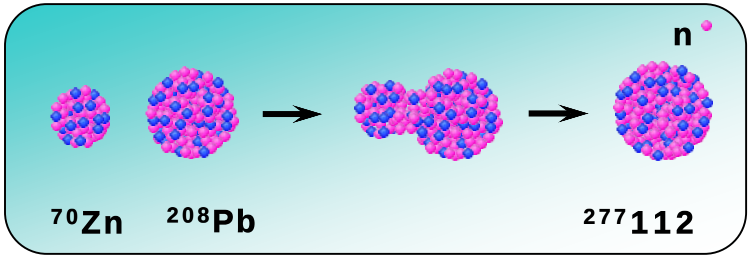
liquid drop + shell corrections





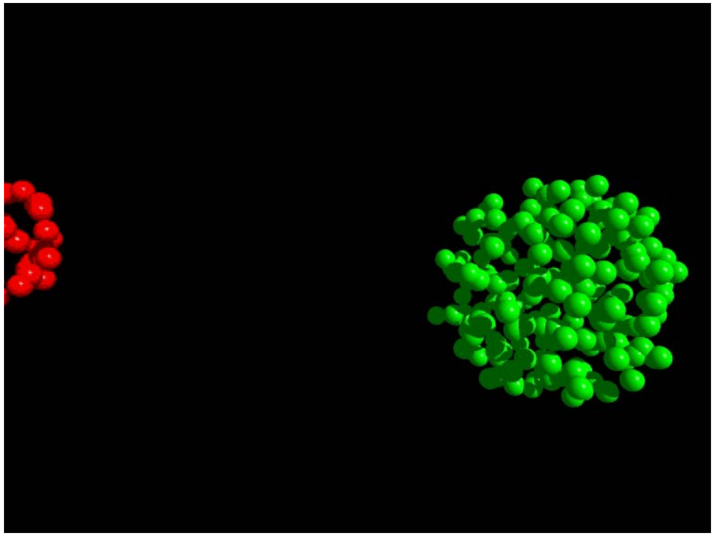
A. Sobiczewski et al.

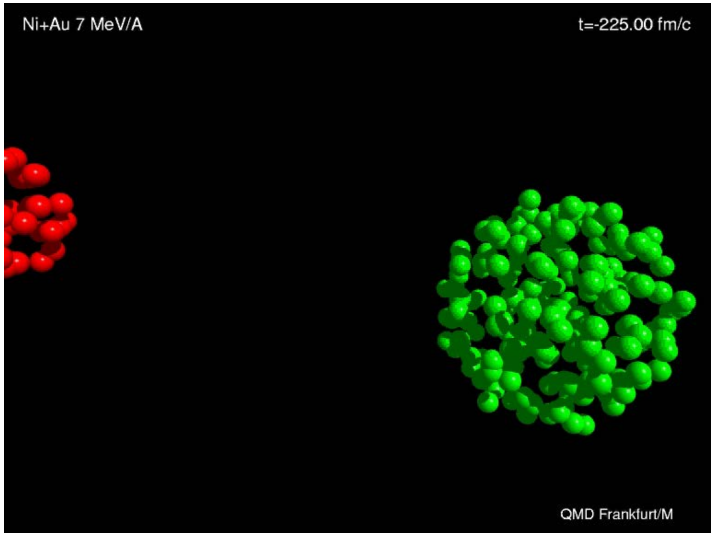
Synthesis of heavy elements



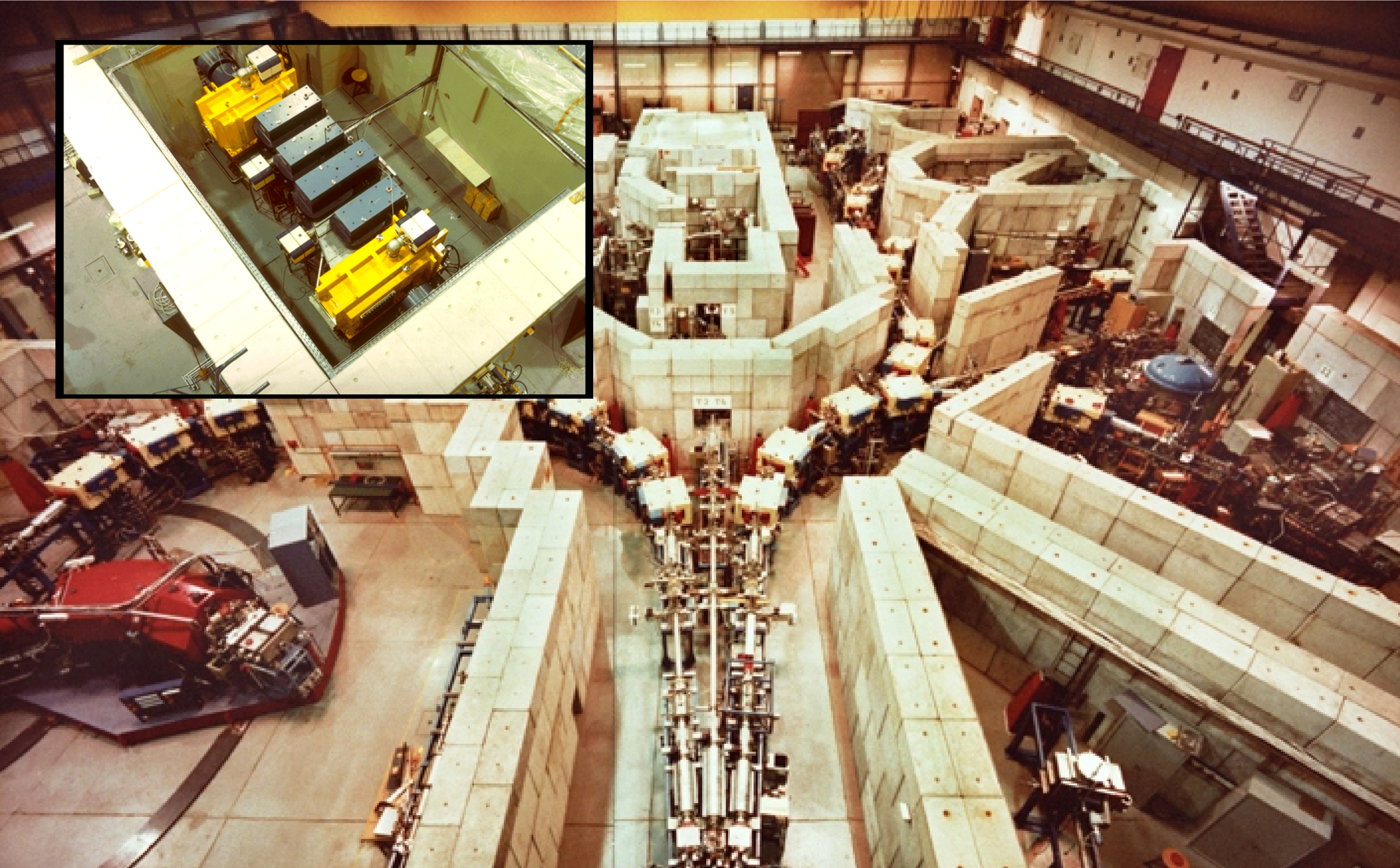
Fusion

$\frac{1}{10^{12}}$





Separator for Heavy Ion Products (SHIP)



