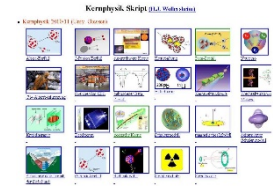


Outline: Nuclear rotation of odd-even nuclei

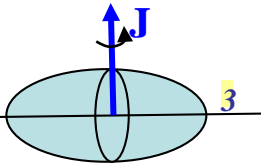
Lecturer: Hans-Jürgen Wollersheim

e-mail: h.j.wollersheim@gsi.de

web-page: <https://web-docs.gsi.de/~wolle/> and click on

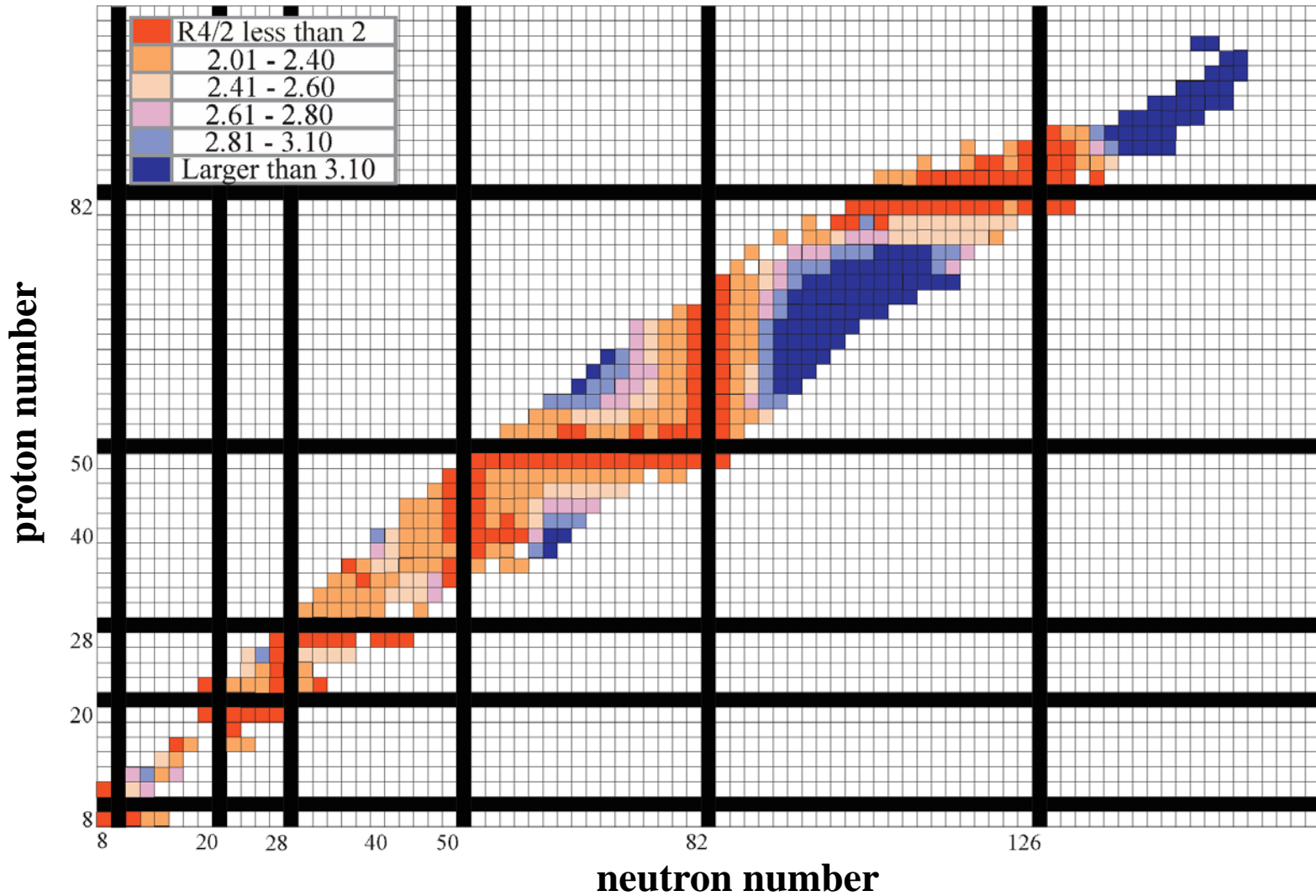


1. particle-rotor model
2. Euler angles
3. Example: ^{181}Ta
4. reduced transition probabilities



Broad perspective on structural evolution:

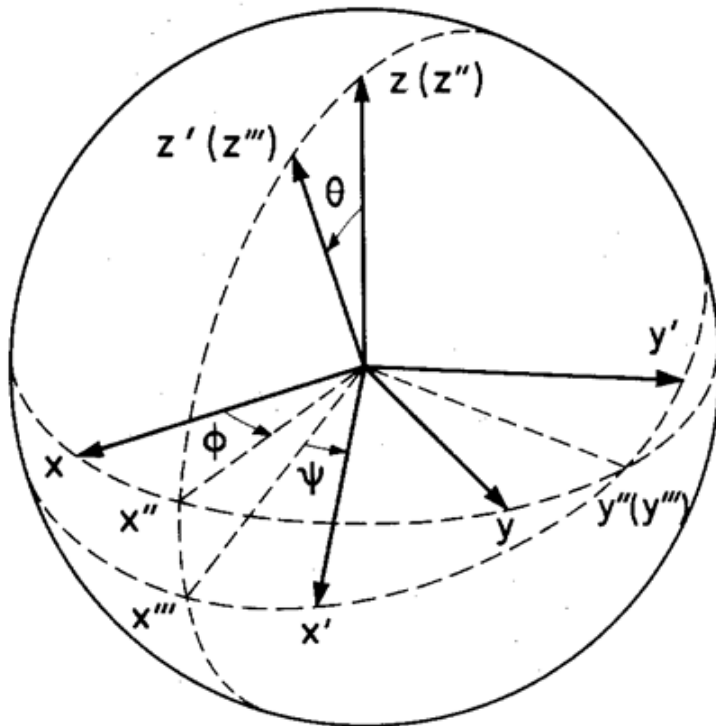
$$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)}$$



for even-even nuclei

The Euler angles

- It is important to recognize that for nuclei the intrinsic reference frame can have any orientation with respect to the lab reference frame as we can hardly control orientation of nuclei (although it is possible in some cases).
- One way to specify the mutual orientation of two reference frames of the common origin is to use Euler angles.



(x, y, z) axes of lab frame
 $(1, 2, 3)$ axes of intrinsic frame

- The rotation from (x, y, z) to (x', y', z') can be decomposed into three parts: a rotation by ϕ about the z axis to (x'', y'', z'') , a rotation of θ about the new y axis (y'') to (x''', y''', z''') , and finally a rotation of ψ about the new z axis (z'').

Quantization

states : $|I, M, K\rangle$

laboratory axes : $[J_x, J_y] = i \cdot \hbar \cdot J_z$ and cyclic permutations

$$[J^2, J_k] = 0 \quad k = x, y, z$$

quantum numbers : $J_z \rightarrow \hbar \cdot M \quad J^2 \rightarrow \hbar^2 \cdot I(I + 1)$

body fixed axes : $[J_1, J_2] = i \cdot \hbar \cdot J_3$ and cyclic permutations

$$[J^2, J_i] = 0 \quad i = 1, 2, 3 \quad [J_z, J_3] = 0$$

quantum numbers : $J_3 \rightarrow \hbar \cdot K \quad J^2 \rightarrow \hbar^2 \cdot I(I + 1)$

Quantization

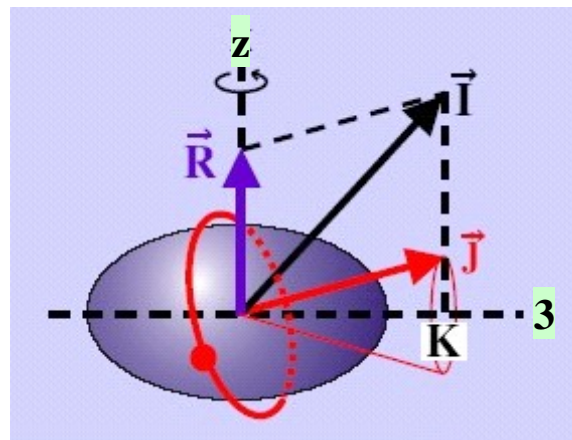
eigenstates : $| I, M, K \rangle$

probability amplitude

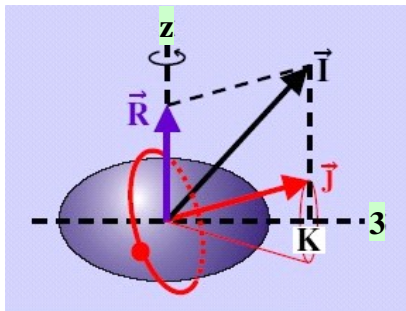
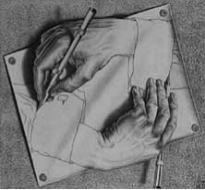
for orientation of rotor : $\langle \psi, \theta, \phi | I, M, K \rangle = \left(\frac{2I+1}{8\pi^2} \right)^{1/2} D_{MK}^I(\psi, \theta, \phi)$

Wigner D – function

$$D_{MK}^I(\psi, \theta, \phi) = e^{iM\psi} d_{MK}^I(\theta) e^{iK\phi}$$



Rotational motion of a deformed nucleus



$$H_{rot} = \sum_{i=1}^3 \frac{\hat{R}_i^2}{2 \cdot \mathfrak{I}_i} = \frac{(\hat{R}^2 - \hat{R}_3^2)}{2 \cdot \mathfrak{I}_1} + \cancel{\frac{\hat{R}_3^2}{2 \cdot \mathfrak{I}_3}}$$

$\mathfrak{I}_1 = \mathfrak{I}_2$

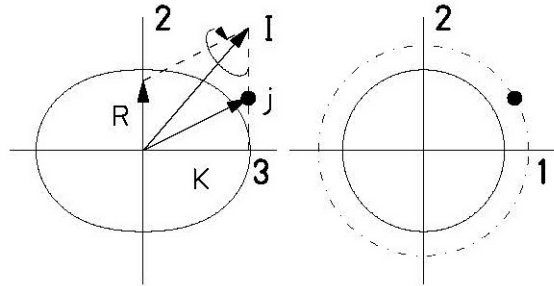
The nucleus does not have an orientation degree of freedom with respect to the symmetry axis

States with projections K and $-K$ are degenerated

$$\Psi_{IMK} = \left(\frac{2 \cdot I + 1}{16 \cdot \pi^2} \right)^{1/2} \cdot \left[D_{MK}^I \cdot \chi_K + (-1)^{I-K} D_{M-K}^I \cdot \chi_{-K} \right]$$

Particle-rotor model

K+4 ———
 K+3 ———
 K+2 ———
 K+1 ———
 K ———



$$H_{rot} = \sum_{i=1}^3 \frac{\hat{R}_i^2}{2 \cdot \mathfrak{I}_i} = \frac{(\hat{R}^2 - \hat{R}_3^2)}{2 \cdot \mathfrak{I}_1} + \cancel{\frac{\hat{R}_3^2}{2 \cdot \mathfrak{I}_3}}$$

$\mathfrak{I}_1 = \mathfrak{I}_2$

The nucleus does not have an orientation degree of freedom with respect to the symmetry axis

with $\vec{R} = \vec{I} - \vec{j}$

$$H_{rot} = \frac{\hbar^2}{2\mathfrak{I}_0} \{(I_1 - j_1)^2 + (I_2 - j_2)^2\}$$

$$H_{rot} = \frac{\hbar^2}{2\mathfrak{I}_0} (I^2 - I_3^2) + \frac{\hbar^2}{2\mathfrak{I}_0} (j_1^2 + j_2^2 - j_3^2) - \frac{\hbar^2}{2\mathfrak{I}_0} (j_+ I_- + j_- I_+)$$

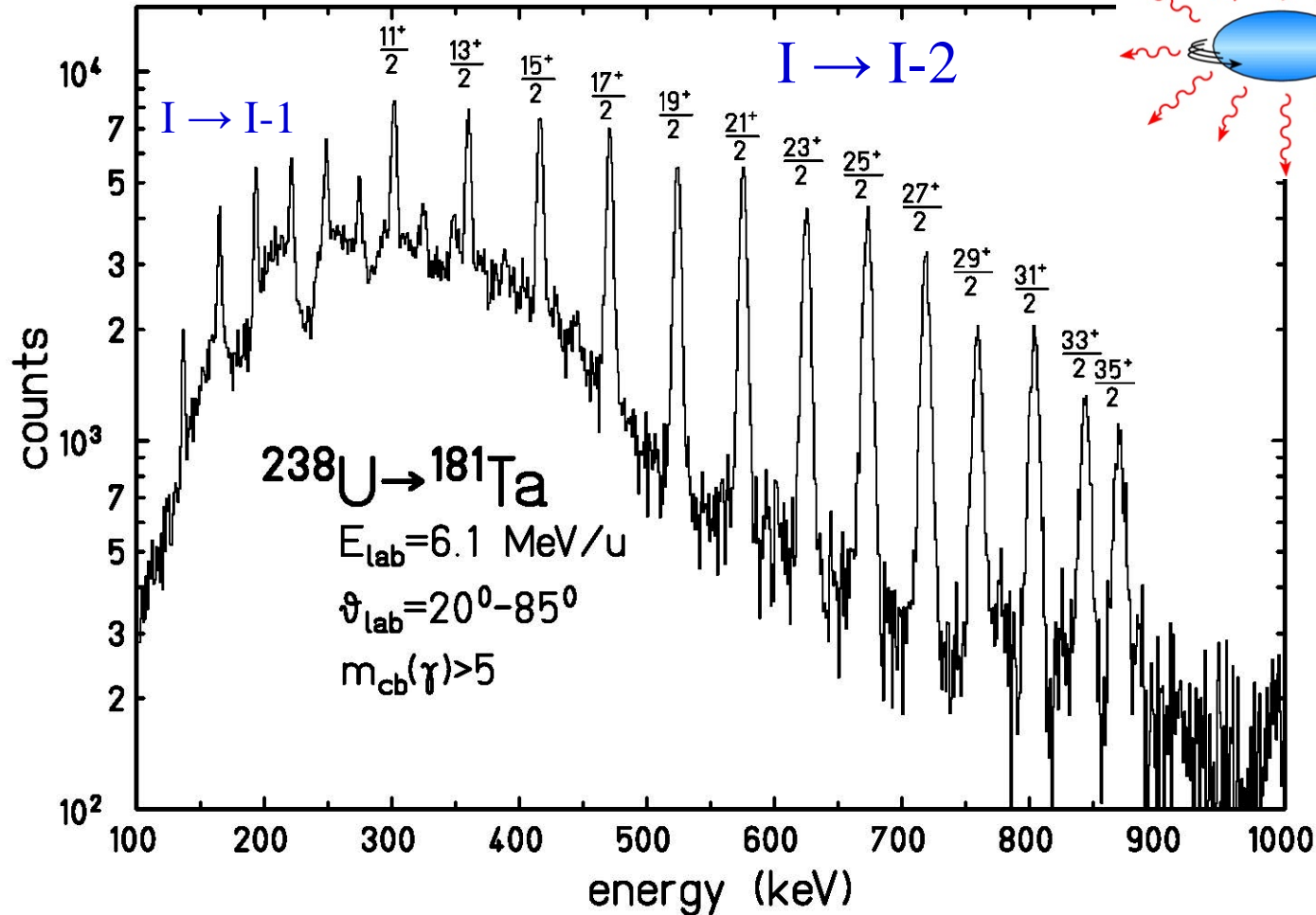
$$I_{\pm} = I_1 \pm i \cdot I_2 \quad j_{\pm} = j_1 \pm i \cdot j_2$$

$$E_K(I) = \epsilon_K + \frac{\hbar^2}{2\mathfrak{I}} [I(I+1) - K^2 + \delta_{K,1/2} \cdot a \cdot (-1)^{I+1/2} (I+1/2)]$$

where a is the so-called *decoupling parameter*

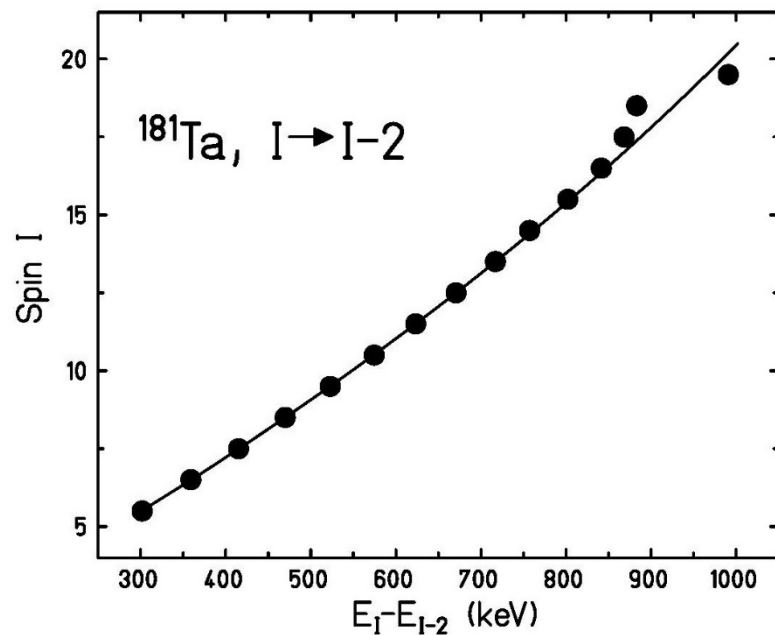
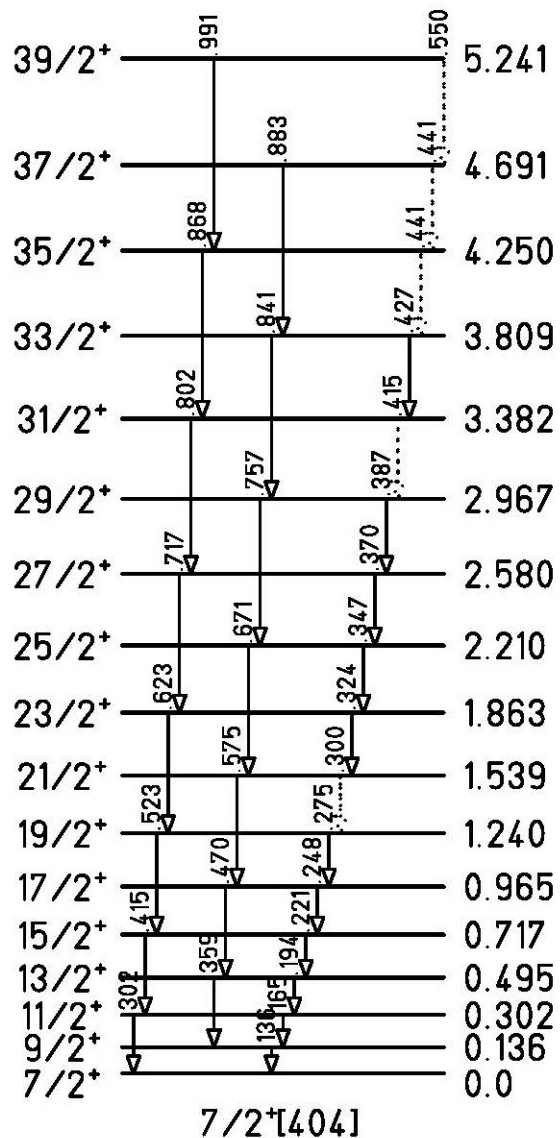
γ -rays from a deformed band in ^{181}Ta

