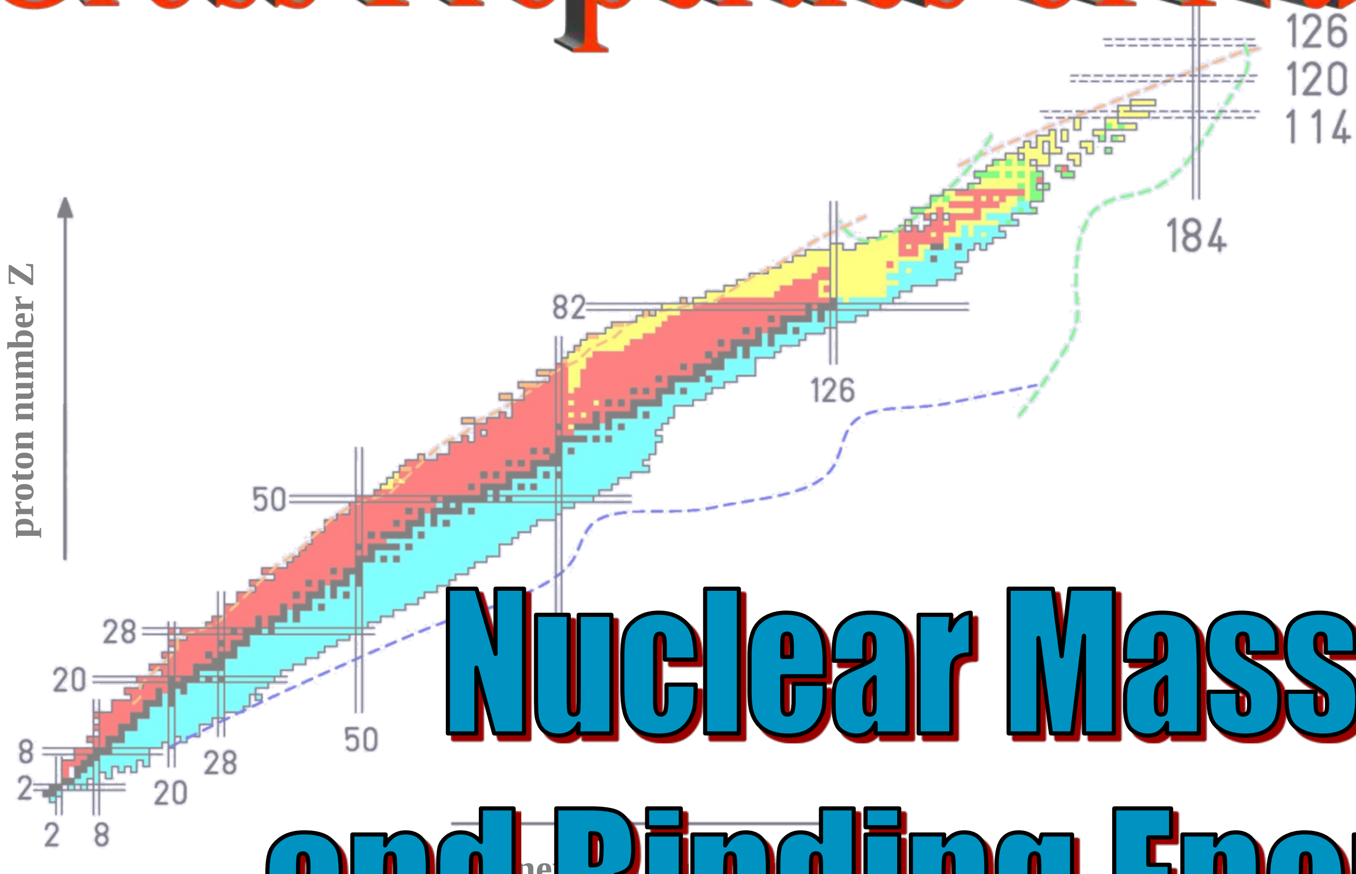
Gross Properties of Nuclei

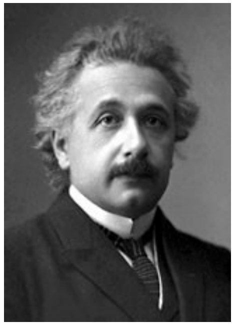


Nuclear Masses and Binding Energies

Nuclear Masses

- In 1905 Albert Einstein following his derivation of the Special Theory of Relativity identifies relation between mass and energy of an object at rest:

$$E = mc^2$$



Albert Einstein
(1879 - 1955)

- The corresponding relation for moving object is

$$E = \frac{m_0 c^2}{\sqrt{1 + \left(\frac{v}{c}\right)^2}}$$

- This discovery explains the energy powering nuclear decay. The question of energy release in nuclear decay was a major scientific puzzle from the time of the discovery of natural radioactivity by Henry Becquerel (1896) until Einstein’s postulate of mass-energy equivalence.

Chemical and nuclear reactions

Heat is evolved in the **chemical reaction** in which hydrogen and oxygen are combined to be water and generates 3.0 eV energy emission:

$$H_2 + \frac{1}{2}O_2 = H_2O + 3.0\ eV$$

Such chemical reaction in which heat is evolved is called exothermic reaction.