Separator for Heavy Ion Products (SHIP)

- Fusion products are slower than scattered or transfer particles

$$v_{CN} = [m_p/(m_p + m_t)] \cdot v_p$$

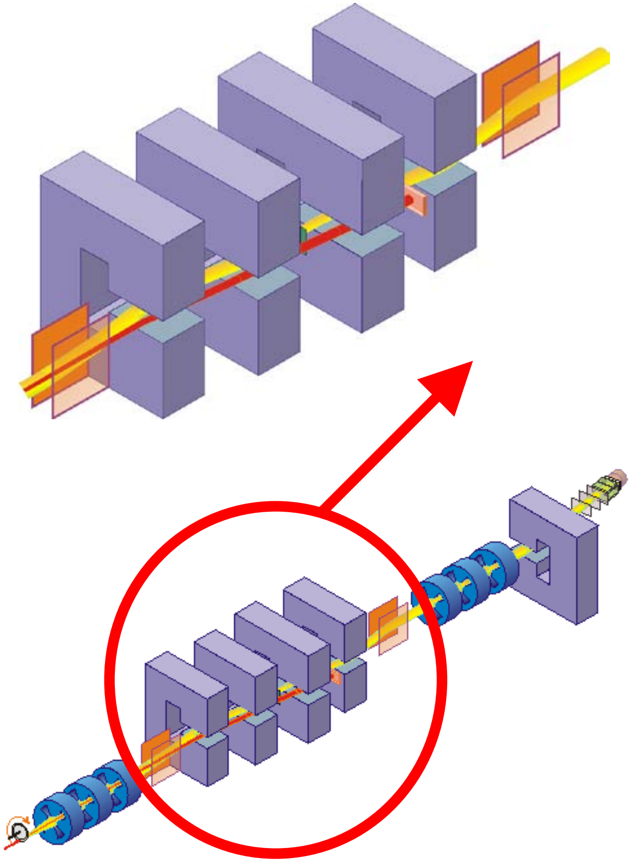
$$e.q.\, v_p \approx 10.3\% \quad \rightarrow \quad v_{CN} \approx 2.2\%$$

- E- and B-field are perpendicular to each other

$$B \cdot \rho = \frac{m \cdot v}{e \cdot q}$$

$$E \cdot \rho = \frac{m \cdot v^2}{e \cdot q}$$

$$F_{mag} = F_{el} \Rightarrow F_{tot} = 0$$



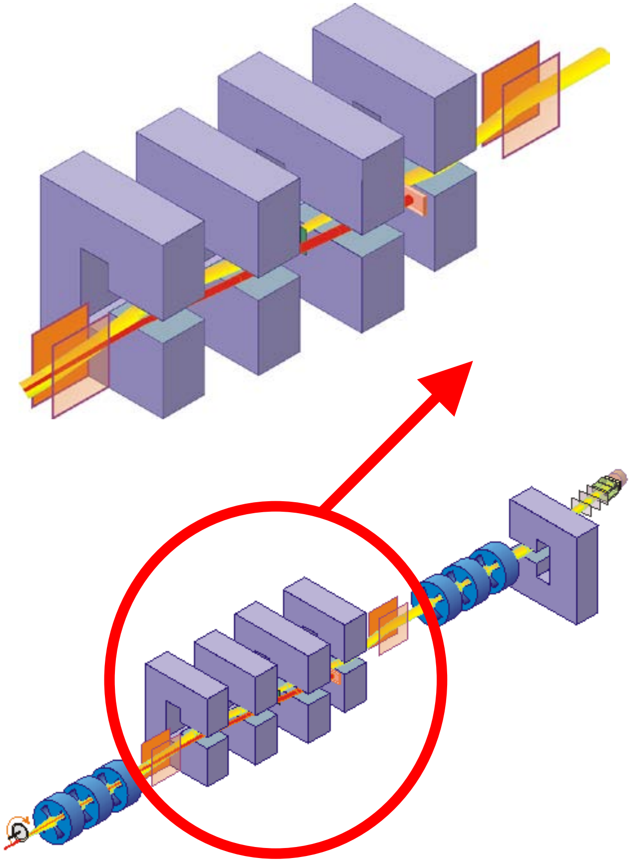
electric deflectors: ±330 kV   dipole magnets: 0.7 T max

Separator for Heavy Ion Products (SHIP)

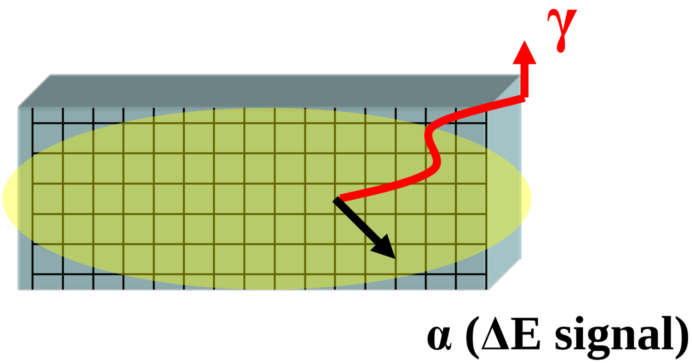
- The choice of E and B determines the transmitted velocity