74	W	W150	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190		
73	Ia	Ia154	Ia155	Ia156	Ia157	Ia158	Ia159	Ia160	Ia161	Ia162	Ia163	Ia164	Ia165	Ia166	Ia167	Ia168	Ia169	Ia170	Ia171	Ia172	Ia173	Ia174	Ia175	Ia176	Ia177	Ia178	Ia179	Ia180	Ia181	Ia182	Ia183	Ia184	Ia185	Ia186	Ia187	Ia188
72	Hf	Hf154	Hf155	Hf156	Hf157	Hf158	Hf159	Hf160	Hf161	Hf162	Hf163	Hf164	Hf165	Hf166	Hf167	Hf168	Hf169	Hf170	Hf171	Hf172	Hf173	Hf174	Hf175	Hf176	Hf177	Hf178	Hf179	Hf180	Hf181	Hf182	Hf183	Hf184	Hf185	Hf186	Hf187	Hf188

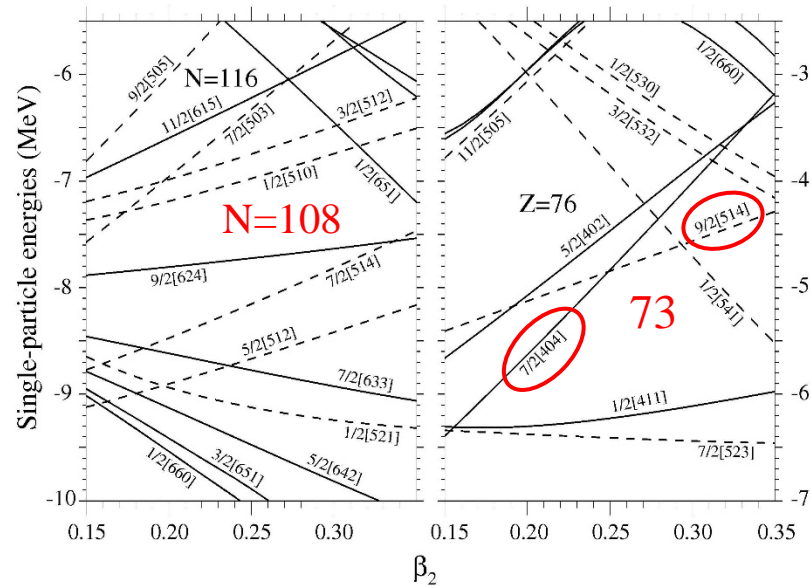
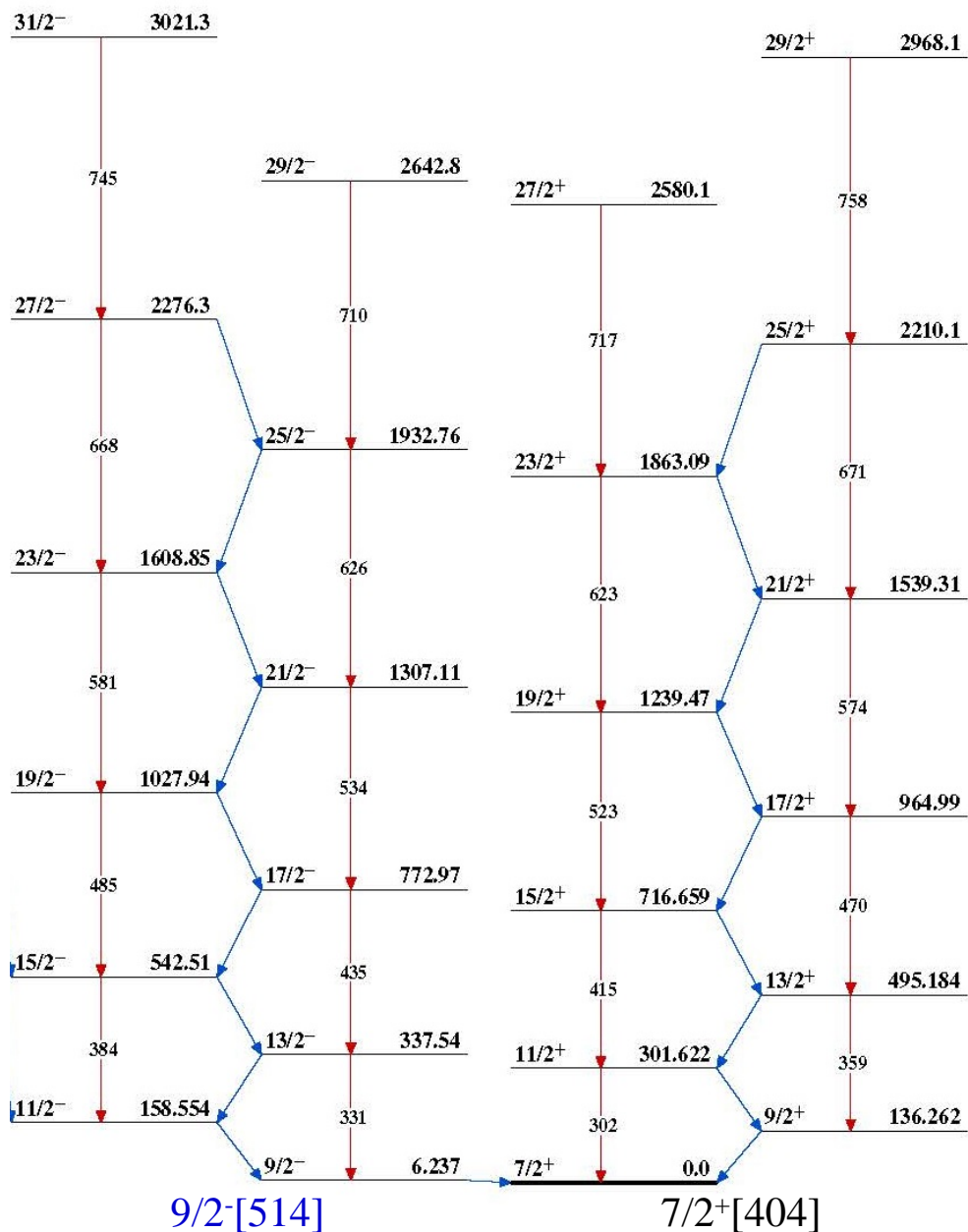


Nuclear level scheme of ^{181}Ta

^{181}Ta



Nilsson diagram of ^{181}Ta



Coriolis band mixing calculation for the $7/2^+[404]$ band in ^{181}Ta

The Hamiltonian H_{rot} is the diagonal part of the rotational Hamiltonian. The eigenvalues are assumed to be given by

$$E_K(I) = \epsilon_K + \frac{\hbar^2}{2\mathfrak{I}} \left[I(I+1) - K^2 + \delta_{K,1/2} \cdot a \cdot (-1)^{I+1/2} (I+1/2) \right]$$

Off-diagonal terms are given by the Coriolis matrix elements

$$V_{K+1,K} = -\frac{\hbar^2}{2\mathfrak{I}} \sqrt{(I-K)(I+K+1)} \cdot \langle K+1 | j_+ | K \rangle$$

Results of Coriolis band mixing calculation for the ground-state rotational band in ^{181}Ta

State I^π	E_{calc} (keV)	Expansion coefficients						
		$\frac{3}{2}^+[422]$	$\frac{3}{2}^+[411]$	$\frac{3}{2}^+[402]$	$\frac{5}{2}^+[413]$	$\frac{5}{2}^+[402]$	$\frac{7}{2}^+[404]$	$\frac{9}{2}^+[404]$
$\frac{7}{2}^+$	0.0	0.0020	-0.0027	0.0	0.0417	-0.0304	0.9987	0.0
$\frac{9}{2}^+$	136.9	0.0040	-0.0053	0.0001	0.0630	-0.0459	0.9969	-0.0025
$\frac{11}{2}^+$	302.9	0.0065	-0.0085	0.0001	0.0818	-0.0594	0.9948	-0.0038
$\frac{13}{2}^+$	497.1	0.0093	-0.0123	0.0002	0.0995	-0.0720	0.9923	-0.0048
$\frac{15}{2}^+$	718.7	0.0127	-0.0167	0.0002	0.1167	-0.0840	0.9894	-0.0058
$\frac{17}{2}^+$	966.5	0.0163	-0.0216	0.0003	0.1335	-0.0955	0.9860	-0.0067
$\frac{19}{2}^+$	1239.3	0.0209	-0.0270	0.0	0.1501	-0.1067	0.9822	-0.0076
$\frac{21}{2}^+$	1535.7	0.0251	-0.0331	0.0004	0.1664	-0.1176	0.9781	-0.0084

Reduced transition probability

expectation value $\langle \widehat{M}_{\lambda m}^{lab} \rangle = \int \Psi^* \widehat{M}_{\lambda m}^{lab} \Psi d\tau$

$$\widehat{M}_{\lambda m}^{lab} = \sum_{m'} D_{mm'}^{\lambda} \widehat{M}_{\lambda m'}^{intr}$$

wave function $\Psi_{IMK} = \sqrt{\frac{2I+1}{16 \cdot \pi^2}} \cdot [D_{MK}^I \cdot X_K + (-1)^{I-K} D_{M-K}^I \cdot X_{-K}]$

$$\langle I_f M_f K | \widehat{M}_{\lambda m}^{lab} | I_i M_i K \rangle = \frac{\sqrt{(2I_i+1)(2I_f+1)}}{8 \cdot \pi^2} \iiint D_{M_f K}^{I_f} X_K \sum_{m'=0} D_{mm'}^{\lambda} \widehat{M}_{\lambda m'}^{intr} D_{M_i K}^{I_i} X_K d\tau$$

$$\iiint D_{M_1 M_1'}^{I_1} D_{M_2 M_2'}^{I_2} D_{M_3 M_3'}^{I_3} d\tau = \frac{8\pi^2}{2I_3+1} \cdot (I_1 I_2 M_1 M_2 | I_3 M_3) \cdot (I_1 I_2 M_1' M_2' | I_3 M_3')$$

$$\langle I_f M_f K | \widehat{M}_{\lambda m}^{lab} | I_i M_i K \rangle = \sqrt{\frac{2I_i+1}{2I_f+1}} \cdot (I_i \lambda M_i (M_f - M_i) | I_f M_f) \cdot (I_i \lambda K 0 | I_f K) \cdot \langle X_K | \widehat{M}_{\lambda 0}^{intr} | X_K \rangle$$

Reduced transition probability

$$\langle I_f M_f K | \widehat{M}_{\lambda m}^{lab} | I_i M_i K \rangle = \sqrt{\frac{2I_i + 1}{2I_f + 1}} \cdot (I_i \lambda M_i (M_f - M_i) | I_f M_f) \cdot (I_i \lambda K 0 | I_f K) \cdot \langle X_K | \widehat{M}_{\lambda 0}^{intr} | X_K \rangle$$

Wigner-Eckart-Theorem (reduction of an expectation value):

$$\langle I_f M_f K | \widehat{M}_{\lambda m}^{lab} | I_i M_i K \rangle = \frac{(I_i \lambda M_i (M_f - M_i) | I_f M_f)}{\sqrt{2I_f + 1}} \cdot \langle I_f K | | \widehat{M}_{\lambda}^{lab} | | I_i K \rangle$$

$$\langle I_f K | | M(E\lambda) | | I_i K \rangle = \sqrt{2I_i + 1} (I_i \lambda K 0 | I_f K) \cdot \langle X_K | \widehat{M}_{\lambda 0}^{intr} | X_K \rangle$$

special case: E2 transition I → I-2

$$\langle I - 2, K | | M(E2) | | I, K \rangle = \sqrt{\frac{15}{32\pi}} \cdot \sqrt{\frac{(I + K - 1) \cdot (I + K) \cdot (I - K - 1) \cdot (I - K)}{(I - 1) \cdot (2I - 1) \cdot I}} \cdot Q_2 e$$

reduced transition probability:

$$B(E\lambda; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |\langle I_f K | | M(E\lambda) | | I_i K \rangle|^2$$

Odd-even nucleus: ^{181}Ta

74	W150	W151	W152	W153	W154	W155	W156	W157	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190
73	I154	I155	I156	I157	I158	I159	I160	I161	I162	I163	I164	I165	I166	I167	I168	I169	I170	I171	I172	I173	I174	I175	I176	I177	I178	I179	I180	I181	I182	I183	I184	I185	I186	I187	I188	I189	I190				
72	H154	H155	H156	H157	H158	H159	H160	H161	H162	H163	H164	H165	H166	H167	H168	H169	H170	H171	H172	H173	H174	H175	H176	H177	H178	H179	H180	H181	H182	H183	H184	H185	H186	H187	H188	H189	H190				

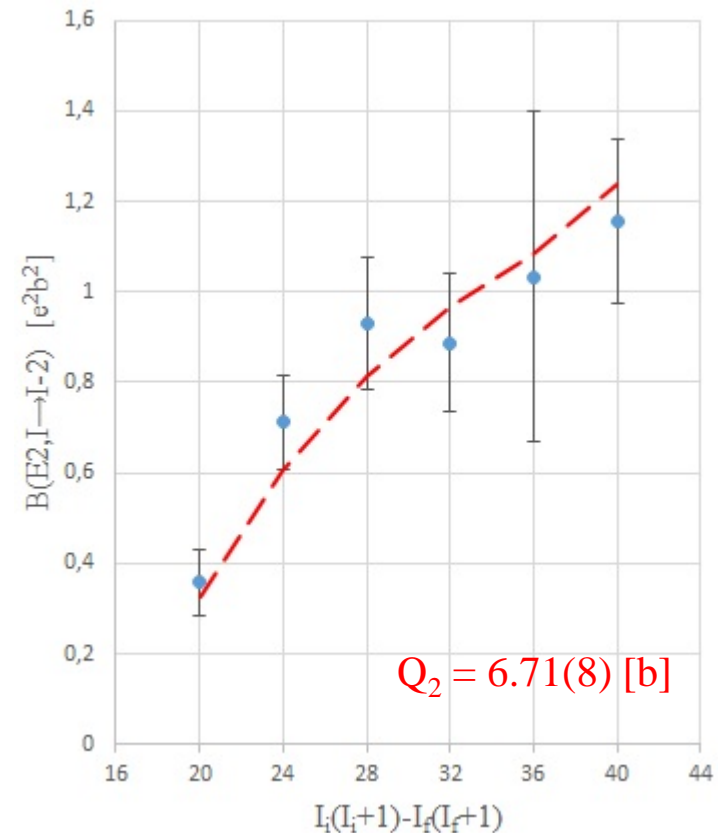
special case: E2 transition $I \rightarrow I-2$

$$\langle I-2, K \| M(E2) \| I, K \rangle = \sqrt{\frac{15}{32\pi}} \cdot \sqrt{\frac{(I+K-1) \cdot (I+K) \cdot (I-K-1) \cdot (I-K)}{(I-1) \cdot (2I-1) \cdot I}} \cdot Q_2 e$$

reduced transition probability:

$$B(E\lambda; I_i \rightarrow I_f) = \frac{1}{2I_i + 1} |\langle I_f K \| M(E\lambda) \| I_i K \rangle|^2$$

$$\frac{Q_2(9/2^-)}{Q_2(7/2^+)} = (0.9681 \pm 0.0002)$$



Matrix elements

$$\langle I - 2, K \| M(E2) \| I, K \rangle = \sqrt{\frac{15}{32\pi}} \cdot \sqrt{\frac{(I + K - 1) \cdot (I + K) \cdot (I - K - 1) \cdot (I - K)}{(I - 1) \cdot (2I - 1) \cdot I}} \cdot Q_2 e$$

$$\langle I - 1, K \| M(E2) \| I, K \rangle = -\sqrt{\frac{5}{16\pi}} \cdot \sqrt{\frac{3 \cdot (I + K) \cdot (I - K) \cdot K^2}{(I - 1) \cdot I \cdot (I + 1)}} \cdot Q_2 e$$