Another example is where one mol of carbon is oxidized to be carbon dioxide with producing 4.1 eV energy:

$$C + O_2 = CO_2 + 4.1\ eV$$

The **nuclear reaction** in which two deuterons bind with each other is an example of nuclear fusion. This exoergic reaction is written as

$$^{2}H + {}^{2}H \rightarrow {}^{3}He + n + 3.27\ MeV$$

If a neutron is absorbed in the uranium-235 nucleus, it would split into two fragments of almost equal masses and evolves some number of neutrons and energy. One of the equations for the processes is written

$$^{235}_{92}U + n \rightarrow {}^{137}_{56}Ba + {}^{97}_{36}Kr + 2n + 215\ MeV$$

Nuclear Masses

	mass (kg)	mass (MeV/c^2)	mass (u)
electron	$9.109 \cdot 10^{-31}$	0.511	0.000549
proton	$1.673 \cdot 10^{-27}$	938.2794	1.007276
neutron	$1.675 \cdot 10^{-27}$	939.5728	1.008665
^{1}H atom	$1.674 \cdot 10^{-27}$	938.7904	1.007825

nuclear and atomic masses are expressed in ATOMIC MASS UNITS (u)

definition: 1/12 of mass of neutral $^{12}C \rightarrow M(^{12}C) = 12$ u

$1\ u = 1.6605 \cdot 10^{-27}$ kg or $931.494\ MeV/c^2$ ($E = mc^2$)

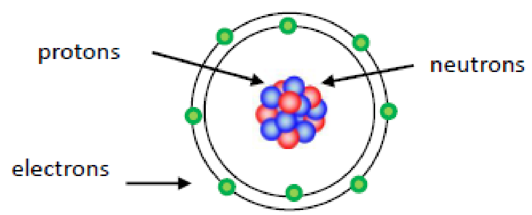
Expect: $M\left({}^{A}_{Z}X_{N}\right) = Z \cdot m_p + N \cdot m_n + Z \cdot m_e$

Find: $M\left({}^{A}_{Z}X_{N}\right) < Z \cdot m_p + N \cdot m_n + Z \cdot m_e$

$\Delta m \cdot c^2 = BE$

mass defect

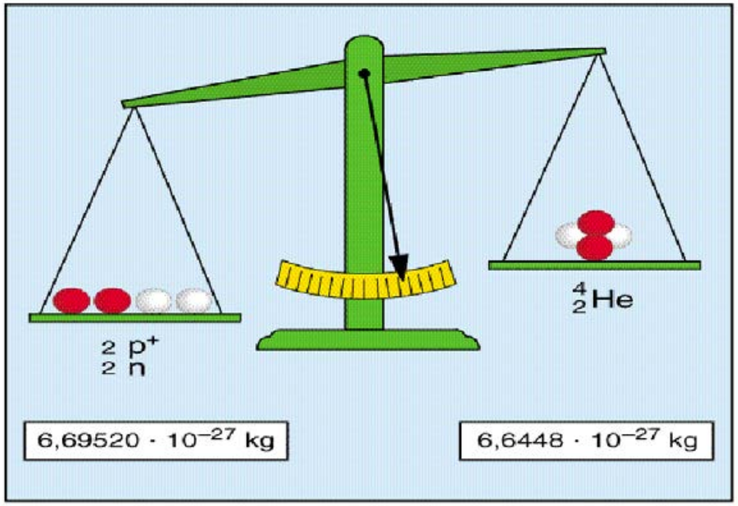
We typically use ATOMIC and not NUCLEAR masses → mass of electrons also included



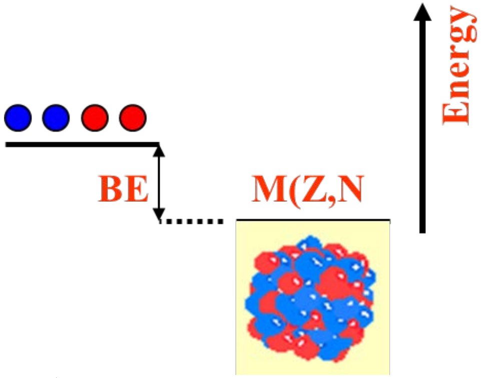
Nuclear Masses and Binding Energy

	mass (kg)	mass (MeV/c²)	mass (u)
electron	$9.109\cdot10^{-31}$	0.511	0.000549
proton	$1.673\cdot10^{-27}$	938.2794	1.007276
neutron	$1.675\cdot10^{-27}$	939.5728	1.008665
^{1}H atom	$1.674\cdot10^{-27}$	938.7904	1.007825

1 u = $1.6605\cdot10^{-27}$ kg or 931.494 MeV/c²



A bound system has a lower potential energy than its constituents !



Binding energy:

$$BE = \{[Z\cdot m_H + N\cdot m_n] - M({}^{A}_{Z}X_N)\}\cdot c^2 > 0$$

BE ($^{4}_{2}He$) = [2·1.007825 + 2·1.008665] – 4.002603 = 0.030377 u (= 28.296 MeV)

Binding Energy and Mass Excess

	mass (kg)	mass (MeV/c^2)	mass (u)
electron	$9.109 \cdot 10^{-31}$	0.511	0.000549
proton	$1.673 \cdot 10^{-27}$	938.2794	1.007276
neutron	$1.675 \cdot 10^{-27}$	939.5728	1.008665
^{1}H atom	$1.674 \cdot 10^{-27}$	938.7904	1.007825

