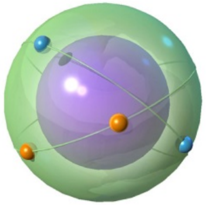
Shell model with residual interaction

$$H = H_0 + H_{residual}$$

Start with 2-particle system, that is a nucleus „doubly magic nucleus + 2 nucleons“

$$H_{residual} = H_{12}(r_{12})$$

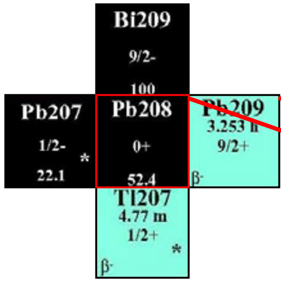
Consider two identical valence nucleons with j_1 and j_2



Two questions:

What total angular momenta $j_1 + j_2 = J$ can be formed?

What are the energies of states with these J values?



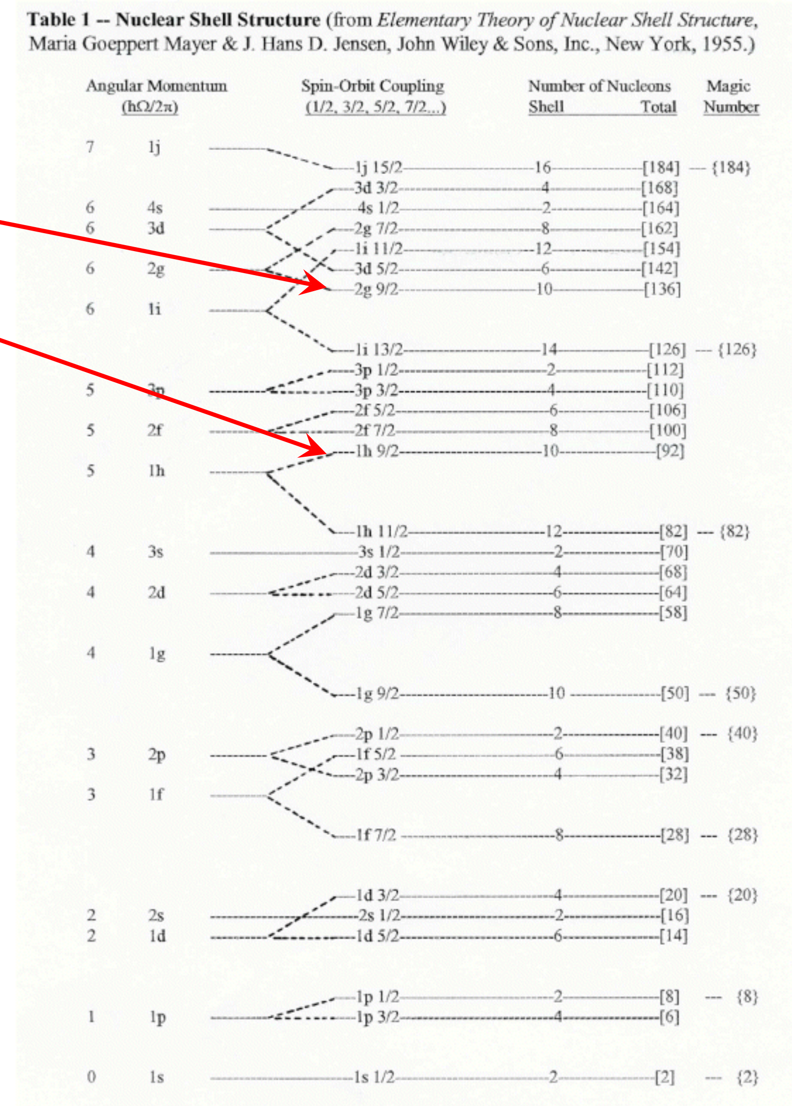


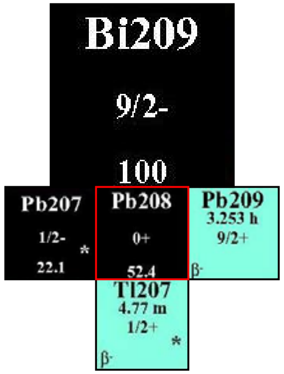
Table 1 -- Nuclear Shell Structure (from *Elementary Theory of Nuclear Shell Structure*, Maria Goeppert Mayer & J. Hans D. Jensen, John Wiley & Sons, Inc., New York, 1955.)

Angular Momentum ($h\Omega/2\pi$)		Spin-Orbit Coupling (1/2, 3/2, 5/2, 7/2...)	Number of Nucleons Shell	Total	Magic Number
7	1j	1j 15/2	16	[184]	{184}
		3d 3/2	4	[168]	
6	4s	4s 1/2	2	[164]	
6	3d	2g 7/2	8	[162]	
		1i 11/2	12	[154]	
		3d 5/2	6	[142]	
6	2g	2g 9/2	10	[136]	
6	1i				
		1i 13/2	14	[126]	{126}
		3p 1/2	2	[112]	
5	3p	3p 3/2	4	[110]	
		2f 5/2	6	[106]	
5	2f	2f 7/2	8	[100]	
		1h 9/2	10	[92]	
5	1h				
		1h 11/2	12	[82]	{82}
4	3s	3s 1/2	2	[70]	
		2d 3/2	4	[68]	
4	2d	2d 5/2	6	[64]	
		1g 7/2	8	[58]	
4	1g				
		1g 9/2	10	[50]	{50}
		2p 1/2	2	[40]	{40}
3	2p	1f 5/2	6	[38]	
		2p 3/2	4	[32]	
3	1f				
		1f 7/2	8	[28]	{28}
		1d 3/2	4	[20]	{20}
2	2s	2s 1/2	2	[16]	
2	1d	1d 5/2	6	[14]	
		1p 1/2	2	[8]	{8}
1	1p	1p 3/2	4	[6]	
0	1s	1s 1/2	2	[2]	{2}