$$v = \frac{E}{B}$$

- The rejected beam will be stopped on a cooled Cu plate



SHIP – stop detector



SHE will be measured in a pixel

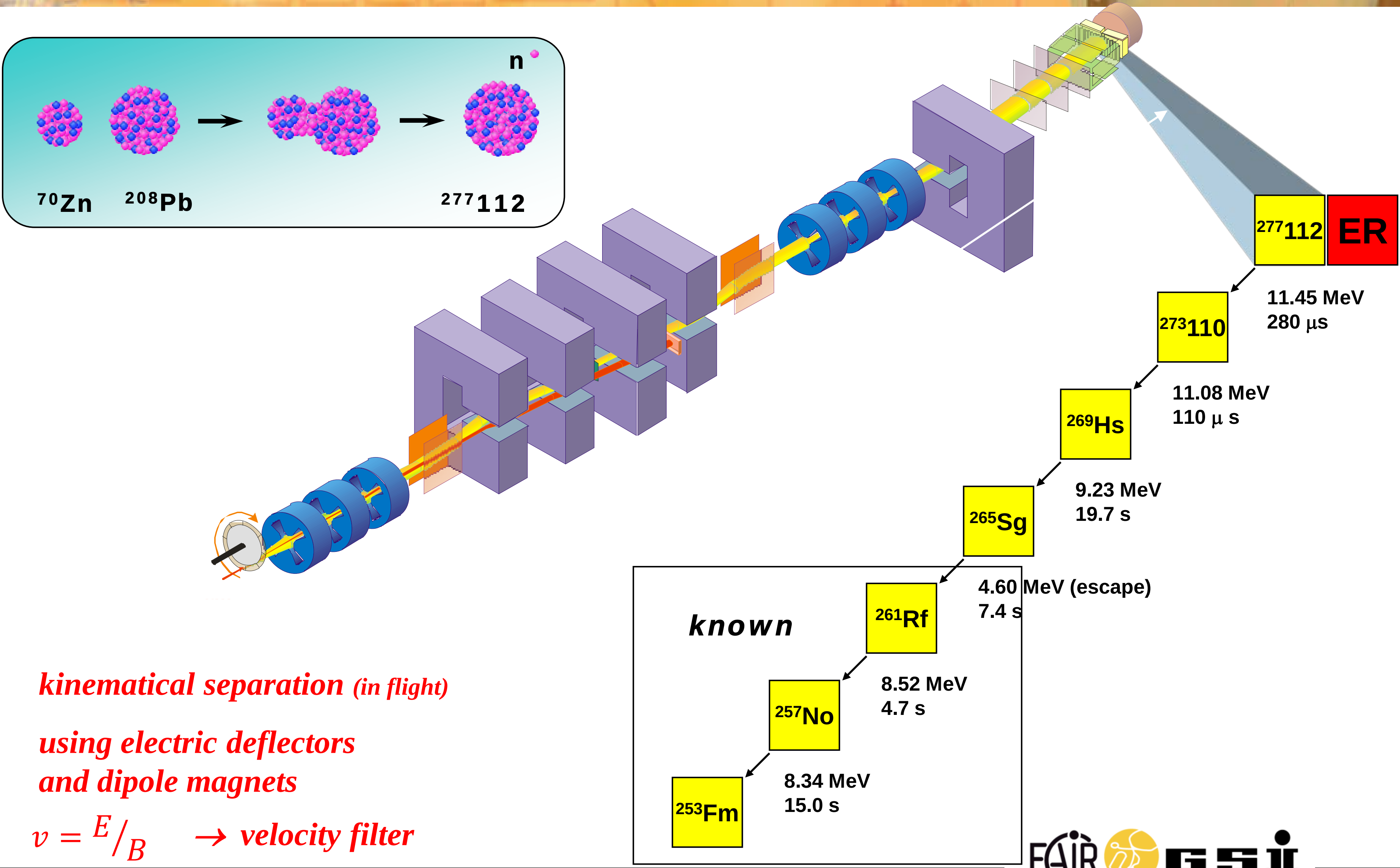
- position sensitive Silicon detector determines the position an energy of SHE and α, β, ...

area: 27*87mm^2, thickness: 0.3mm, 16 strips
energy resolution ΔE=18-20 keV @ E_α> 6MeV (cooling 260K)
position resolution Δx=0.3mm (FWHM)

Wait for the emission of an α-particle (or β-particle)

correlation method: implantation and decay event in the same pixel

Synthesis and identification of heavy elements with SHIP



^{277}112 ER

11.45 MeV
280 µs

^{273}110

11.08 MeV
110 µ s

^{269}Hs

9.23 MeV
19.7 s

^{265}Sg

4.60 MeV (escape)
7.4 s

known

^{261}Rf

8.52 MeV
4.7 s

^{257}No

8.34 MeV
15.0 s

^{253}Fm

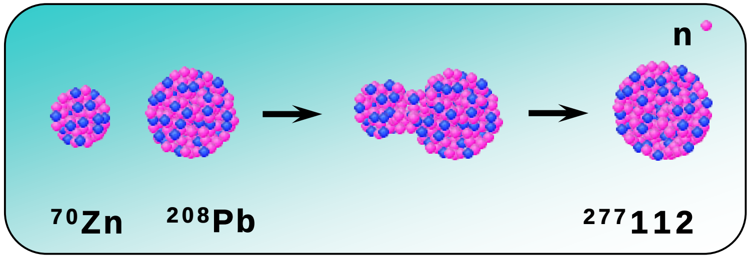
Date: 09-Feb-1996
Time: 22:37 h

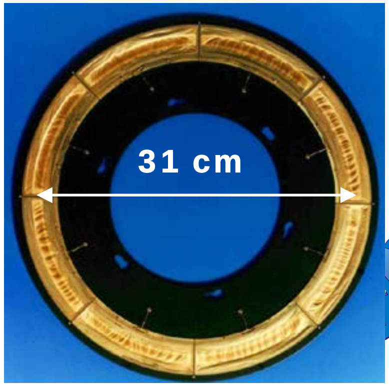
kinematical separation (in flight)

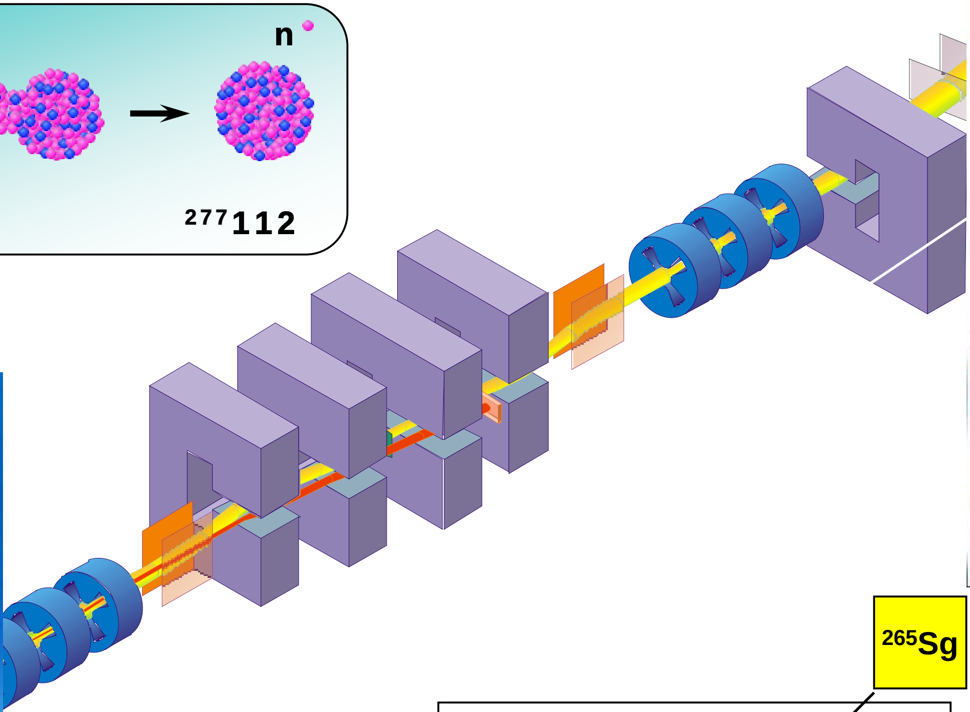
using electric deflectors
and dipole magnets

