Outline: Nuclear rotation of odd-even nuclei

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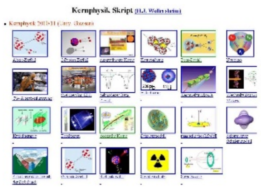
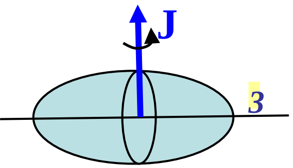
web-page: https://web-docs.gsi.de/~wolle/ and click on

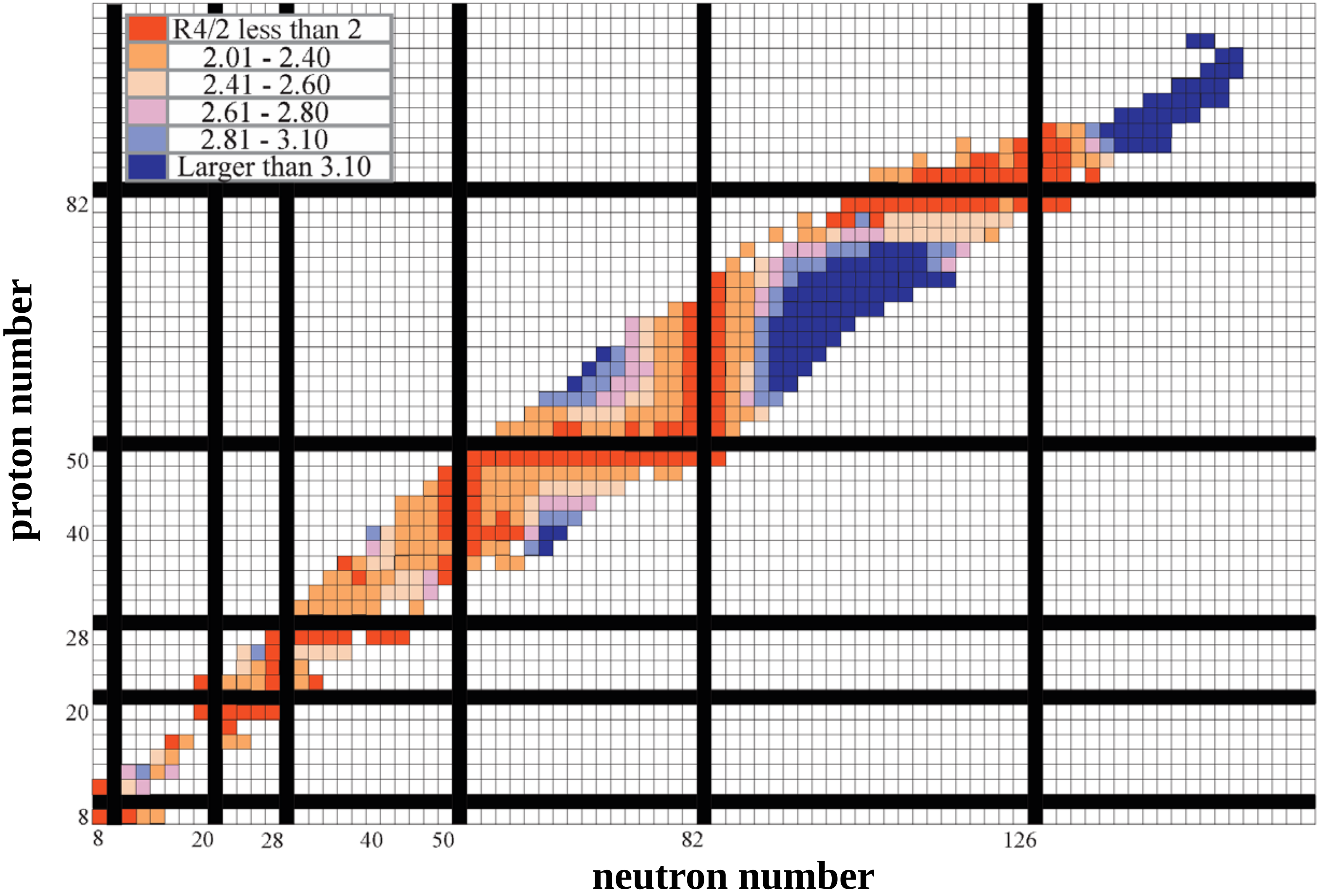


1. particle-rotor model
2. Euler angles
3. Example: ^{181}Ta
4. reduced transition probabilities



Broad perspective on structural evolution:

$$R_{4/2} = \frac{E(4_1^+)}{E(2_1^+)}$$

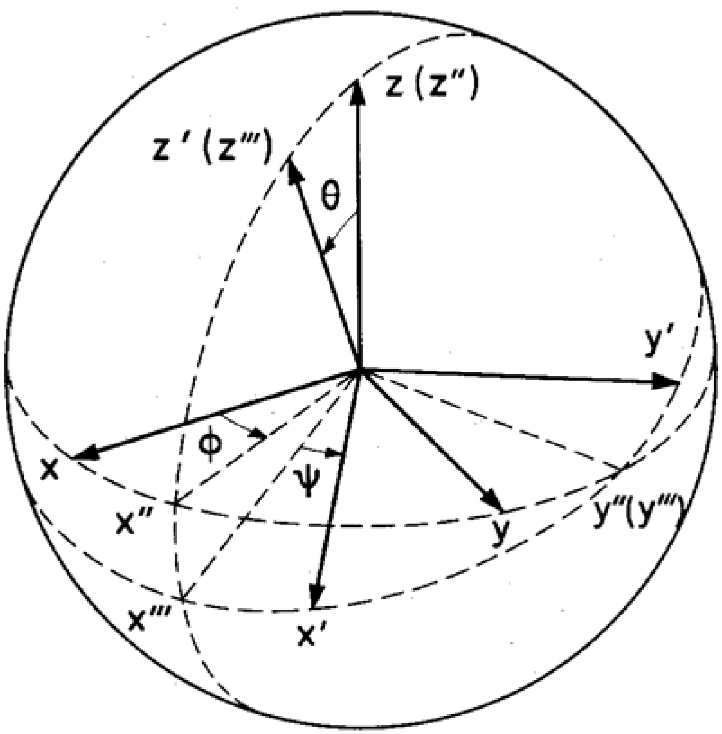


for even-even nuclei

The Euler angles

- It is important to recognize that for nuclei the intrinsic reference frame can have any orientation with respect to the lab reference frame as we can hardly control orientation of nuclei (although it is possible in some cases).

- One way to specify the mutual orientation of two reference frames of the common origin is to use Euler angles.



(x, y, z) axes of lab frame
(1,2,3) axes of intrinsic frame

- The rotation from (x,y,z) to (x´,y´,z´) can be decomposed into three parts: a rotation by ϕ about the z axis to (x'', y'', z''), a rotation of θ about the new y axis (y'') to (x''', y''', z'''), and finally a rotation of ψ about the new z axis (z''') .

Quantization

states: $|I, M, K>$

laboratory axes: $[J_x, J_y] = i \cdot \hbar \cdot J_z$ and cyclic permutations

$$[J^2, J_k] = 0 \qquad k = x, y, z$$

quantum numbers: $J_z \longrightarrow \hbar \cdot M \quad J^2 \longrightarrow \hbar^2 \cdot I(I+1)$

body fixed axes: $[J_1, J_2] = i \cdot \hbar \cdot J_3$ and cyclic permutations

$$[J^2, J_i] = 0 \qquad i = 1,2,3 \qquad [J_z, J_3] = 0$$

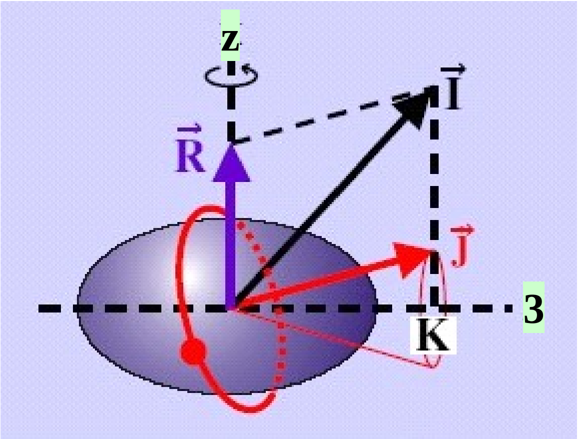
quantum numbers: $J_3 \longrightarrow \hbar \cdot K \quad J^2 \longrightarrow \hbar^2 \cdot I(I+1)$

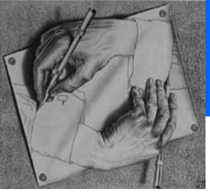
eigenstates: $|I, M, K>$

probability amplitude

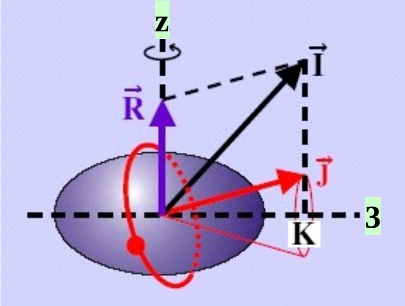
for orientation of rotor: $$<\psi,\theta,\phi|I,M,K> = \left(\frac{2I+1}{8\pi^2}\right)^{1/2} D^I_{MK}(\psi,\theta,\phi)$$

Wigner D – function $$D^I_{MK}(\psi,\theta,\phi) = e^{iM\psi} d^I_{MK}(\theta) e^{iK\phi}$$





Rotational motion of a deformed nucleus



$$H_{rot} = \sum_{i=1}^{3} \frac{\hat{R}_i^2}{2\cdot\Im_i} = \frac{\left(\hat{R}^2 - \hat{R}_3^2\right)}{2\cdot\Im_1} + \frac{\hat{R}_3^2}{2\cdot\Im_3}$$

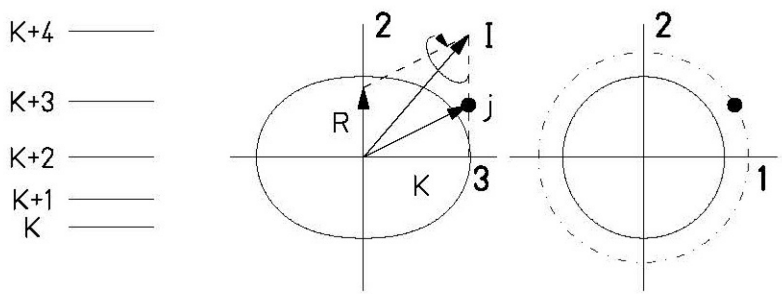
$\Im_1 = \Im_2$

The nucleus does not have an orientation degree of freedom with respect to the symmetry axis

States with projections K and –K are degenerated

$$\Psi_{IMK} = \left(\frac{2\cdot I+1}{16\cdot\pi^2}\right)^{1/2} \cdot \left[D^I_{M\,K}\cdot\chi_K + (-1)^{I-K} D^I_{M\,-K}\cdot\chi_{-K}\right]$$

Particle-rotor model



$$H_{rot} = \sum_{i=1}^{3} \frac{\hat{R}_i^2}{2\cdot\Im_i} = \frac{\left(\hat{R}^2 - \hat{R}_3^2\right)}{2\cdot\Im_1} + \cancel{\frac{\hat{R}_3^2}{2\cdot\Im_3}}$$

$$\Im_1 = \Im_2$$

The nucleus does not have an orientation degree of freedom with respect to the symmetry axis

with $\vec{R} = \vec{I} - \vec{j}$

$$H_{rot} = \frac{\hbar^2}{2\Im_0}\{(I_1 - j_1)^2 + (I_2 - j_2)^2\}$$

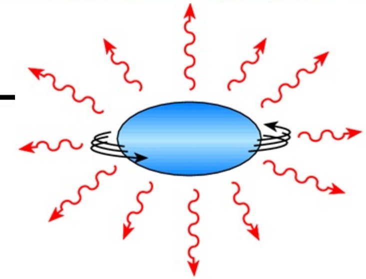
$$H_{rot} = \frac{\hbar^2}{2\Im_0}(I^2 - I_3^2) + \frac{\hbar^2}{2\Im_0}(j_1^2 + j_2^2 - j_3^2) - \frac{\hbar^2}{2\Im_0}(j_+ I_- + j_- I_+)$$

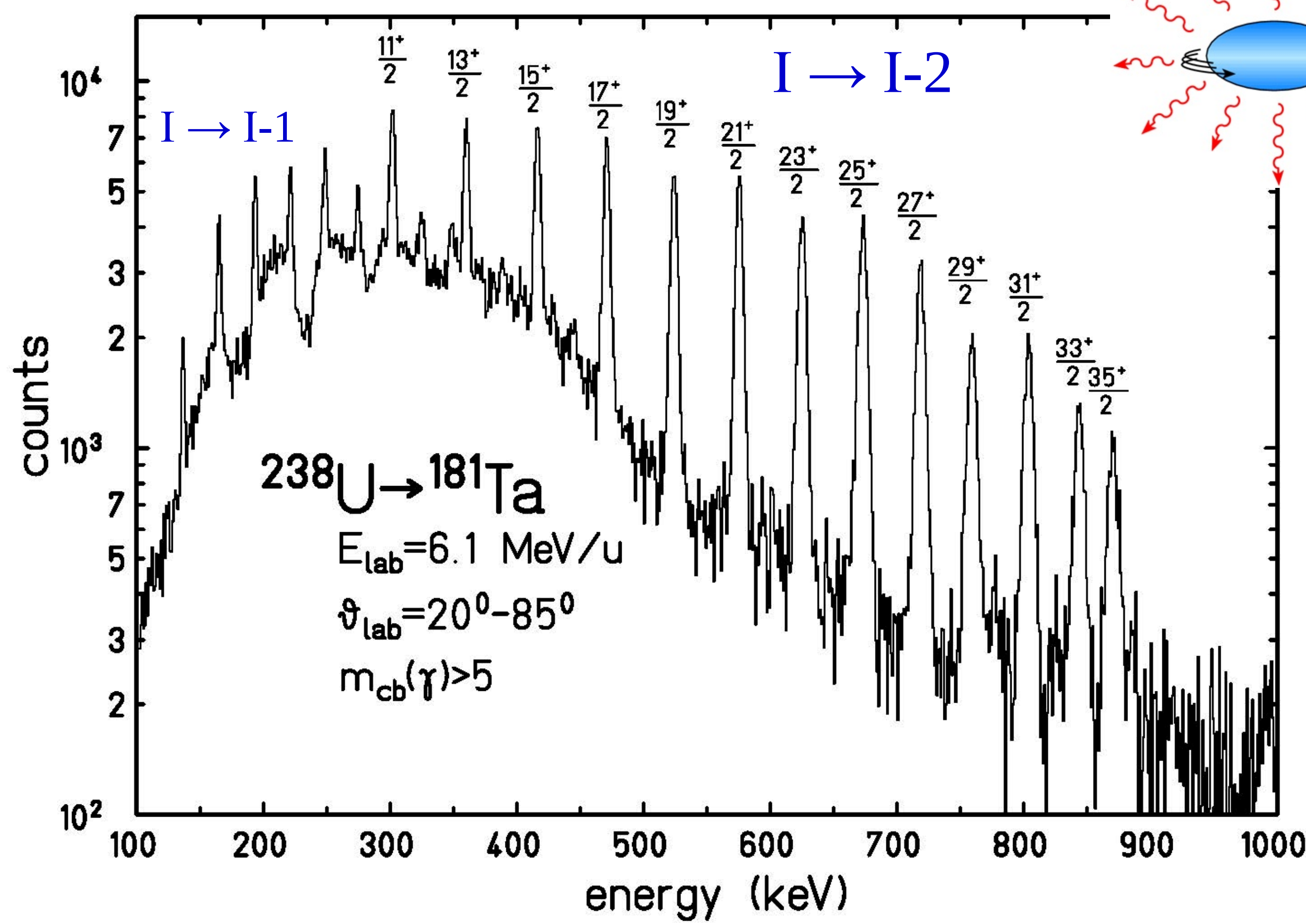
$$I_\pm = I_1 \pm i\cdot I_2 \qquad j_\pm = j_1 \pm i\cdot j_2$$

$$E_K(I) = \epsilon_K + \frac{\hbar^2}{2\Im}\left[I(I+1) - K^2 + \delta_{K,1/2}\cdot a\cdot(-1)^{I+1/2}(I+1/2)\right]$$