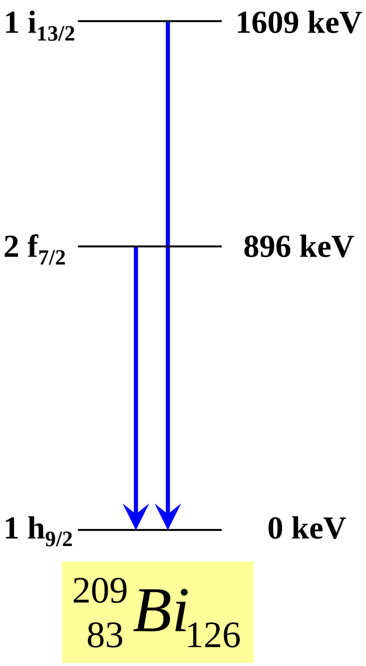
Maria Goeppert-Mayer

J. Hans D. Jensen

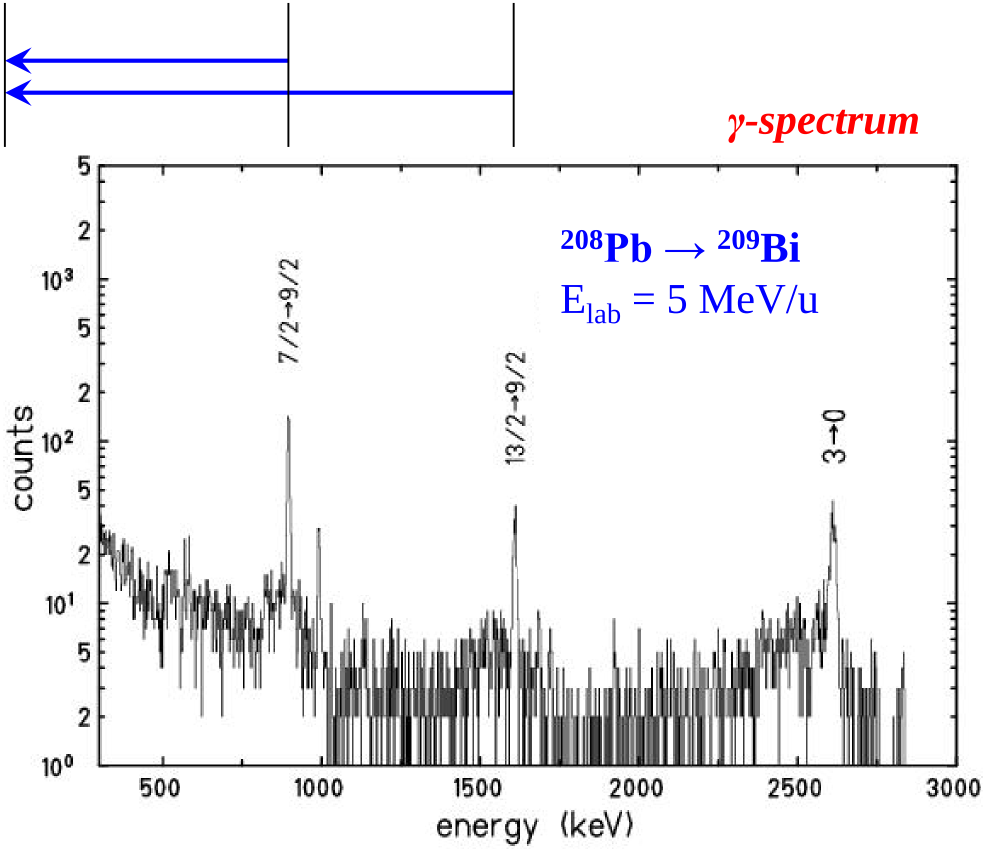
Experimental single-particle energies



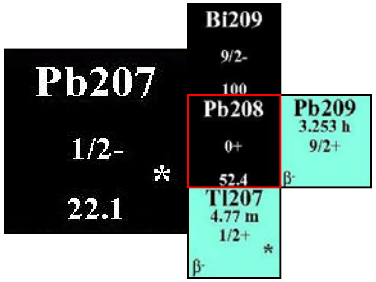
single-particle energies



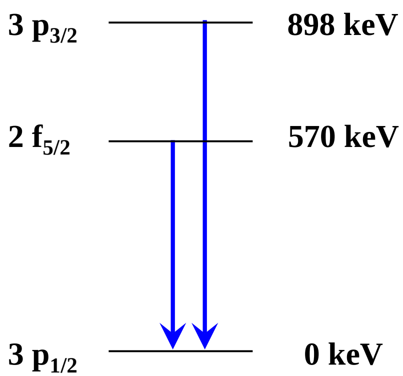
γ-spectrum



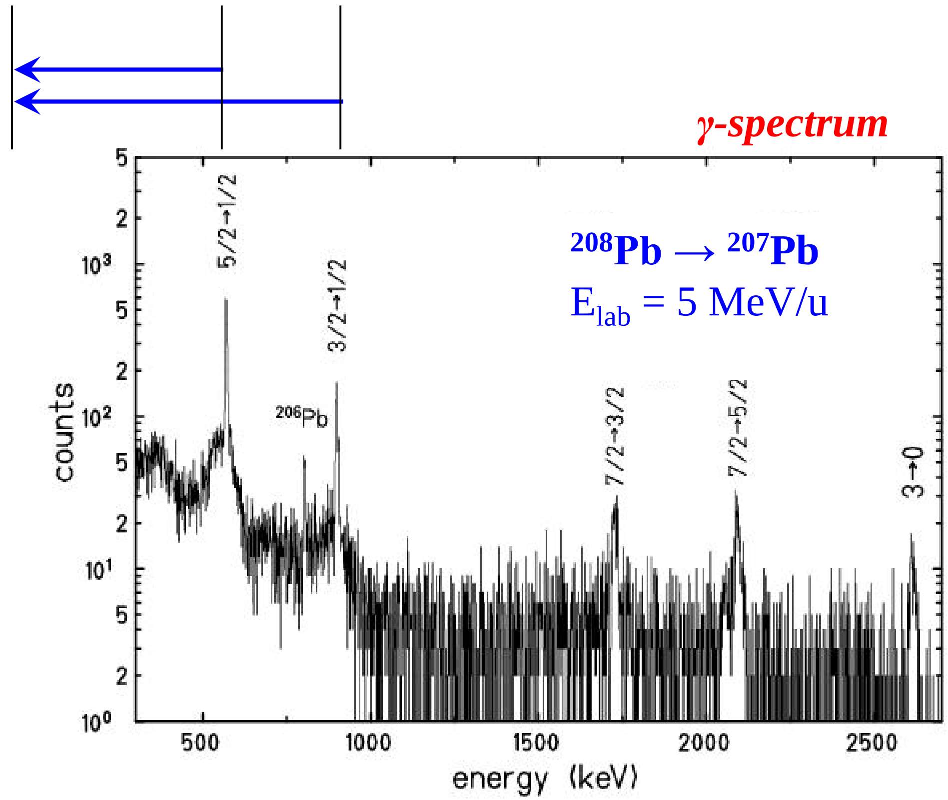
Experimental single-particle energies



single-hole energies

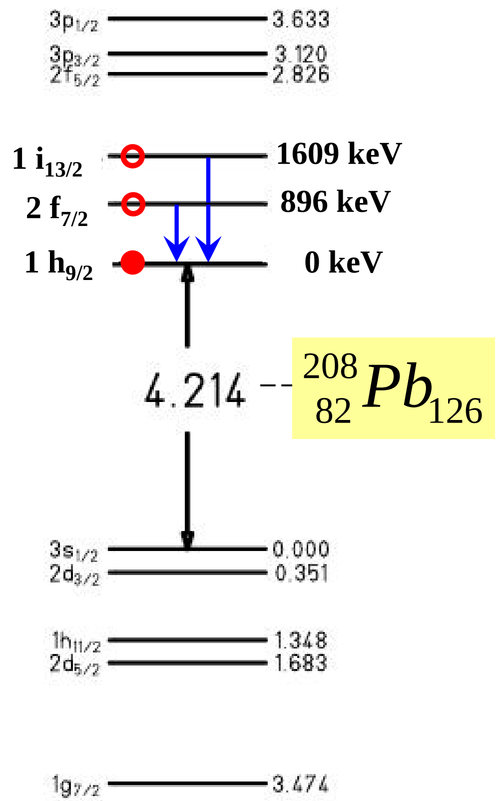


$^{207}_{82}Pb_{125}$



particle states

^{209}Bi



^{207}Tl

hole states

protons

^{209}Pb

energy of shell closure:

$$BE(^{209}Bi) - BE(^{208}Pb) = E(1h_{9/2})$$

$$BE(^{207}Tl) - BE(^{208}Pb) = -E(3s_{1/2})$$