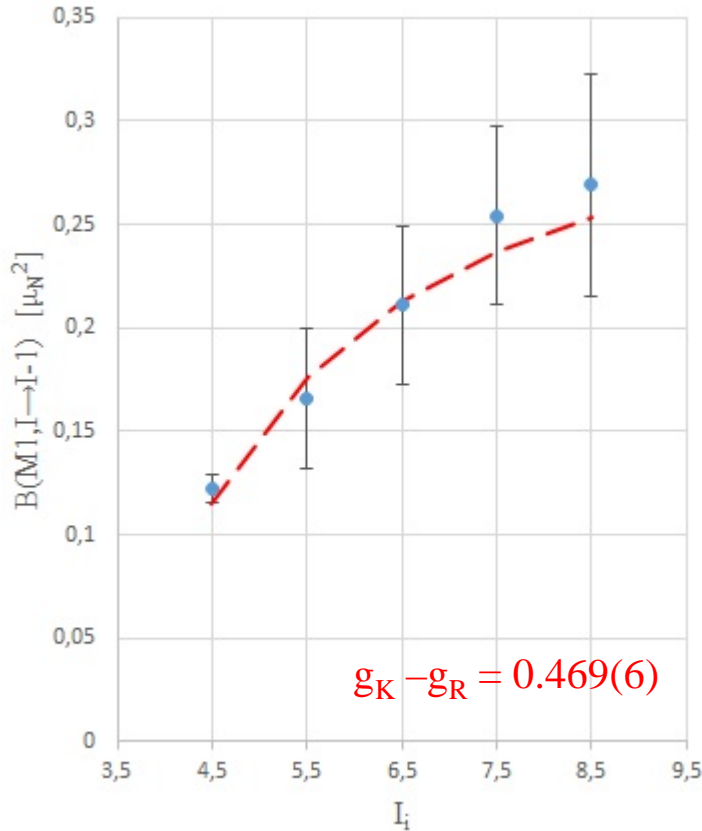
$$\langle I, K \| M(E2) \| I, K \rangle = -\sqrt{\frac{5}{16\pi}} \cdot \sqrt{\frac{2I + 1}{(2I - 1) \cdot I \cdot (I + 1) \cdot (2I + 3)}} \cdot (I^2 - 3K^2 + I) \cdot Q_2 e$$

Odd-even nucleus: ^{181}Ta

	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190
74	183.94 0.0 0-	184.00 7.3 0-	184.07 14.5 0-	184.14 21.7 0-	184.21 28.9 0-	184.28 36.1 0-	184.35 43.3 0-	184.42 50.5 0-	184.49 57.7 0-	184.56 64.9 0-	184.63 72.1 0-	184.70 79.3 0-	184.77 86.5 0-	184.84 93.7 0-	184.91 100.9 0-	184.98 108.1 0-	185.05 115.3 0-	185.12 122.5 0-	185.19 129.7 0-	185.26 136.9 0-	185.33 144.1 0-	185.40 151.3 0-	185.47 158.5 0-	185.54 165.7 0-	185.61 172.9 0-	185.68 180.1 0-	185.75 187.3 0-	185.82 194.5 0-	185.89 201.7 0-	185.96 208.9 0-	186.03 216.1 0-	186.10 223.3 0-	
73	184.00 0.0 0-	184.07 7.3 0-	184.14 14.5 0-	184.21 21.7 0-	184.28 28.9 0-	184.35 36.1 0-	184.42 43.3 0-	184.49 50.5 0-	184.56 57.7 0-	184.63 64.9 0-	184.70 72.1 0-	184.77 79.3 0-	184.84 86.5 0-	184.91 93.7 0-	184.98 100.9 0-	185.05 108.1 0-	185.12 115.3 0-	185.19 122.5 0-	185.26 129.7 0-	185.33 136.9 0-	185.40 144.1 0-	185.47 151.3 0-	185.54 158.5 0-	185.61 165.7 0-	185.68 172.9 0-	185.75 180.1 0-	185.82 187.3 0-	185.89 194.5 0-	185.96 201.7 0-	186.03 208.9 0-	186.10 216.1 0-	186.17 223.3 0-	
72	184.00 0.0 0-	184.07 7.3 0-	184.14 14.5 0-	184.21 21.7 0-	184.28 28.9 0-	184.35 36.1 0-	184.42 43.3 0-	184.49 50.5 0-	184.56 57.7 0-	184.63 64.9 0-	184.70 72.1 0-	184.77 79.3 0-	184.84 86.5 0-	184.91 93.7 0-	184.98 100.9 0-	185.05 108.1 0-	185.12 115.3 0-	185.19 122.5 0-	185.26 129.7 0-	185.33 136.9 0-	185.40 144.1 0-	185.47 151.3 0-	185.54 158.5 0-	185.61 165.7 0-	185.68 172.9 0-	185.75 180.1 0-	185.82 187.3 0-	185.89 194.5 0-	185.96 201.7 0-	186.03 208.9 0-	186.10 216.1 0-	186.17 223.3 0-	

$$\langle I-1, K \| M(M1) \| I, K \rangle = -\sqrt{\frac{3}{4\pi}} \sqrt{\frac{(I+K)(I-K)}{I}} \cdot K \cdot (g_K - g_R) [1 + \delta_{K,1/2} (-1)^{I+1/2} b_0] \mu_N$$

The quantity b_0 depends on the magnetic decoupling parameter



$$\mu(7/2^+) = 2.3705 \pm 0.0007$$

$$\mu = \frac{K}{I+1} \cdot (g_K - g_R) \cdot K + g_R \cdot I$$

$$g_R = 0.313(5)$$

$$g_K = 0.782(2)$$

$$\mu(9/2^-) = 5.28 \pm 0.09$$

$${}^{9/2^-} (g_K - g_R) = \frac{22}{81} \left({}^{9/2^-} \mu - {}^{7/2^+} \mu \frac{9}{7} + {}^{7/2^+} (g_K - g_R) \frac{7}{2} \right)$$

$${}^{9/2^-} (g_K - g_R) \approx {}^{7/2^+} (g_K - g_R) \frac{77}{81} + 0.606 = 1.052$$

Odd-even nucleus: ^{181}Ta

74	W150	W151	W152	W153	W154	W155	W156	W157	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190
73	Ta150	Ta151	Ta152	Ta153	Ta154	Ta155	Ta156	Ta157	Ta158	Ta159	Ta160	Ta161	Ta162	Ta163	Ta164	Ta165	Ta166	Ta167	Ta168	Ta169	Ta170	Ta171	Ta172	Ta173	Ta174	Ta175	Ta176	Ta177	Ta178	Ta179	Ta180	Ta181	Ta182	Ta183	Ta184	Ta185	Ta186	Ta187	Ta188	Ta189	Ta190
72	Hf150	Hf151	Hf152	Hf153	Hf154	Hf155	Hf156	Hf157	Hf158	Hf159	Hf160	Hf161	Hf162	Hf163	Hf164	Hf165	Hf166	Hf167	Hf168	Hf169	Hf170	Hf171	Hf172	Hf173	Hf174	Hf175	Hf176	Hf177	Hf178	Hf179	Hf180	Hf181	Hf182	Hf183	Hf184	Hf185	Hf186	Hf187	Hf188	Hf189	Hf190

$$13/2^+ \quad 0.495 \quad \tau = 9.1 \pm 1.2 \text{ ps}$$

$$\tau = \left\{ \sum_K \sum_\ell [\varepsilon_{N \rightarrow K}^2(\lambda) + \delta_{N \rightarrow K}^2(\lambda)] \right\}^{-1}$$

$$11/2^+ \quad 0.302 \quad \tau = 23.1 \pm 4.3 \text{ ps}$$

$$\tau = T_{1/2} / \ln 2$$

$$9/2^+ \quad 0.136 \quad \tau = 57.0 \pm 2.3 \text{ ps}$$

$$7/2^+ \quad 0.0$$

^{181}Ta

$$\delta_{N \rightarrow M}(\lambda) = \left\{ \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \frac{1}{\hbar} \left(\frac{\hbar\omega}{\hbar c} \right)^{2\lambda+1} \right\}^{1/2} \cdot (2I_N + 1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(\lambda) \| I_N \rangle$$

$$\delta_{N \rightarrow M}(E2) = \{ 1.225 \cdot 10^{13} \cdot E_\gamma^5(\text{MeV})^5 \}^{1/2} \cdot (2I_n + 1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(E2) \| I_N \rangle$$

$$\delta_{N \rightarrow M}(M1) = \{ 1.758 \cdot 10^{13} \cdot E_\gamma^3(\text{MeV})^3 \}^{1/2} \cdot (2I_n + 1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(M1) \| I_N \rangle$$

$$\varepsilon_{N \rightarrow M}^2(\ell) = \delta_{N \rightarrow M}^2(\ell) \cdot \alpha_{N \rightarrow M}(\ell)$$

conversion coefficient.: bricc.anu.edu.au

Odd-even nucleus: ^{181}Ta

$$13/2^+ \quad 0.495 \quad \tau = 9.1 \pm 1.2 \text{ ps}$$

$$\tau = \left\{ \sum_K \sum_{\ell} [\varepsilon_{N \rightarrow K}^2(\lambda) + \delta_{N \rightarrow K}^2(\lambda)] \right\}^{-1}$$

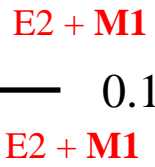
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$$9/2^+ \quad 0.136 \quad \tau = 57.0 \pm 2.3 \text{ ps}$$

$$7/2^+ \quad 0.0$$

^{181}Ta



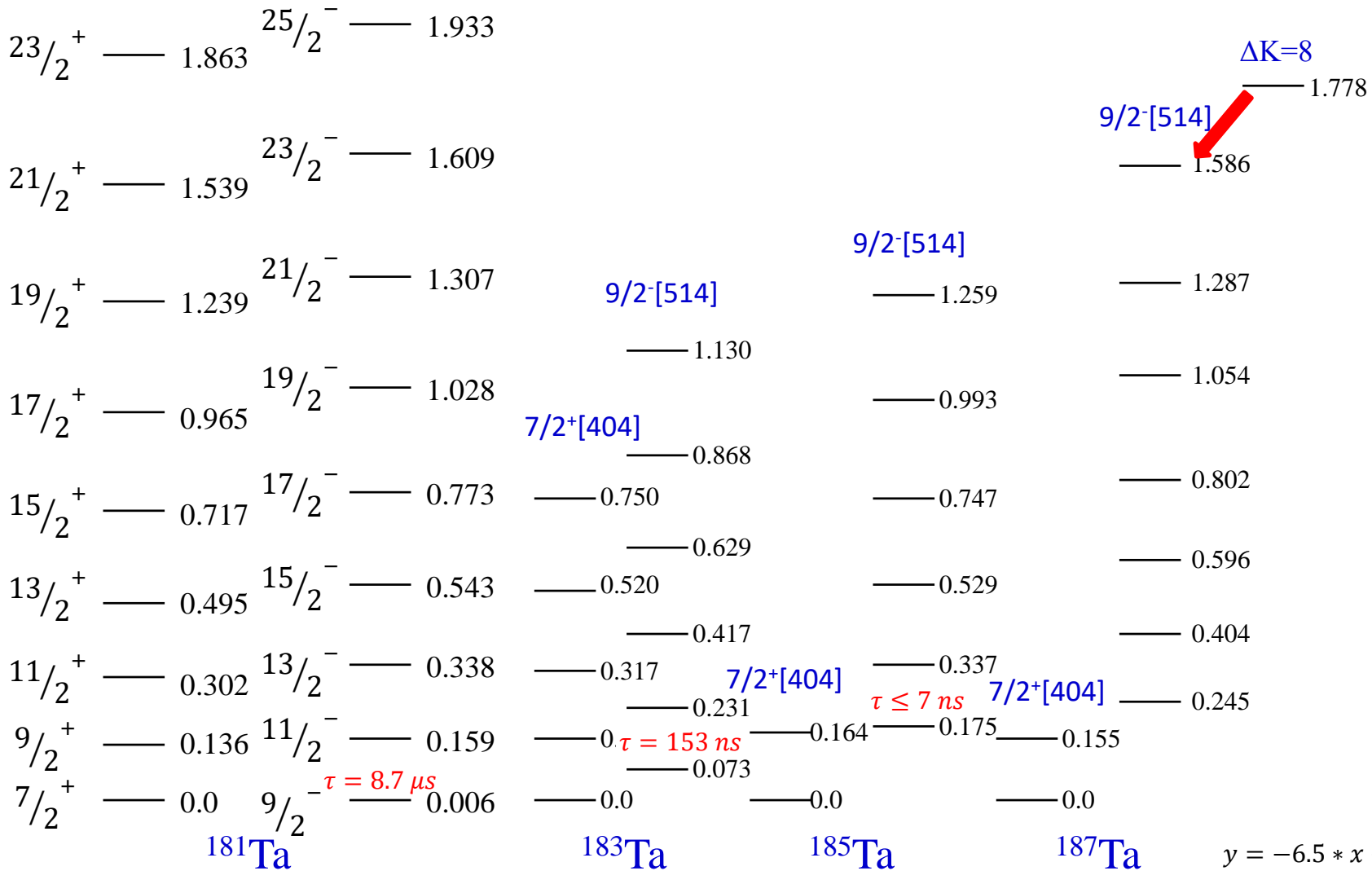
Spin I	E_{γ} (MeV)	$(2I+1)^{-1/2}$	$\langle I-1//M()//I \rangle$	delta	α_T	ε^2	τ (ps)
9/2	0.1365	0.3162	-3.899 (E2)	-29594.	1.1	$9.98 \cdot 10^8$	533
			1.103 (M1)	73591	1.8	$9.75 \cdot 10^9$	58.7
11/2	0.1654	0.2887	-4.291 (E2)	-48238	0.6	$1.33 \cdot 10^9$	274
			1.413 (M1)	115044	1.0	$1.38 \cdot 10^{10}$	32.6
11/2	0.3017	0.2887	1.977 (E2)	99868	0.1	$8.08 \cdot 10^8$	24.1

Odd-proton-even nuclei

		μ	$(g_K - g_R)$	B(M1)W.u.	$(g_K - g_R)$
^{153}Eu	$5/2^+$	+1.5324(3)	0.551	0.00608(28)	0.185
^{159}Tb	$3/2^+$	+2.014(4)	1.556	0.173(8)	1.471
^{165}Ho	$7/2^-$	+4.177(5)	1.012	0.275(14)	0.973
^{169}Tm	$1/2^+$	-0.2316(15)		0.0342(8)	
^{175}Lu	$7/2^+$	+2.2327(11)	0.298	0.0354(14)	0.349
^{181}Ta	$7/2^+$	+2.3705(7)	0.352	0.068(4)	0.484
^{185}Re	$5/2^+$	+3.1871(3)	1.217	0.28(5)	1.252
^{187}Re	$5/2^+$	+3.2197(3)	1.242	0.260(18)	1.206

Ta-nuclei: level schemes

74	W150	W151	W152	W153	W154	W155	W156	W157	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190						
73	Ia154	Ia155	Ia156	Ia157	Ia158	Ia159	Ia160	Ia161	Ia162	Ia163	Ia164	Ia165	Ia166	Ia167	Ia168	Ia169	Ia170	Ia171	Ia172	Ia173	Ia174	Ia175	Ia176	Ia177	Ia178	Ia179	Ia180	Ia181	Ia182	Ia183	Ia184	Ia185	Ia186	Ia187	Ia188	Ia189	Ia190	Ia191	Ia192	Ia193	Ia194	Ia195	Ia196	Ia197	Ia198	Ia199	Ia200
72	Hf154	Hf155	Hf156	Hf157	Hf158	Hf159	Hf160	Hf161	Hf162	Hf163	Hf164	Hf165	Hf166	Hf167	Hf168	Hf169	Hf170	Hf171	Hf172	Hf173	Hf174	Hf175	Hf176	Hf177	Hf178	Hf179	Hf180	Hf181	Hf182	Hf183	Hf184	Hf185	Hf186	Hf187	Hf188	Hf189	Hf190	Hf191	Hf192	Hf193	Hf194	Hf195	Hf196	Hf197	Hf198	Hf199	Hf200