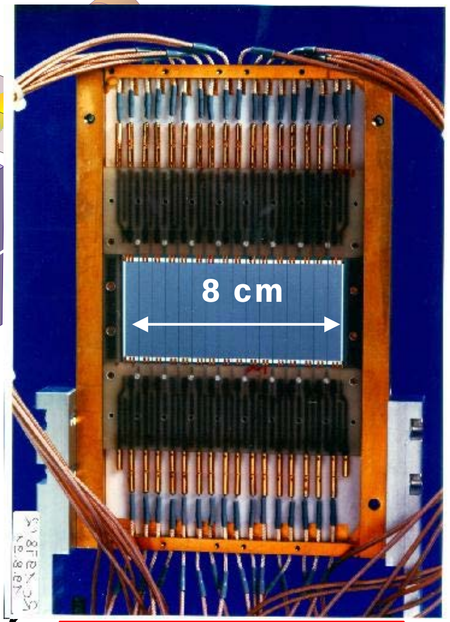
$v = {}^{E}/_{B}$ → *velocity filter*

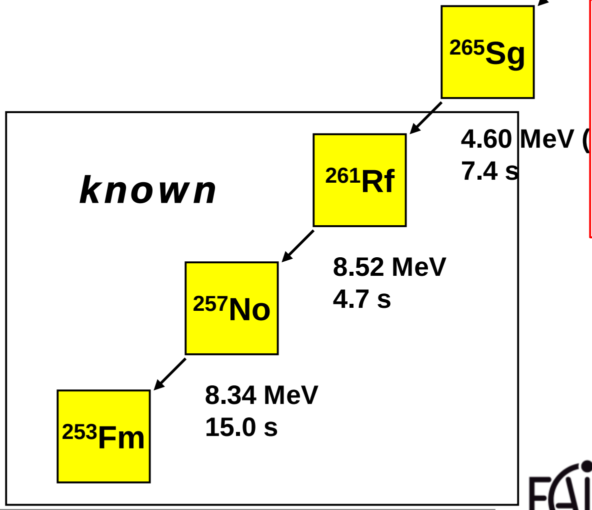
Synthesis and identification of heavy elements with SHIP











*Identification by α-α correlations down to **known** isotopes*

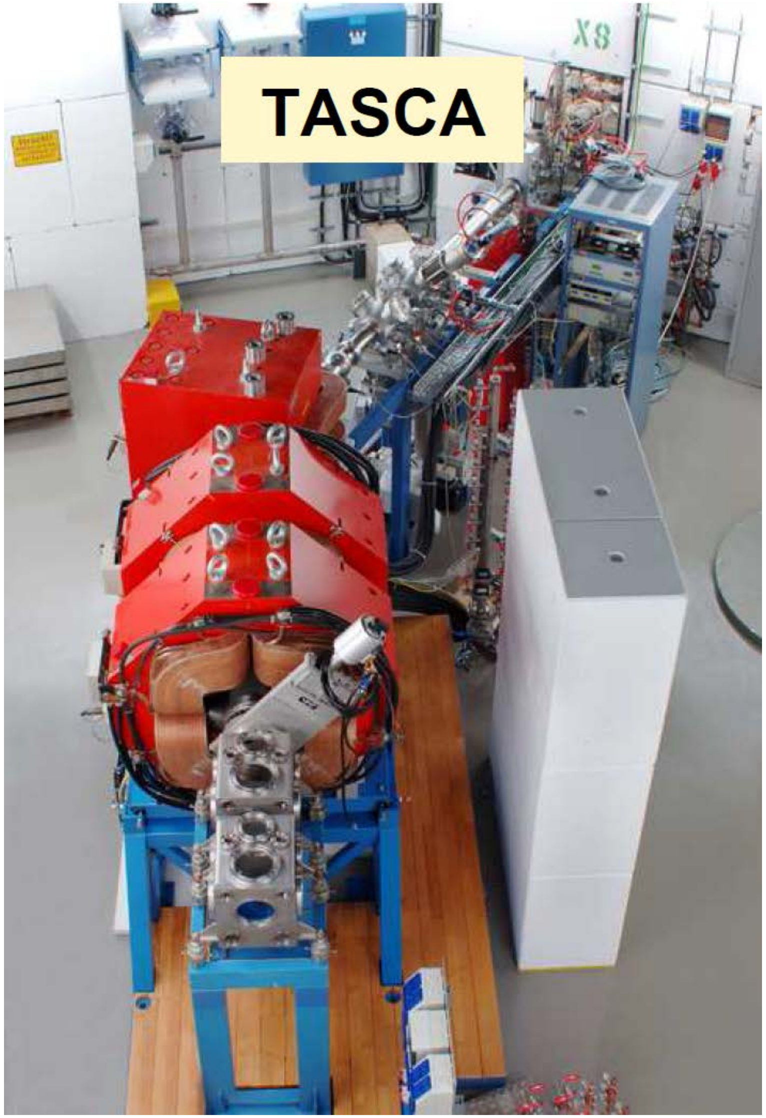
kinematical separation (in flight)

using electric deflectors
and dipole magnets

$v = E/B$ → *velocity filter*

Date: 09-Feb-1996
Time: 22:37 h

Trans Actinide Separator and Chemistry Apparatur



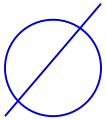
New gas-filled Separator for SHE

Actinide targets (U, Pu, Am, …)
Highest UNILAC beam intensities

high transmission
~60%

5 cm x 12 cm

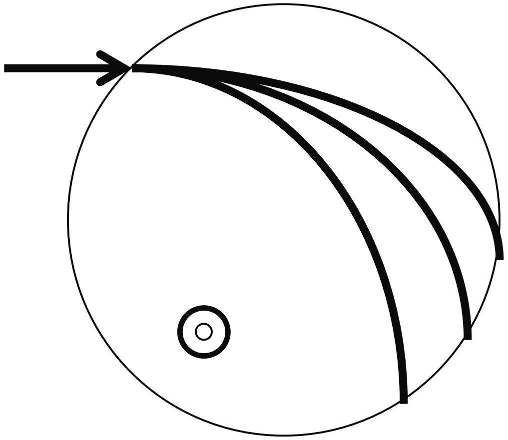
small image size
~40%


3 cm

The gas-filled recoil-separator comprises a dipole magnet for spatial separation of ions according to their magnetic rigidity, followed by a quadrupole duplet for focusing the separated superheavy elements into the focal plane.