$$E(1h_{9/2}) - E(3s_{1/2}) = BE(^{209}Bi) + BE(^{207}Tl) - 2 \cdot BE(^{208}Pb)$$
$$= -4.211\,MeV$$

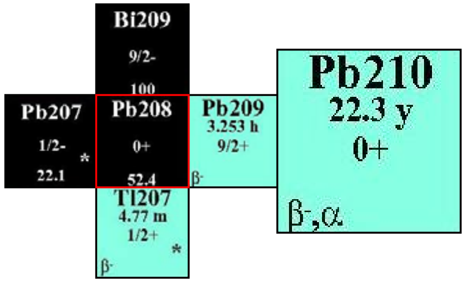
^{207}Pb

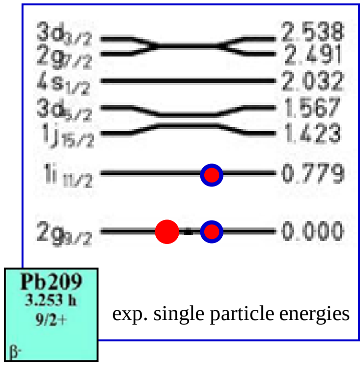
$$BE(^{209}Pb) - BE(^{208}Pb) = E(2g_{9/2})$$

$$BE(^{207}Pb) - BE(^{208}Pb) = -E(3p_{1/2})$$

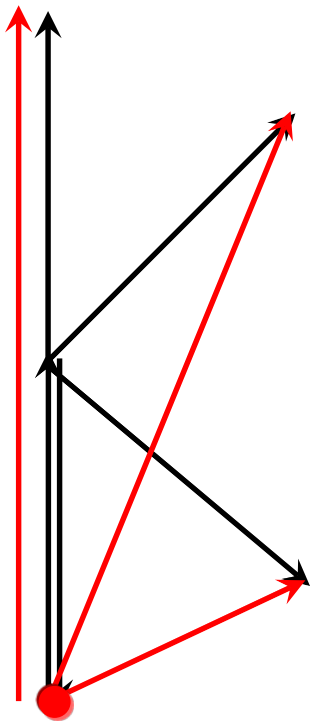
$$E(2g_{9/2}) - E(3p_{1/2}) = BE(^{209}Pb) + BE(^{207}Pb) - 2 \cdot BE(^{208}Pb)$$
$$= -3.432$$

Level scheme of ^{210}Pb





Coupling of two angular momenta



$j_1 + j_2$ all values from: $j_1 - j_2$ to $j_1 + j_2$ ($j_1 = j_2$)

Example: $j_1 = 3$, $j_2 = 5$: $J = 2, 3, 4, 5, 6, 7, 8$

BUT: For $j_1 = j_2$: $J = 0, 2, 4, 6, \ldots (2j - 1)$ (Why these?)