^{187}Ta level scheme

74	W150	W151	W152	W153	W154	W155	W156	W157	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190							
73	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia		
72	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf

expected lifetimes

$\Delta K=8$
 $(25/2^-)$ — 1.778 $\tau = 10.5(13) \text{ s}$
 $9/2^+[514]$

$(21/2^-)$ — 1.586 $\tau = 0.6 \text{ ps}$

$(19/2^-)$ — 1.287 $\tau = 1.1 \text{ ps}$

$(17/2^-)$ — 1.054 $\tau = 1.1 \text{ ps}$

$(15/2^-)$ — 0.802 $\tau = 2.2 \text{ ps}$

$(13/2^-)$ — 0.596 $\tau = 3.4 \text{ ps}$

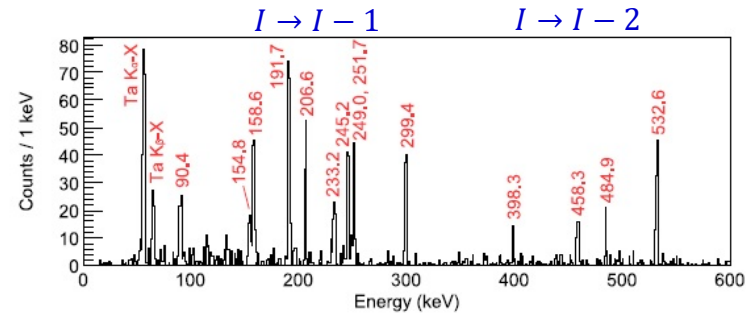
$(11/2^-)$ — 0.404 $\tau = 7.9 \text{ ps}$

$7/2^+[404]$

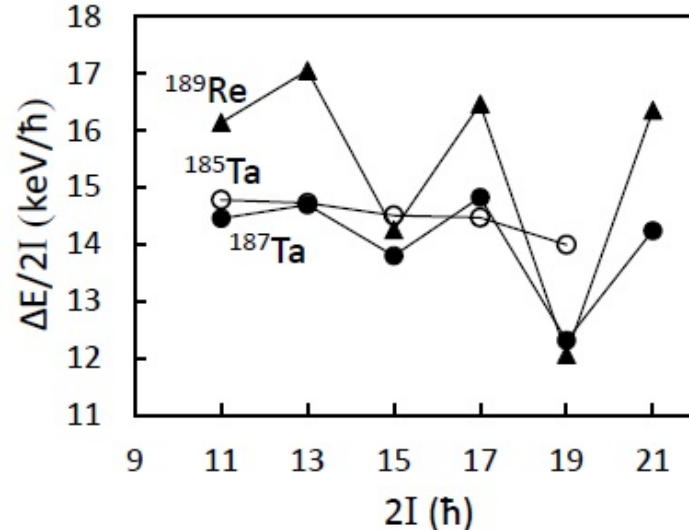
$(9/2^-)$ — 0.245 $\tau \sim 40 \text{ ns}$

$(9/2^+)$ — 0.155

$(7/2^+)$ — 0.0

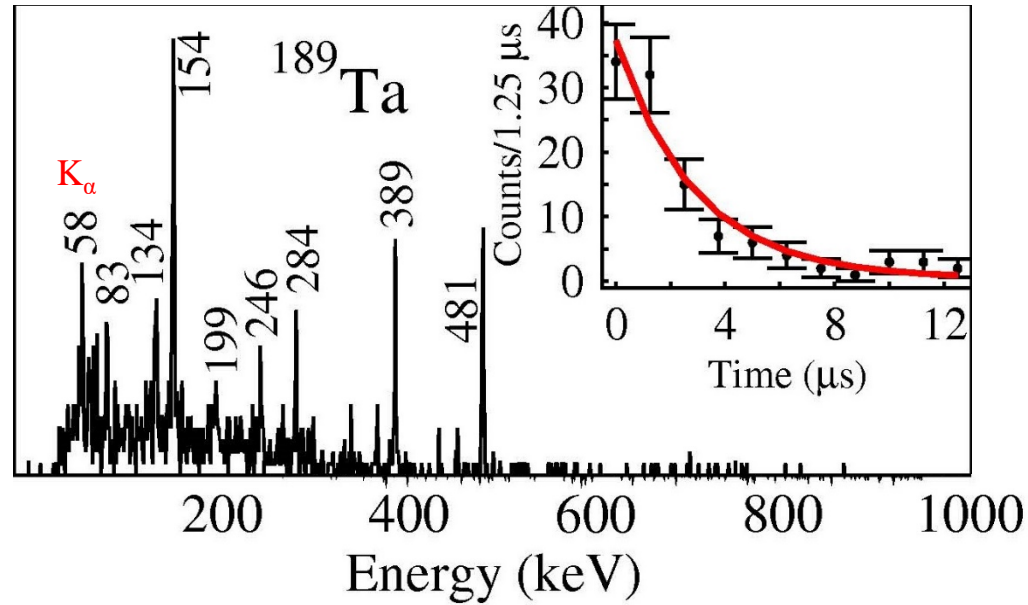


Sum of $\gamma\gamma$ -coincidence energy spectra

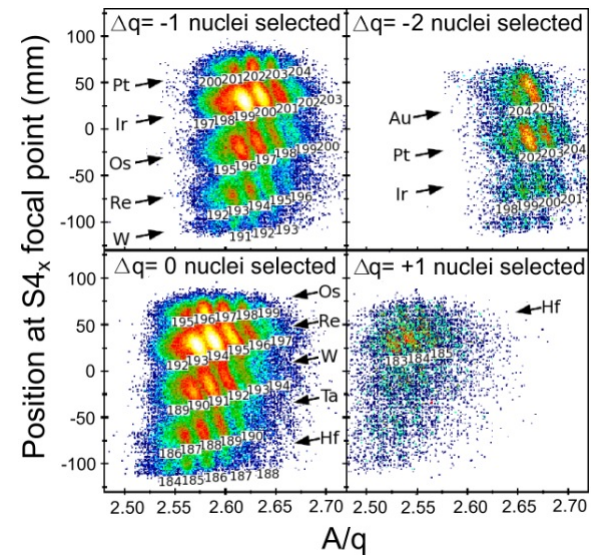
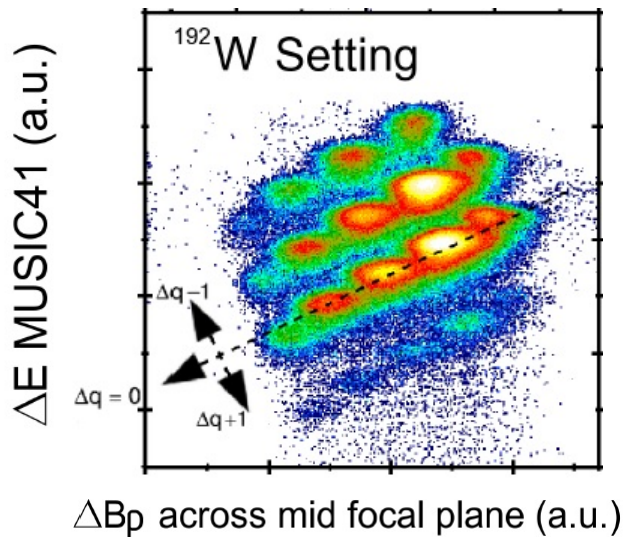


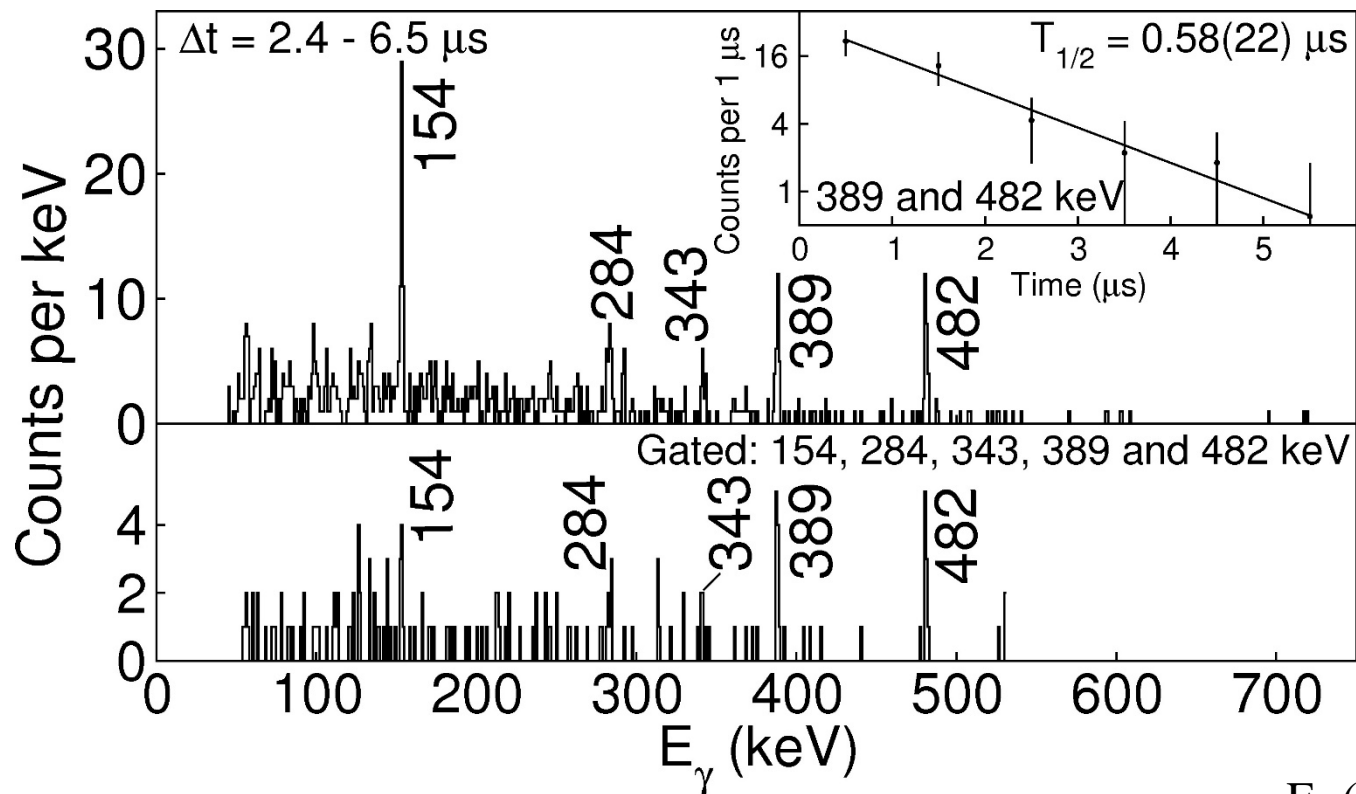
Signature splitting versus angular momentum

Experimental information on ^{189}Ta



^{208}Pb primary beam



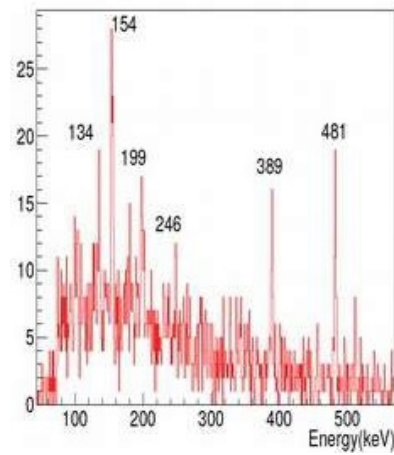
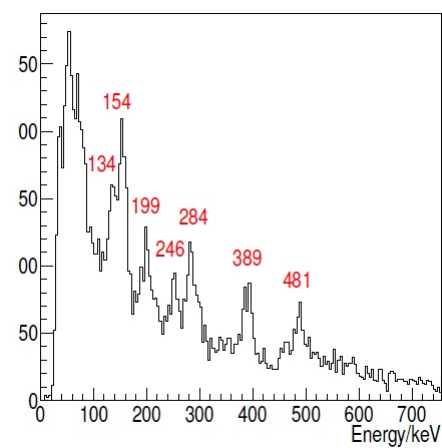


E_γ (keV)	intensity
153.9	100(19)
283.7	73(17)
342.5	47(13)
388.7	80(19)
481.6	97(21)

Experimental information on ^{189}Ta

74	W	W156	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190
73	Ia	Ta154	Ta157	Ta158	Ta159	Ta160	Ta161	Ta162	Ta163	Ta164	Ta165	Ta166	Ta167	Ta168	Ta169	Ta170	Ta171	Ta172	Ta173	Ta174	Ta175	Ta176	Ta177	Ta178	Ta179	Ta180	Ta181	Ta182	Ta183	Ta184	Ta185	Ta186	Ta187	Ta188
72	Hf	Hf154	Hf155	Hf156	Hf157	Hf158	Hf159	Hf160	Hf161	Hf162	Hf163	Hf164	Hf165	Hf166	Hf167	Hf168	Hf169	Hf170	Hf171	Hf172	Hf173	Hf174	Hf175	Hf176	Hf177	Hf178	Hf179	Hf180	Hf181	Hf182	Hf183	Hf184	Hf185	Hf186

116



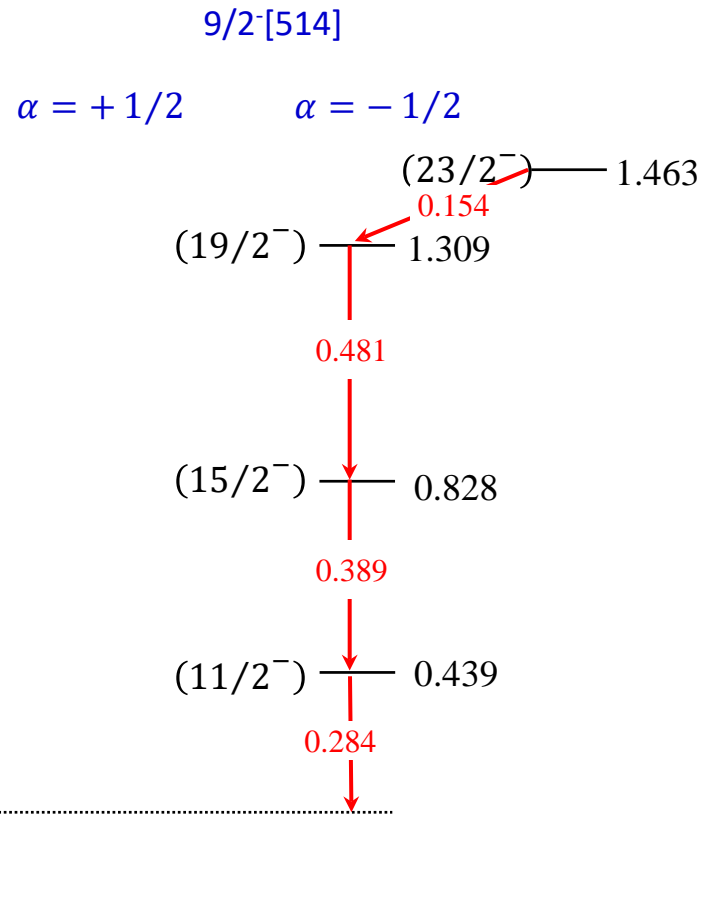
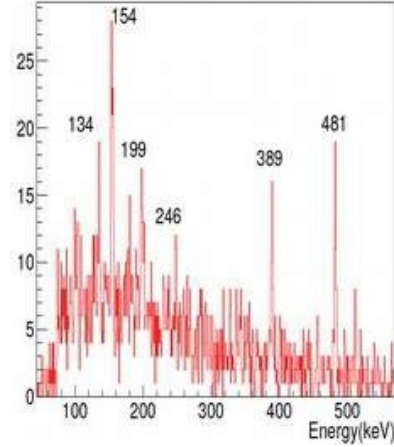
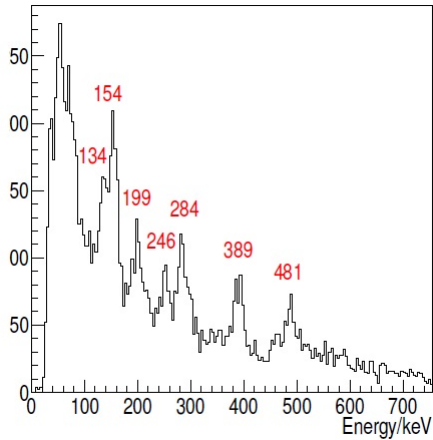
E_γ	ΔE_γ	γ -intensity	α_{tot}	intensity	S.J. Steer	$T_{1/2}$ (μs)
154	3.2	149.5±14.1	100%	7.34	E1:119/E2:173	100%
199	4.2	67.1±9.0	44.9±7.4	0.303	58.5±9.6	
198	2.5	32.3±9.6	21.6±6.7	0.308	28.3±8.8	
200.5	2.5	31.1±9.6	20.8±6.7	0.295	26.9±8.7	
247.2	1.8	36.9±7.1	24.7±5.3	0.149	28.4±6.1	
284.6	2.4	37.5±8.3	25.5±6.0	0.025	E1:26.1±6.2	73±22
342.7	2.6	29.4±7.0	19.7±5.0	0.056	20.8±5.3	47±16
389.3	2.5	110.5±10.9	73.9±10.1	0.039	76.8±10.4	80±24
481	3.2	131.6±12.5	88.0±11.8	0.022	90±12	97±28
83						0.202(12)
134						0.284(11)

gate

481	154			284	389			-
389	154			284				481
284	154						389	481
343	154	199	246					
246	154	199		343	370			453
199	154						389	481
154	134	154		246	343		284	389
134								481