1 u = $1.6605 \cdot 10^{-27}$ kg or 931.494 MeV/c^2

Binding energy: $BE = \{[Z \cdot m_H + N \cdot m_n] - M({}^A_Z X_N)\} \cdot c^2$

Mass excess: $\Delta M = M(A,Z) - A \cdot M(u)$

Example:

mass of ^{56}Fe: M(56,26) = 55.934942 [u]

mass excess: $\Delta M = M(A,Z) - A \cdot M(u) = 55.934942 - 56 = -0.06506$ [u]
$= -0.06506 \cdot 931.494 = -60.601$ [MeV/c^2]

binding energy: $BE(Z,A) = [Z \cdot M(^1H) + N \cdot M_n - M(Z,A)] \cdot c^2$
$= [26 \cdot 1.007825 + 30 \cdot 1.008665 - 55.934942] \cdot 931.494 = 492.25$ [MeV]

Binding Energy

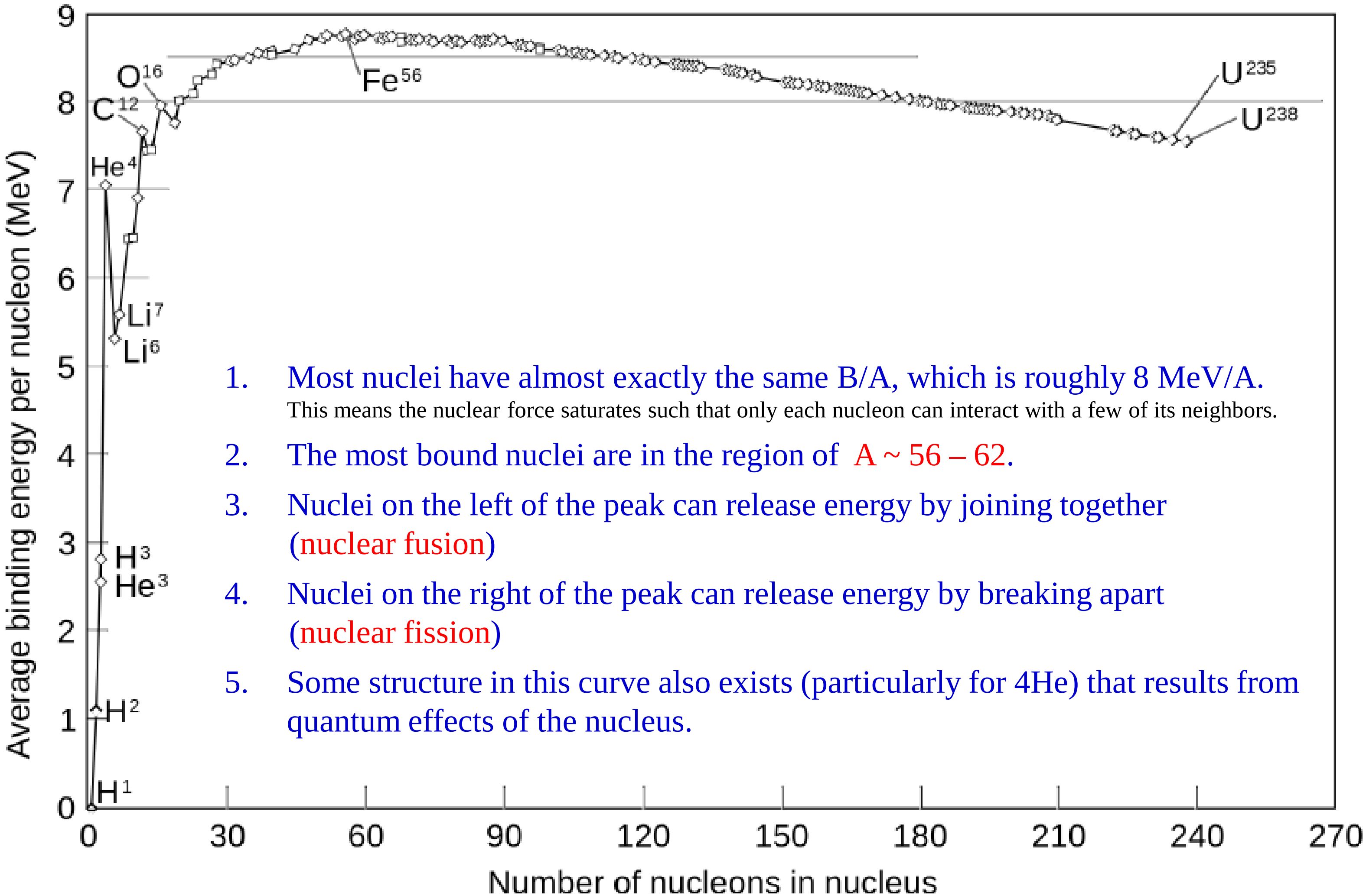
	mass (kg)	mass (MeV/c²)	mass (u)
electron	$9.109\cdot10^{-31}$	0.511	0.000549
proton	$1.673\cdot10^{-27}$	938.2794	1.007276
neutron	$1.675\cdot10^{-27}$	939.5728	1.008665
^{1}H atom	$1.674\cdot10^{-27}$	938.7904	1.007825