Coupling of two angular momenta

How can we know which total angular momenta J are observed for the coupling of two identical nucleons in the same orbit with angular momentum j?

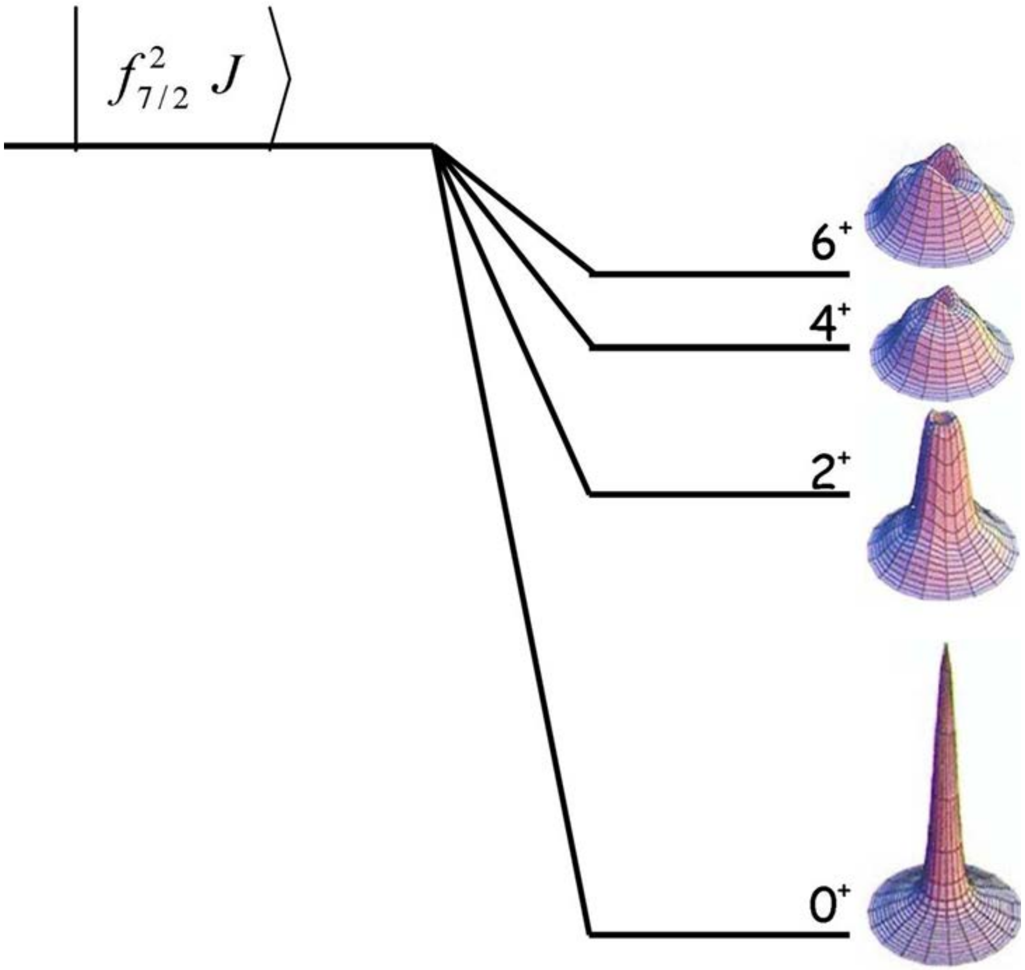
Several methods: easiest is the "m-scheme".

Table 5.1 *m scheme for the configuration* $|(7/2)^2 J\rangle$*

$j_1 = 7/2$ m_1	$j_2 = 7/2$ m_2	M	J
7/2	5/2	6	
7/2	3/2	5	
7/2	1/2	4	
7/2	–1/2	3	6
7/2	–3/2	2	
7/2	–5/2	1	
7/2	–7/2	0	
5/2	3/2	4	
5/2	1/2	3	
5/2	–1/2	2	4
5/2	–3/2	1	
5/2	–5/2	0	
3/2	1/2	2	
3/2	–1/2	1	2
3/2	–3/2	0	
1/2	–1/2	0	0

* Only positive total M values are shown. The table is symmetric for $M < 0$.