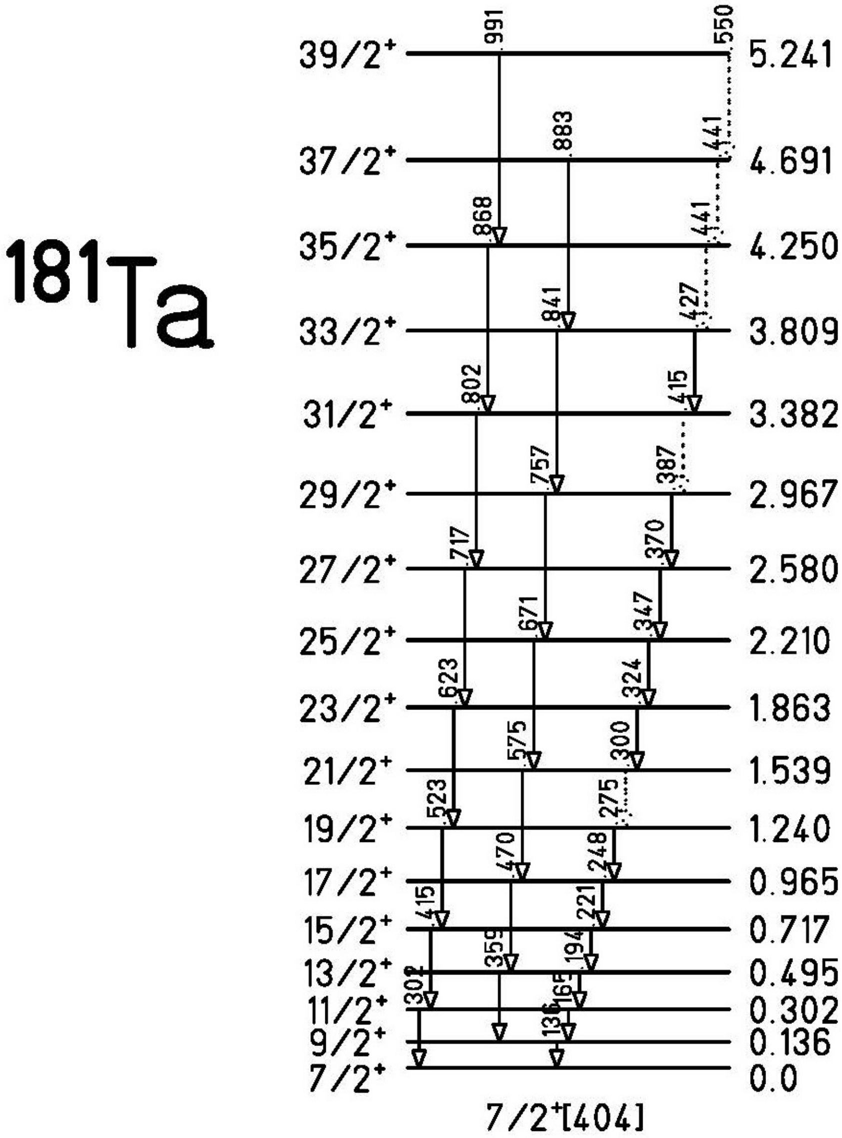
where a is the so-called *decoupling parameter*

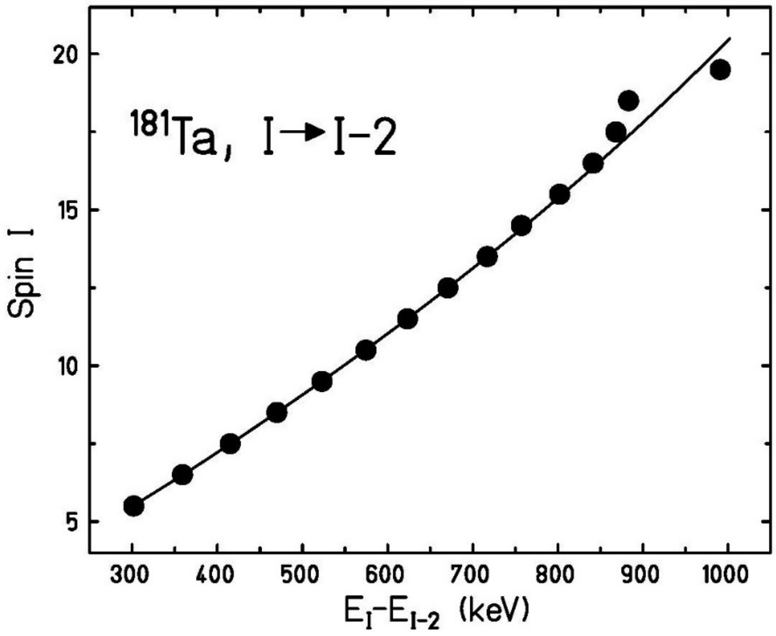
γ-rays from a deformed band in ^{181}Ta



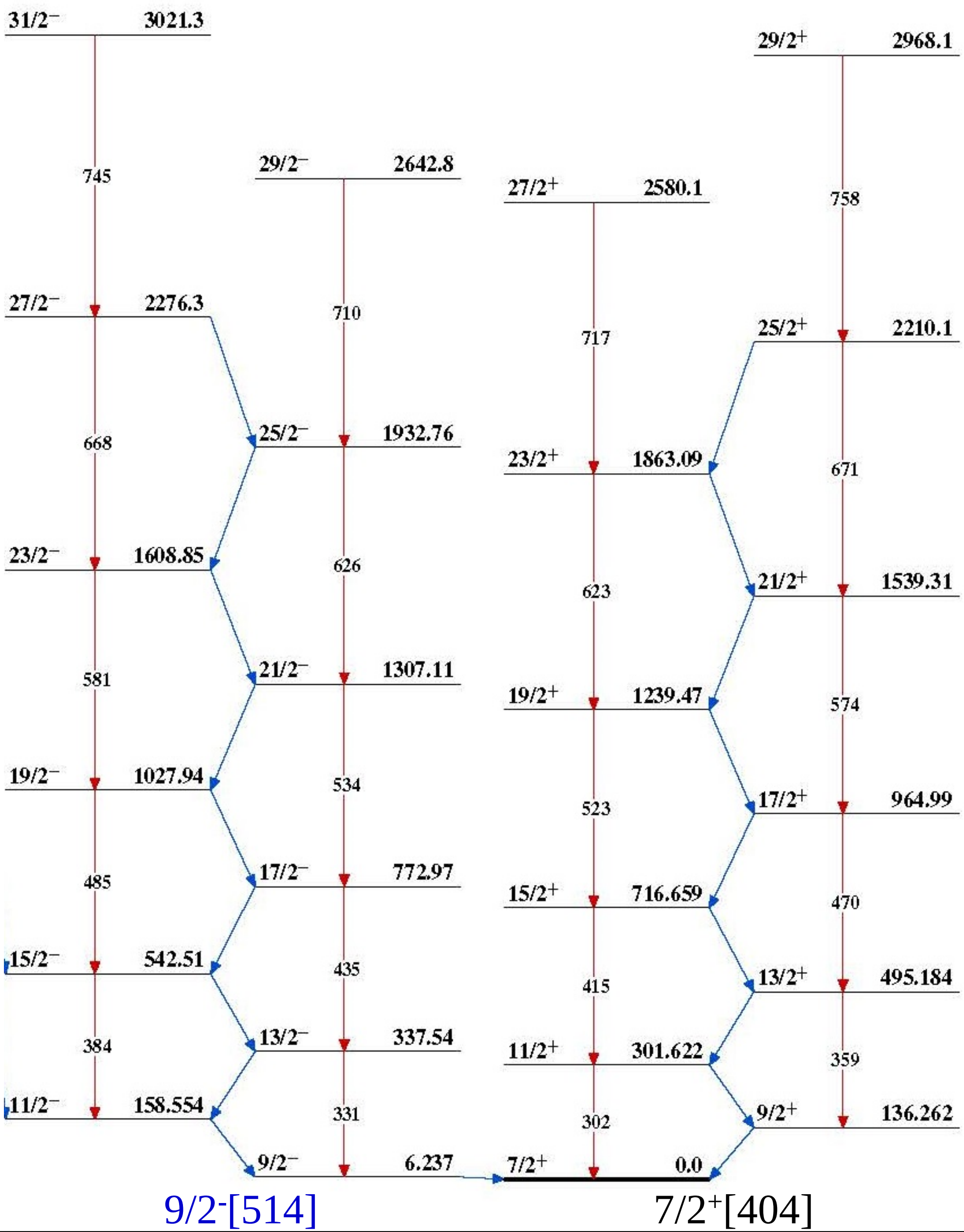


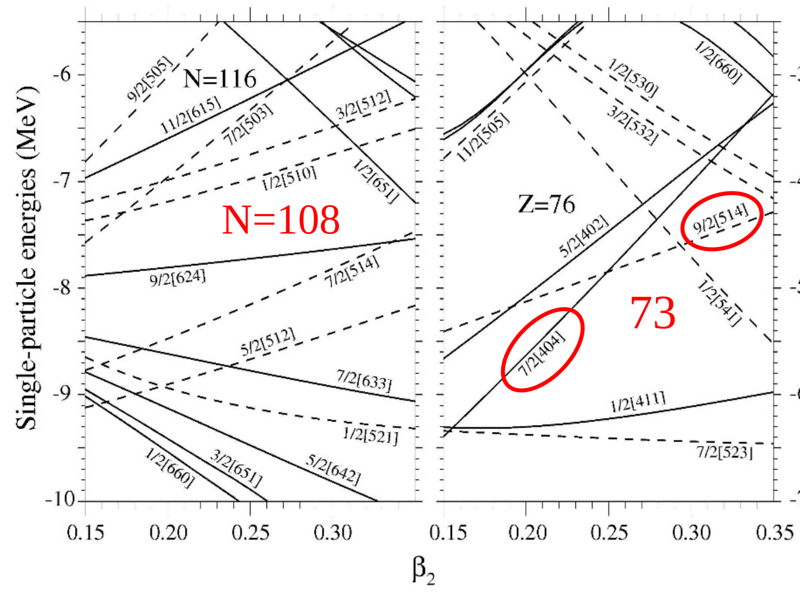
Nuclear level scheme of ^{181}Ta





Nilsson diagram of ^{181}Ta





Coriolis band mixing calculation for the $7/2^+[404]$ band in ^{181}Ta

The Hamiltonian H_{rot} is the diagonal part of the rotational Hamiltonian. Tha eigenvalues are assumed to be given by

$$E_K(I) = \epsilon_K + \frac{\hbar^2}{2\mathfrak{I}}\left[I(I+1) - K^2 + \delta_{K,1/2} \cdot a \cdot (-1)^{I+1/2}(I+1/2)\right]$$

Off-diagonal terms are given by the Coriolis matrix elements

$$V_{K+1,K} = -\frac{\hbar^2}{2\mathfrak{I}}\sqrt{(I-K)(I+K+1)} \cdot \langle K+1|j_+|K\rangle$$

Results of Coriolis band mixing calculation for the ground-state rotational band in ^{181}Ta

State I^π	E_{calc} (keV)	Expansion coefficients						
		$\frac{3}{2}^+[422]$	$\frac{3}{2}^+[411]$	$\frac{3}{2}^+[402]$	$\frac{5}{2}^+[413]$	$\frac{5}{2}^+[402]$	$\frac{7}{2}^+[404]$	$\frac{9}{2}^+[404]$
$\frac{7}{2}^+$	0.0	0.0020	−0.0027	0.0	0.0417	−0.0304	0.9987	0.0
$\frac{9}{2}^+$	136.9	0.0040	−0.0053	0.0001	0.0630	−0.0459	0.9969	−0.0025
$\frac{11}{2}^+$	302.9	0.0065	−0.0085	0.0001	0.0818	−0.0594	0.9948	−0.0038
$\frac{13}{2}^+$	497.1	0.0093	−0.0123	0.0002	0.0995	−0.0720	0.9923	−0.0048
$\frac{15}{2}^+$	718.7	0.0127	−0.0167	0.0002	0.1167	−0.0840	0.9894	−0.0058
$\frac{17}{2}^+$	966.5	0.0163	−0.0216	0.0003	0.1335	−0.0955	0.9860	−0.0067
$\frac{19}{2}^+$	1239.3	0.0209	−0.0270	0.0	0.1501	−0.1067	0.9822	−0.0076
$\frac{21}{2}^+$	1535.7	0.0251	−0.0331	0.0004	0.1664	−0.1176	0.9781	−0.0084

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Reduced transition probability

expectation value $\langle \hat{M}^{lab}_{\lambda m} \rangle = \int \Psi^* \hat{M}^{lab}_{\lambda m} \Psi d\tau$

$$\hat{M}^{lab}_{\lambda m} = \sum_{m'} D^{\lambda}_{mm'} \hat{M}^{intr}_{\lambda m'}$$

wave function $\Psi_{IMK} = \sqrt{\frac{2I+1}{16 \cdot \pi^2}} \cdot [D^I_{MK} \cdot \mathrm{X}_K + (-1)^{I-K} D^I_{M-K} \cdot \mathrm{X}_{-K}]$

$$\langle I_f M_f K | \hat{M}^{lab}_{\lambda m} | I_i M_i K \rangle = \frac{\sqrt{(2I_i+1)(2I_f+1)}}{8 \cdot \pi^2} \iiint D^{I_f}_{M_f K} \mathrm{X}_K \sum_{m'=0} D^{\lambda}_{mm'} \hat{M}^{intr}_{\lambda m'} D^{I_i}_{M_i K} \mathrm{X}_K d\tau$$

$$\iiint D^{I_1}_{M_1 M_1'} D^{I_2}_{M_2 M_2'} D^{I_3}_{M_3 M_3'} d\tau = \frac{8\pi^2}{2I_3+1} \cdot (I_1 I_2 M_1 M_2 \mid I_3 M_3) \cdot (I_1 I_2 M_1' M_2' \mid I_3 M_3')$$

$$\langle I_f M_f K | \hat{M}^{lab}_{\lambda m} | I_i M_i K \rangle = \sqrt{\frac{2I_i+1}{2I_f+1}} \cdot (I_i \lambda M_i (M_f - M_i) | I_f M_f) \cdot (I_i \lambda K 0 | I_f K) \cdot \langle \mathrm{X}_K | \hat{M}^{intr}_{\lambda 0} | \mathrm{X}_K \rangle$$

Reduced transition probability

$$\langle I_f M_f K | \widehat{M}_{\lambda m}^{lab} | I_i M_i K \rangle = \sqrt{\frac{2I_i + 1}{2I_f + 1}} \cdot \left(I_i \lambda M_i (M_f - M_i) | I_f M_f \right) \cdot \left(I_i \lambda K 0 | I_f K \right) \cdot \langle \mathrm{X}_K | \widehat{M}_{\lambda 0}^{intr} | \mathrm{X}_K \rangle$$

Wigner-Eckart-Theorem (reduction of an expectation value):

$$\langle I_f M_f K | \widehat{M}_{\lambda m}^{lab} | I_i M_i K \rangle = \frac{\left(I_i \lambda M_i (M_f - M_i) | I_f M_f \right)}{\sqrt{2I_f + 1}} \cdot \langle I_f K \| \widehat{M}_{\lambda}^{lab} \| I_i K \rangle$$

$$\langle I_f K \| M(E\lambda) \| I_i K \rangle = \sqrt{2I_i + 1} \left(I_i \lambda K 0 | I_f K \right) \cdot \langle \mathrm{X}_K | \widehat{M}_{\lambda 0}^{intr} | \mathrm{X}_K \rangle$$

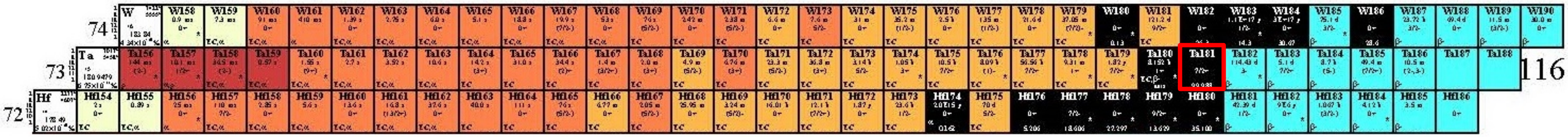
special case: E2 transition I→I-2

$$\langle I - 2, K \| M(E2) \| I, K \rangle = \sqrt{\frac{15}{32\pi}} \cdot \sqrt{\frac{(I + K - 1) \cdot (I + K) \cdot (I - K - 1) \cdot (I - K)}{(I - 1) \cdot (2I - 1) \cdot I}} \cdot Q_2 e$$

reduced transition probability:

$$B(E\lambda; I_i \to I_f) = \frac{1}{2I_i + 1} \left| \langle I_f K \| M(E\lambda) \| I_i K \rangle \right|^2$$