1 u = $1.6605\cdot10^{-27}$ kg or 931.494 MeV/c^2

$$BE = \{[Z \cdot m_H + N \cdot m_n] - M(^A_Z X_N)\} \cdot c^2$$

element	mass of nucleons [u]	nuclear mass [u]	binding energy [MeV]	BE/A [MeV/u]
^{2}H	2.01594	2.01355	2.23	1.12
^{4}He	4.03188	4.00151	28.29	7.07
^{7}Li	7.05649	7.01336	40.15	5.74
^{9}Be	9.07243	9.00999	58.13	6.46
^{56}Fe	56.44913	55.92069	492.24	8.79
^{107}Ag	107.86187	106.87934	915.23	8.55
^{127}I	128.02684	126.87544	1072.53	8.45
^{206}Pb	207.67109	205.92952	1622.27	7.88
^{210}Po	211.70297	209.93683	1645.16	7.73
^{238}U	239.93448	238.00037	1801.63	7.57

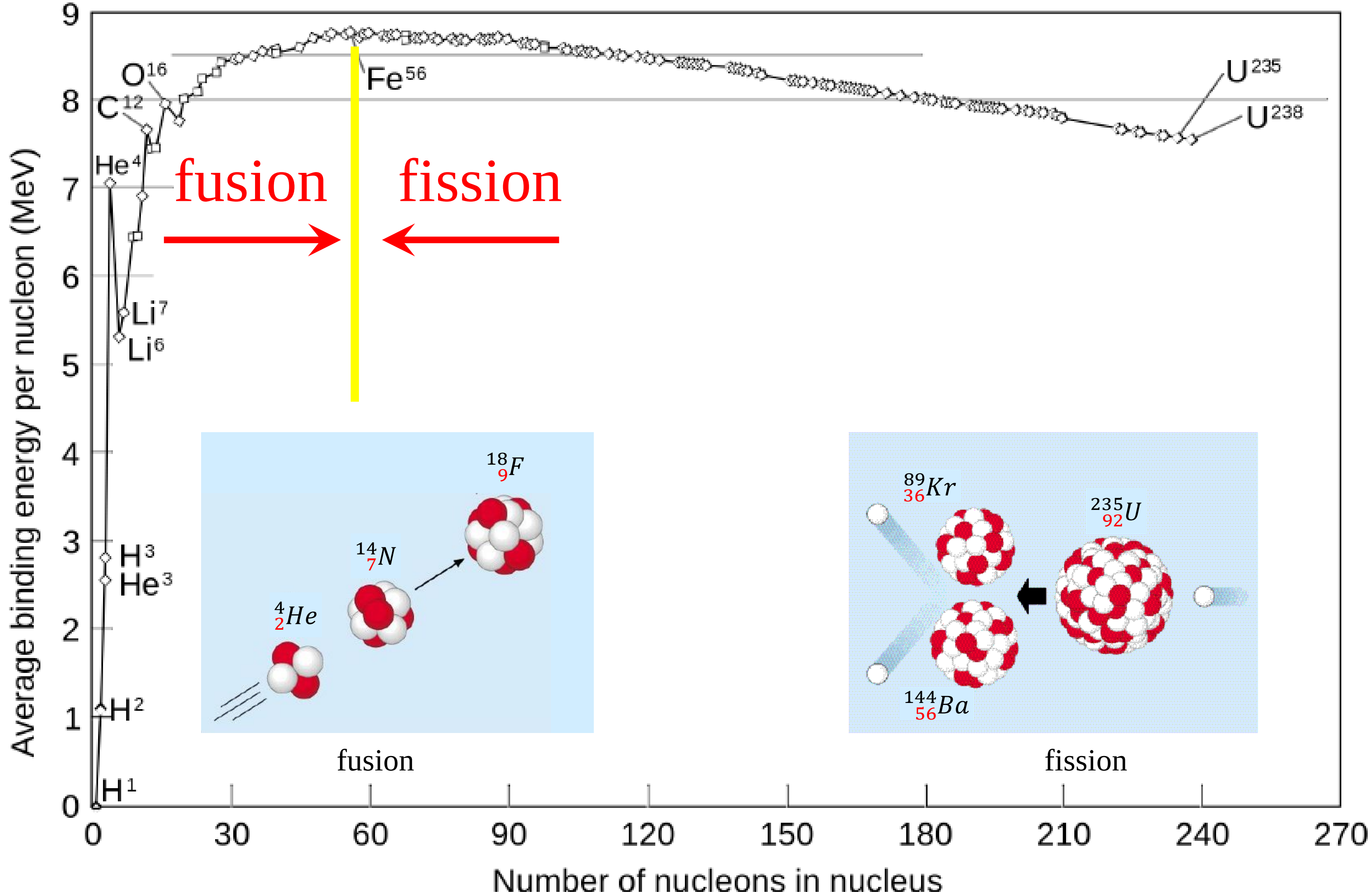
Binding energy per nucleon (BE/A)



1. Most nuclei have almost exactly the same B/A, which is roughly 8 MeV/A.
   This means the nuclear force saturates such that only each nucleon can interact with a few of its neighbors.
2. The most bound nuclei are in the region of A ~ 56 – 62.
3. Nuclei on the left of the peak can release energy by joining together (nuclear fusion)
4. Nuclei on the right of the peak can release energy by breaking apart (nuclear fission)
5. Some structure in this curve also exists (particularly for 4He) that results from quantum effects of the nucleus.