Experimental information on ^{189}Ta

74	W154	W155	W156	W157	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190	
73	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	
72	Hf	Hf154	Hf155	Hf156	Hf157	Hf158	Hf159	Hf160	Hf161	Hf162	Hf163	Hf164	Hf165	Hf166	Hf167	Hf168	Hf169	Hf170	Hf171	Hf172	Hf173	Hf174	Hf175	Hf176	Hf177	Hf178	Hf179	Hf180	Hf181	Hf182	Hf183	Hf184	Hf185	Hf186	Hf187	Hf188	Hf189	Hf190



gate

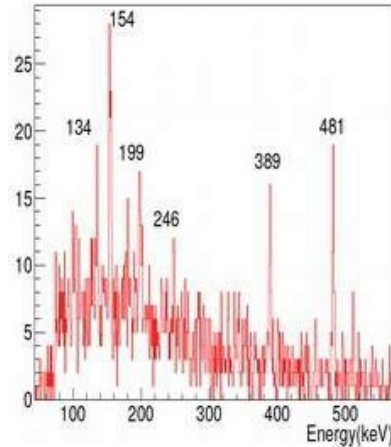
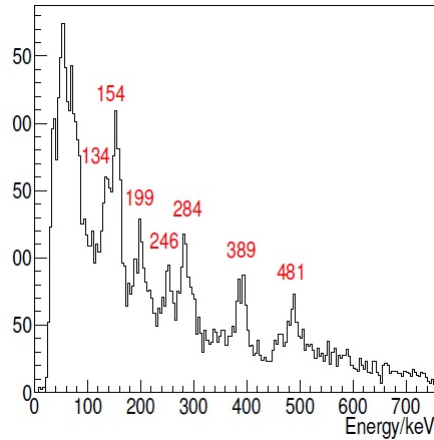
481	154			284	389				
389	154			284			481		
284	154				389		481		
343	154	199	246						
246	154	199		343	370		453	481	
199	154					389		481	
154	134	154		246	343		284	389	481
134									

$7/2^+ [404]$

^{189}Ta

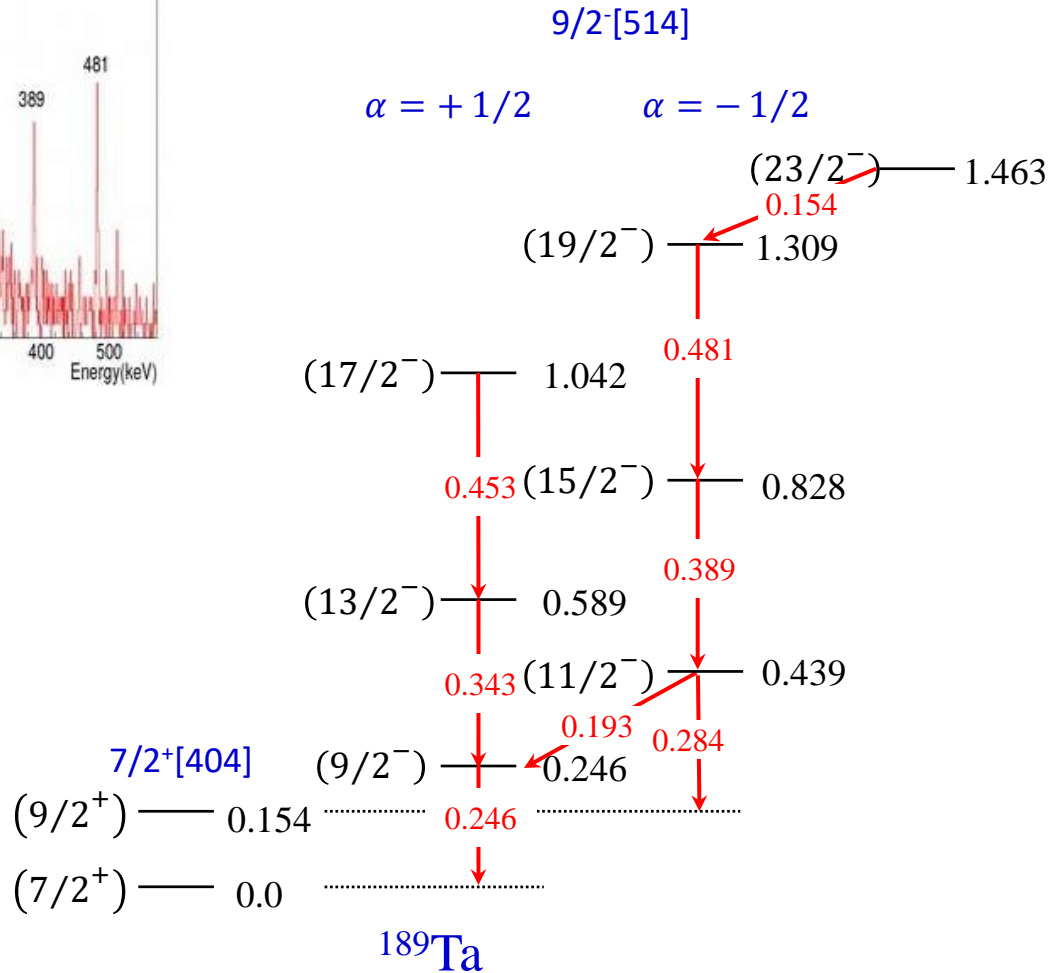
Experimental information on ^{189}Ta

74	W154	W155	W156	W157	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190
73	Ta154	Ta155	Ta156	Ta157	Ta158	Ta159	Ta160	Ta161	Ta162	Ta163	Ta164	Ta165	Ta166	Ta167	Ta168	Ta169	Ta170	Ta171	Ta172	Ta173	Ta174	Ta175	Ta176	Ta177	Ta178	Ta179	Ta180	Ta181	Ta182	Ta183	Ta184	Ta185	Ta186	Ta187	Ta188	Ta189	Ta190
72	Hf154	Hf155	Hf156	Hf157	Hf158	Hf159	Hf160	Hf161	Hf162	Hf163	Hf164	Hf165	Hf166	Hf167	Hf168	Hf169	Hf170	Hf171	Hf172	Hf173	Hf174	Hf175	Hf176	Hf177	Hf178	Hf179	Hf180	Hf181	Hf182	Hf183	Hf184	Hf185	Hf186	Hf187	Hf188	Hf189	Hf190



gate

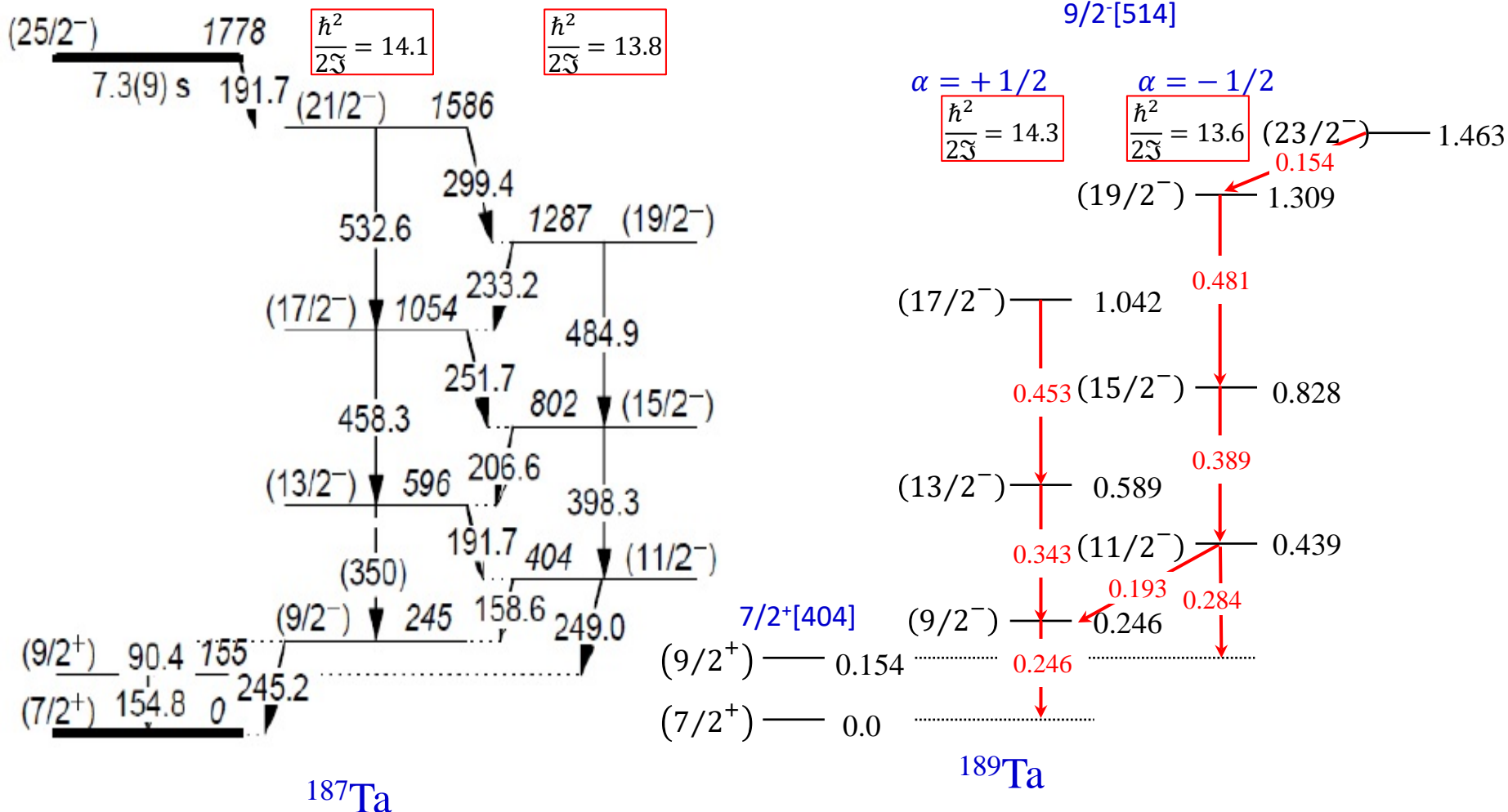
481	154			284	389				
389	154			284				481	
284	154							481	
343	154	199	246						
246	154	199		343	370			453	481
199	154							389	481
154	134	154		246	343			284	389
134									481



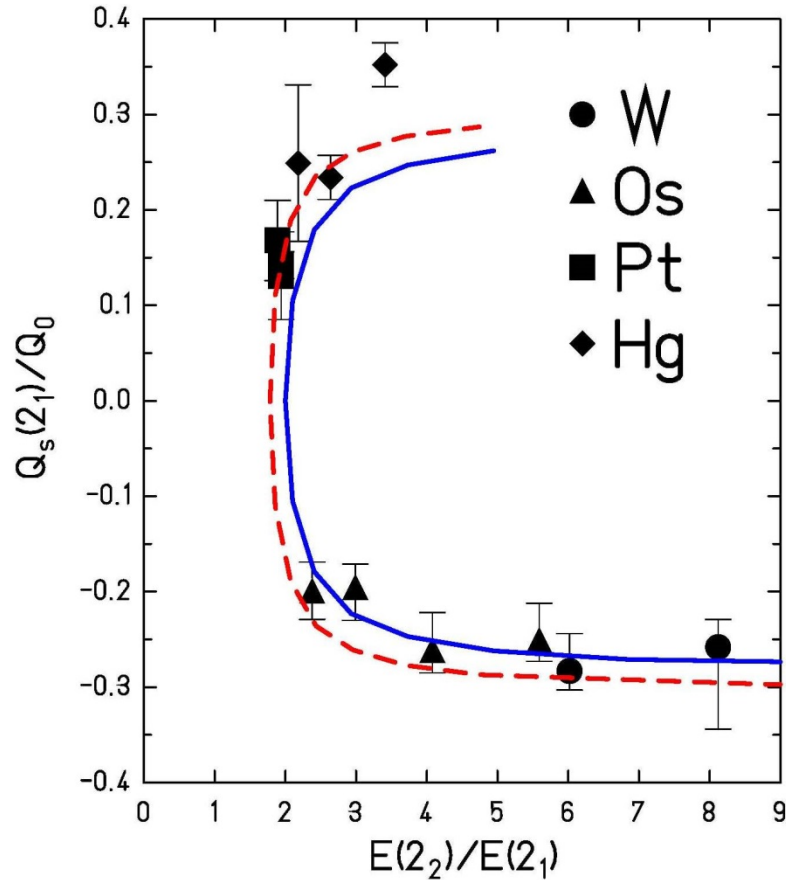
Ta-nuclei: level schemes

74 W	W150 0.9 ms 0 ⁻	W159 7.3 ms 0 ⁻	W160 9.1 ms 0 ⁻	W161 4.0 ms 0 ⁻	W162 1.39 s 0 ⁻	W163 2.95 s 0 ⁻	W164 6.8 s 0 ⁻	W165 5.1 s 0 ⁻	W166 1.83 s 0 ⁻	W167 1.99 s 0 ⁻	W168 3.3 s 0 ⁻	W169 7.6 s 0 ⁻	W170 2.42 ms 0 ⁻	W171 2.33 ms 0 ⁻	W172 6.0 ms 0 ⁻	W173 7.8 ms 0 ⁻	W174 31 ms 0 ⁻	W175 25.2 ms 0 ⁻	W176 2.53 ms 0 ⁻	W177 1.95 ms 0 ⁻	W178 31.64 ms 0 ⁻	W179 37.65 ms 0 ⁻	W180 0 ms 0 ⁻	W181 151.2 s 0 ⁻	W182 1.17 s 0 ⁻	W183 1.17 s 0 ⁻	W184 3.8 s 0 ⁻	W185 75.1 s 0 ⁻	W186 23.7 s 0 ⁻	W187 38.3 s 0 ⁻	W188 99.4 s 0 ⁻	W189 11.5 s 0 ⁻	W190 36.0 s 0 ⁻		
73 Ia	Ia154 180.949 s 0 ⁻	Ia155 181 ms 0 ⁻	Ia156 143 ms 0 ⁻	Ia157 9.97 s 0 ⁻	Ia158 1.56 s 0 ⁻	Ia159 2.7 s 0 ⁻	Ia160 3.92 s 0 ⁻	Ia161 18.5 s 0 ⁻	Ia162 14.5 s 0 ⁻	Ia163 14.5 s 0 ⁻	Ia164 14.5 s 0 ⁻	Ia165 31.0 s 0 ⁻	Ia166 34.4 s 0 ⁻	Ia167 1.4 ms 0 ⁻	Ia168 2.0 ms 0 ⁻	Ia169 4.9 ms 0 ⁻	Ia170 6.76 ms 0 ⁻	Ia171 23.3 ms 0 ⁻	Ia172 35.8 ms 0 ⁻	Ia173 3.14 s 0 ⁻	Ia174 1.05 s 0 ⁻	Ia175 10.5 s 0 ⁻	Ia176 4.09 s 0 ⁻	Ia177 56.5 s 0 ⁻	Ia178 0.31 ms 0 ⁻	Ia179 1.82 s 0 ⁻	Ia180 9.99 s 0 ⁻	Ia181 4.26 s 0 ⁻	Ia182 9.97 s 0 ⁻	Ia183 1.07 s 0 ⁻	Ia184 4.1 s 0 ⁻	Ia185 49.4 ms 0 ⁻	Ia186 10.5 ms 0 ⁻	Ia187 10.5 ms 0 ⁻	Ia188 0 ⁻
72 HF	HF154 2 s 0 ⁻	HF155 0.39 s 0 ⁻	HF156 25 ms 0 ⁻	HF157 110 ms 0 ⁻	HF158 5.25 s 0 ⁻	HF159 5.6 s 0 ⁻	HF160 1.24 s 0 ⁻	HF161 15.8 s 0 ⁻	HF162 37.5 s 0 ⁻	HF163 68.3 s 0 ⁻	HF164 111 s 0 ⁻	HF165 7.5 s 0 ⁻	HF166 4.97 ms 0 ⁻	HF167 5.05 ms 0 ⁻	HF168 28.8 ms 0 ⁻	HF169 32.6 ms 0 ⁻	HF170 1610 s 0 ⁻	HF171 15.1 s 0 ⁻	HF172 1.25 s 0 ⁻	HF173 21.6 s 0 ⁻	HF174 211.5 s 0 ⁻	HF175 714 s 0 ⁻	HF176 5.206 s 0 ⁻	HF177 14.406 s 0 ⁻	HF178 22.297 s 0 ⁻	HF179 13.679 s 0 ⁻	HF180 35.100 s 0 ⁻	HF181 0 ⁻	HF182 0 ⁻	HF183 0 ⁻	HF184 0 ⁻	HF185 0 ⁻	HF186 0 ⁻	HF187 0 ⁻	HF188 0 ⁻