Odd-even nucleus: ^{181}Ta

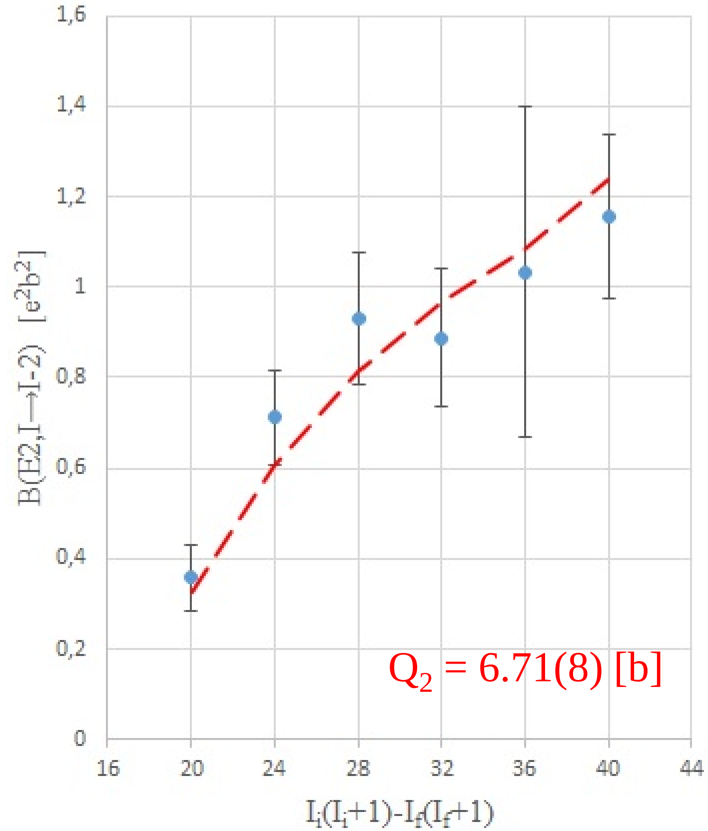


special case: E2 transition I→I-2

$$\langle I-2,K\|M(E2)\|I,K\rangle = \sqrt{\frac{15}{32\pi}}\cdot\sqrt{\frac{(I+K-1)\cdot(I+K)\cdot(I-K-1)\cdot(I-K)}{(I-1)\cdot(2I-1)\cdot I}}\cdot Q_2 e$$

reduced transition probability:

$$B\left(E\lambda; I_i \to I_f\right) = \frac{1}{2I_i+1}\left|\langle I_f K\|M(E\lambda)\|I_i K\rangle\right|^2$$



$$\frac{Q_2(9/2^-)}{Q_2(7/2^+)} = (0.9681 \pm 0.0002)$$

M. Loewe; dissertation

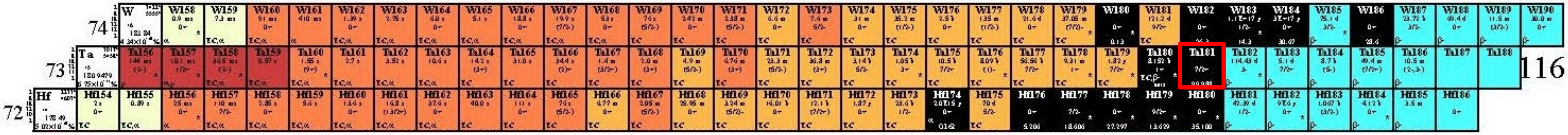
Matrix elements

$$\langle I-2,K\|M(E2)\|I,K\rangle = \sqrt{\frac{15}{32\pi}} \cdot \sqrt{\frac{(I+K-1)\cdot(I+K)\cdot(I-K-1)\cdot(I-K)}{(I-1)\cdot(2I-1)\cdot I}} \cdot Q_2 e$$

$$\langle I-1,K\|M(E2)\|I,K\rangle = -\sqrt{\frac{5}{16\pi}} \cdot \sqrt{\frac{3\cdot(I+K)\cdot(I-K)\cdot K^2}{(I-1)\cdot I\cdot(I+1)}} \cdot Q_2 e$$

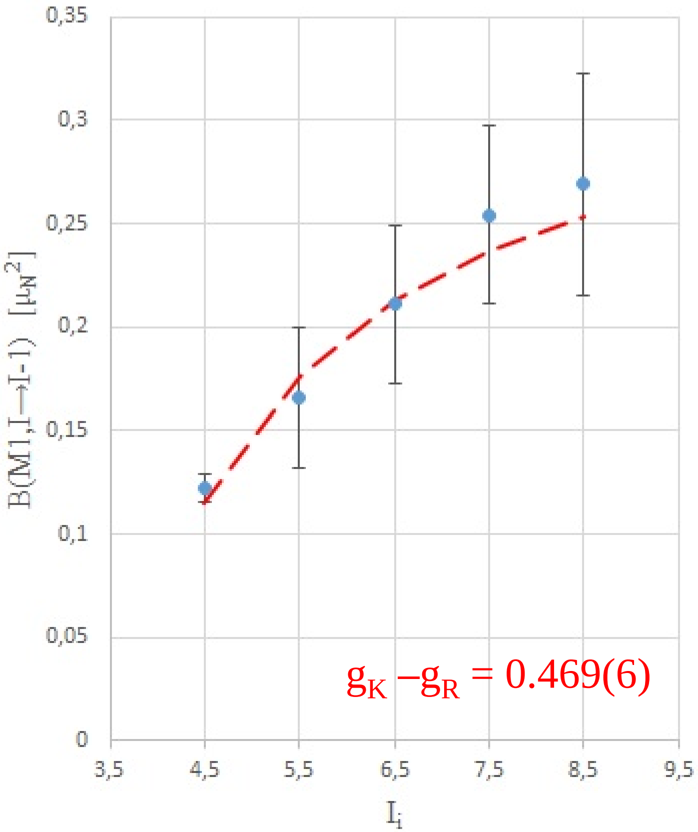
$$\langle I,K\|M(E2)\|I,K\rangle = -\sqrt{\frac{5}{16\pi}} \cdot \sqrt{\frac{2I+1}{(2I-1)\cdot I\cdot(I+1)\cdot(2I+3)}} \cdot (I^2-3K^2+I)\cdot Q_2 e$$

Odd-even nucleus: ^{181}Ta



$$\langle I-1,K\|M(M1)\|I,K\rangle = -\sqrt{\frac{3}{4\pi}}\sqrt{\frac{(I+K)(I-K)}{I}}\cdot K\cdot(g_K-g_R)\left[1+\delta_{K,1/2}(-1)^{I+1/2}b_0\right]\mu_N$$

The quantity b_0 depends on the magnetic decoupling parameter



M. Loewe; dissertation

$$\mu(7/2^+) = 2.3705 \pm 0.0007$$

$$\mu = \frac{K}{I+1}\cdot(g_K-g_R)\cdot K + g_R\cdot I$$

$g_R = 0.313(5)$

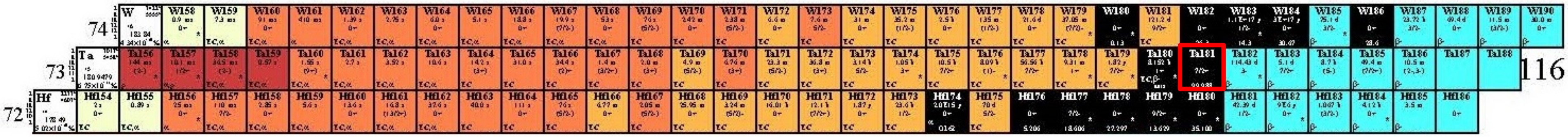
$g_K = 0.782(2)$

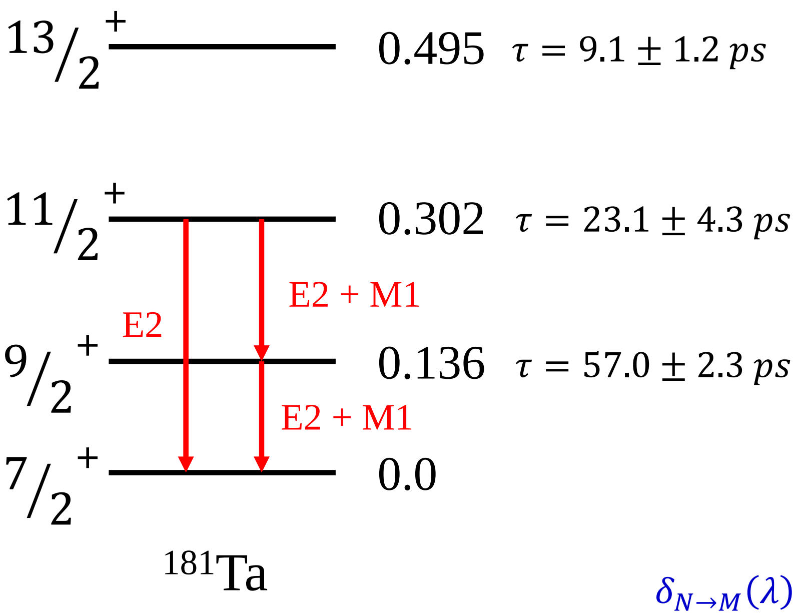
$$\mu(9/2^-) = 5.28 \pm 0.09$$

$$^{9/2^-}(g_K-g_R) = \frac{22}{81}\left({}^{9/2^-}\mu - {}^{7/2^+}\mu\frac{9}{7} + {}^{7/2^+}(g_K-g_R)\frac{7}{2}\right)$$

$$^{9/2^-}(g_K-g_R) \approx {}^{7/2^+}(g_K-g_R)\frac{77}{81} + 0.606 = 1.052$$

Odd-even nucleus: ^{181}Ta





$$\tau = \left\{ \sum_K \sum_\ell [\varepsilon^2_{N\to K}(\lambda) + \delta^2_{N\to K}(\lambda)] \right\}^{-1}$$

$$\tau = T_{1/2}/ln2$$

$$\delta_{N\to M}(\lambda) = \left\{ \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \frac{1}{\hbar} \left(\frac{\hbar\omega}{\hbar c}\right)^{2\lambda+1} \right\}^{1/2} \cdot (2I_N+1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(\lambda) \| I_N \rangle$$

$$\delta_{N\to M}(E2) = \{1.225 \cdot 10^{13} \cdot E_\gamma^5 (MeV)^5\}^{1/2} \cdot (2I_n+1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(E2) \| I_N \rangle$$

$$\delta_{N\to M}(M1) = \{1.758 \cdot 10^{13} \cdot E_\gamma^3 (MeV)^3\}^{1/2} \cdot (2I_n+1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(M1) \| I_N \rangle$$

$$\varepsilon^2_{N\to M}(\ell) = \delta^2_{N\to M}(\ell) \cdot \alpha_{N\to M}(\ell)$$

conversion coefficient.: bricc.anu.edu.au

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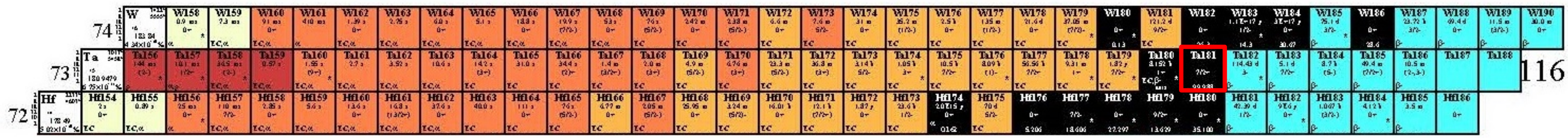
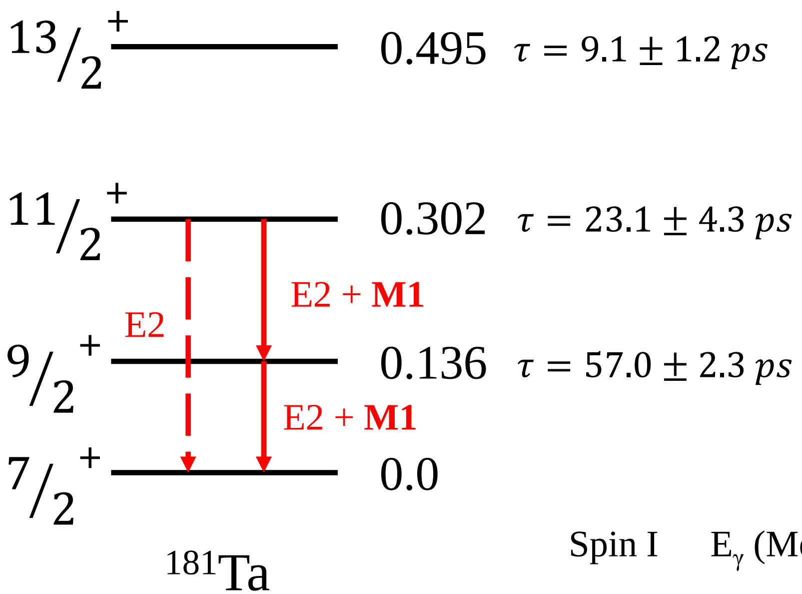
Odd-even nucleus: ^{181}Ta





$$\tau = \left\{ \sum_K \sum_\ell \left[\varepsilon^2_{N\to K}(\lambda) + \delta^2_{N\to K}(\lambda) \right] \right\}^{-1}$$

$$\tau = T_{1/2} / ln2$$

Spin I	E_γ (MeV)	$(2I+1)^{-1/2}$	<I-1//M()//I>	delta	α_T	ε^2	τ (ps)
9/2	0.1365	0.3162	-3.899 (E2)	-29594.	1.1	$9.98 \cdot 10^8$	533
			1.103 (M1)	73591	1.8	$9.75 \cdot 10^9$	**58.7**
11/2	0.1654	0.2887	-4.291 (E2)	-48238	0.6	$1.33 \cdot 10^9$	274
			1.413 (M1)	115044	1.0	$1.38 \cdot 10^{10}$	32.6
11/2	0.3017	0.2887	1.977 (E2)	99868	0.1	$8.08 \cdot 10^8$	**24.1**

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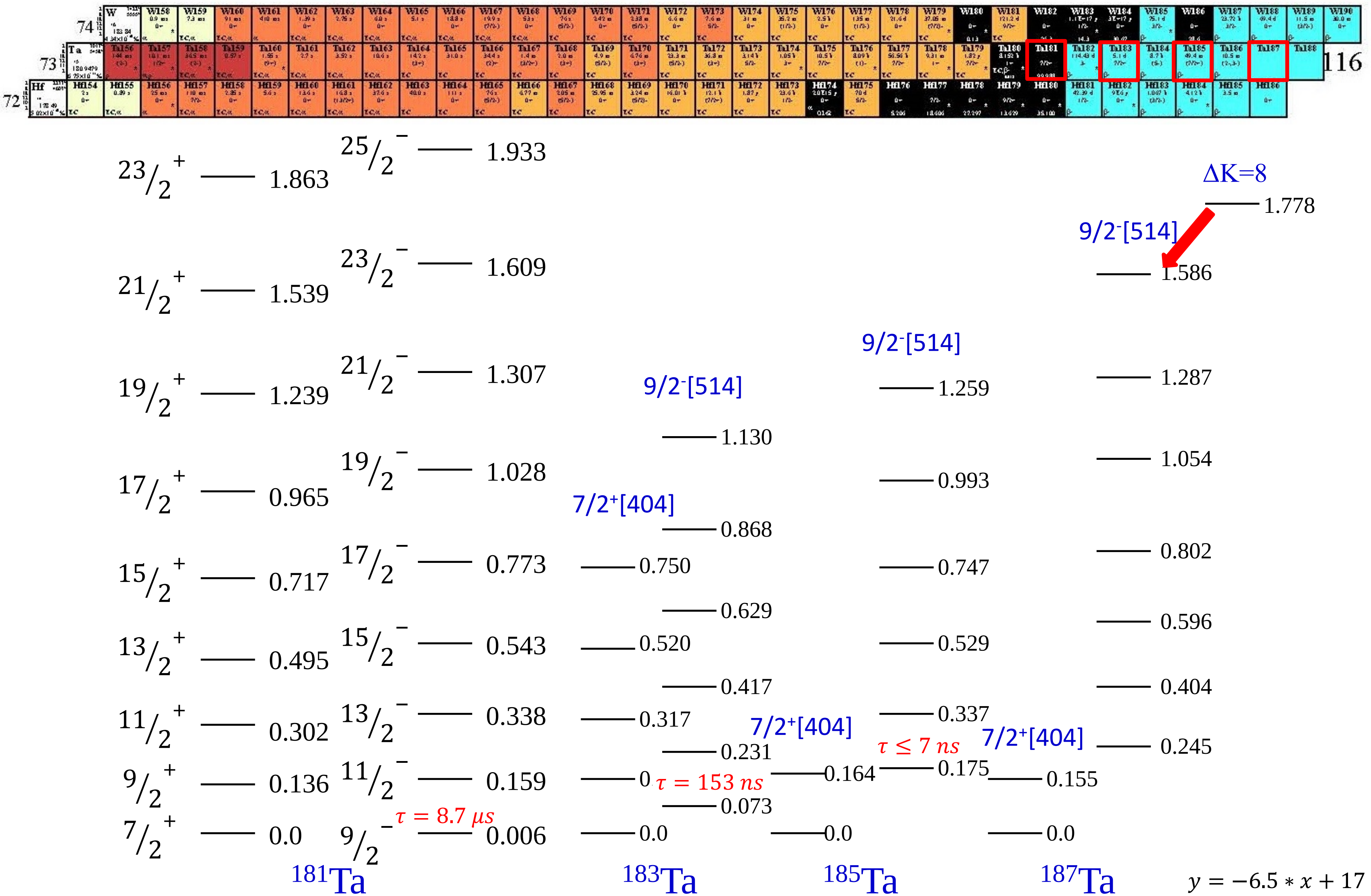
Odd proton-even nuclei