Characteristics of Nuclear Binding

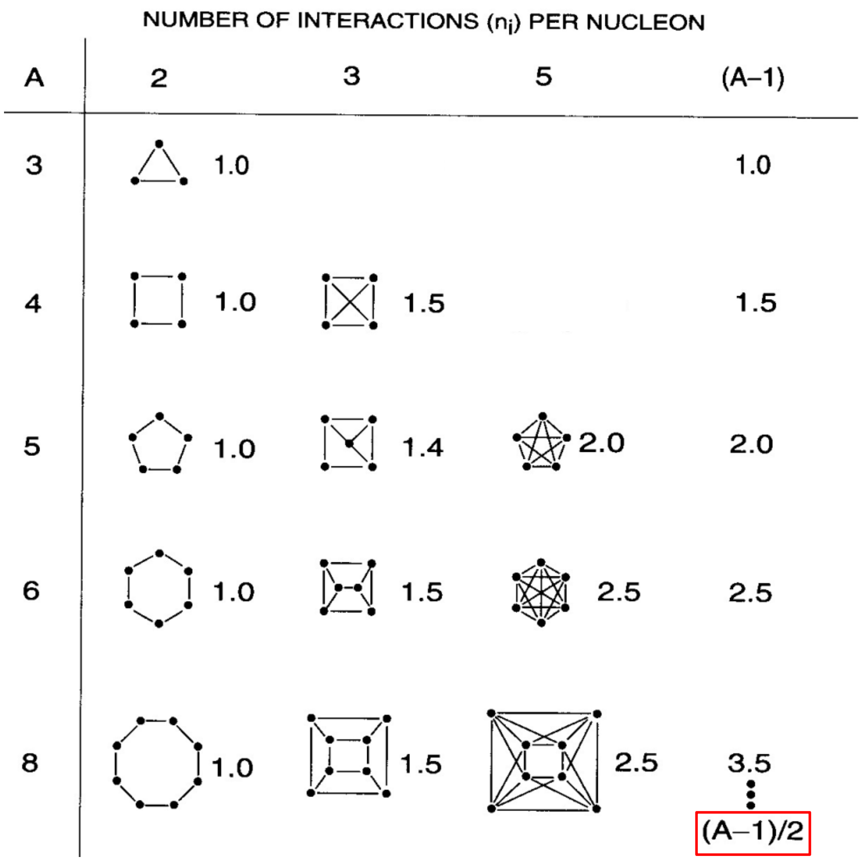
Binding energy per nucleon (BE/A)

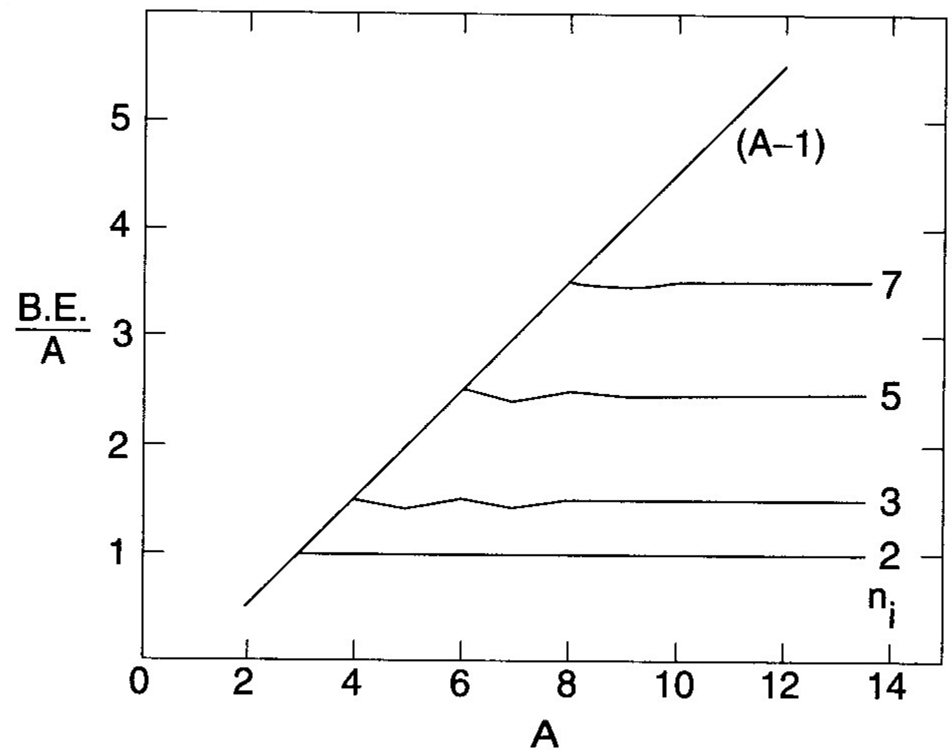


Long vs short range interaction

Long range force: $B \propto \frac{A(A-1)}{2}$ $\curvearrowright$ $B/A \propto A$