116



Prolate-oblate shape transition

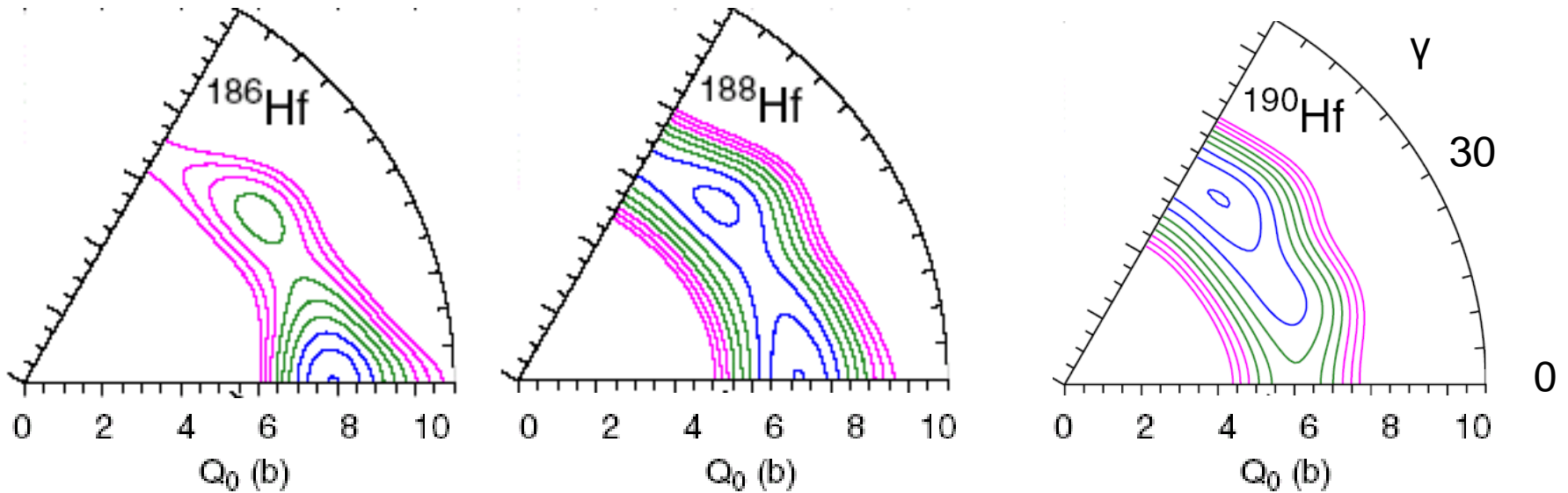


Pb190 12 m 0+	Pb191 13 m (0 ⁺)	Pb192 15 m 0+	Pb193 1 m (0 ⁺)	Pb194 110 m 0+	Pb195 12 m 32 ⁺	Pb196 37 m 0+	Pb197 1 m 32 ⁺	Pb198 240 h 0+	Pb199 90 m 32 ⁺	Pb200 11.5 h 0+	Pb201 5.227 y 52 ⁺	Pb202 2.327 y 52 ⁺	Pb203 51.73 h 52 ⁺	Pb204 1.4 0+	Pb205 1.5277 y 52 ⁺	Pb206 0+	Pb207 241 22 ⁺	Pb208 283.828 y 0+
Tl189 3.2 m (0 ⁺)	Tl190 1.6 m (0 ⁺)	Tl191 9.8 m (0 ⁺)	Tl192 1.6 m (3 ⁺)	Tl193 2.15 m 12 ⁺	Tl194 33.0 m 2 ⁺	Tl195 1.16 h 12 ⁺	Tl196 1.54 h 12 ⁺	Tl197 1.34 h 12 ⁺	Tl198 5.3 h 12 ⁺	Tl199 7.47 h 12 ⁺	Tl200 1.1 h 12 ⁺	Tl201 12.24 d 12 ⁺	Tl202 12.24 d 12 ⁺	Tl203 1.4 12 ⁺	Tl204 3.73 y 2 ⁺	Tl205 4.77 m 6 ⁺	Tl206 4.77 m 12 ⁺	Tl207 4.77 m 12 ⁺
Hg188 15 m 0+	Hg189 7 m 32 ⁺	Hg190 360 m 0+	Hg191 0 m (0 ⁺)	Hg192 125 h 32 ⁺	Hg193 730 h 32 ⁺	Hg194 447 32 ⁺	Hg195 59 h 32 ⁺	Hg196 0.15 32 ⁺	Hg197 42.1 h 2 ⁺	Hg198 9.97 1687	Hg199 12.2 2110	Hg200 35.7 32 ⁺	Hg201 35.7 16	Hg202 29.524 52 ⁺	Hg203 46.812 d 52 ⁺	Hg204 70.476 52 ⁺	Hg205 52 m 12 ⁺	Hg206 83 m 0+
Au187 14 m 12 ⁺	Au188 134 m 16	Au189 237 m 12 ⁺	Au190 42.8 m 1 ⁺	Au191 2.18 h 32 ⁺	Au192 1.94 h 32 ⁺	Au193 17.65 h 32 ⁺	Au194 30.03 h 32 ⁺	Au195 186.59 d 32 ⁺	Au196 4.13 d 2 ⁺	Au197 3.9574 d 2 ⁺	Au198 2.139 d 32 ⁺	Au199 22.4 m 16	Au200 42.4 m 16	Au201 28 m 32 ⁺	Au202 281 (5)	Au203 51 (5)	Au204 51 32 ⁺	Au205 31 (0 ⁺)
Pt186 22 h 0+	Pt187 2.25 h 32 ⁺	Pt188 10.2 d 32 ⁺	Pt189 10.87 h 32 ⁺	Pt190 6.221 y 32 ⁺	Pt191 2.802 d 32 ⁺	Pt192 0+ 32 ⁺	Pt193 58 y 12 ⁺	Pt194 32.9 32 ⁺	Pt195 12.2 32 ⁺	Pt196 4.13 d 2 ⁺	Pt197 19.8915 h 32 ⁺	Pt198 12.2 h 32 ⁺	Pt199 30.8 m 50 ⁺	Pt200 112.5 h 0+	Pt201 1.25 m 44 h (5)	Pt202 44 h (5)	Pt203 44 h (5)	Pt204 44 h (5)
Ir185 14.4 h 52 ⁺	Ir186 18.4 h 52 ⁺	Ir187 10.5 h 32 ⁺	Ir188 41.5 h 1 ⁺	Ir189 13.2 d 32 ⁺	Ir190 11.78 d (4)	Ir191 53.3 d 42 ⁺	Ir192 53.3 d 42 ⁺	Ir193 19.28 h 1 ⁺	Ir194 19.28 h 1 ⁺	Ir195 15 h 32 ⁺	Ir196 51 (8)	Ir197 5.8 m 32 ⁺	Ir198 8 m 32 ⁺	Ir199 112.5 h 8 m	Ir200 112.5 h 8 m	Ir201 112.5 h 8 m	Ir202 112.5 h 8 m	Ir203 112.5 h 8 m
O184 56213 y 0+	O185 93.4 d 0+	O186 2.0215 y 12 ⁺	O187 1.2 0+	O188 0+ 32 ⁺	O189 202 0+	O190 202 0+	O191 154.4 0+	O192 154.4 0+	O193 30.1 h 6.0 y 32 ⁺	O194 6.95 m 34 m 0+	O195 111 h 34 m 0+	O196 111 h 34 m 0+	O197 111 h 34 m 0+	O198 111 h 34 m 0+	O199 111 h 34 m 0+	O200 111 h 34 m 0+	O201 111 h 34 m 0+	O202 111 h 34 m 0+
Re183 0.0 d 52 ⁺	Re184 38.0 d 52 ⁺	Re185 363 52 ⁺	Re186 3.7183 d 1 ⁺	Re187 4.3320 y 52 ⁺	Re188 17.005 h 1 ⁺	Re189 24.2 h 52 ⁺	Re190 24.2 h 52 ⁺	Re191 24.2 h (9)	Re192 24.2 h (9)	Re193 24.2 h (9)	Re194 24.2 h (9)	Re195 24.2 h (9)	Re196 24.2 h (9)	Re197 24.2 h (9)	Re198 24.2 h (9)	Re199 24.2 h (9)	Re200 24.2 h (9)	Re201 24.2 h (9)
W182 0+ 26.3	W183 1.16977 y 15 ⁺	W184 36017 y 0+	W185 7.14 32 ⁺	W186 7.14 32 ⁺	W187 13.72 h 32 ⁺	W188 13.72 h 32 ⁺	W189 11.2 m 30.0 m 0+	W190 11.2 m 30.0 m 0+	W191 11.2 m 30.0 m 0+	W192 11.2 m 30.0 m 0+	W193 11.2 m 30.0 m 0+	W194 11.2 m 30.0 m 0+	W195 11.2 m 30.0 m 0+	W196 11.2 m 30.0 m 0+	W197 11.2 m 30.0 m 0+	W198 11.2 m 30.0 m 0+	W199 11.2 m 30.0 m 0+	W200 11.2 m 30.0 m 0+

isotope	β	γ
¹⁸² W	0.274	11.4 ⁰
¹⁸⁴ W	0.258	13.8 ⁰
¹⁸⁶ W	0.223	15.9 ⁰
¹⁸⁶ Os	0.196	16.5 ⁰
¹⁸⁸ Os	0.185	19.2 ⁰
¹⁹⁰ Os	0.184	22.3 ⁰
¹⁹² Os	0.168	25.2 ⁰
¹⁹² Pt	0.146	-
¹⁹⁴ Pt	0.134	-
¹⁹⁶ Pt	0.135	-
¹⁹⁸ Hg	0.106	36.3 ⁰
²⁰⁰ Hg	0.098	39.1 ⁰
²⁰² Hg	0.082	33.4 ⁰
²⁰⁴ Hg	0.068	31.5 ⁰

n-rich hafnium ground states

critical point
N = 116



Band crossing prediction in ^{180}Hf

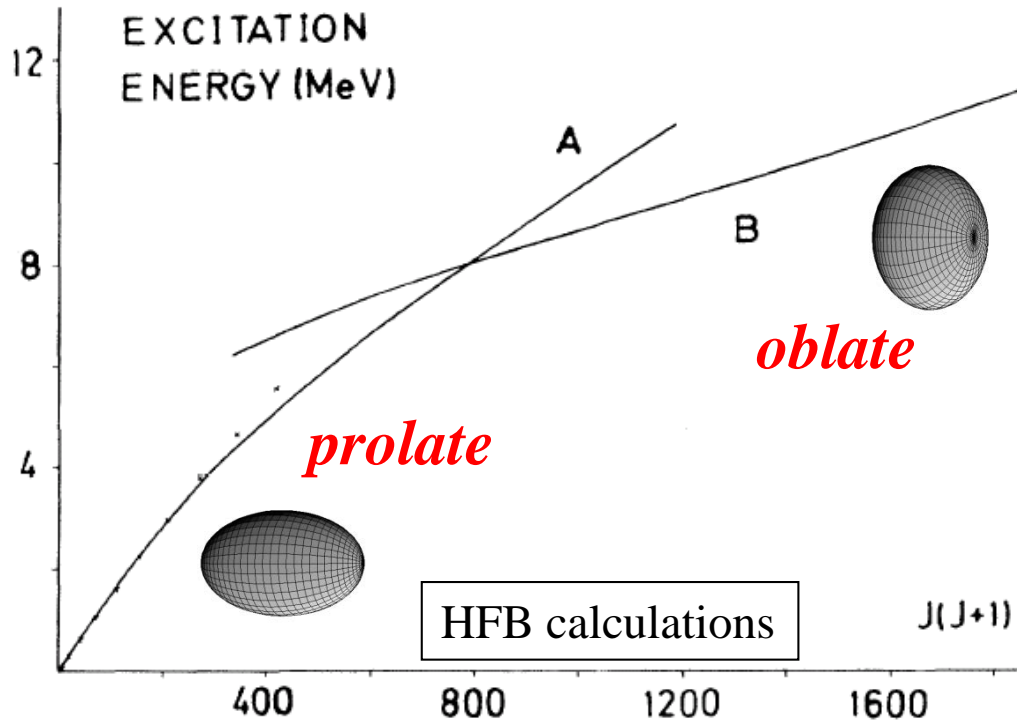
74	W150	W151	W152	W153	W154	W155	W156	W157	W158	W159	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190
73	Ta150	Ta151	Ta152	Ta153	Ta154	Ta155	Ta156	Ta157	Ta158	Ta159	Ta160	Ta161	Ta162	Ta163	Ta164	Ta165	Ta166	Ta167	Ta168	Ta169	Ta170	Ta171	Ta172	Ta173	Ta174	Ta175	Ta176	Ta177	Ta178	Ta179	Ta180	Ta181	Ta182	Ta183	Ta184	Ta185	Ta186	Ta187	Ta188		
72	Hf150	Hf151	Hf152	Hf153	Hf154	Hf155	Hf156	Hf157	Hf158	Hf159	Hf160	Hf161	Hf162	Hf163	Hf164	Hf165	Hf166	Hf167	Hf168	Hf169	Hf170	Hf171	Hf172	Hf173	Hf174	Hf175	Hf176	Hf177	Hf178	Hf179	Hf180	Hf181	Hf182	Hf183	Hf184	Hf185	Hf186				116

R. R. Hilton and H. J. Mang

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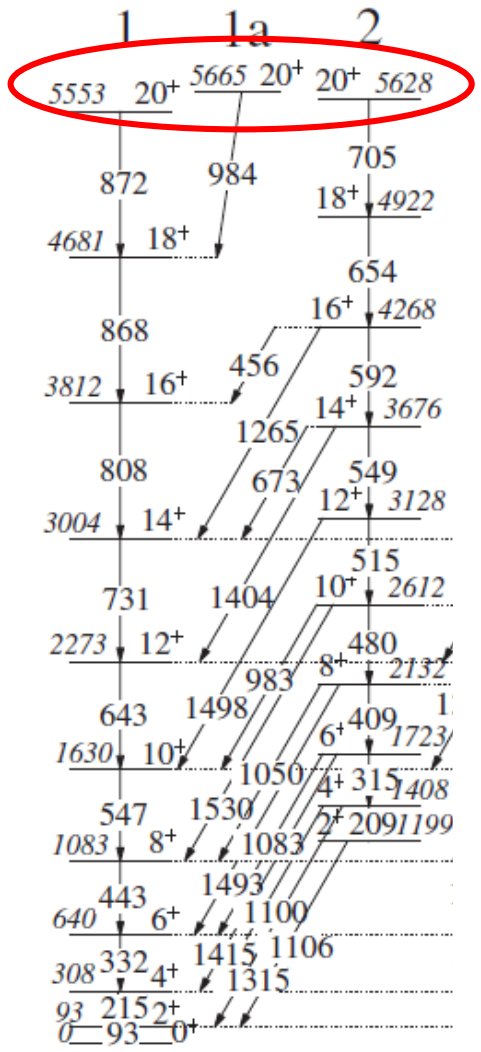
(Received 6 September 1979)

Giant backbending is predicted to occur in ^{180}Hf at $J \approx 26\hbar$. The effect is clearly seen to be the result of the crossing of two bands with very different intrinsic structure.



^{180}Hf oblate band?