		μ	(g_K-g_R)	B(M1)W.u.	(g_K-g_R)
^{153}Eu	$5/2^+$	+1.5324(3)	0.551	0.00608(28)	0.185
^{159}Tb	$3/2^+$	+2.014(4)	1.556	0.173(8)	1.471
^{165}Ho	$7/2^-$	+4.177(5)	1.012	0.275(14)	0.973
^{169}Tm	$1/2^+$	-0.2316(15)		0.0342(8)	
^{175}Lu	$7/2^+$	+2.2327(11)	0.298	0.0354(14)	0.349
^{181}Ta	$7/2^+$	+2.3705(7)	0.352	0.068(4)	0.484
^{185}Re	$5/2^+$	+3.1871(3)	1.217	0.28(5)	1.252
^{187}Re	$5/2^+$	+3.2197(3)	1.242	0.260(18)	1.206

Ta-nuclei: level schemes



^{187}Ta level scheme

expected lifetimes

ΔK=8

$(25/2^-)$ —— 1.778 $\tau = 10.5(13)\ s$

9/2⁻[514]

$(21/2^-)$ —— 1.586 $\tau = 0.6\ ps$

$(19/2^-)$ —— 1.287 $\tau = 1.1\ ps$

$(17/2^-)$ —— 1.054 $\tau = 1.1\ ps$

$(15/2^-)$ —— 0.802 $\tau = 2.2\ ps$

$(13/2^-)$ —— 0.596 $\tau = 3.4\ ps$

$(11/2^-)$ —— 0.404 $\tau = 7.9\ ps$

7/2⁺[404]

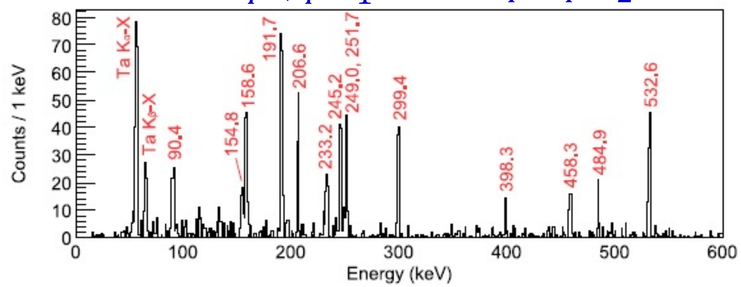
$(9/2^-)$ —— 0.245 $\tau \sim 40\ ns$

$(9/2^+)$ —— 0.155

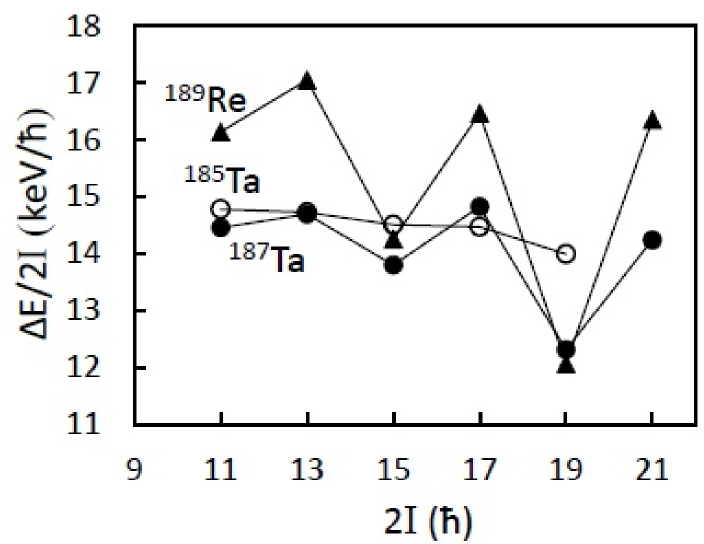
$(7/2^+)$ —— 0.0

^{187}Ta P.M. Walker; Phys. Rev. Lett. 125 192505

$I \rightarrow I-1$ $I \rightarrow I-2$



Sum of γγ-coincidence energy spectra

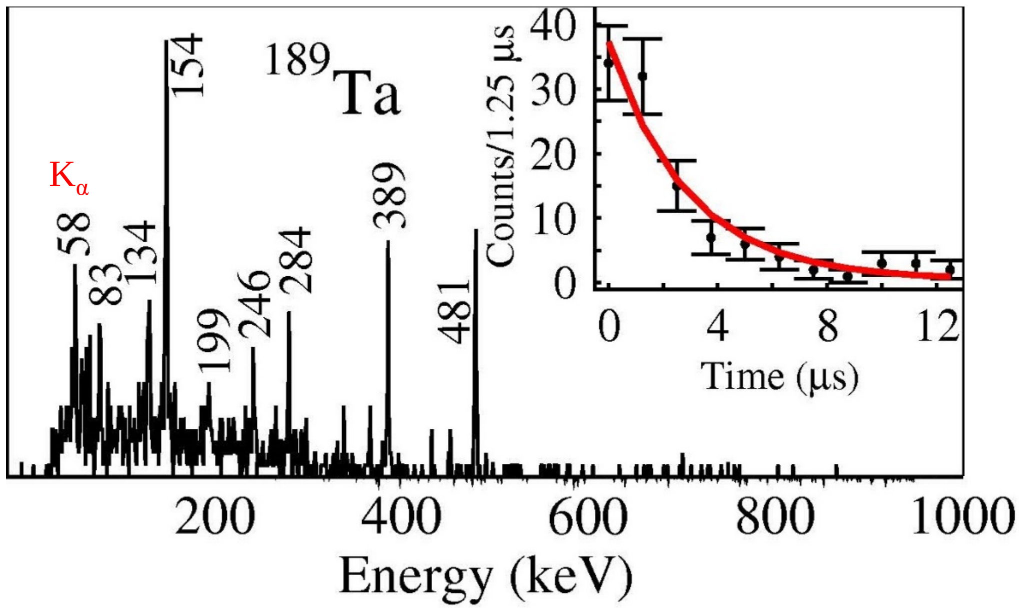


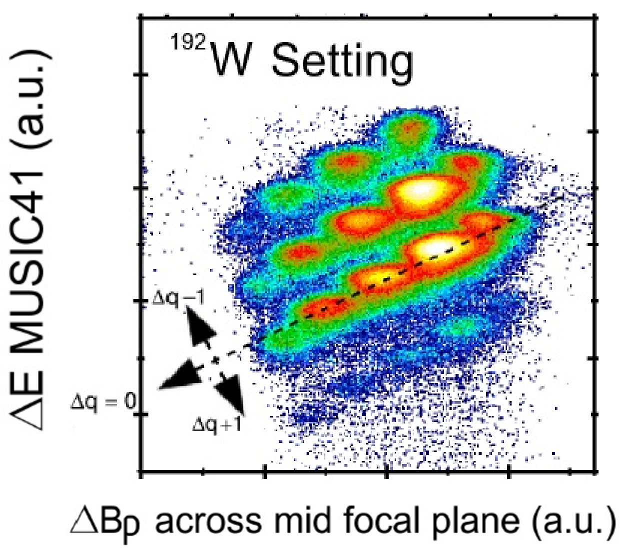
Signature splitting versus angular momentum

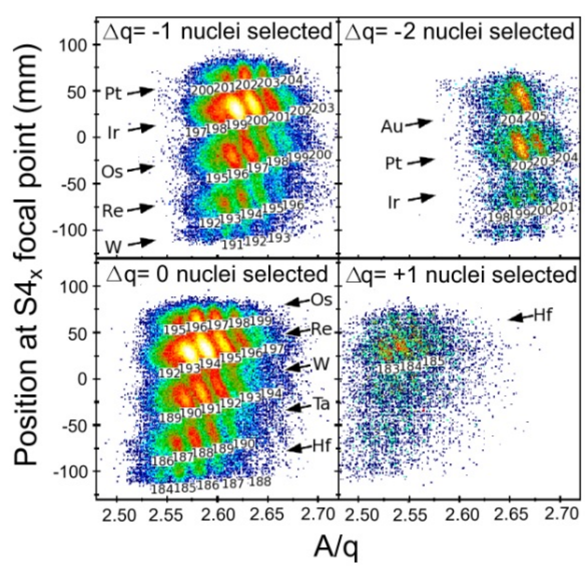
$y = -6.5 * x + 17$

Experimental information on ^{189}Ta

^{208}Pb primary beam

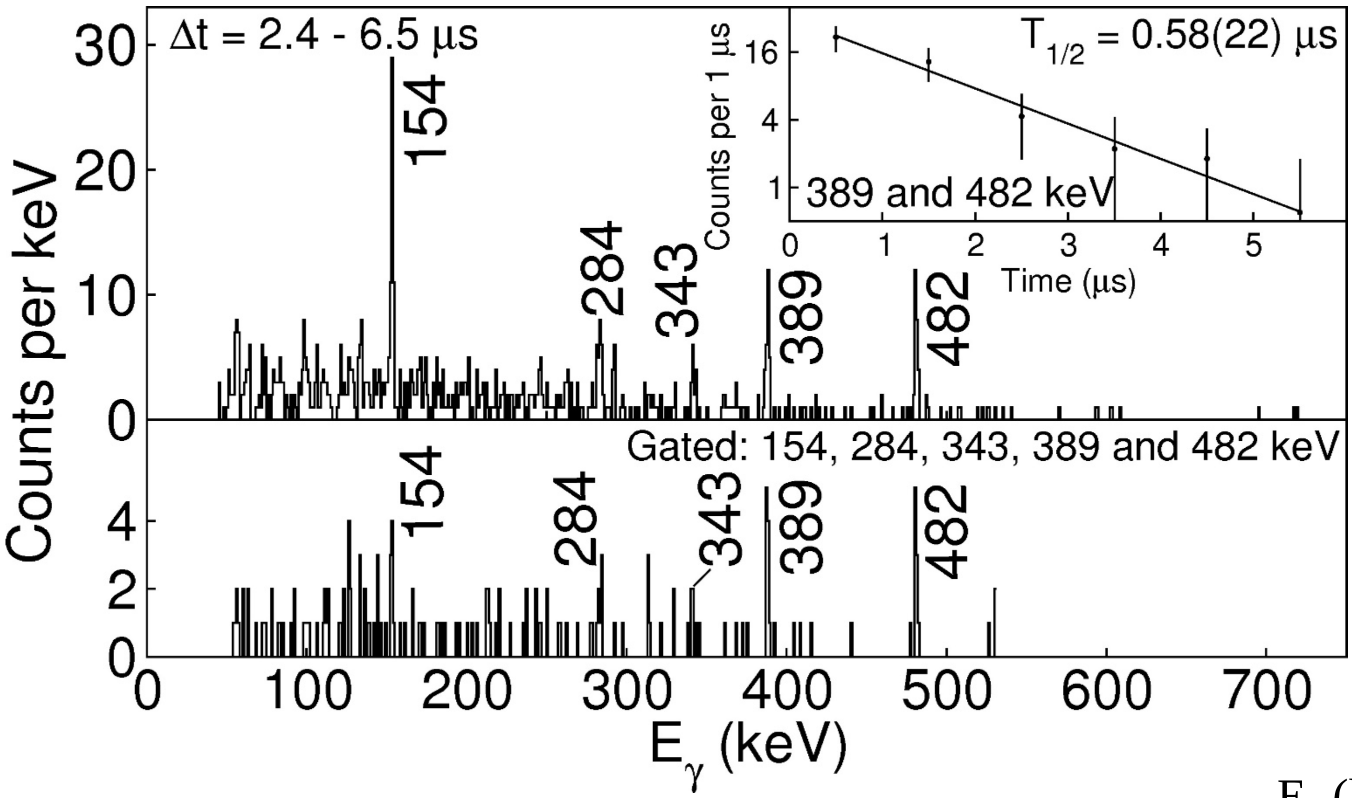






PhD-thesis: Steven John Steer

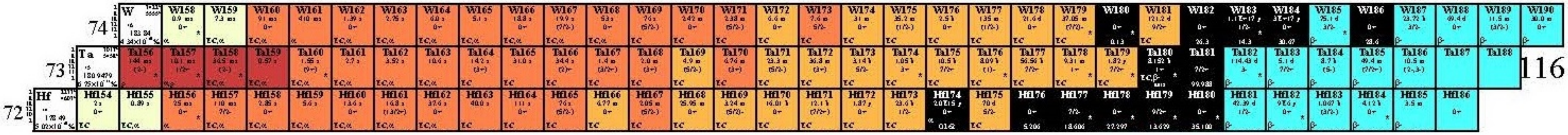
Experimental information on ^{189}Ta

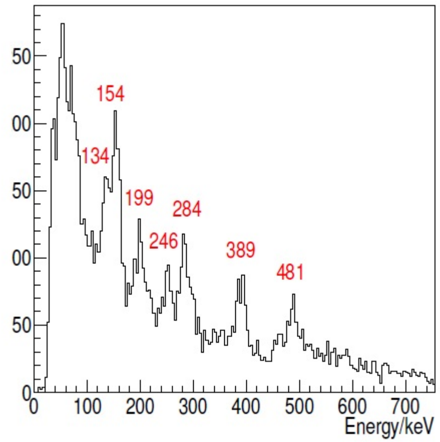


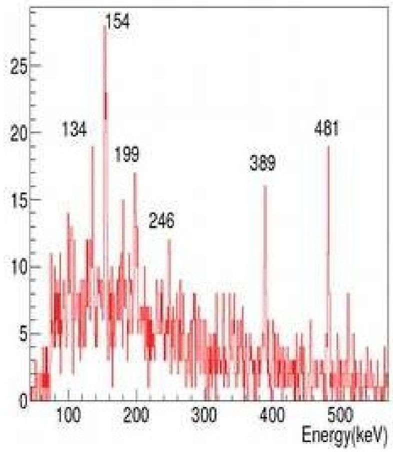
E_γ (keV)	intensity
153.9	100(19)
283.7	73(17)
342.5	47(13)
388.7	80(19)
481.6	97(21)