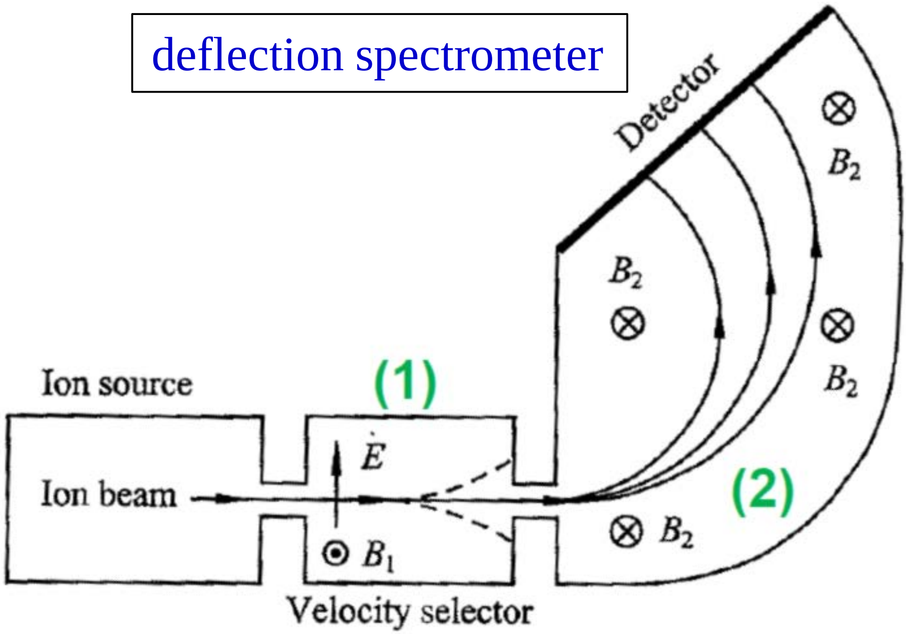
Short range force: saturation





Mass spectroscopy

Mass of nuclei are measured by passing ion beams through magnetic and electric field (2 steps)



1) select a constant velocity with a velocity (Wien) filter
E and **B_1** are at right angles

$$q \cdot E = q \cdot v \cdot B_1$$

$$v = {}^{E}/_{B_1}$$

2) Measure a trajectory of ions in a uniform magnetic **B_2** field

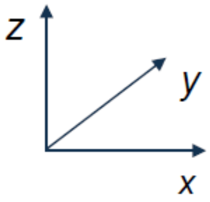
$$\frac{m \cdot v^2}{r} = q \cdot v \cdot B_2$$

If $B_1 = B_2 = B$ $$\frac{q}{m} = \frac{v}{B \cdot r} = \frac{E}{B^2 \cdot r}$$

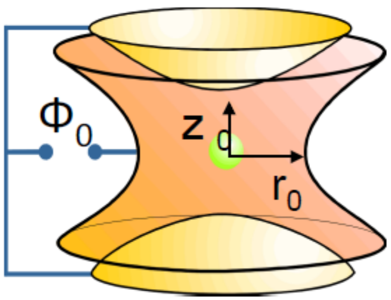
Penning trap measurement

The most precise measurements come from storage devices that confine ions in three dimensions by means of well-controlled electromagnetic fields.

Examples: storage rings, Penning trap, Paul trap



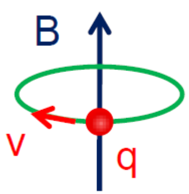
$$\Phi = \frac{\Phi_0 (r^2 - 2z^2)}{2 r_0^2}$$



In a Penning trap,

1) Ions are confined in 2D with a strong magnetic field B
2) A weak electrostatic field is used to trap ions in 3D (along the Z-axis)

1) Cyclotron oscillations (ω_c)



$$\frac{m \cdot v^2}{r} = q \cdot v \cdot B$$

$$v = \frac{q \cdot B \cdot r}{m}$$

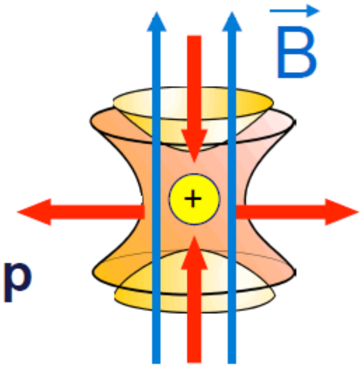
$$t = \frac{2\pi \cdot r}{v} = \frac{2\pi \cdot m}{q \cdot B}$$

$$\omega_c = 2\pi \cdot f = \frac{2\pi}{t} = \frac{qB}{m}$$

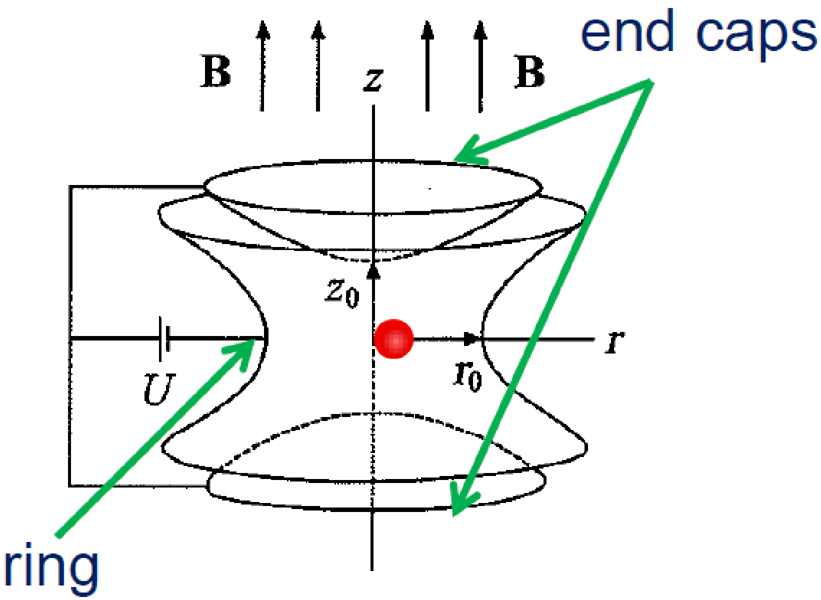
$\vec{E}(\Phi) + \vec{B}$

↓

Penning trap



2) A weak axially symmetric electrostatic potential is superimposed to produce a saddle point at the center.