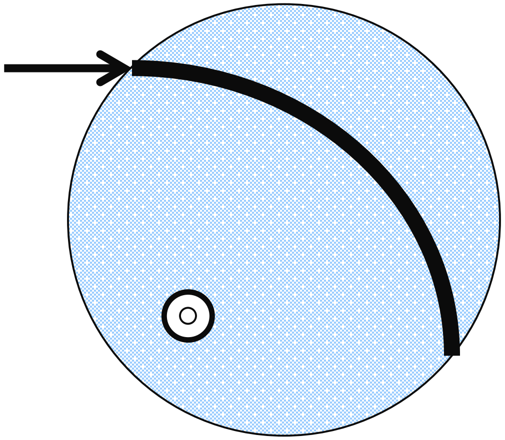
Vacuum or gas-filled separator

vacuum



$$B \cdot \rho = \frac{m \cdot v}{q_i}$$

gas (He: 0.5 – 2 mbar)



$$B \cdot \rho = \frac{m \cdot v}{\overline{q}}$$

$$\overline{q} \sim (v / v_0) \cdot Z^{1/3}$$

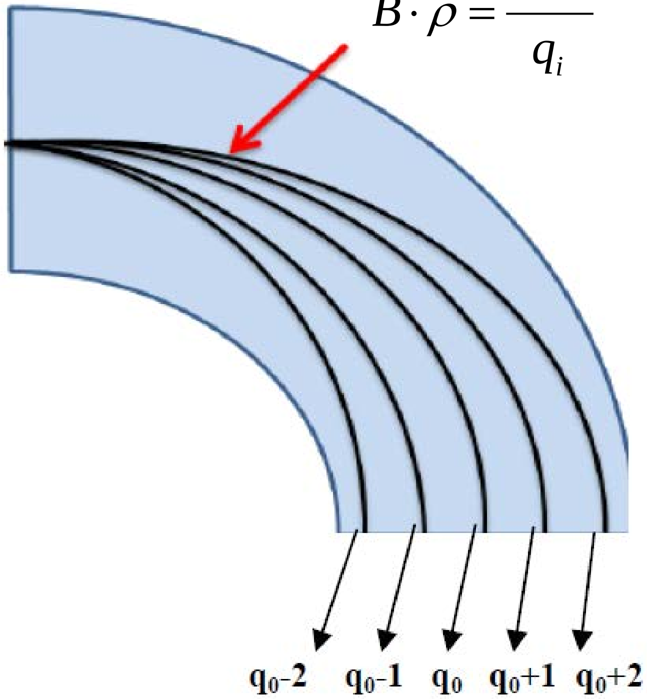
$$B \cdot \rho \sim 0.0227 \cdot A / Z^{1/3} \quad [Tm]$$

Fused nuclei leave the target in different charge states. In a gas-filled separator one obtains very fast an average charge state. (**B·ρ** is independent of **v**), so that the transmission is increased substantially.

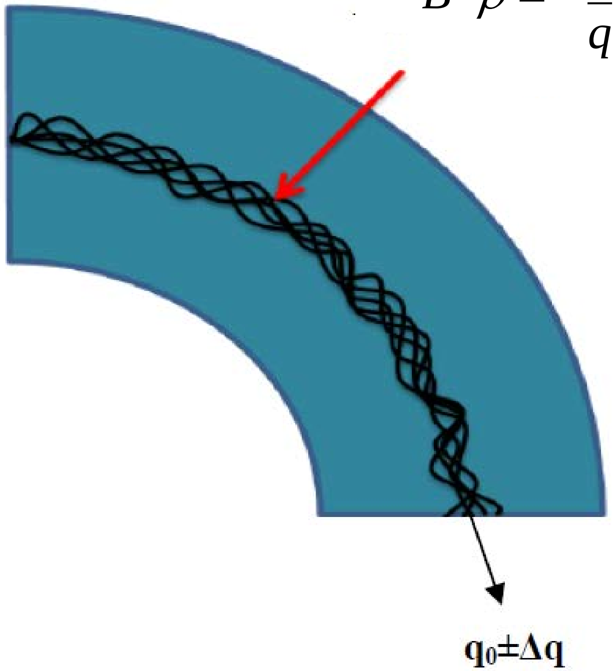
Bohr velocity v_0=c/137

Vacuum or gas-filled separator

$$B \cdot \rho = \frac{m \cdot v}{q_i}$$



$$B \cdot \rho = \frac{m \cdot v}{\bar{q}}$$

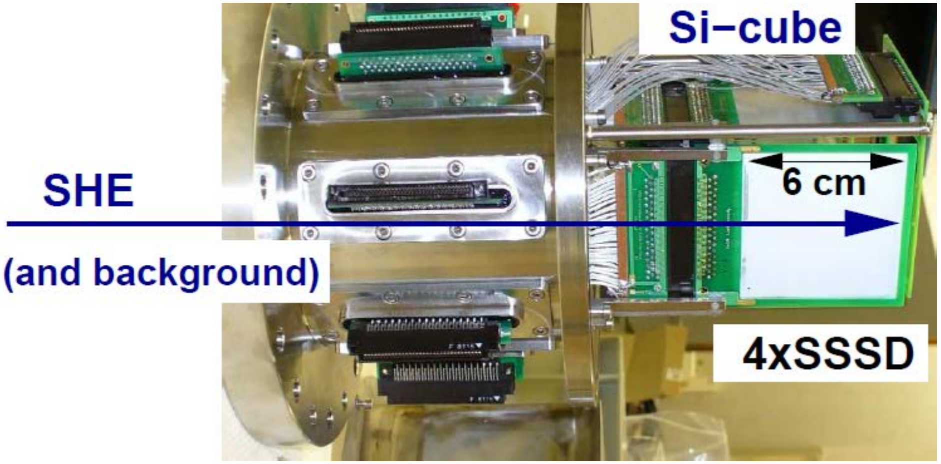


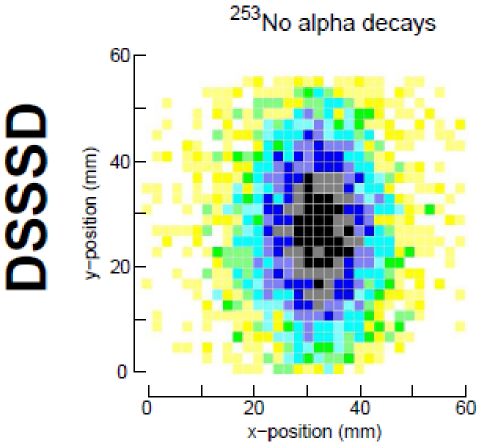
- Heavy ions leave the target in a charge distribution
- Vacuum systems accept only a few charge states
- **excellent resolution**

- Ion scattering with the gas (velocity of ions and electrons are almost equal)
- magnetic rigidity Bρ is independent of the velocity, since also the average charge state depends on the velocity
- **large acceptance**