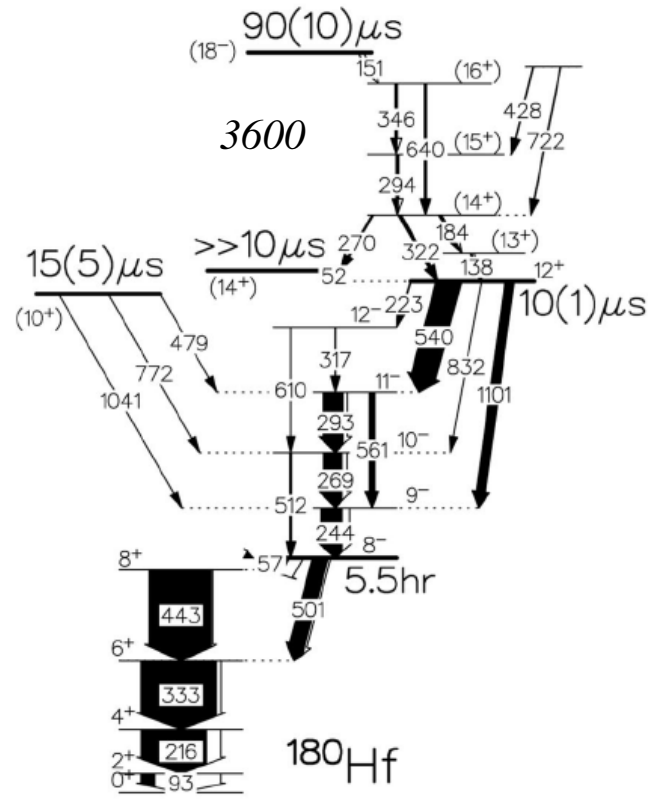
74	W150	W151	W160	W161	W162	W163	W164	W165	W166	W167	W168	W169	W170	W171	W172	W173	W174	W175	W176	W177	W178	W179	W180	W181	W182	W183	W184	W185	W186	W187	W188	W189	W190
73	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	Ia	
72	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	Hf	



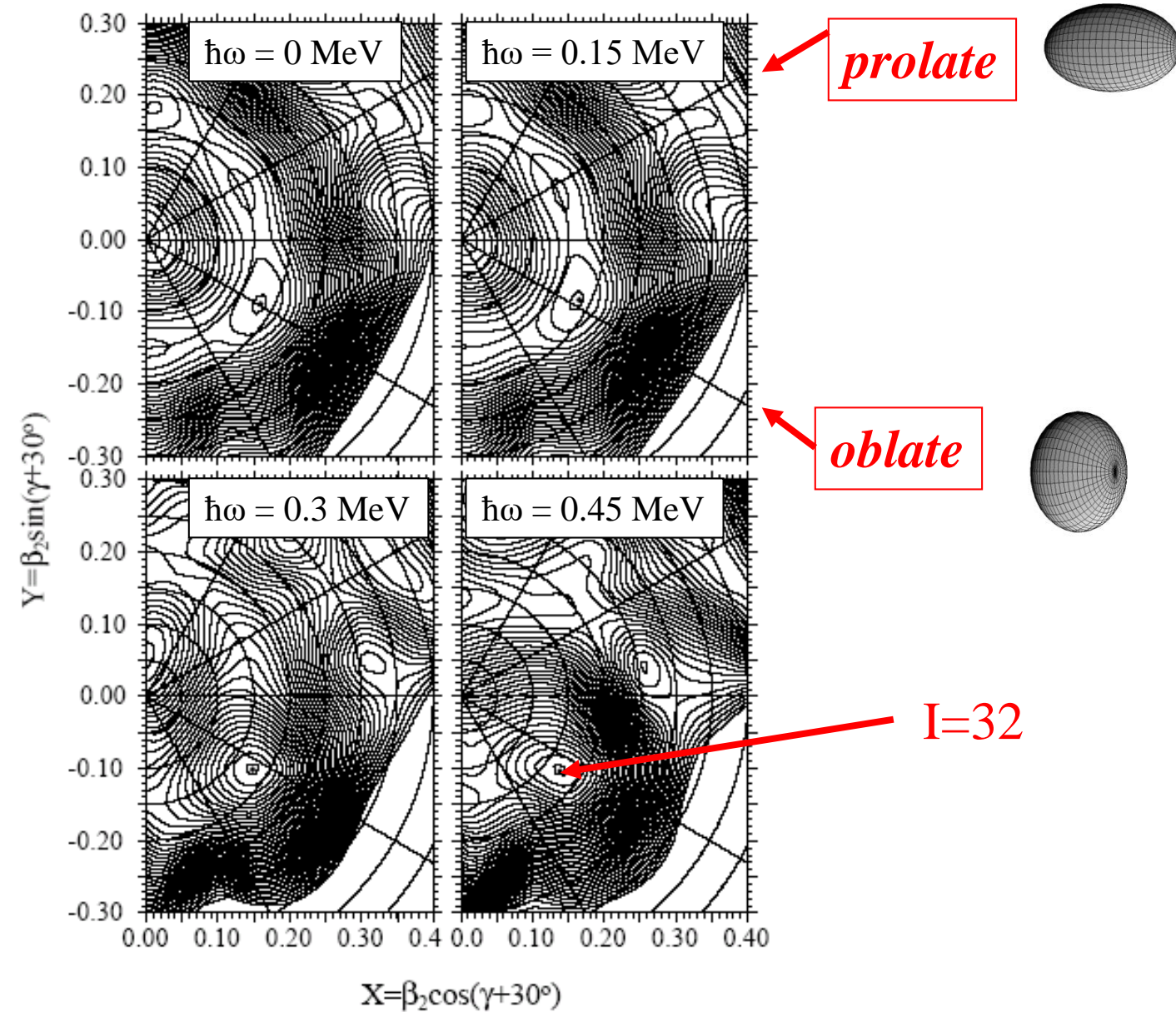
three 20⁺ states

Tandel et al.,
Phys. Rev. Lett. 101 (2008) 182503
with Gammasphere

pre-Gammasphere
high-K yrast isomers:
d'Alarcao et al., Phys. Rev. C59 (1999) 1227(R)

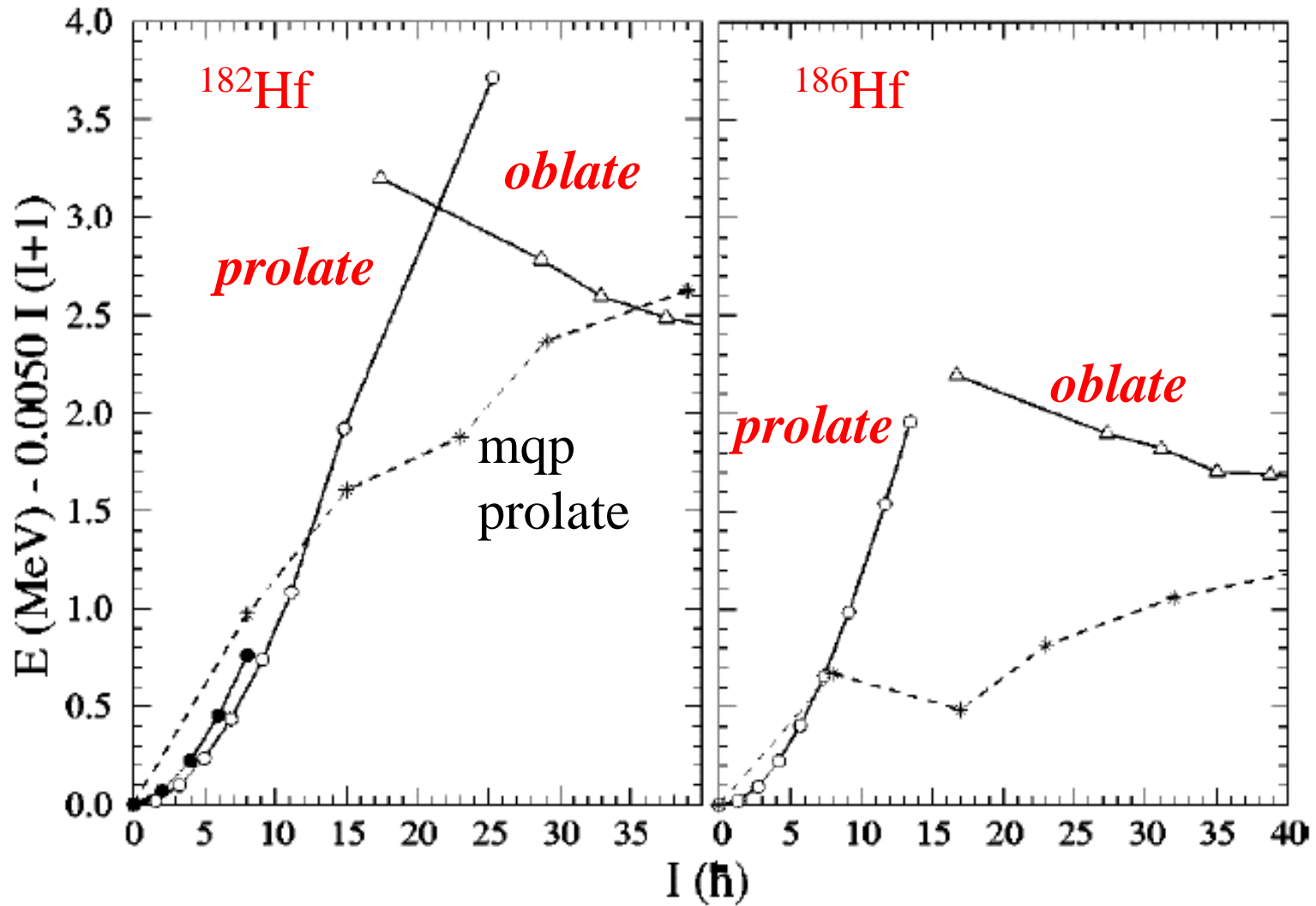


^{190}Hf TRS oblate rotor beyond the critical point



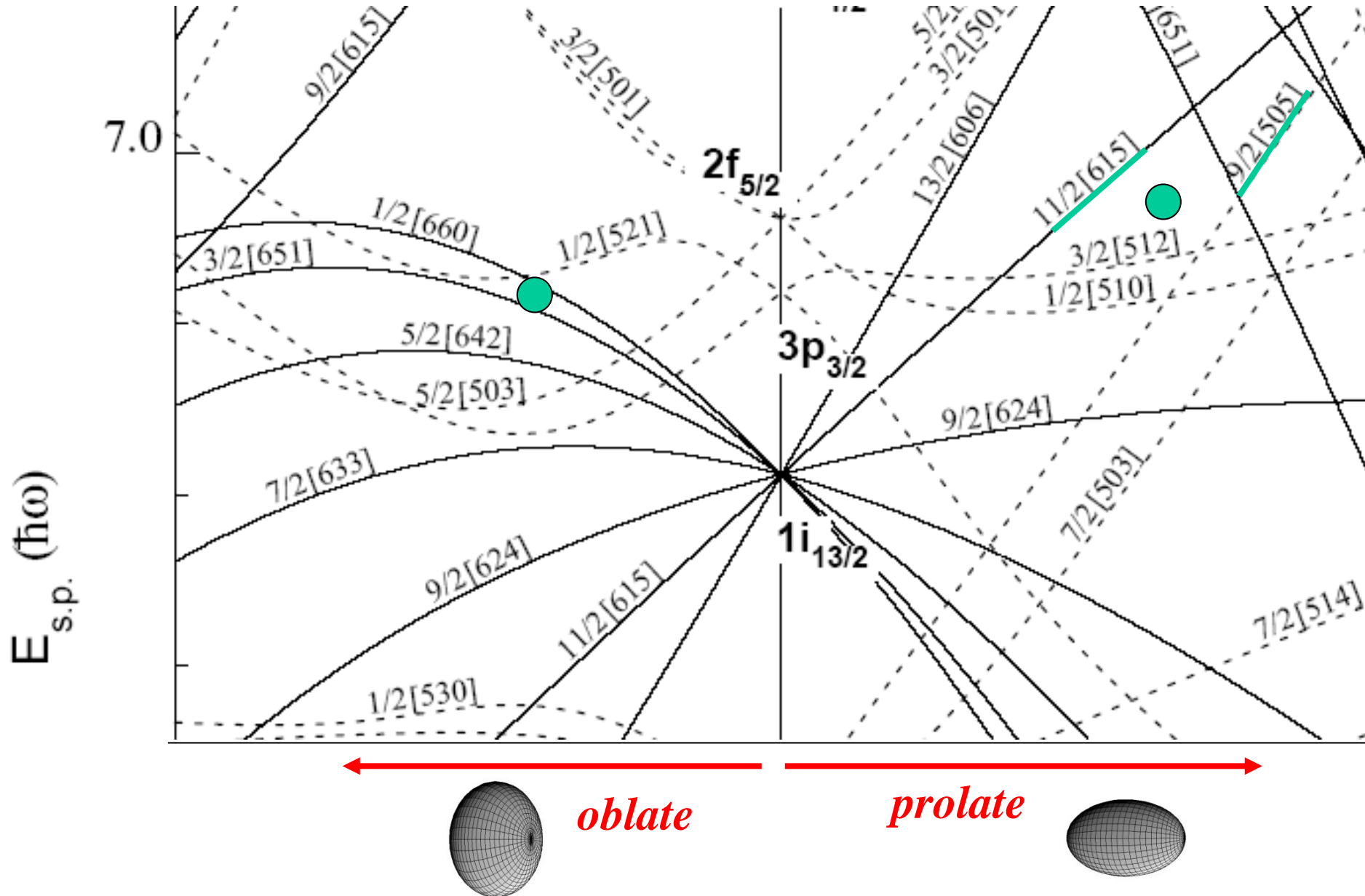
Xu et al., unpublished

Hf prolate vs oblate



Nilsson single-particle diagram

● N = 116 (^{188}Hf , ^{190}W , ^{192}Os)



Appendix: Odd-even nuclei

Erl50 18.5 s 0+	Erl51 23.5 s (7/2-)	Erl52 10.3 s 0+	Erl53 37.1 s (7/2-)	Erl54 3.73 m 0+	Erl55 5.3 m 7/2-	Erl56 19.5 m 0+	Erl57 18.65 m 3/2-	Erl58 2.29 h 0+	Erl59 36 m 3/2-	Erl60 28.58 h 0+	Erl61 3.21 h 3/2-	Erl62 0+	Erl63 75.0 m 5/2-	Erl64 0+	Erl65 10.36 h 5/2-	Erl66 0+	Erl67 7/2+ ±	Erl68 0+	Erl69 9.40 d 1/2-	Erl70 0+	Erl71 7.516 h 5/2-	Erl72 49.3 h 0+	Erl73 1.4 m (7/2-)	Erl74 3.3 m 0+	Erl75 1.2 m (9/2+)	
EC	EC	EC,α	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	0.14	EC	1.61	EC	33.6	22.95	26.8	β	14.9	β	β	β	β	β	
Hol49 21.1 s (11/2-)	Hol50 72 s 2-	Hol51 35.2 s (11/2-)	Hol52 161.8 s 2-	Hol53 2.01 m 11/2-	Hol54 11.76 m (2-)	Hol55 48 m 5/2+	Hol56 56 m (4+)	Hol57 12.6 m 7/2-	Hol58 11.3 m 5+	Hol59 33.05 m 7/2-	Hol60 25.6 m 5+	Hol61 2.48 h 7/2-	Hol62 15.0 m 1+	Hol63 4570 y 7/2-	Hol64 29 m 1+	Hol65 7/2-	Hol66 26.83 h 0-	Hol67 3.1 h 7/2-	Hol68 2.99 m 3+	Hol69 4.7 m 7/2-	Hol70 2.76 m (6+)	Hol71 53 s (7/2-)	Hol72 25 s	Hol73	Hol74	
EC	EC	EC,α	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC,β	100	β	β	β	β	β	β	β	β	β	β
Dy148 3.1 m 0+	Dy149 4.20 m (7/2-)	Dy150 7.17 m 0+	Dy151 17.9 m 7/2(-)	Dy152 2.35 h 0+	Dy153 6.4 h 7/2(-)	Dy154 3.02+6 y 0+	Dy155 9.9 h 3/2-	Dy156 0+	Dy157 8.14 h 3/2-	Dy158 0+	Dy159 144.4 d 3/2-	Dy160 0+	Dy161 5/2+	Dy162 0+	Dy163 5/2-	Dy164 0+	Dy165 2.334 h 7/2+	Dy166 81.6 h 0+	Dy167 6.20 m (1/2-)	Dy168 8.7 m 0+	Dy169 39 s (5/2-)	Dy170 0+	Dy171	Dy172 0+	Dy173	
EC	EC	EC,α	EC,α	EC,α	α	α	EC	0.06	EC	0.10	EC	2.34	18.9	25.5	24.9	28.2	β	β	β	β	β	β	β	β	β	

$$\langle I - 2, K \| M(E2) \| I, K \rangle = \sqrt{\frac{15}{32\pi}} \cdot \sqrt{\frac{(I + K - 1) \cdot (I + K) \cdot (I - K - 1) \cdot (I - K)}{(I - 1) \cdot (2I - 1) \cdot I}} \cdot Q_2 e$$

	$\langle I_f \ M(E2) \ I_i \rangle$	Q_2 (b)
^{162}Dy	2.32±0.02 eb	7.36±0.03
^{163}Dy	2.31±0.02 eb	7.29±0.12
^{164}Dy	2.38±0.01 eb	7.54±0.04
^{166}Er	2.42±0.01 eb	7.67±0.03
^{167}Er	2.24±0.01 eb	7.60±0.10
^{168}Er	2.40±0.02 eb	7.61±0.06