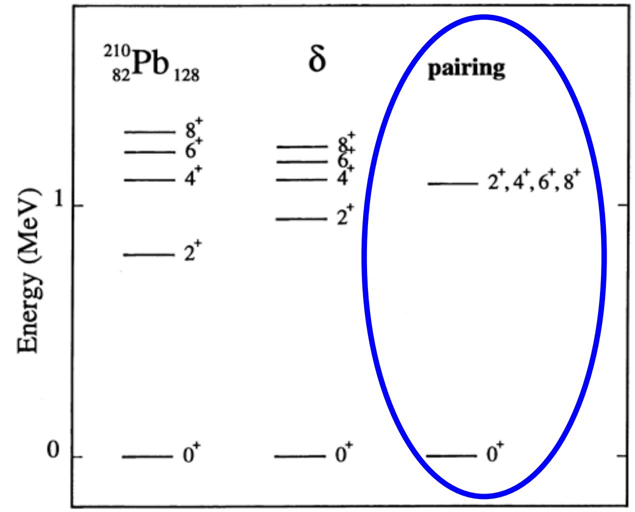
Coupling of two angular momenta



Residual interaction - pairing

- ***Spectrum of ^{210}Pb:*** $^{208}_{82}Pb_{126}$ core + 2 neutrons

  $|g^2_{9/2}; J = 2,4,6,8\rangle \quad \nu = 2$ (two unpaired nucleons)

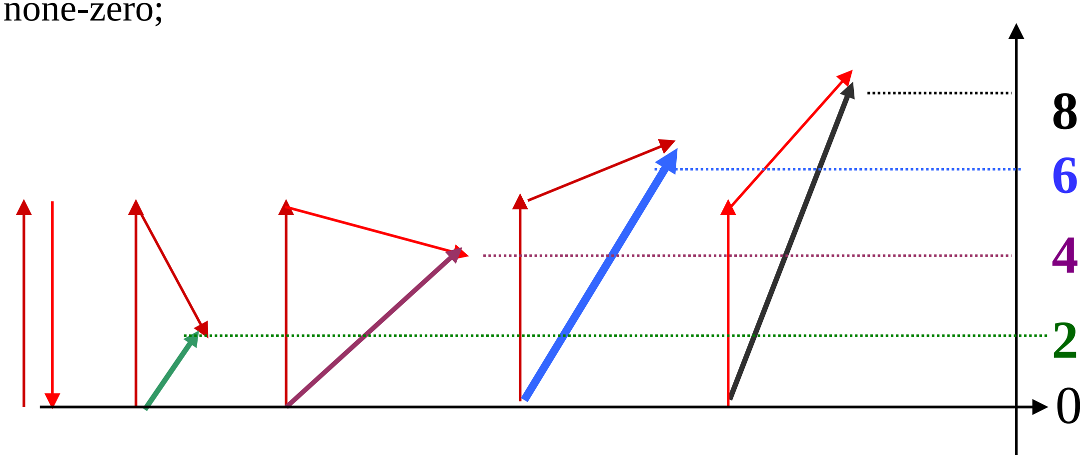


- ***Assume pairing interaction in a single-j shell***

$$\langle j^2\, JM_J | V_{pairing}(r_1, r_2) | j^2\, JM_J \rangle = \begin{cases} -\frac{1}{2}(2j+1)\cdot g & \nu = 0, J = 0 \\ 0, & \nu = 2, J \neq 0 \end{cases}$$

For the ground state the energy eigenvalue is none-zero;
all nucleons paired (ν=0) and spin J=0.

- ***The δ-interaction yields a simple geometrical expression for the coupling of two nucleons***



Pairing – δ-interaction

$$\Delta E(j_1 j_2\, J) = \langle j_1 j_2\, JM | V_{12} | j_1 j_2\, JM \rangle = \frac{1}{\sqrt{2J+1}} \langle j_1 j_2\, J \| V_{12} \| j_1 j_2\, J \rangle$$

wave function: $\varphi(n\ell m) = \frac{1}{r} R_{n\ell}(r) \cdot Y_{\ell m}(\theta, \phi)$

interaction: $V_{12}(\delta) = \frac{-V_0}{r_1 r_2} \delta(r_1 - r_2)\, \delta(cos\theta_1 - cos\theta_2)\, \delta(\phi_1 - \phi_2)$

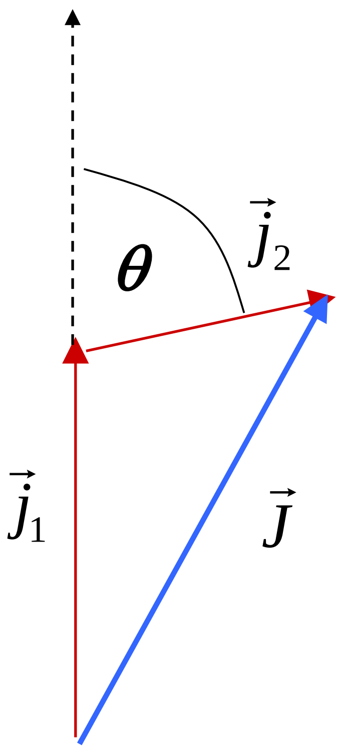
$$\Delta E(j_1 j_2\, J) = -V_0 \cdot F_R(n_1 \ell_1 n_2 \ell_2) \cdot A(j_1 j_2\, J)$$

with $F_R(n_1 \ell_1 n_2 \ell_2) = \frac{1}{4\pi} \int \frac{1}{r^2} R^2_{n_1 \ell_1}(r) R^2_{n_2 \ell_2}(r)\, dr$

and $A(j_1 j_2\, J) = (2j_1 + 1) \cdot (2j_2 + 1) \cdot \begin{pmatrix} j_1 & j_2 & J \\ 1/2 & -1/2 & 0 \end{pmatrix}^2$

A. de-Shalit & I. Talmi: Nuclear Shell Theory, p.200

δ-interaction (semiclassical concept)



$$J^2 = j_1^2 + j_2^2 + 2|j_1||j_2|cos\theta$$

$$cos\theta = \frac{J^2 - j_1^2 - j_2^2}{2|j_1||j_2|} = \frac{J(J+1) - j_1(j_1+1) - j_2(j_2+1)}{2\sqrt{j_1(j_1+1)j_2(j_2+1)}}$$

$$cos\theta \cong \frac{J^2 - 2j^2}{2j^2} \quad for \quad j_1 = j_2 = j \quad and \quad j, J \gg 1$$