PhD-thesis: Steven John Steer

Experimental information on ^{189}Ta





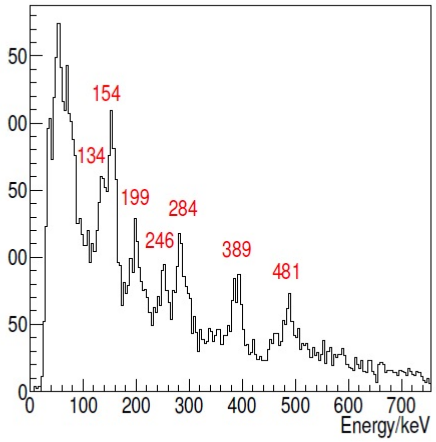


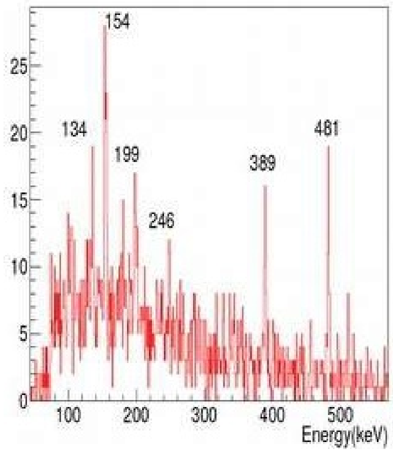
E_γ	ΔE_γ	γ-intensity		α_{tot}	intensity	S.J. Steer	$T_{1/2}$ (µs)
154	3.2	149.5±14.1	100%	0.734	E1:119/E2:173	100%	1.09(4)
199	4.2	67.1±9.0	44.9±7.4	0.303	58.5±9.6		
198	2.5	32.3±9.6	21.6±6.7	0.308	28.3±8.8		
200.5	2.5	31.1±9.6	20.8±6.7	0.295	26.9±8.7		
247.2	1.8	36.9±7.1	24.7±5.3	0.149	28.4±6.1		
284.6	2.4	37.5±8.3	25.5±6.0	0.025	E1:26.1±6.2	73±22	1.16(7)
342.7	2.6	29.4±7.0	19.7±5.0	0.056	20.8±5.3	47±16	
389.3	2.5	110.5±10.9	73.9±10.1	0.039	76.8±10.4	80±24	1.08(6)
481	3.2	131.6±12.5	88.0±11.8	0.022	90±12	97±28	1.12(7)
83							0.202(12)
134							0.284(11)

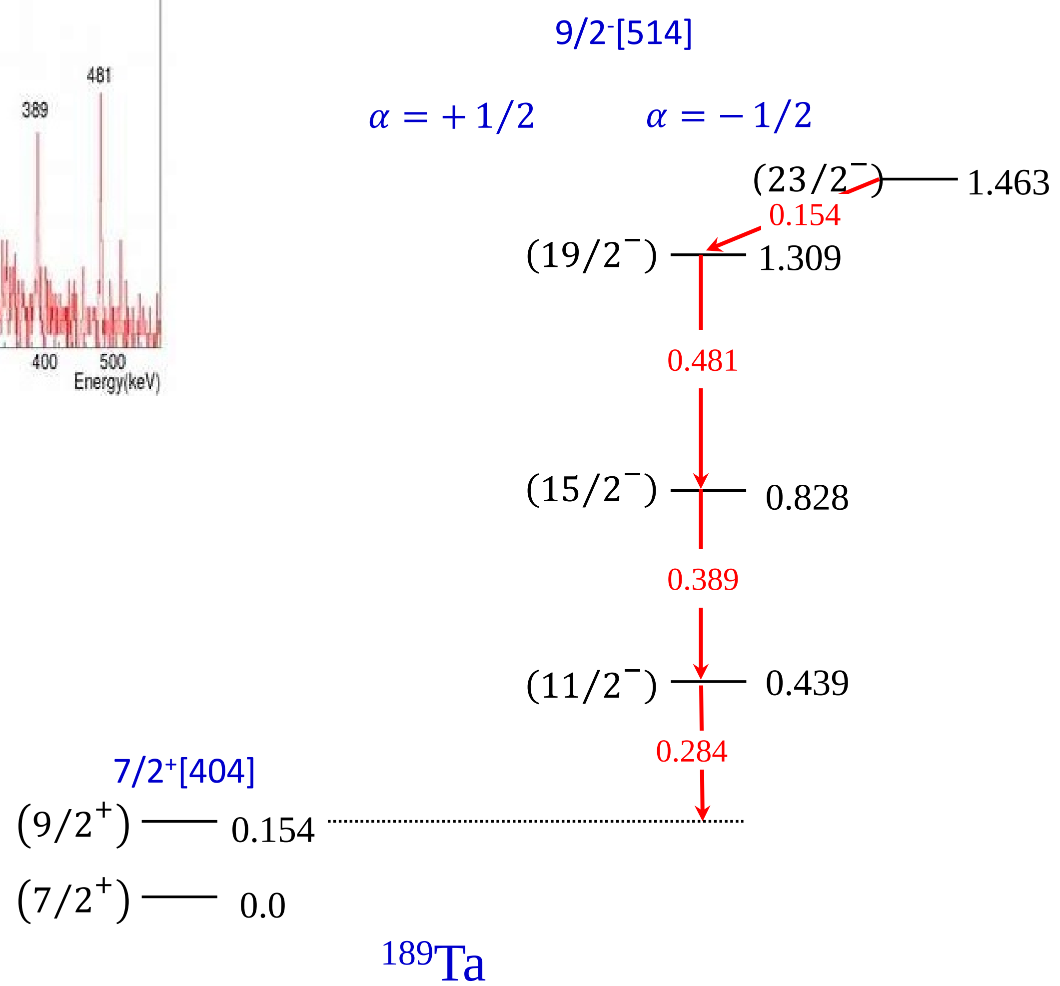
gate										
481		154					284	389		-
389		154					284	-		481
284		154					-	389		481
343		154	199	246	-					
246		154	199	-	343	370			453	481
199		154	-					389		481
154	134	154		246	343		284	389		481
134										

PhD-thesis: Sultan Alhomaidhi

Experimental information on ^{189}Ta





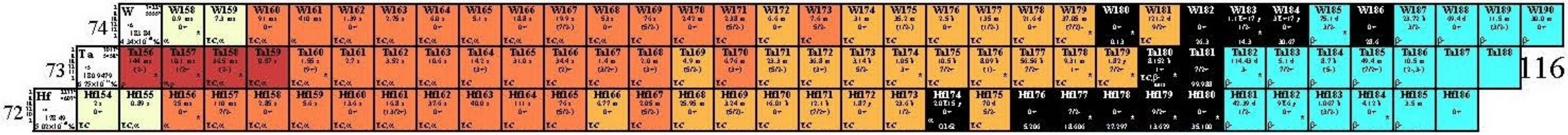


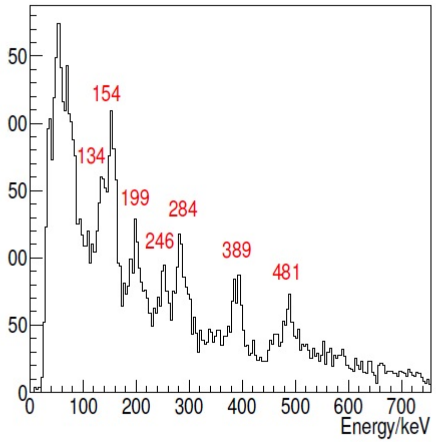
gate										
481		154					284	389		-
389		154					284	-		481
284		154					-	389		481
343		154	199	246	-					
246		154	199	-	343	370			453	481
199		154	-					389		481
154	134	154		246	343		284	389		481
134										

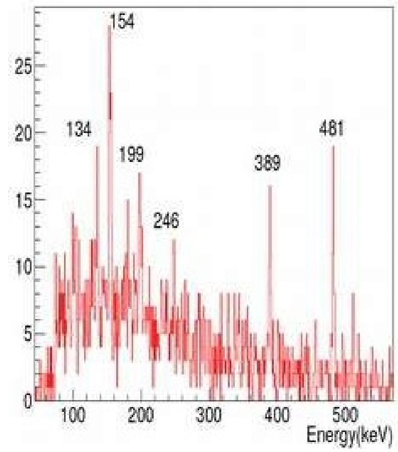
PhD-thesis: Sultan Alhomaidhi

$y = -6.5 * x + 16.6$

Experimental information on ^{189}Ta







gate										
481		154					284	389		-
389		154					284	-		481
284		154					-	389		481
343		154	199	246	-					
246		154	199	-	343	370			453	481
199		154	-					389		481
154	134	154		246	343		284	389		481
134										

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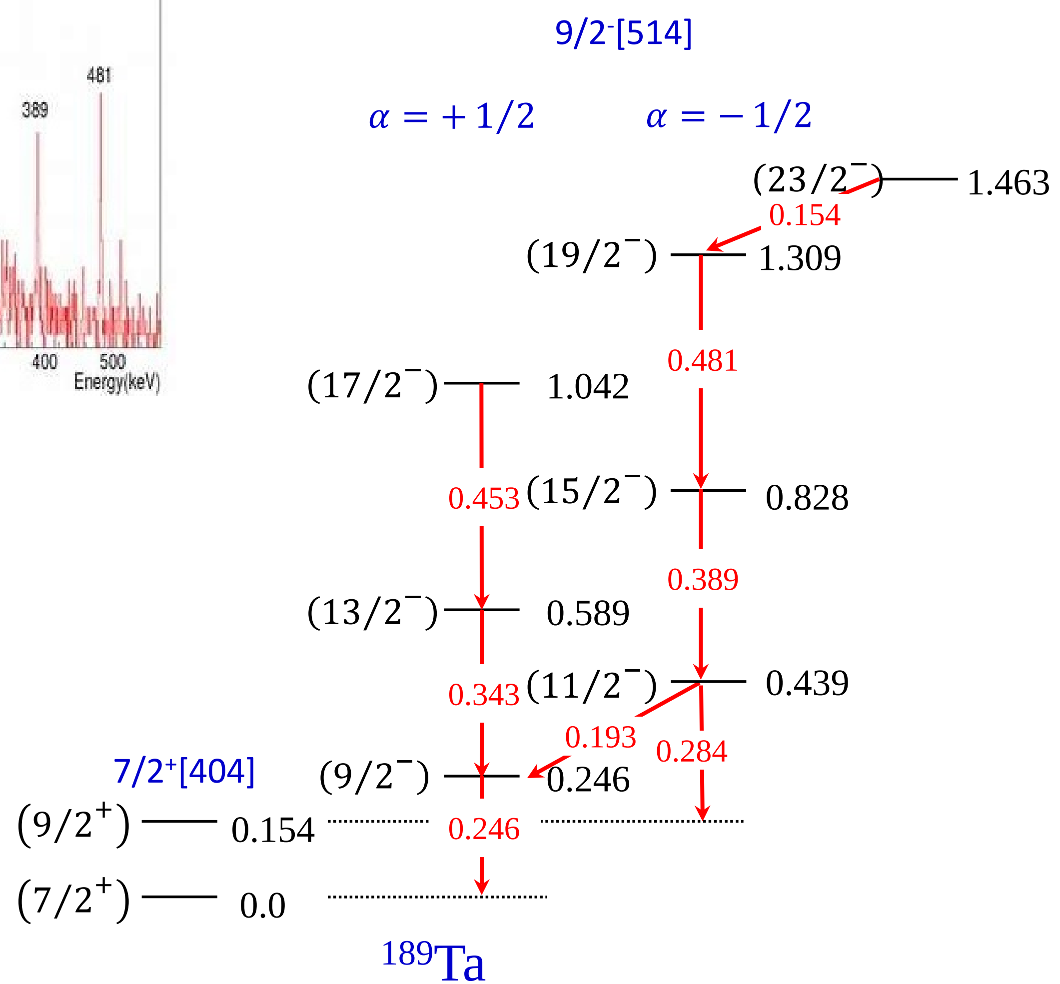
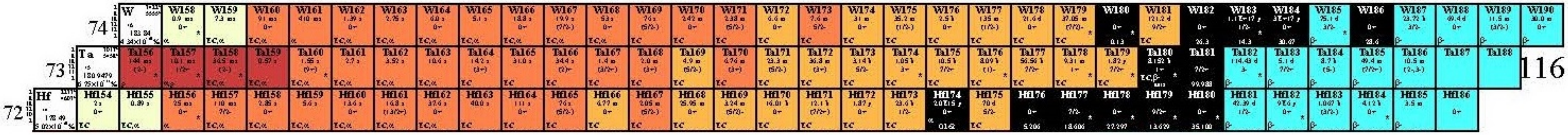
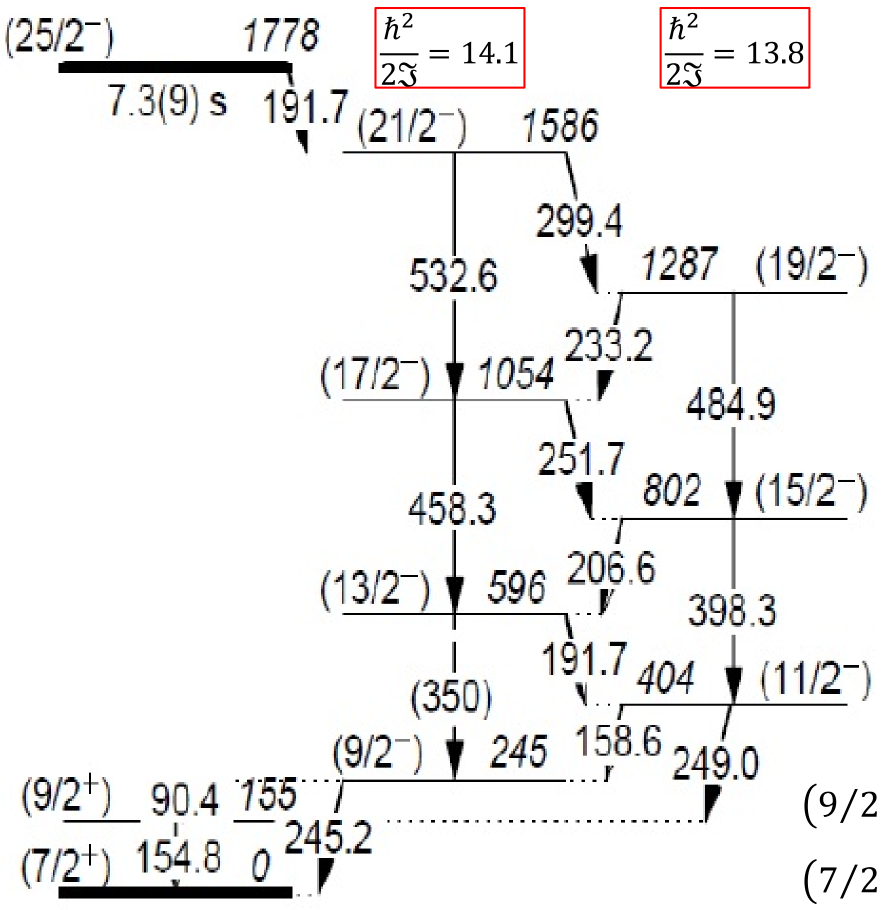