but: problems with resolution, background suppression

TASISPEC – TASCA Small Image Spectrometer





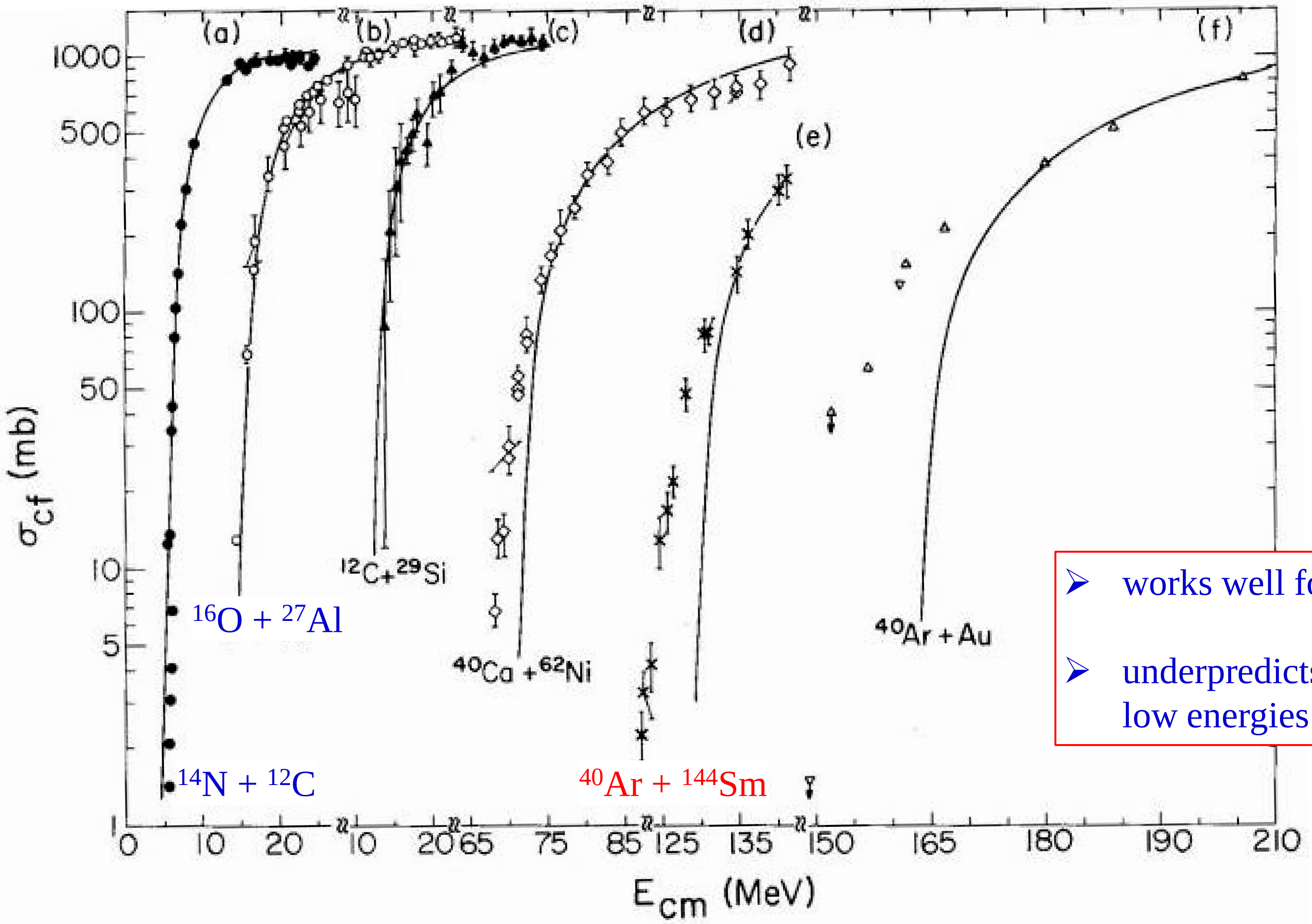
DSSSD: 0.5mm, 32x32 strips
4 SSSD: 1.0mm, 4x32 strips

Efficiency: ~80% (alpha)
~50-100 keV energy threshold

Ge-cube:
4 Clover, 1 Cluster: 23 Ge crystals
Efficiency: ~40% at 250 keV
Multi-coincidence options

Comparison between potential model and exp. data

Fusion cross sections calculated with a static, energy independent potential

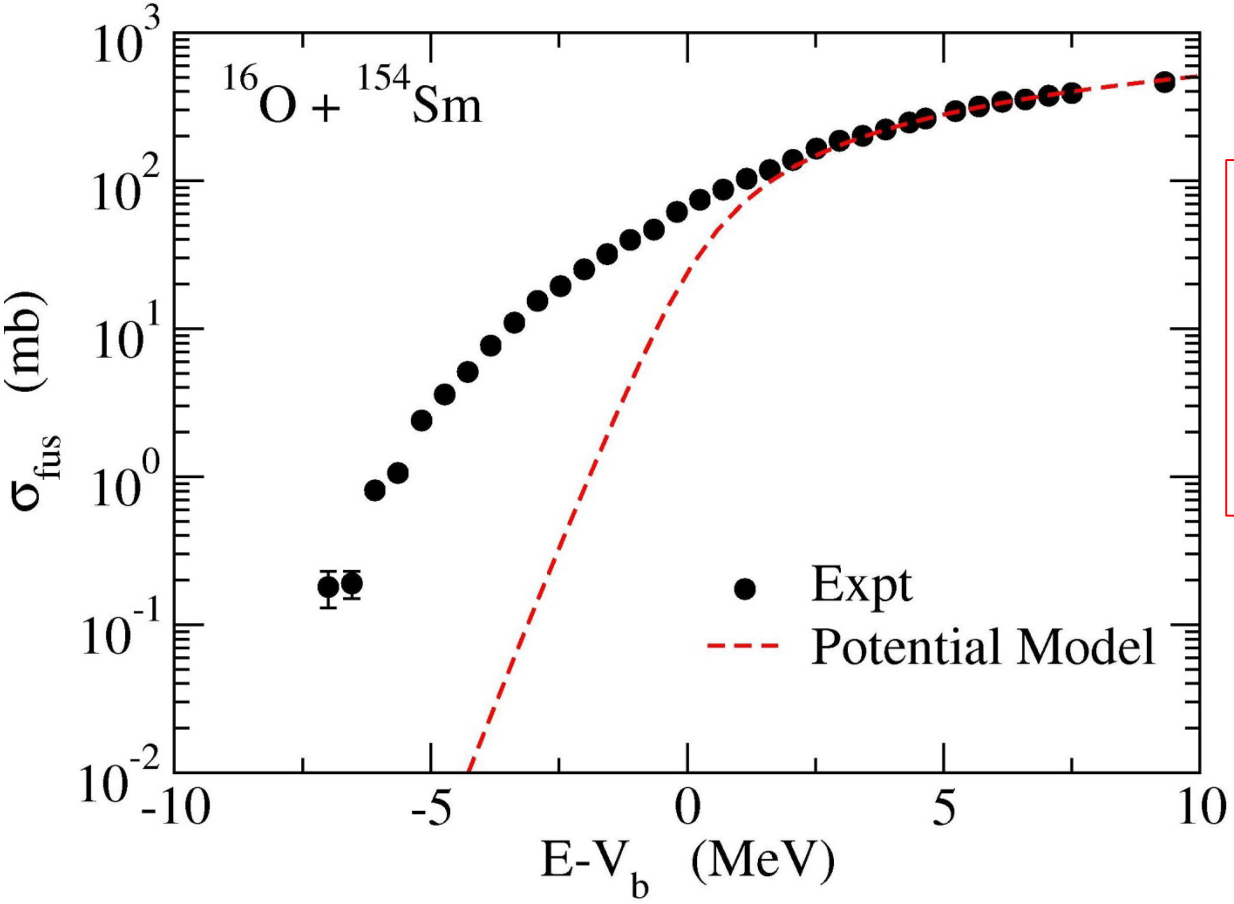


- works well for relatively light systems
- underpredicts σ_{fus} for heavy systems at low energies