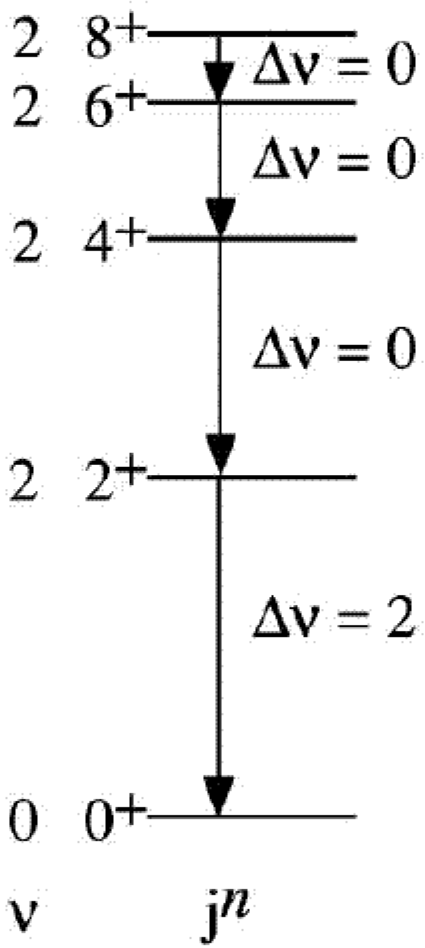
θ* = 0⁰** belongs to **large *J, ***θ* = 180⁰** belongs to **small *J***

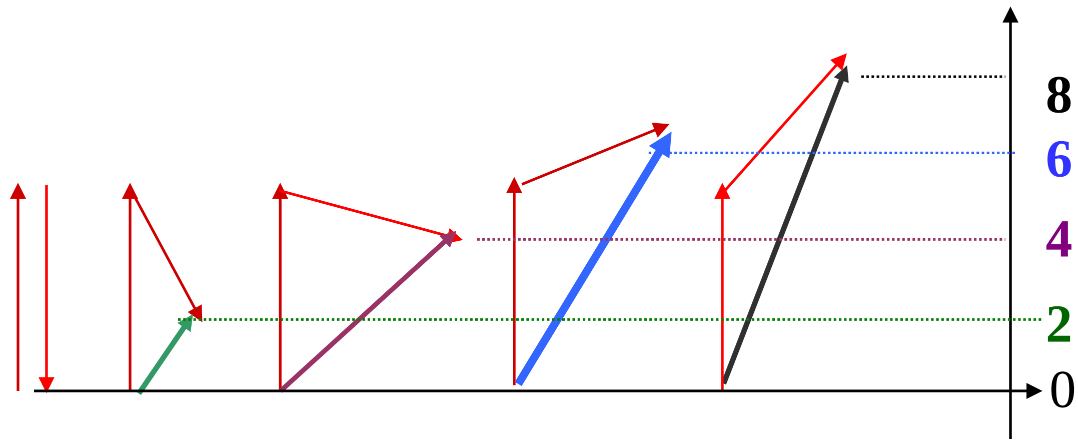
example $h_{11/2}^2$: J=0 θ =180⁰, J=2 θ ~159⁰, J=4 θ ~137⁰,
J=6 θ ~114⁰, J=8 θ ~ 87⁰, J=10 θ ~ 49⁰

$$sin\theta = \sqrt{1 - cos^2\theta} = \frac{J}{j}\left[1 - \frac{J^2}{4j^2}\right]^{1/2} \qquad sin\frac{\theta}{2} = [(1 - cos\theta)/2]^{1/2} = \left(1 - \frac{J^2}{4j^2}\right)^{1/2}$$

$$\begin{pmatrix} j & j & J \\ 1/2 & -1/2 & 0 \end{pmatrix}^2 \approx \left(1 - \frac{J^2}{4j^2}\right)\frac{1}{\pi}\frac{1}{Jj\left(1 - \frac{J^2}{4j^2}\right)^{1/2}} = \frac{sin^2(\theta/2)}{\pi \cdot j^2 \cdot sin\theta} = \frac{tan(\theta/2)}{\pi \cdot j^2}$$

Pairing δ-interaction





δ-interaction yields a simple geometrical explanation for **Seniority-Isomers:**

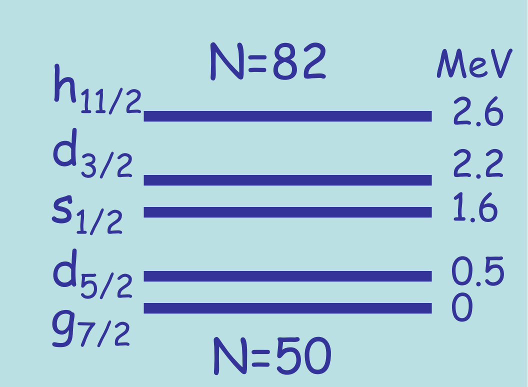
$$\Delta E \sim -V_o \cdot F_R \cdot \tan\left(\theta/_2\right)$$

for $T=1$, even J

energy intervals between states 0^+, 2^+, 4^+, ...$(2j\text{-}1)^+$ decrease with increasing spin.