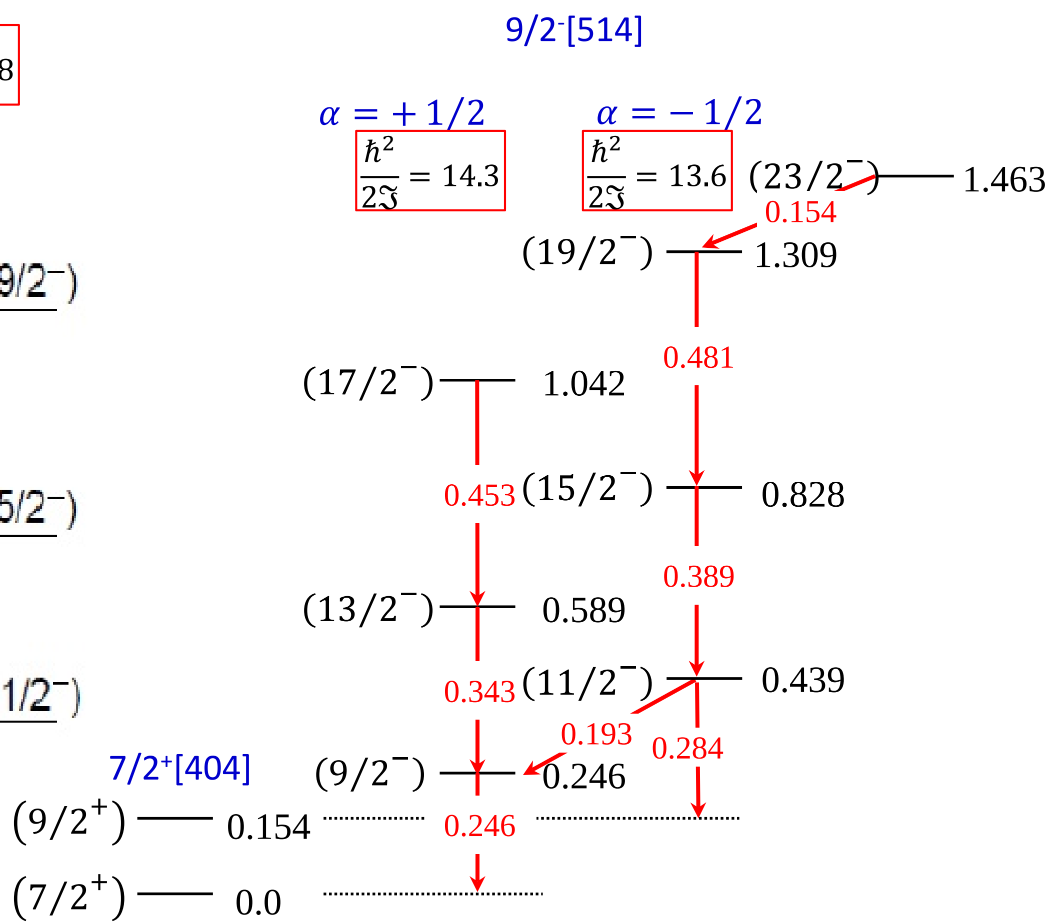
$y = -6.5 * x + 16.6$

Ta-nuclei: level schemes





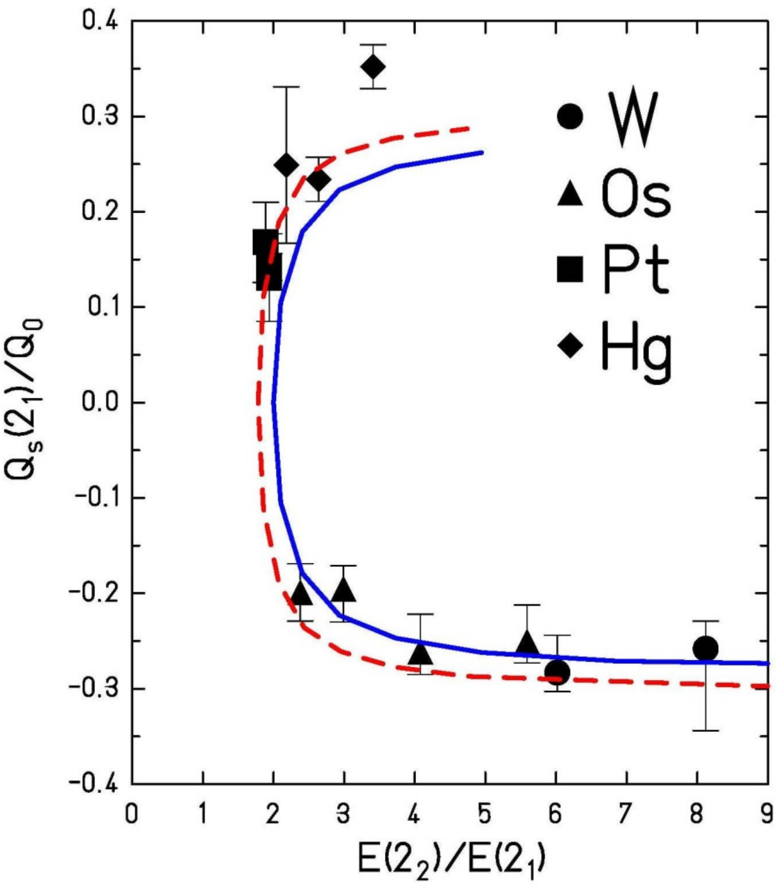
^{187}Ta

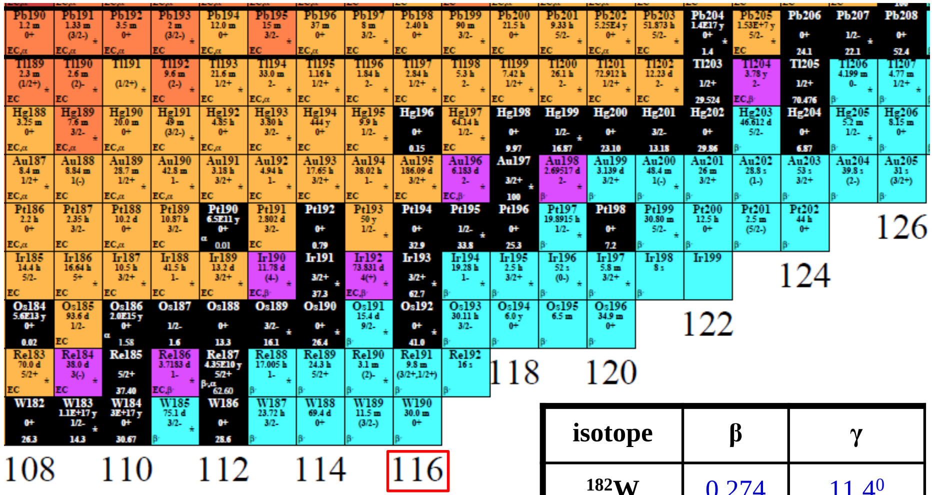


^{189}Ta

$y = -6.5 * x + 17$

Prolate-oblate shape transition

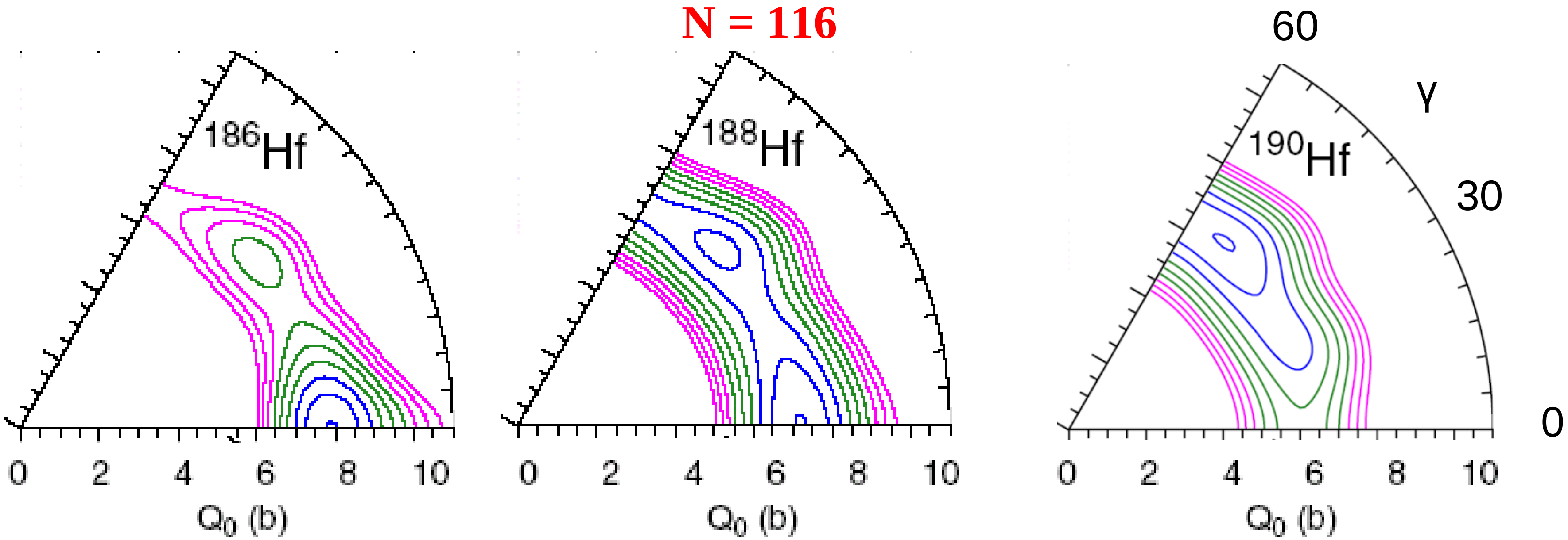




isotope	β	γ
^{182}W	0.274	11.4^0
^{184}W	0.258	13.8^0
^{186}W	0.223	15.9^0
^{186}Os	0.196	16.5^0
^{188}Os	0.185	19.2^0
^{190}Os	0.184	22.3^0
^{192}Os	0.168	25.2^0
^{192}Pt	0.146	-
^{194}Pt	0.134	-
^{196}Pt	0.135	-
^{198}Hg	0.106	36.3^0
^{200}Hg	0.098	39.1^0
^{202}Hg	0.082	33.4^0
^{204}Hg	0.068	31.5^0

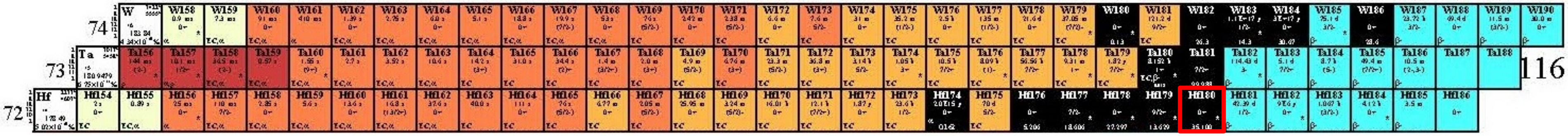
n-rich hafnium ground states

critical point
N = 116



Robledo et al., *J. Phys. G: Nucl. Part. Phys.* **36**, 115104 (2009).

Band crossing prediction in ^{180}Hf



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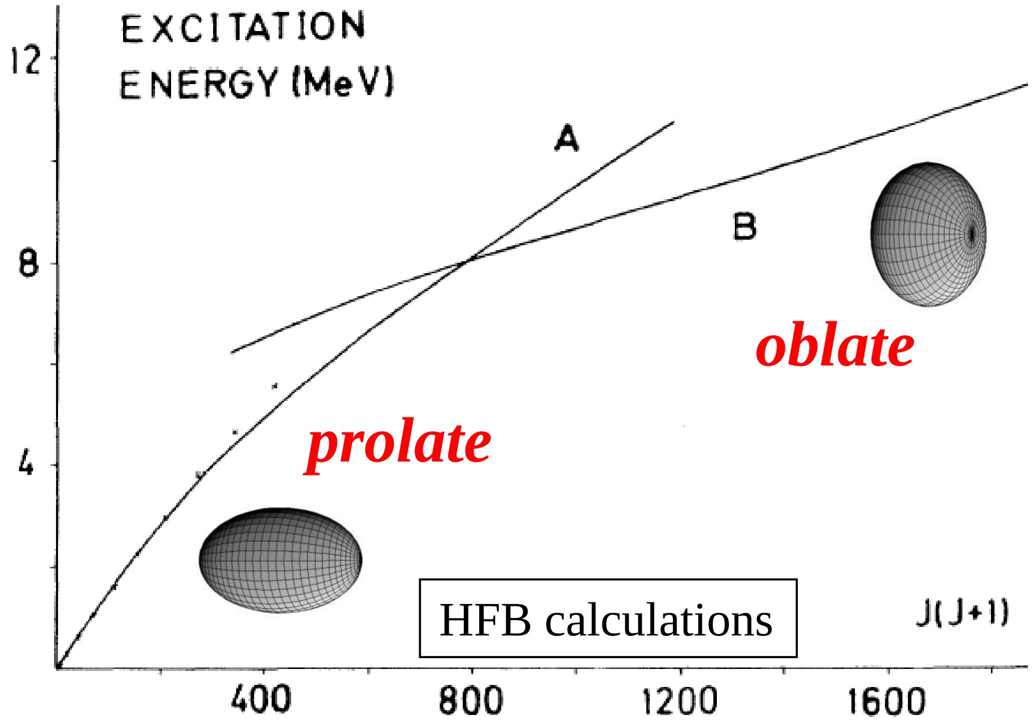
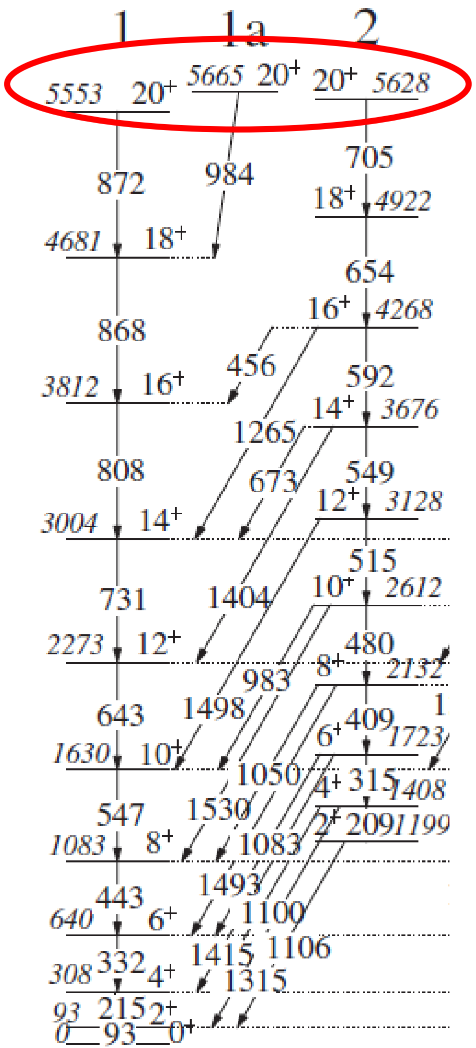
(Received 6 September 1979)

Giant backbending is predicted to occur in ^{180}Hf at $J \approx 26\hbar$. The effect is clearly seen to be the result of the crossing of two bands with very different intrinsic structure.



Phys. Rev. Lett. 43 (1979) 1979

^{180}Hf oblate band?



three 20^{+} states

Tandel et al.,

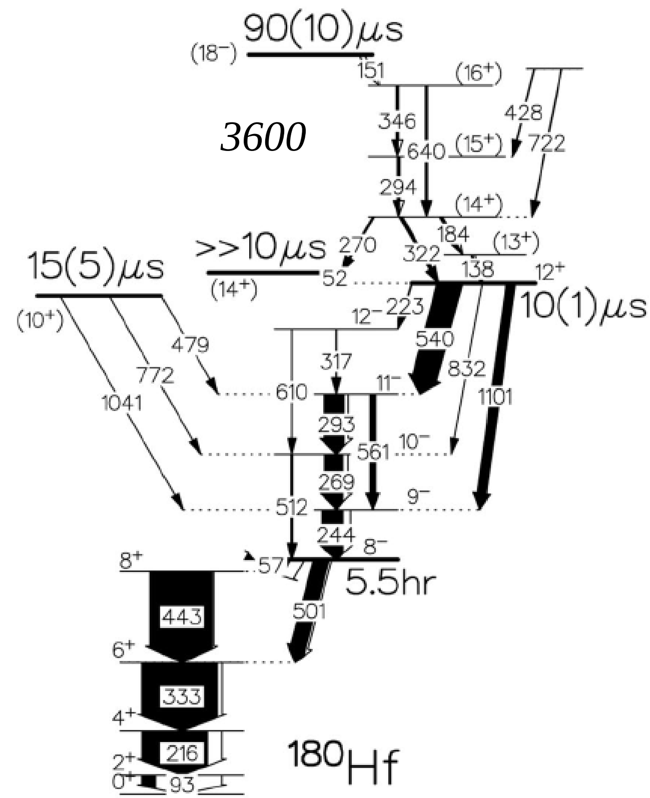
Phys. Rev. Lett. 101 (2008) 182503

with Gammasphere

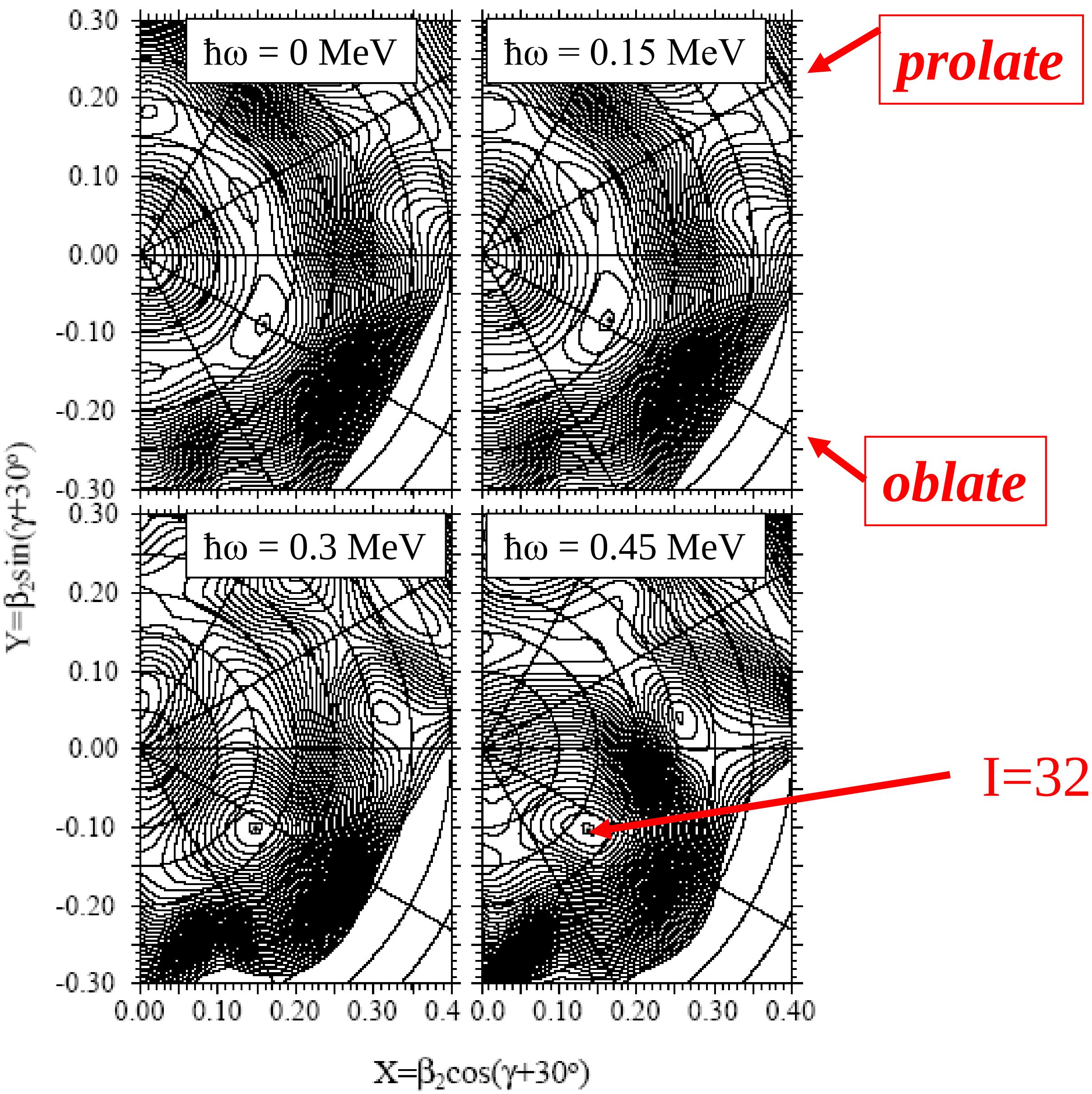
pre-Gammasphere

high-K yrast isomers:

d'Alarcao et al., Phys. Rev. C59 (1999) 1227(R)