This requires the quadrupole potential:

$$\Phi(z,r) = \frac{U}{4d^2} \cdot (2z^2 - r^2)$$

$$d = \frac{1}{2}(2z_0^2 - r_0^2)^{1/2}$$

(so that U is the potential difference between the endcap and the ring electrodes)

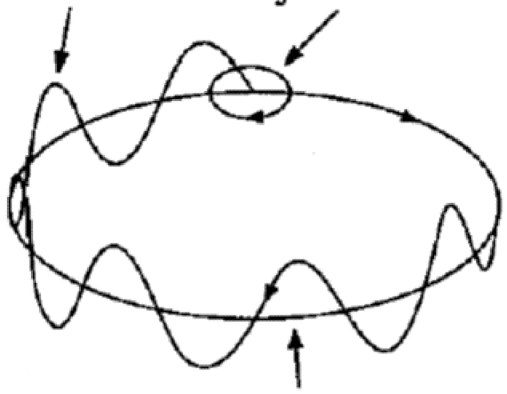
Penning trap measurement

Solving the equations of motions result in three independent motional modes with frequencies;

$$\omega_z = \sqrt{q \cdot U /_{m \cdot d^2}} \qquad \text{(axial motion)}$$

$$\omega_+ = {}^{\omega_c}/_2 + \sqrt{{}^{\omega_c^2}/_4 - {}^{\omega_z^2}/_2} \qquad \text{(cyclotron motion)}$$

$$\omega_- = {}^{\omega_c}/_2 - \sqrt{{}^{\omega_c^2}/_4 - {}^{\omega_z^2}/_2} \qquad \text{(magnetron motion)}$$



The condition on the magnetic field,

$${}^{\omega_c^2}/_4 - {}^{\omega_z^2}/_2 > 0 \qquad B^2 > 2 \cdot m \cdot U/q \cdot d^2$$

$$\omega_\pm = {}^{\omega_c}/_2 \cdot \left[1 \pm \left(1 - {}^{2 \cdot \omega_z^2}/_{\omega_c^2}\right)^{1/2}\right]$$

$$= {}^{\omega_c}/_2 \cdot \left[1 \pm \left(1 - {}^{\omega_z^2}/_{\omega_c^2}\right)\right]$$