The ^{100}Sn / ^{132}Sn region, a brief background

Single particle energies



The ^{100}Sn / ^{132}Sn region

Z = 50

$g_{7/2}$ →

Sn102	Sn103	Sn104	Sn105	Sn106	Sn107	Sn108	Sn109	Sn110	Sn111
	7 s	20.8 s	31 s	115 s	2.90 m	10.30 m	18.0 m	4.11 h	35.3 m
0+		0+		0+	(5/2+)	0+	5/2(+)	0+	7/2+
	EC	EC	ECp	EC	EC	EC	EC	EC	EC

$d_{5/2}$ → $s_{1/2}$ → $d_{3/2}$ →

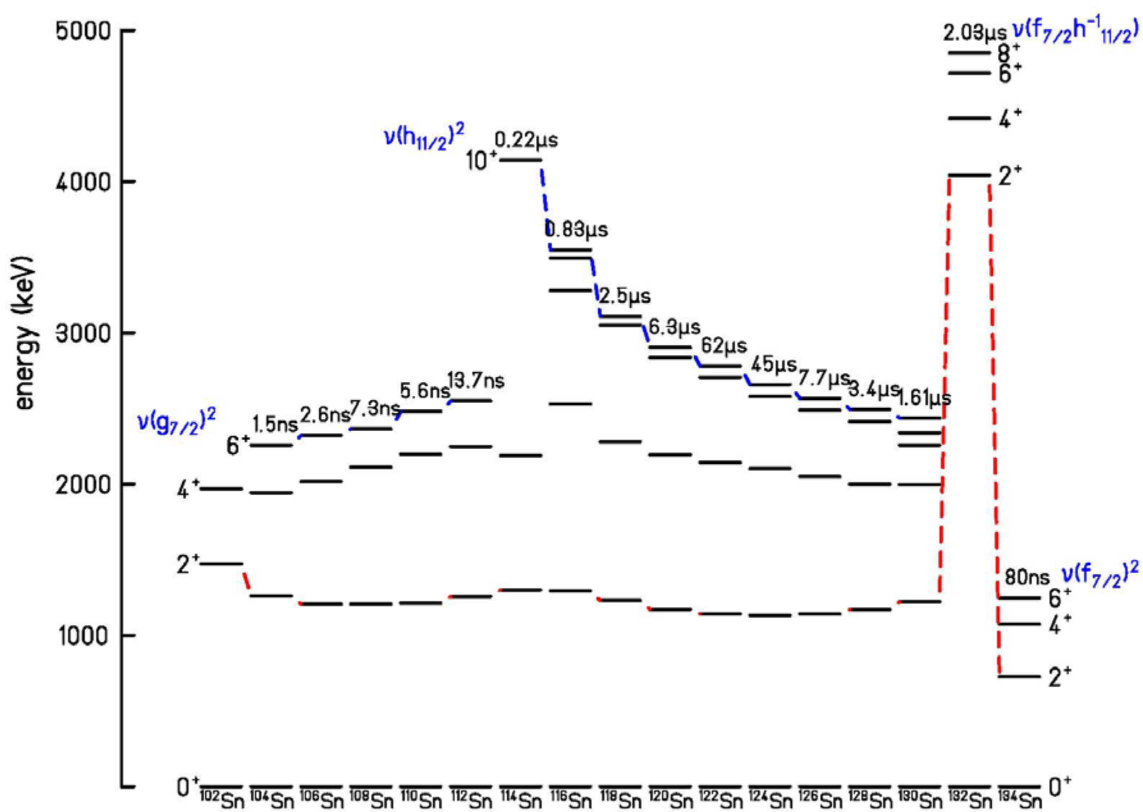
Sn112	Sn113	Sn114	Sn115	Sn116	Sn117	Sn118	Sn119	Sn120
	115.09 d							
0+	1/2+	0+	1/2+	0+	1/2+	0+	1/2+	0+
0.97	EC	0.65	0.34	14.53	7.68	24.23	8.59	32.59

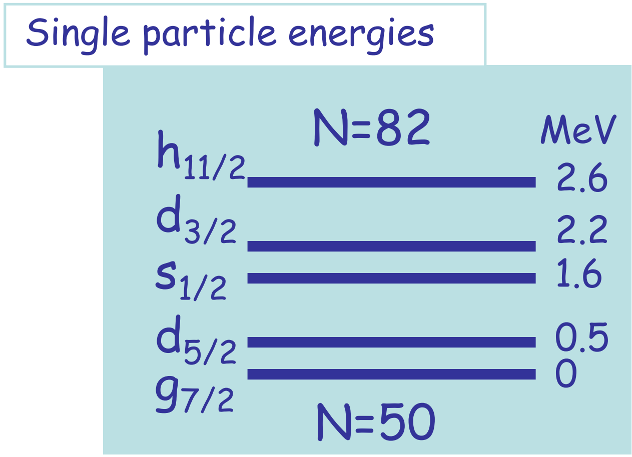
Naïve single particle filling

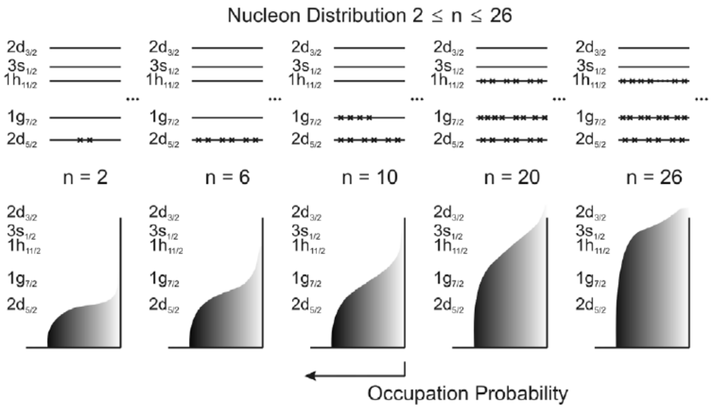
$h_{11/2}$ →

Sn121	Sn122	Sn123	Sn124	Sn125	Sn126	Sn127	Sn128	Sn129	Sn130	Sn131	Sn132
27.06 h		129.2 d		9.64 d	1E+5 y	2.10 h	59.07 m	2.23 m	3.72 m	56.0 s	39.7 s
3/2+	0+	11/2-	0+	11/2-	0+	(11/2-)	0+	(3/2+)	0+	(3/2+)	0+
β-	4.63	β-	5.79	β-	β-	β-	β-	β-	β-	β-	β-