L.C. Vaz, J.M. Alexander and G.R. Satchler, Phys.Rep. 69 (1981) 373

Comparison between potential model and exp. data

Fusion cross sections calculated with a static, energy independent potential



Potential model:

Reproduces the data reasonably well

for $E > V_B$

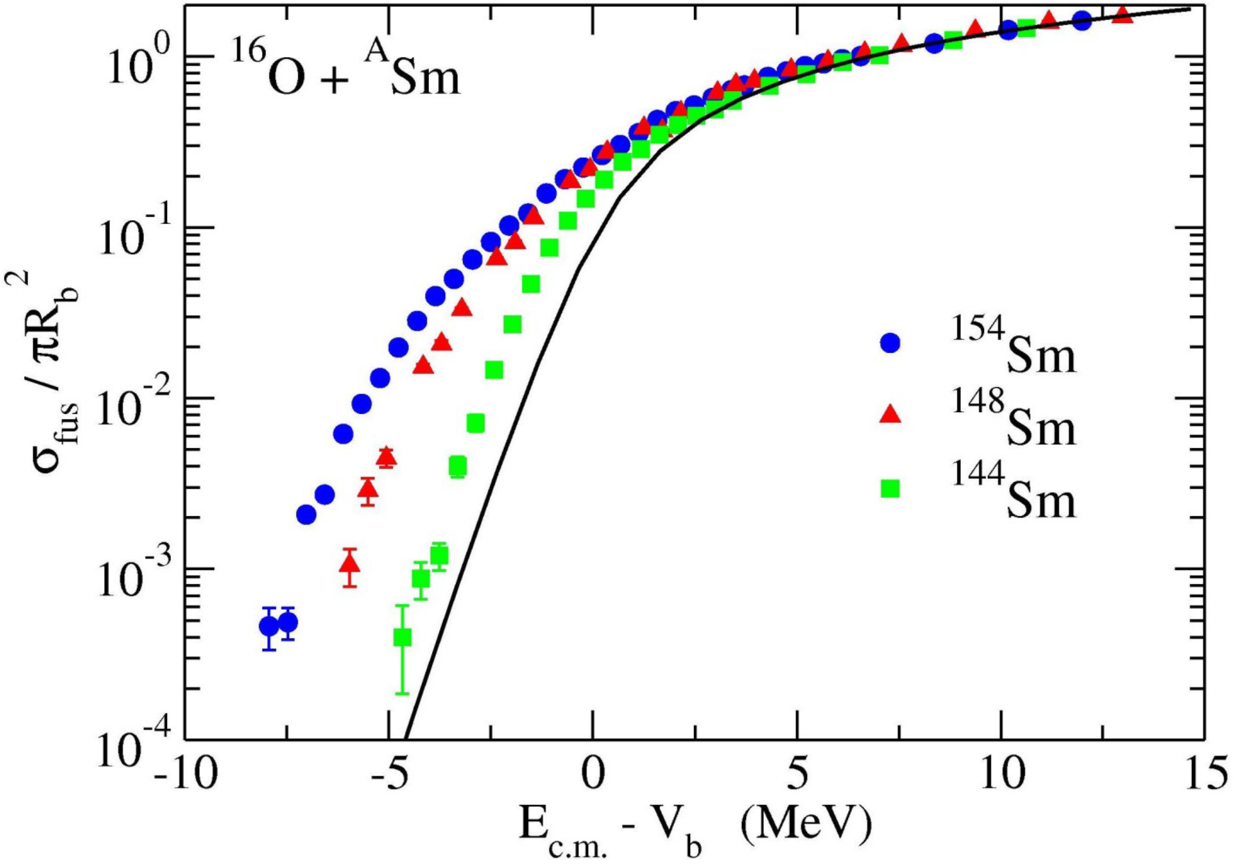
Underpredicts σ_{fus}

for $E < V_B$

L.C. Vaz, J.M. Alexander and G.R. Satchler, Phys.Rep. 69 (1981) 373

Comparison between potential model and exp. data

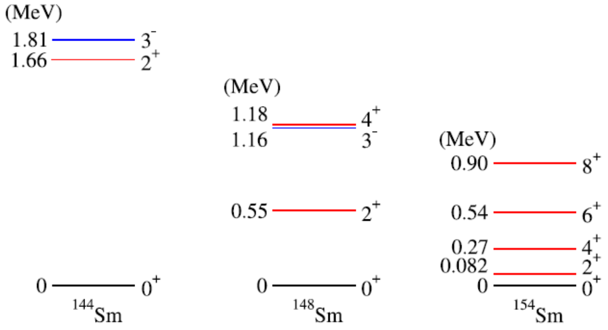
Fusion cross sections calculated with a static, energy independent potential



Effect of collective excitation

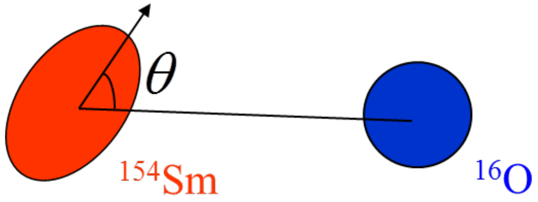
$$\beta_2 \cong \frac{4\pi}{3\cdot Z_t \cdot R_t^2}\cdot\sqrt{\frac{B(E2\uparrow)}{e^2}}$$

$$\beta_4 \cong \frac{4\pi}{3\cdot Z_t \cdot R_t^4}\cdot\sqrt{\frac{B(E4\uparrow)}{e^2}}$$