$$\omega_- = {}^{\omega_z^2}/_{2\omega_c} = {}^{U}/_{2 \cdot B \cdot d^2}$$

$$\omega_+ = \omega_c - {}^{U}/_{2 \cdot B \cdot d^2}$$

In the experiment one needs to measure the sum

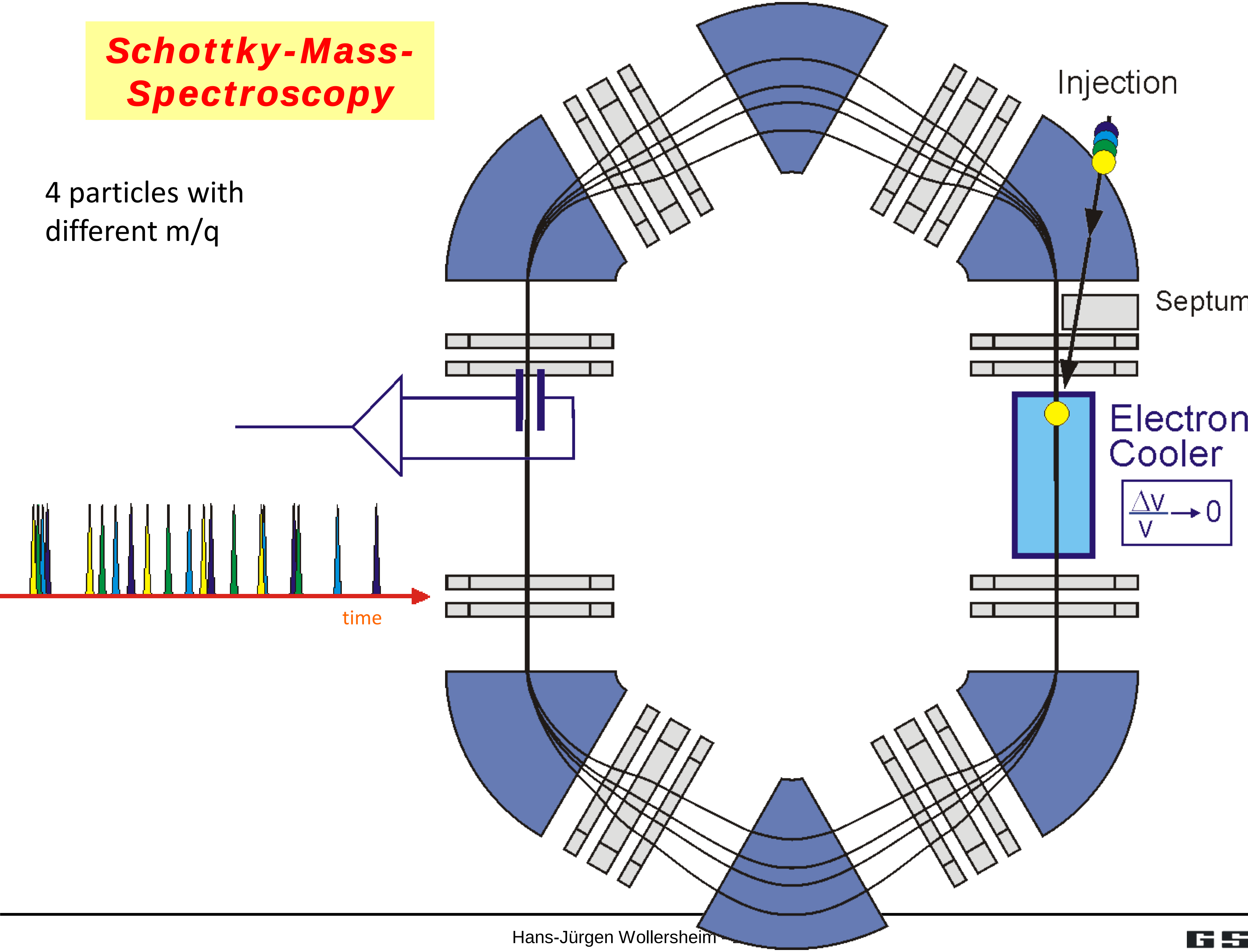
$$\omega_+ + \omega_-$$

(not cyclotron frequency from ω_+ only)

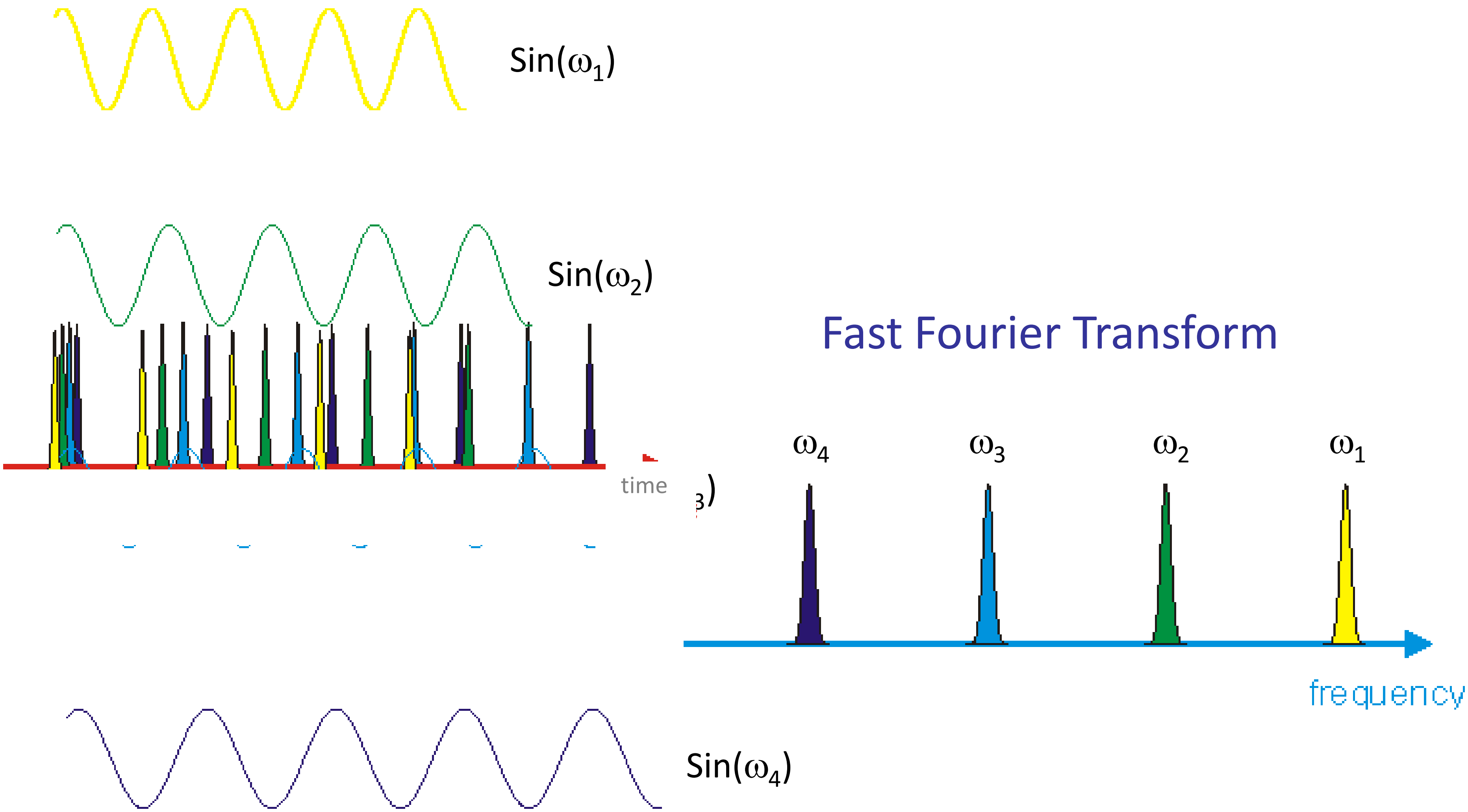
$$\rightarrow \omega_c = \omega_+ + \omega_- \qquad \omega_c^2 = \omega_+^2 + \omega_-^2 + \omega_z^2 \qquad (\omega_+ \cdot \omega_- = \omega_z^2/2)$$

Schottky-Mass-Spectroscopy

4 particles with
different m/q



Schottky-Mass-Spectroscopy



Small-band Schottky frequency spectra