Appendix: Odd-even nuclei

Erl50 18.5 s 0+	Erl51 23.5 s (7/2-)	Erl52 10.3 s 0+	Erl53 37.1 s (7/2-)	Erl54 3.73 m 0+	Erl55 5.3 m 7/2-	Erl56 19.5 m 0+	Erl57 18.65 m 3/2-	Erl58 2.29 h 0+	Erl59 36 m 3/2-	Erl60 28.58 h 0+	Erl61 3.21 h 3/2-	Erl62 0+	Erl63 75.0 m 5/2-	Erl64 0+	Erl65 10.36 h 5/2-	Erl66 0+	Erl67 7/2+ ±	Erl68 0+	Erl69 9.40 d 1/2-	Erl70 0+	Erl71 7.516 h 5/2-	Erl72 49.3 h 0+	Erl73 1.4 m (7/2-)	Erl74 3.3 m 0+	Erl75 1.2 m (9/2+)		
EC	EC	EC,α	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	0.14	EC	1.61	EC	33.6	22.95	26.8	β	14.9	β	β	β	β	β	β	
Hol49 21.1 s (11/2-)	Hol50 72 s 2-	Hol51 35.2 s (11/2-)	Hol52 161.8 s 2-	Hol53 2.01 m 11/2-	Hol54 11.76 m (2-)	Hol55 48 m 5/2+	Hol56 56 m (4+)	Hol57 12.6 m 7/2-	Hol58 11.3 m 5+	Hol59 33.05 m 7/2-	Hol60 25.6 m 5+	Hol61 2.48 h 7/2-	Hol62 15.0 m 1+	Hol63 4570 y 7/2-	Hol64 29 m 1+	Hol65 7/2-	Hol66 26.83 h 0-	Hol67 3.1 h 7/2-	Hol68 2.99 m 3+	Hol69 4.7 m 7/2-	Hol70 2.76 m (6+)	Hol71 53 s (7/2-)	Hol72 25 s	Hol73	Hol74	Hol75	
EC	EC	EC,α	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC,β	100	β	β	β	β	β	β	β	β	β	β	β
Dy148 3.1 m 0+	Dy149 4.20 m (7/2-)	Dy150 7.17 m 0+	Dy151 17.9 m 7/2(-)	Dy152 2.38 h 0+	Dy153 6.4 h 7/2(-)	Dy154 3.02+6 y 0+	Dy155 9.9 h 3/2-	Dy156 0+	Dy157 8.14 h 3/2-	Dy158 0+	Dy159 144.4 d 3/2-	Dy160 0+	Dy161 52+	Dy162 0+	Dy163 5/2-	Dy164 0+	Dy165 2.334 h 7/2+	Dy166 81.6 h 0+	Dy167 6.20 m (1/2-)	Dy168 8.7 m 0+	Dy169 39 s (5/2-)	Dy170 0+	Dy171	Dy172 0+	Dy173		
EC	EC	EC,α	EC,α	EC,α	α	α	EC	0.06	EC	0.10	EC	2.34	18.9	25.5	24.9	28.2	β	β	β	β	β	β	β	β	β	β	β

$$\langle I - 2, K \| M(E2) \| I, K \rangle = \sqrt{\frac{15}{32\pi}} \cdot \sqrt{\frac{(I + K - 1) \cdot (I + K) \cdot (I - K - 1) \cdot (I - K)}{(I - 1) \cdot (2I - 1) \cdot I}} \cdot Q_2 e$$

$$\langle I - 1, K \| M(E2) \| I, K \rangle = -\sqrt{\frac{5}{16\pi}} \cdot \sqrt{\frac{3 \cdot (I + K) \cdot (I - K) \cdot K^2}{(I - 1) \cdot I \cdot (I + 1)}} \cdot Q_2 e$$

$$^{163}\text{Dy}: \frac{B(E2; 5/2 \rightarrow 7/2)}{B(E2; 5/2 \rightarrow 9/2)} = 2.76 \pm 0.14 (2.86_{theo})$$

$$^{167}\text{Er}: \frac{B(E2; 7/2 \rightarrow 9/2)}{B(E2; 7/2 \rightarrow 11/2)} = 3.81 \pm 0.15 (3.89_{theo})$$

Appendix: Odd-even nuclei

Erl50 18.5 s 0+	Erl51 23.5 s (7/2)-	Erl52 10.3 s 0+	Erl53 37.1 s (7/2)-	Erl54 3.73 m 0+	Erl55 5.3 m 7/2-	Erl56 19.5 m 0+	Erl57 18.65 m 3/2-	Erl58 2.29 h 0+	Erl59 36 m 3/2-	Erl60 28.58 h 0+	Erl61 3.21 h 3/2-	Erl62 0+	Erl63 75.0 m 5/2-	Erl64 0+	Erl65 10.36 h 5/2-	Erl66 0+	Erl67 7/2+ ±	Erl68 0+	Erl69 9.40 d 1/2-	Erl70 0+	Erl71 7.516 h 5/2-	Erl72 49.3 h 0+	Erl73 1.4 m (7/2)-	Erl74 3.3 m 0+	Erl75 1.2 m (9/2+)		
EC	EC	EC,α	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	0.14	EC	1.61	EC	33.6	22.95	26.8	β	14.9	β	β	β	β	β		
Hol49 21.1 s (11/2)-	Hol50 72 s 2-	Hol51 35.2 s (11/2)-	Hol52 161.8 s 2-	Hol53 2.01 m 11/2-	Hol54 11.76 m (2)-	Hol55 48 m 5/2+	Hol56 56 m (4+)	Hol57 12.6 m 7/2-	Hol58 11.3 m 5+	Hol59 33.05 m 7/2-	Hol60 25.6 m 5+	Hol61 2.48 h 7/2-	Hol62 15.0 m 1+	Hol63 4570 y 7/2-	Hol64 29 m 1+	Hol65 7/2-	Hol66 26.83 h 0-	Hol67 3.1 h 7/2-	Hol68 2.99 m 3+	Hol69 4.7 m 7/2-	Hol70 2.76 m (6+)	Hol71 53 s (7/2)-	Hol72 25 s	Hol73	Hol74	Hol75	
EC	EC	EC,α	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC,β	100	β	β	β	β	β	β	β	β	β	β	β
Dy148 3.1 m 0+	Dy149 4.20 m (7/2)-	Dy150 7.17 m 0+	Dy151 17.9 m 7/2(-)	Dy152 2.38 h 0+	Dy153 6.4 h 7/2(-)	Dy154 3.02+6 y 0+	Dy155 9.9 h 3/2-	Dy156 0+	Dy157 8.14 h 3/2-	Dy158 0+	Dy159 144.4 d 3/2-	Dy160 0+	Dy161 5/2+	Dy162 0+	Dy163 5/2-	Dy164 0+	Dy165 2.334 h 7/2+	Dy166 81.6 h 0+	Dy167 6.20 m (1/2)-	Dy168 8.7 m 0+	Dy169 39 s (5/2)-	Dy170 0+	Dy171	Dy172 0+	Dy173		
EC	EC	EC,α	EC,α	EC,α	α	α	EC	0.06	EC	0.10	EC	2.34	18.9	25.5	24.9	28.2	β	β	β	β	β	β	β	β	β	β	β

$$11/2^- \text{ ————— } 0.282 \quad \tau = 0.38 \pm 0.07 \text{ ns}$$

$$\tau = \left\{ \sum_K \sum_{\ell} [\varepsilon_{N \rightarrow K}^2(\ell) + \delta_{N \rightarrow K}^2(\ell)] \right\}^{-1}$$

$$9/2^- \text{ ————— } 0.167 \quad \tau = 0.49 \pm 0.09 \text{ ns}$$

$$7/2^- \text{ ————— } 0.073 \quad \tau = 2.18 \pm 0.07 \text{ ns}$$

$$5/2^- \text{ ————— } 0.0$$

¹⁶³Dy

$$\delta_{N \rightarrow M}(\ell) = \left\{ \frac{8\pi(\ell + 1)}{\ell[(2\ell + 1)!!]^2} \frac{1}{\hbar} \left(\frac{\hbar\omega}{\hbar c} \right)^{2\ell+1} \right\}^{1/2} \cdot (2I_N + 1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(\ell) \| I_N \rangle$$

$$\delta_{N \rightarrow M}(E2) = \{1.225 \cdot 10^{13} \cdot E_{\gamma}^5(\text{MeV})^5\}^{1/2} \cdot (2I_N + 1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(E2) \| I_N \rangle$$

$$\delta_{N \rightarrow M}(M1) = \{1.758 \cdot 10^{13} \cdot E_{\gamma}^3(\text{MeV})^3\}^{1/2} \cdot (2I_N + 1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(M1) \| I_N \rangle$$

$$\varepsilon_{N \rightarrow M}^2(\ell) = \delta_{N \rightarrow M}^2(\ell) \cdot \alpha_{N \rightarrow M}(\ell) \quad \text{conversion coefficient.: bricc.anu.edu.au}$$

Appendix: Odd-even nuclei

Erl50 18.5 s 0+	Erl51 23.5 s (7/2-)	Erl52 10.3 s 0+	Erl53 37.1 s (7/2-)	Erl54 3.73 m 0+	Erl55 5.3 m 7/2-	Erl56 19.5 m 0+	Erl57 18.65 m 3/2-	Erl58 2.29 h 0+	Erl59 36 m 3/2-	Erl60 28.58 h 0+	Erl61 3.21 h 3/2-	Erl62 0+	Erl63 75.0 m 5/2-	Erl64 0+	Erl65 10.36 h 5/2-	Erl66 0+	Erl67 7/2+ ±	Erl68 0+	Erl69 9.40 d 1/2-	Erl70 0+	Erl71 7.516 h 5/2-	Erl72 49.3 h 0+	Erl73 1.4 m (7/2-)	Erl74 3.3 m 0+	Erl75 1.2 m (9/2+)	
EC	EC	EC,α	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	0.14	EC	1.61	EC	33.6	22.95	26.8	β	14.9	β	β	β	β	β	
Hol49 21.1 s (11/2-)	Hol50 72 s 2-	Hol51 35.2 s (11/2-)	Hol52 161.8 s 2-	Hol53 2.01 m 11/2-	Hol54 11.76 m (2-)	Hol55 48 m 5/2+	Hol56 56 m (4+)	Hol57 12.6 m 7/2-	Hol58 11.3 m 5+	Hol59 33.05 m 7/2-	Hol60 25.6 m 5+	Hol61 2.48 h 7/2-	Hol62 15.0 m 1+	Hol63 4570 y 7/2-	Hol64 29 m 1+	Hol65 7/2-	Hol66 26.83 h 0-	Hol67 3.1 h 7/2-	Hol68 2.99 m 3+	Hol69 4.7 m 7/2-	Hol70 2.76 m (6+)	Hol71 53 s (7/2-)	Hol72 25 s	Hol73	Hol74	
EC	EC	EC,α	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC,β	100	β	β	β	β	β	β	β	β	β	β
Dy148 3.1 m 0+	Dy149 4.20 m (7/2-)	Dy150 7.17 m 0+	Dy151 17.9 m 7/2(-)	Dy152 2.38 h 0+	Dy153 6.4 h 7/2(-)	Dy154 3.02+6 y 0+	Dy155 9.9 h 3/2-	Dy156 0+	Dy157 8.14 h 3/2-	Dy158 0+	Dy159 144.4 d 3/2-	Dy160 0+	Dy161 5/2+	Dy162 0+	Dy163 5/2-	Dy164 0+	Dy165 2.334 h 7/2+	Dy166 81.6 h 0+	Dy167 6.20 m (1/2-)	Dy168 8.7 m 0+	Dy169 39 s (5/2-)	Dy170 0+	Dy171	Dy172 0+	Dy173	
EC	EC	EC,α	EC,α	EC,α	α	α	EC	0.06	EC	0.10	EC	2.34	18.9	25.5	24.9	28.2	β	β	β	β	β	β	β	β	β	β

$$11/2^- \text{ ————— } 0.282$$

$$9/2^- \text{ ————— } 0.167 \quad \tau = 0.49 \pm 0.09 \text{ ns}$$

$$7/2^- \text{ ————— } 0.073 \quad \tau = 2.18 \pm 0.07 \text{ ns}$$

$$5/2^- \text{ ————— } 0.0$$

¹⁶³Dy

$$\tau = \left\{ \sum_K \sum_{\ell} [\varepsilon_{N \rightarrow K}^2(\ell) + \delta_{N \rightarrow K}^2(\ell)] \right\}^{-1}$$

Appendix: Odd-even nuclei

Erl50 18.5 s 0+	Erl51 23.5 s (7/2-)	Erl52 10.3 s 0+	Erl53 37.1 s (7/2-)	Erl54 3.73 m 0+	Erl55 5.3 m 7/2-	Erl56 19.5 m 0+	Erl57 18.65 m 3/2-	Erl58 2.29 h 0+	Erl59 36 m 3/2-	Erl60 28.58 h 0+	Erl61 3.21 h 3/2-	Erl62 0+	Erl63 75.0 m 5/2-	Erl64 0+	Erl65 10.36 h 5/2-	Erl66 0+	Erl67 7/2+ +	Erl68 0+	Erl69 9.40 d 1/2-	Erl70 0+	Erl71 7.516 h 5/2-	Erl72 49.3 h 0+	Erl73 1.4 m (7/2-)	Erl74 3.3 m 0+	Erl75 1.2 m (9/2+)	
EC	EC	EC,α	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	0.14	EC	1.61	EC	33.6	22.95	26.8	β	14.9	β	β	β	β	β	
Hol49 21.1 s (11/2-)	Hol50 72 s 2-	Hol51 35.2 s (11/2-)	Hol52 161.8 s 2-	Hol53 2.01 m 11/2-	Hol54 11.76 m (2-)	Hol55 48 m 5/2+	Hol56 56 m (4+)	Hol57 12.6 m 7/2-	Hol58 11.3 m 5+	Hol59 33.05 m 7/2-	Hol60 25.6 m 5+	Hol61 2.48 h 7/2-	Hol62 15.0 m 1+	Hol63 4570 y 7/2-	Hol64 29 m 1+	Hol65 7/2-	Hol66 26.83 h 0-	Hol67 3.1 h 7/2-	Hol68 2.99 m 3+	Hol69 4.7 m 7/2-	Hol70 2.76 m (6+)	Hol71 53 s (7/2-)	Hol72 25 s	Hol73	Hol74	
EC	EC	EC,α	EC,α	EC,α	EC,α	EC	EC	EC	EC	EC	EC	EC	EC	EC	EC,β	100	β	β	β	β	β	β	β	β	β	β
Dy148 3.1 m 0+	Dy149 4.20 m (7/2-)	Dy150 7.17 m 0+	Dy151 17.9 m 7/2(-)	Dy152 2.38 h 0+	Dy153 6.4 h 7/2(-)	Dy154 3.02+6 y 0+	Dy155 9.9 h 3/2-	Dy156 0+	Dy157 8.14 h 3/2-	Dy158 0+	Dy159 144.4 d 3/2-	Dy160 0+	Dy161 5/2+	Dy162 0+	Dy163 5/2-	Dy164 0+	Dy165 2.334 h 7/2+	Dy166 81.6 h 0+	Dy167 6.20 m (1/2-)	Dy168 8.7 m 0+	Dy169 39 s (5/2-)	Dy170 0+	Dy171	Dy172 0+	Dy173	
EC	EC	EC,α	EC,α	EC,α	α	α	EC	0.06	EC	0.10	EC	2.34	18.9	25.5	24.9	28.2	β	β	β	β	β	β	β	β	β	

$$13/2^+ \text{---} 0.295 \quad \tau = 0.38 \pm 0.07 \text{ ns}$$

$$\tau = \left\{ \sum_K \sum_{\ell} [\varepsilon_{N \rightarrow K}^2(\ell) + \delta_{N \rightarrow K}^2(\ell)] \right\}^{-1}$$

$$11/2^+ \text{---} 0.178 \quad \tau = 0.49 \pm 0.09 \text{ ns}$$

$$9/2^+ \text{---} 0.079 \quad \tau = 2.18 \pm 0.07 \text{ ns}$$

$$7/2^+ \text{---} 0.0$$

¹⁶⁷Er

Spin I	E _γ (MeV)	(2I+1) ^{-1/2}	<I-1//M()/I>	delta	α _T	ε ²	τ (ns)
7/2	0.0734	0.3536	-3.886 (E2)	-7019.1	8.9	4.38·10 ⁸	2.05
			0.108 (M1)	3183.7	5.7	5.78·10 ⁷	1.80
9/2	0.0939	0.3162	-4.002 (E2)	-11968	3.4	4.87·10 ⁸	1.59
			0.153 (M1)	5837.1	2.9	9.88·10 ⁷	1.31
	0.167	0.3162	2.299 (E2)	29133	0.4	3.65·10 ⁸	0.51

Appendix: Spherical harmonics



$$Y_{00}(\theta, \phi) = \frac{1}{\sqrt{4\pi}}$$

$$Y_{10}(\theta, \phi) = \frac{1}{2} \cdot \sqrt{\frac{3}{\pi}} \cdot \cos \theta$$

$$Y_{1\pm 1}(\theta, \phi) = m \frac{1}{2} \cdot \sqrt{\frac{3}{2\pi}} \cdot \sin \theta \cdot e^{\pm i\phi}$$

$$Y_{20}(\theta, \phi) = \sqrt{\frac{5}{16\pi}} \cdot (3 \cdot \cos^2 \theta - 1)$$

$$Y_{2\pm 1}(\theta, \phi) = m \sqrt{\frac{15}{8\pi}} \cdot \sin \theta \cdot \cos \theta \cdot e^{\pm i\phi}$$

$$Y_{2\pm 2}(\theta, \phi) = \sqrt{\frac{15}{32\pi}} \cdot \sin^2 \theta \cdot e^{\pm 2i\phi}$$

$$Y_{30}(\theta, \phi) = \sqrt{\frac{7}{16\pi}} \cdot (2 \cos^3 \theta - 3 \cos \theta \sin^2 \theta)$$

$$Y_{3\pm 1}(\theta, \phi) = m \sqrt{\frac{21}{64\pi}} \cdot (4 \cos^2 \theta \sin \theta - \sin^3 \theta) \cdot e^{\pm i\phi}$$

$$Y_{3\pm 2}(\theta, \phi) = \sqrt{\frac{105}{32\pi}} \cdot \cos \theta \sin^2 \theta \cdot e^{(\pm 2)i\phi}$$

$$Y_{3\pm 3}(\theta, \phi) = m \sqrt{\frac{35}{64\pi}} \cdot \sin^3 \theta \cdot e^{(\pm 3)i\phi}$$