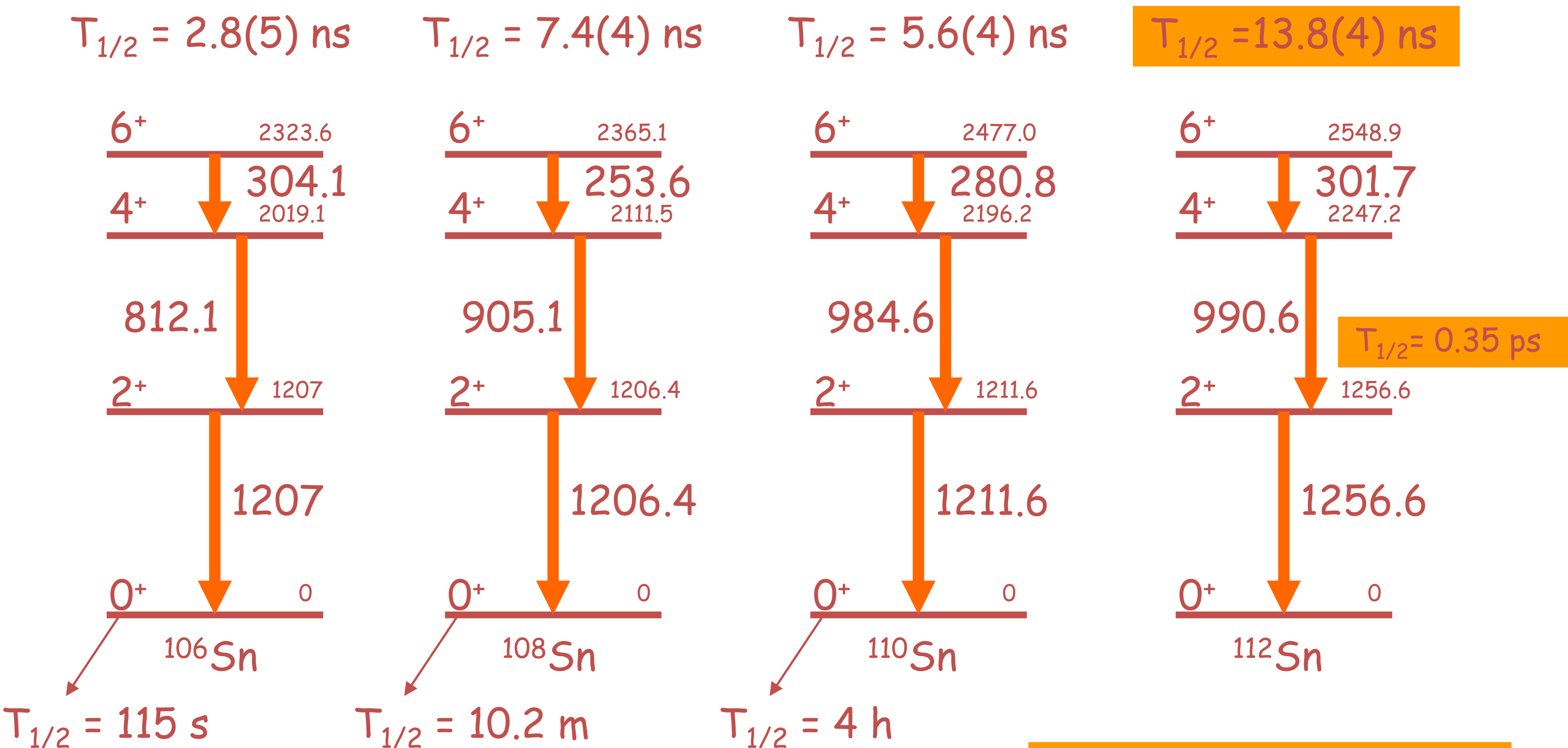
The ^{100}Sn / ^{132}Sn region, isomeric states







Isomeric states in ^{106}Sn – ^{112}Sn



$B_{exp}(E2,6^+\text{->}4^+)$ = 0.49(+-0.02) W.u

$B_{exp}(E2,2^+\text{->}0^+)$ = 16.0 W.u

Generalized seniority scheme

Seniority quantum number **ν** is equal to the number of unpaired particles in the **j^n** configuration, where **n** is the number of valence nucleons.

E2 transition rates: $B\left(E2; J_i \rightarrow J_f\right) = \frac{1}{2 \cdot J_i + 1} \cdot \langle J_f \| Q \| J_i \rangle^2$

$$\langle j^n\, J=2 \| Q \| j^n\, J=0 \rangle^2 = \left[\frac{n \cdot (2j+1-n)}{2 \cdot (2j-1)}\right] \cdot \langle j^2\, J=2 \| Q \| j^2\, J=0 \rangle^2$$

$$= \left[\frac{(2j+1)^2}{2 \cdot (2j-1)}\right] \cdot f \cdot (1-f) \cdot \langle j^2\, J=2 \| Q \| j^2\, J=0 \rangle^2 \qquad f = \frac{n}{2j+1} \rightarrow 1 \;\; for\ large\ n$$

$$B(E2; 2_1^+ \rightarrow 0_1^+) \approx f \cdot (1-f)$$

$\approx N_{particles} * N_{holes}$