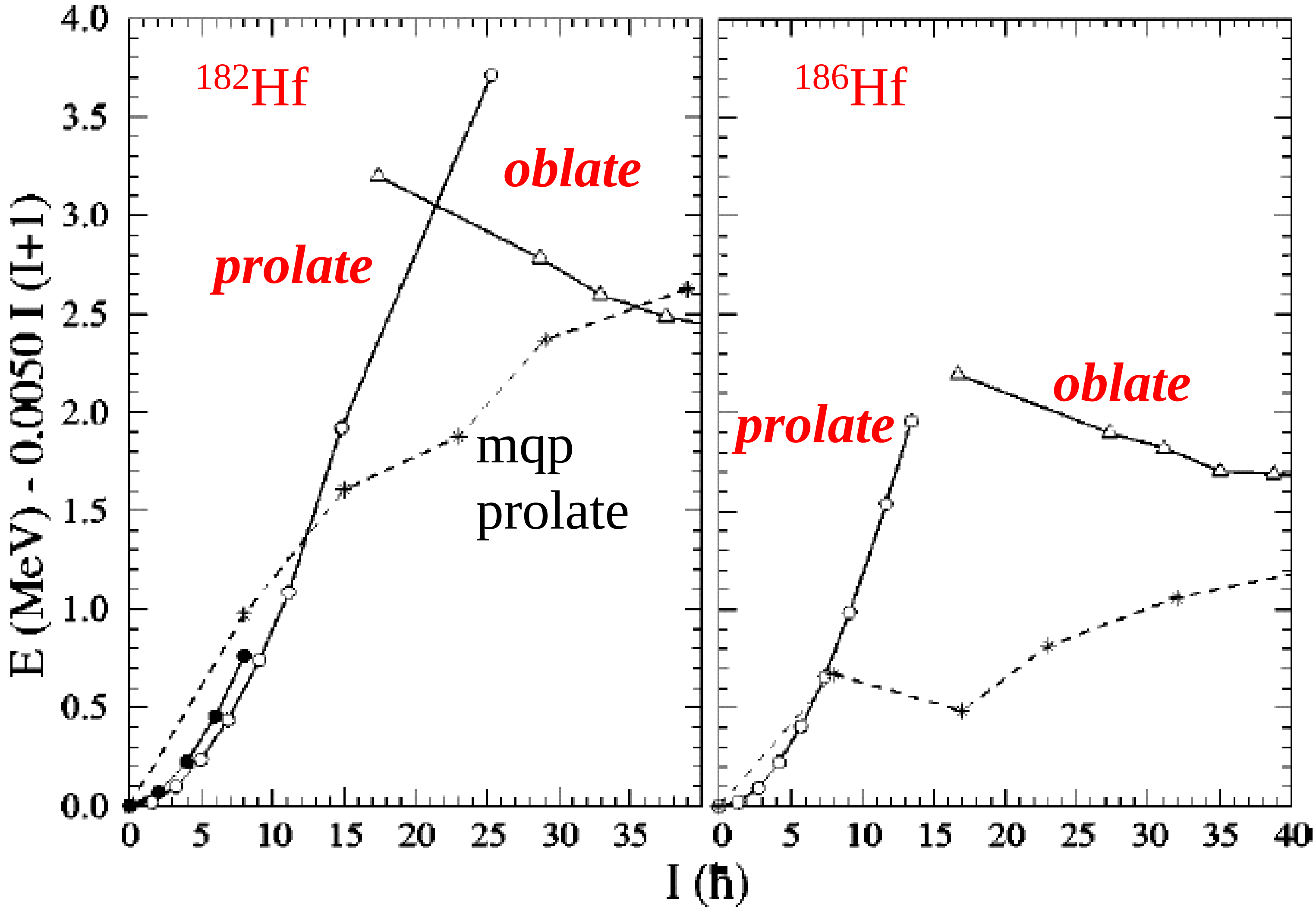


^{190}Hf TRS oblate rotor beyond the critical point



Xu et al., unpublished

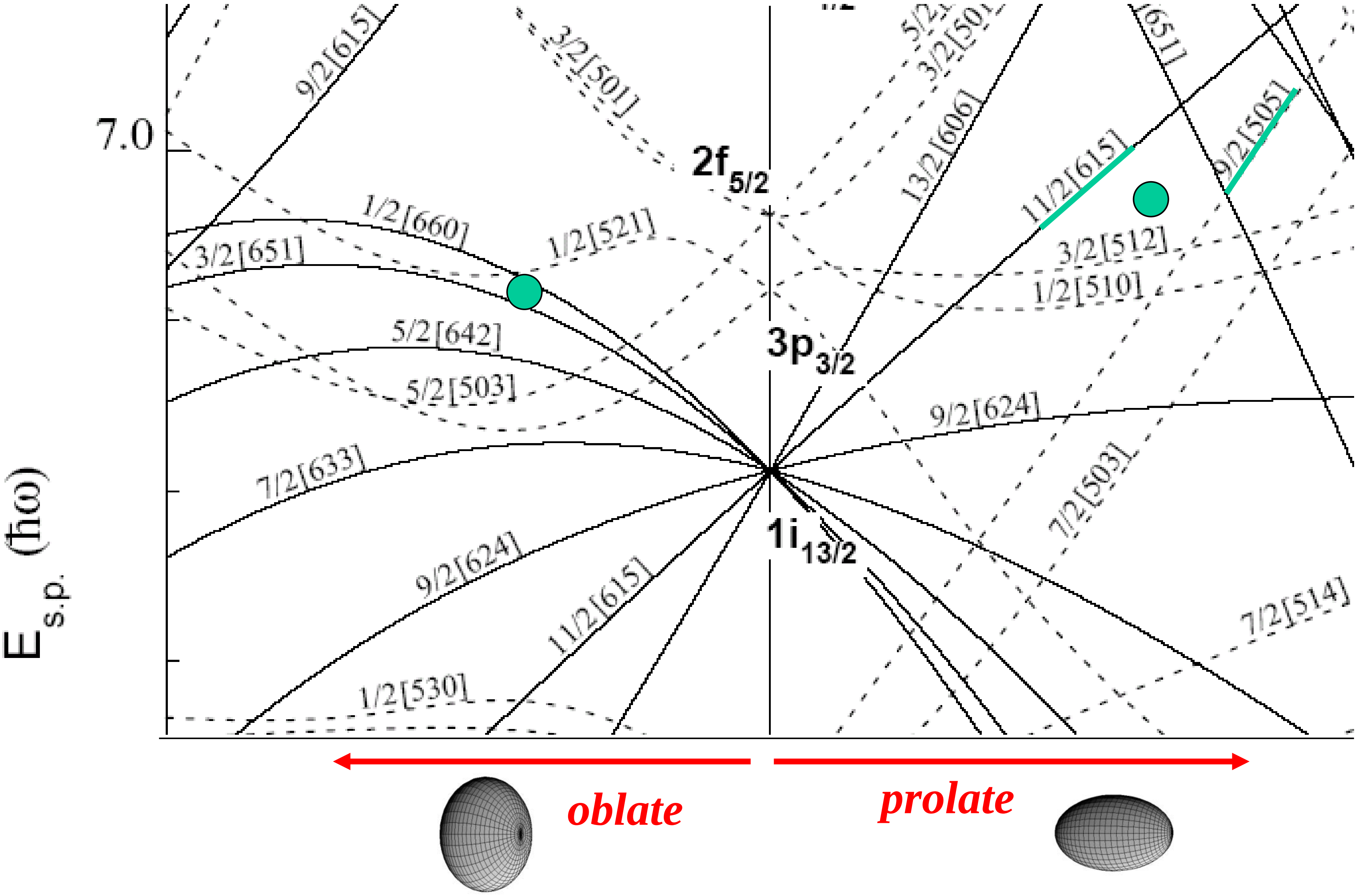
Hf prolate vs oblate



Xu et al., Phys. Rev. C62 (2000) 014301

Nilsson single-particle diagram

N = 116 (^{188}Hf, ^{190}W, ^{192}Os)



Appendix: Odd-even nuclei

Er150 18.5 s 0+ EC	Er151 23.5 s (7/2-) EC	Er152 10.3 s 0+ EC,α	Er153 37.1 s (7/2-) EC,α	Er154 3.73 m 0+ EC,α	Er155 5.3 m 7/2- EC,α	Er156 19.5 m 0+ EC	Er157 18.65 m 3/2- EC	Er158 2.29 h 0+ EC	Er159 36 m 3/2- EC	Er160 28.58 h 0+ EC	Er161 3.21 h 3/2- EC	Er162 0+ 0.14
Ho149 21.1 s (11/2-) EC	Ho150 72 s 2- EC	Ho151 35.2 s (11/2-) EC,α	Ho152 161.8 s 2- EC,α	Ho153 2.01 m 11/2- EC,α	Ho154 11.76 m (2)- EC,α	Ho155 48 m 5/2+ EC	Ho156 56 m (4+) EC	Ho157 12.6 m 7/2- EC	Ho158 11.3 m 5+ EC	Ho159 33.05 m 7/2- EC	Ho160 25.6 m 5+ EC	Ho161 2.48 h 7/2- EC
Dy148 3.1 m 0+ EC	Dy149 4.20 m (7/2-) EC	Dy150 7.17 m 0+ EC,α	Dy151 17.9 m 7/2(-) EC,α	Dy152 2.38 h 0+ EC,α	Dy153 6.4 h 7/2(-) α	Dy154 3.0E+6 y 0+ α	Dy155 9.9 h 3/2- EC	Dy156 0+ 0.06	Dy157 8.14 h 3/2- EC	Dy158 0+ 0.10	Dy159 144.4 d 3/2- EC	Dy160 0+ 2.34

Er163 75.0 m 5/2- EC	Er164 0+ 1.61	Er165 10.36 h 5/2- EC	Er166 0+ 33.6	Er167 7/2+ 22.95	Er168 0+ 26.8	Er169 9.40 d 1/2- β-	Er170 0+ 14.9	Er171 7.516 h 5/2- β-	Er172 49.3 h 0+ β-	Er173 1.4 m (7/2-) β-	Er174 3.3 m 0+ β-	Er175 1.2 m (9/2+) β-
Ho162 15.0 m 1+ EC	Ho163 4570 y 7/2- EC	Ho164 29 m 1+ EC,β-	Ho165 7/2- 100	Ho166 26.83 h 0- β-	Ho167 3.1 h 7/2- β-	Ho168 2.99 m 3+ β-	Ho169 4.7 m 7/2- β-	Ho170 2.76 m (6+) β-	Ho171 53 s (7/2-) β-	Ho172 25 s β-	Ho173	Ho174
Dy161 5/2+ 18.9	Dy162 0+ 25.5	Dy163 5/2- 24.9	Dy164 0+ 28.2	Dy165 2.334 h 7/2+ β-	Dy166 81.6 h 0+ β-	Dy167 6.20 m (1/2-) β-	Dy168 8.7 m 0+ β-	Dy169 39 s (5/2-) β-	Dy170 0+	Dy171	Dy172 0+	Dy173

$$\langle I-2,K\|M(E2)\|I,K\rangle = \sqrt{\frac{15}{32\pi}}\cdot\sqrt{\frac{(I+K-1)\cdot(I+K)\cdot(I-K-1)\cdot(I-K)}{(I-1)\cdot(2I-1)\cdot I}}\cdot Q_2 e$$

	$< I_f\|M(E2)\|I_i >$	Q_2 (b)
^{162}Dy	2.32±0.02 eb	7.36±0.03
^{163}Dy	2.31±0.02 eb	7.29±0.12
^{164}Dy	2.38±0.01 eb	7.54±0.04
^{166}Er	2.42±0.01 eb	7.67±0.03
^{167}Er	2.24±0.01 eb	7.60±0.10
^{168}Er	2.40±0.02 eb	7.61±0.06

Appendix: Odd-even nuclei

Er150 18.5 s 0+ EC	Er151 23.5 s (7/2-) ± EC	Er152 10.3 s 0+ EC,α	Er153 37.1 s (7/2-) EC,α	Er154 3.73 m 0+ EC,α	Er155 5.3 m 7/2- EC,α	Er156 19.5 m 0+ EC	Er157 18.65 m 3/2- ± EC	Er158 2.29 h 0+ EC	Er159 36 m 3/2- EC	Er160 28.58 h 0+ EC	Er161 3.21 h 3/2- EC	Er162 0+ 0.14
Ho149 21.1 s (11/2-) ± EC	Ho150 72 s 2- ± EC	Ho151 35.2 s (11/2-) ± EC,α	Ho152 161.8 s 2- ± EC,α	Ho153 2.01 m 11/2- ± EC,α	Ho154 11.76 m (2)- ± EC,α	Ho155 48 m 5/2+ EC	Ho156 56 m (4+) ± EC	Ho157 12.6 m 7/2- EC	Ho158 11.3 m 5+ ± EC	Ho159 33.05 m 7/2- ± EC	Ho160 25.6 m 5+ ± EC	Ho161 2.48 h 7/2- ± EC
Dy148 3.1 m 0+ EC	Dy149 4.20 m (7/2-) ± EC	Dy150 7.17 m 0+ EC,α	Dy151 17.9 m 7/2(-) EC,α	Dy152 2.38 h 0+ EC,α	Dy153 6.4 h 7/2(-) α	Dy154 3.0E+6 y 0+ α	Dy155 9.9 h 3/2- EC	Dy156 0+ 0.06	Dy157 8.14 h 3/2- ± EC	Dy158 0+ 0.10	Dy159 144.4 d 3/2- EC	Dy160 0+ 2.34