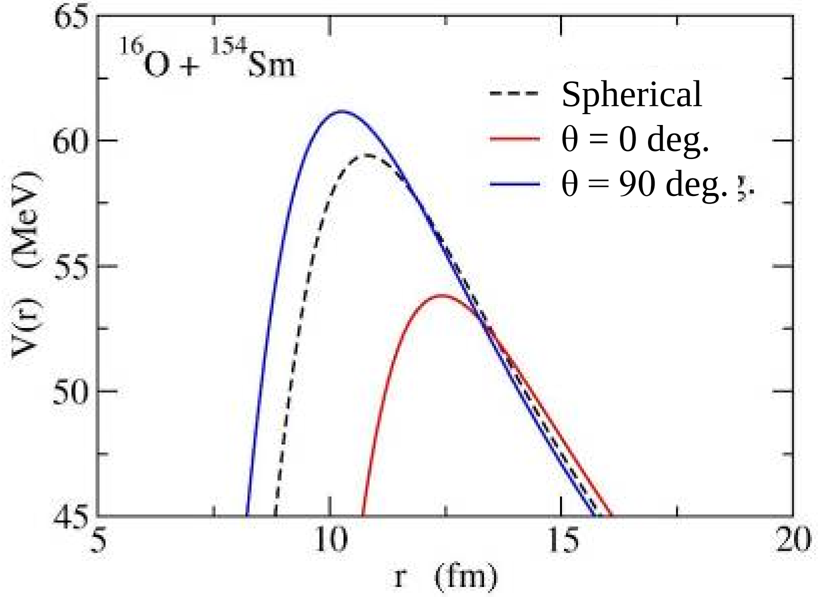


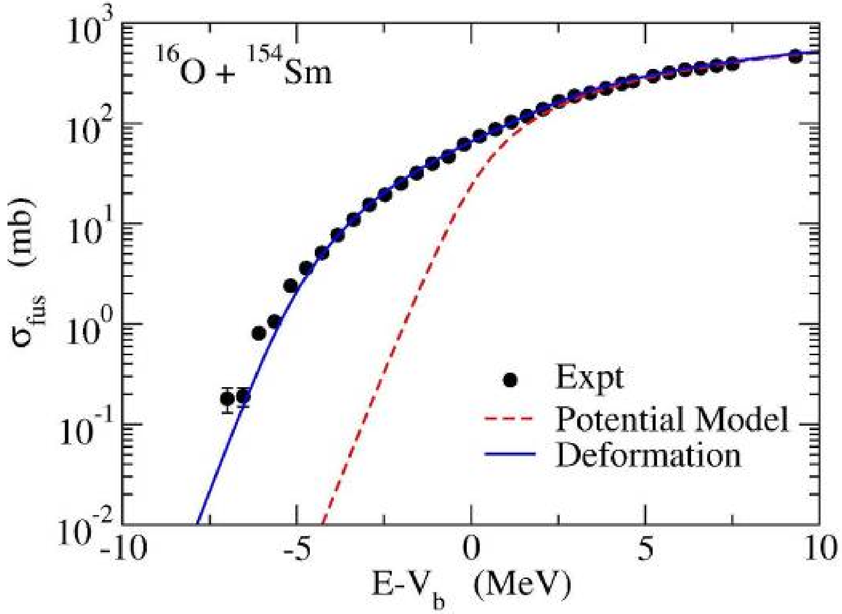
L.C. Vaz, J.M. Alexander and G.R. Satchler, Phys.Rep. 69 (1981) 373

Fusion for a deformed nucleus



$$\sigma_{fus}(E) = \int_0^1 \sigma_{fus}(E,\theta)\,d(\cos\theta)$$





- The barrier is lower for θ = 0
- The barrier is higher for θ = π/2

Deformation enhances σ_{fus} by a factor of 10 - 100

Fusion below and above the barrier inconsistent

■ $^{16}O + ^{208}Pb$

● $^{16}O + ^{204}Pb$

— a = 0.66 fm

— a = 1.18 fm

— a = 1.65 fm

σ(mb)

— a = 0.66 fm

— a = 1.18 fm

— a = 1.65 fm

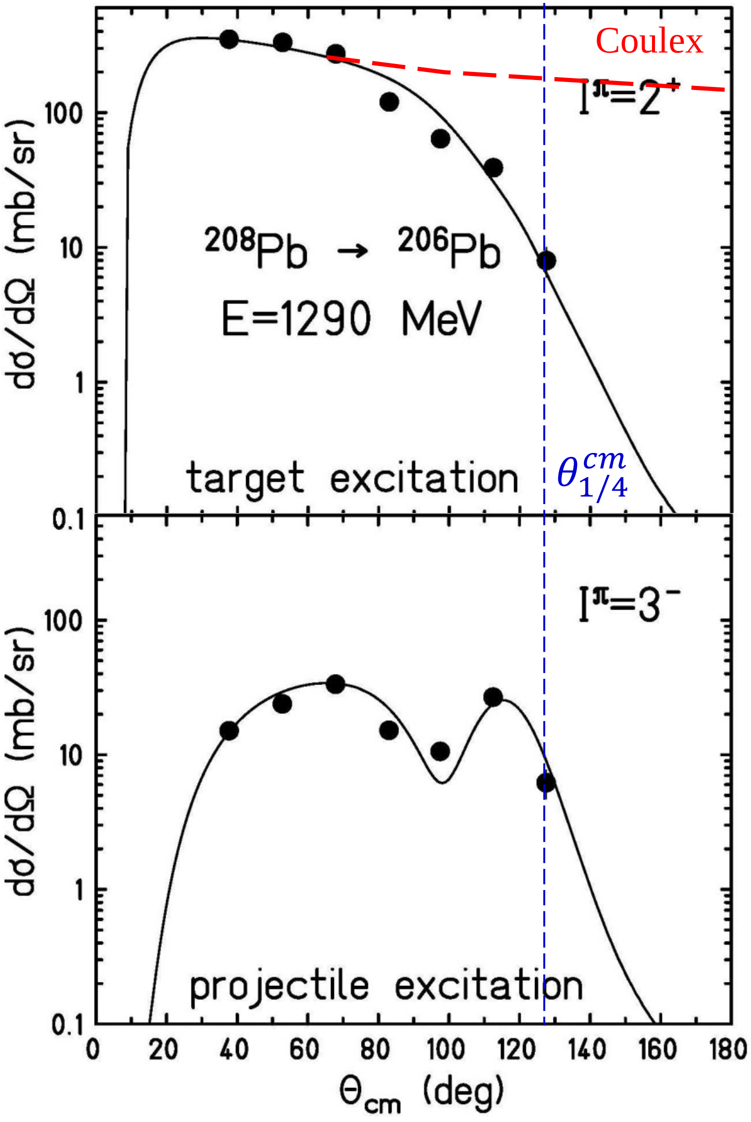
$E_{c.m.} – V_B$ (MeV)

$E_{c.m.} – V_B$ (MeV)

M. Dasgupta et al., Phys Rev Lett 99 (2007) 192701

Inelastic scattering close to the Coulomb barrier

electromagnetic and nuclear excitation



^{208}Pb + ^{206}Pb at E_{cm} = 641.7 MeV

C_p = 6.81 fm, C_t = 6.79 fm, R_{int} = 15.95 fm, $V_C(R_{int})$ = 607,0 MeV

$\theta_{1/4}^{cm} = 127.6^0$

$$\frac{d\sigma_{inel}}{d\Omega_{cm}} = \{1 - P_{abs}(D,\theta_{cm})\}\cdot\frac{d\sigma_{coul}}{d\Omega_{cm}}$$

$$\sigma_{reac} = P_{abs}(D,\theta_{cm})\cdot\sigma_{Ruth}$$

$$[1-P_{abs}(D)] = exp\left\{-\frac{2}{\hbar}\int_{-\infty}^{+\infty} W[r(t)]dt\right\}$$

$$W[r(t)] = W_0\cdot exp\left[-\frac{r(t)-C_1-C_2}{a_I}\right]$$

$$[1-P_{abs}(D)] = exp\left\{-\frac{2}{\hbar}\cdot W_0\cdot exp\left[-\frac{D-C_1-C_2}{a_I}\right]\cdot\frac{D}{v}\right\}$$