Er163 75.0 m 5/2- EC	Er164 0+ 1.61	Er165 10.36 h 5/2- EC	Er166 0+ 33.6	Er167 7/2+ ± 22.95	Er168 0+ 26.8	Er169 9.40 d 1/2- β-	Er170 0+ 14.9	Er171 7.516 h 5/2- β-	Er172 49.3 h 0+ β-	Er173 1.4 m (7/2-) β-	Er174 3.3 m 0+ β-	Er175 1.2 m (9/2+) β-
Ho162 15.0 m 1+ ± EC	Ho163 4570 y 7/2- ± EC	Ho164 29 m 1+ ± EC,β-	Ho165 7/2- 100	Ho166 26.83 h 0- ± β-	Ho167 3.1 h 7/2- β-	Ho168 2.99 m 3+ ± β-	Ho169 4.7 m 7/2- β-	Ho170 2.76 m (6+) ± β-	Ho171 53 s (7/2-) β-	Ho172 25 s β-	Ho173	Ho174
Dy161 5/2+ 18.9	Dy162 0+ 25.5	Dy163 5/2- 24.9	Dy164 0+ 28.2	Dy165 2.334 h 7/2+ ± β-	Dy166 81.6 h 0+ β-	Dy167 6.20 m (1/2-) β-	Dy168 8.7 m 0+ β-	Dy169 39 s (5/2-) β-	Dy170 0+	Dy171	Dy172 0+	Dy173

$$\langle I-2, K \| M(E2) \| I, K \rangle = \sqrt{\frac{15}{32\pi}} \cdot \sqrt{\frac{(I+K-1)\cdot(I+K)\cdot(I-K-1)\cdot(I-K)}{(I-1)\cdot(2I-1)\cdot I}} \cdot Q_2 e$$

$$\langle I-1, K \| M(E2) \| I, K \rangle = -\sqrt{\frac{5}{16\pi}} \cdot \sqrt{\frac{3\cdot(I+K)\cdot(I-K)\cdot K^2}{(I-1)\cdot I\cdot(I+1)}} \cdot Q_2 e$$

^{163}Dy: $$\frac{B(E2;5/2\rightarrow 7/2)}{B(E2;5/2\rightarrow 9/2)} = 2.76 \pm 0.14\ (2.86_{theo})$$

^{167}Er: $$\frac{B(E2;7/2\rightarrow 9/2)}{B(E2;7/2\rightarrow 11/2)} = 3.81 \pm 0.15\ (3.89_{theo})$$

Appendix: Odd-even nuclei

Er150 18.5 s 0+ EC	Er151 23.5 s (7/2-) ± EC	Er152 10.3 s 0+ EC,α	Er153 37.1 s (7/2-) EC,α	Er154 3.73 m 0+ EC,α	Er155 5.3 m 7/2- EC,α	Er156 19.5 m 0+ EC	Er157 18.65 m 3/2- ± EC	Er158 2.29 h 0+ EC	Er159 36 m 3/2- EC	Er160 28.58 h 0+ EC	Er161 3.21 h 3/2- EC	Er162 0+ 0.14
Ho149 21.1 s (11/2-) ± EC	Ho150 72 s 2- ± EC	Ho151 35.2 s (11/2-) ± EC,α	Ho152 161.8 s 2- ± EC,α	Ho153 2.01 m 11/2- ± EC,α	Ho154 11.76 m (2)- ± EC,α	Ho155 48 m 5/2+ EC	Ho156 56 m (4+) ± EC	Ho157 12.6 m 7/2- EC	Ho158 11.3 m 5+ ± EC	Ho159 33.05 m 7/2- ± EC	Ho160 25.6 m 5+ ± EC	Ho161 2.48 h 7/2- ± EC
Dy148 3.1 m 0+ EC	Dy149 4.20 m (7/2-) ± EC	Dy150 7.17 m 0+ EC,α	Dy151 17.9 m 7/2(-) EC,α	Dy152 2.38 h 0+ EC,α	Dy153 6.4 h 7/2(-) α	Dy154 3.0E+6 y 0+ α	Dy155 9.9 h 3/2- EC	Dy156 0+ 0.06	Dy157 8.14 h 3/2- ± EC	Dy158 0+ 0.10	Dy159 144.4 d 3/2- EC	Dy160 0+ 2.34

Er163 75.0 m 5/2- EC	Er164 0+ 1.61	Er165 10.36 h 5/2- EC	Er166 0+ 33.6	Er167 7/2+ ± 22.95	Er168 0+ 26.8	Er169 9.40 d 1/2- β-	Er170 0+ 14.9	Er171 7.516 h 5/2- β-	Er172 49.3 h 0+ β-	Er173 1.4 m (7/2-) β-	Er174 3.3 m 0+ β-	Er175 1.2 m (9/2+) β-
Ho162 15.0 m 1+ ± EC	Ho163 4570 y 7/2- ± EC	Ho164 29 m 1+ ± EC,β-	Ho165 7/2- 100	Ho166 26.83 h 0- ± β-	Ho167 3.1 h 7/2- β-	Ho168 2.99 m 3+ ± β-	Ho169 4.7 m 7/2- β-	Ho170 2.76 m (6+) ± β-	Ho171 53 s (7/2-) β-	Ho172 25 s β-	Ho173	Ho174
Dy161 5/2+ 18.9	Dy162 0+ 25.5	Dy163 5/2- 24.9	Dy164 0+ 28.2	Dy165 2.334 h 7/2+ ± β-	Dy166 81.6 h 0+ β-	Dy167 6.20 m (1/2-) β-	Dy168 8.7 m 0+ β-	Dy169 39 s (5/2-) β-	Dy170 0+	Dy171	Dy172 0+	Dy173

$^{11}/_{2}{}^{-}$ 0.282 $\tau = 0.38 \pm 0.07\ ns$

$^{9}/_{2}{}^{-}$ 0.167 $\tau = 0.49 \pm 0.09\ ns$

$^{7}/_{2}{}^{-}$ 0.073 $\tau = 2.18 \pm 0.07\ ns$

$^{5}/_{2}{}^{-}$ 0.0

^{163}Dy

$$\tau = \left\{ \sum_K \sum_\ell \left[\varepsilon^2_{N\to K}(\ell) + \delta^2_{N\to K}(\ell) \right] \right\}^{-1}$$

$$\delta_{N\to M}(\ell) = \left\{ \frac{8\pi(\ell+1)}{\ell[(2\ell+1)!!]^2} \frac{1}{\hbar} \left(\frac{\hbar\omega}{\hbar c} \right)^{2\ell+1} \right\}^{1/2} \cdot (2I_N+1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(\ell) \| I_N \rangle$$

$$\delta_{N\to M}(E2) = \left\{ 1.225 \cdot 10^{13} \cdot E_\gamma^5 (MeV)^5 \right\}^{1/2} \cdot (2I_n+1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(E2) \| I_N \rangle$$

$$\delta_{N\to M}(M1) = \left\{ 1.758 \cdot 10^{13} \cdot E_\gamma^3 (MeV)^3 \right\}^{1/2} \cdot (2I_n+1)^{-1/2} \cdot \langle I_M \| \mathcal{M}(M1) \| I_N \rangle$$

$$\varepsilon^2_{N\to M}(\ell) = \delta^2_{N\to M}(\ell) \cdot \alpha_{N\to M}(\ell)$$

conversion coefficient.: bricc.anu.edu.au

Appendix: Odd-even nuclei

Er150 18.5 s 0+ EC	Er151 23.5 s (7/2-) EC	Er152 10.3 s 0+ EC,α	Er153 37.1 s (7/2-) EC,α	Er154 3.73 m 0+ EC,α	Er155 5.3 m 7/2- EC,α	Er156 19.5 m 0+ EC	Er157 18.65 m 3/2- EC	Er158 2.29 h 0+ EC	Er159 36 m 3/2- EC	Er160 28.58 h 0+ EC	Er161 3.21 h 3/2- EC	Er162 0+ 0.14
Ho149 21.1 s (11/2-) EC	Ho150 72 s 2- EC	Ho151 35.2 s (11/2-) EC,α	Ho152 161.8 s 2- EC,α	Ho153 2.01 m 11/2- EC,α	Ho154 11.76 m (2)- EC,α	Ho155 48 m 5/2+ EC	Ho156 56 m (4+) EC	Ho157 12.6 m 7/2- EC	Ho158 11.3 m 5+ EC	Ho159 33.05 m 7/2- EC	Ho160 25.6 m 5+ EC	Ho161 2.48 h 7/2- EC
Dy148 3.1 m 0+ EC	Dy149 4.20 m (7/2-) EC	Dy150 7.17 m 0+ EC,α	Dy151 17.9 m 7/2(-) EC,α	Dy152 2.38 h 0+ EC,α	Dy153 6.4 h 7/2(-) α	Dy154 3.0E+6 y 0+ α	Dy155 9.9 h 3/2- EC	Dy156 0+ 0.06	Dy157 8.14 h 3/2- EC	Dy158 0+ 0.10	Dy159 144.4 d 3/2- EC	Dy160 0+ 2.34

Er163 75.0 m 5/2- EC	Er164 0+ 1.61	Er165 10.36 h 5/2- EC	Er166 0+ 33.6	Er167 7/2+ 22.95	Er168 0+ 26.8	Er169 9.40 d 1/2- β-	Er170 0+ 14.9	Er171 7.516 h 5/2- β-	Er172 49.3 h 0+ β-	Er173 1.4 m (7/2-) β-	Er174 3.3 m 0+ β-	Er175 1.2 m (9/2+) β-
Ho162 15.0 m 1+ EC	Ho163 4570 y 7/2- EC	Ho164 29 m 1+ EC,β-	Ho165 7/2- 100	Ho166 26.83 h 0- β-	Ho167 3.1 h 7/2- β-	Ho168 2.99 m 3+ β-	Ho169 4.7 m 7/2- β-	Ho170 2.76 m (6+) β-	Ho171 53 s (7/2-) β-	Ho172 25 s β-	Ho173	Ho174
Dy161 5/2+ 18.9	Dy162 0+ 25.5	Dy163 5/2- 24.9	Dy164 0+ 28.2	Dy165 2.334 h 7/2+ β-	Dy166 81.6 h 0+ β-	Dy167 6.20 m (1/2-) β-	Dy168 8.7 m 0+ β-	Dy169 39 s (5/2-) β-	Dy170 0+	Dy171	Dy172 0+	Dy173

$$\tau = \left\{ \sum_K \sum_\ell \left[\varepsilon^2_{N\to K}(\ell) + \delta^2_{N\to K}(\ell) \right] \right\}^{-1}$$

$^{11}/_2{}^-$ 0.282

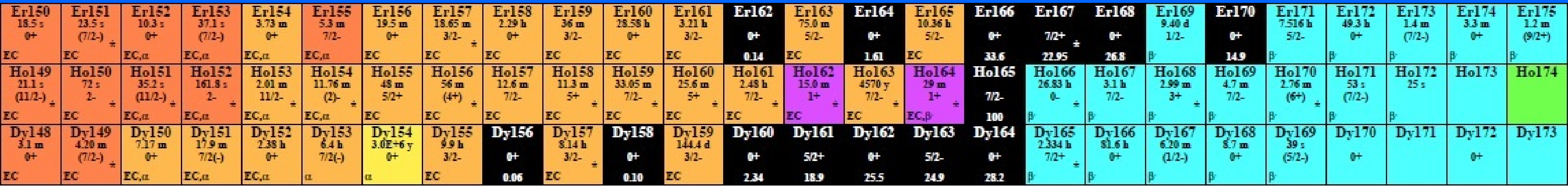
$^{9}/_2{}^-$ 0.167 $\tau = 0.49 \pm 0.09\ ns$

$^{7}/_2{}^-$ 0.073 $\tau = 2.18 \pm 0.07\ ns$

$^{5}/_2{}^-$ 0.0

^{163}Dy

Appendix: Odd-even nuclei



$13/2^+$ — 0.295 $\tau = 0.38 \pm 0.07\ ns$

$11/2^+$ — 0.178 $\tau = 0.49 \pm 0.09\ ns$

$9/2^+$ — 0.079 $\tau = 2.18 \pm 0.07\ ns$

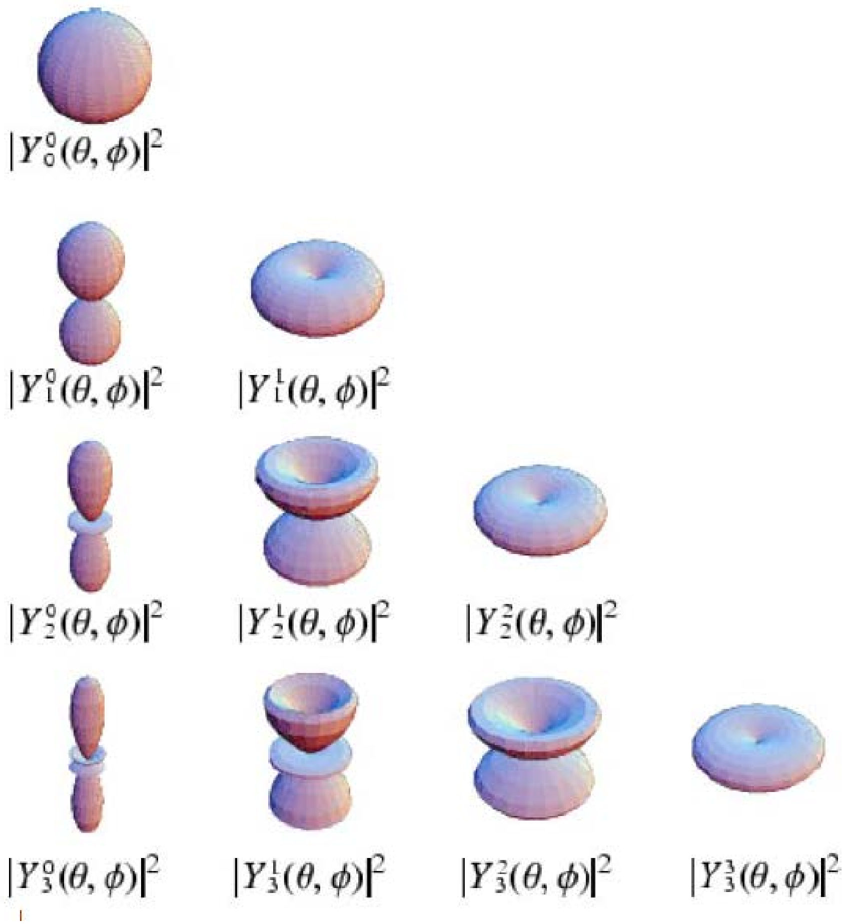
$7/2^+$ — 0.0

^{167}Er

$$\tau = \left\{ \sum_K \sum_\ell \left[\varepsilon^2_{N\to K}(\ell) + \delta^2_{N\to K}(\ell) \right] \right\}^{-1}$$

Spin I	E_γ (MeV)	$(2I+1)^{-1/2}$	<I-1//M()//I>	delta	α_T	ε^2	τ (ns)
7/2	0.0734	0.3536	-3.886 (E2)	-7019.1	8.9	$4.38\cdot10^8$	2.05
			0.108 (M1)	3183.7	5.7	$5.78\cdot10^7$	1.80
9/2	0.0939	0.3162	-4.002 (E2)	-11968	3.4	$4.87\cdot10^8$	1.59
			0.153 (M1)	5837.1	2.9	$9.88\cdot10^7$	1.31
	0.167	0.3162	2.299 (E2)	29133	0.4	$3.65\cdot10^8$	0.51

Appendix: Spherical harmonics



$$Y_{00}(\theta,\phi)=\frac{1}{\sqrt{4\pi}}$$

$$Y_{10}(\theta,\phi)=\frac{1}{2}\cdot\sqrt{\frac{3}{\pi}}\cdot\cos\theta$$

$$Y_{1\pm1}(\theta,\phi)=\mp\frac{1}{2}\cdot\sqrt{\frac{3}{2\pi}}\cdot\sin\theta\cdot e^{\pm i\phi}$$

$$Y_{20}(\theta,\phi)=\sqrt{\frac{5}{16\pi}}\cdot\left(3\cdot\cos^2\theta-1\right)$$

$$Y_{2\pm1}(\theta,\phi)=\mp\sqrt{\frac{15}{8\pi}}\cdot\sin\theta\cdot\cos\theta\cdot e^{\pm i\phi}$$

$$Y_{2\pm2}(\theta,\phi)=\sqrt{\frac{15}{32\pi}}\cdot\sin^2\theta\cdot e^{\pm 2i\phi}$$

$$Y_{30}(\theta,\phi)=\sqrt{\frac{7}{16\pi}}\cdot\left(2\cos^3\theta-3\cos\theta\sin^2\theta\right)$$

$$Y_{3\pm1}(\theta,\phi)=\mp\sqrt{\frac{21}{64\pi}}\cdot\left(4\cos^2\theta\sin\theta-\sin^3\theta\right)\cdot e^{\pm i\phi}$$

$$Y_{3\pm2}(\theta,\phi)=\sqrt{\frac{105}{32\pi}}\cdot\cos\theta\sin^2\theta\cdot e^{(\pm2)i\phi}$$

$$Y_{3\pm3}(\theta,\phi)=\mp\sqrt{\frac{35}{64\pi}}\cdot\sin^3\theta\cdot e^{(\pm3)i\phi}$$