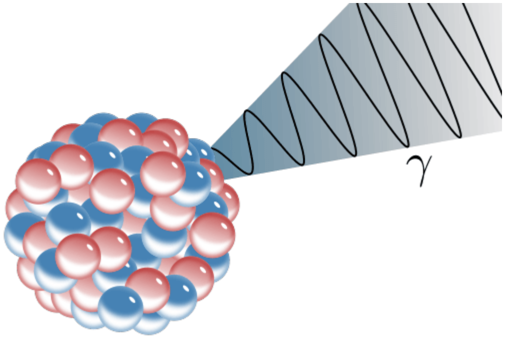
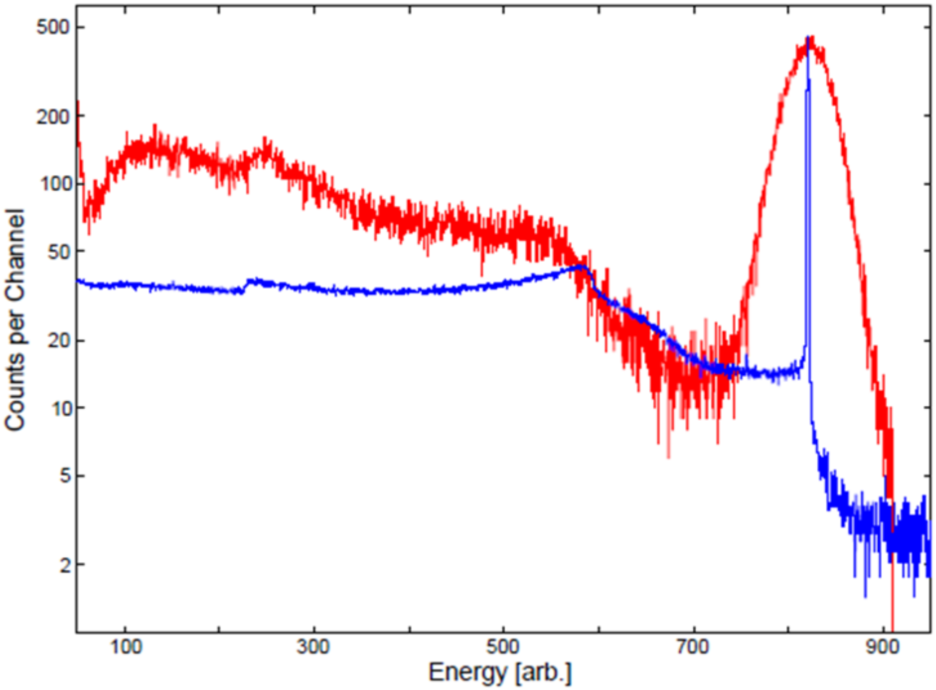
Building a level scheme: γ-decay

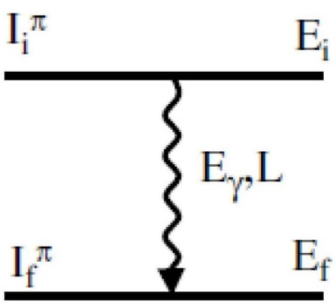
❖ Gamma-ray emission is usually the dominant decay mode

Measurements of γ-rays let us deduce:
Energy, Spin (angular distr. / correl.), Parity (polarization), magnetic moment, lifetime (recoil distance, Doppler shift), …
of the involved nuclear levels.



^{137}Cs detected in red: NaI scintillator
blue: HPGe (high purity Ge semiconductor)

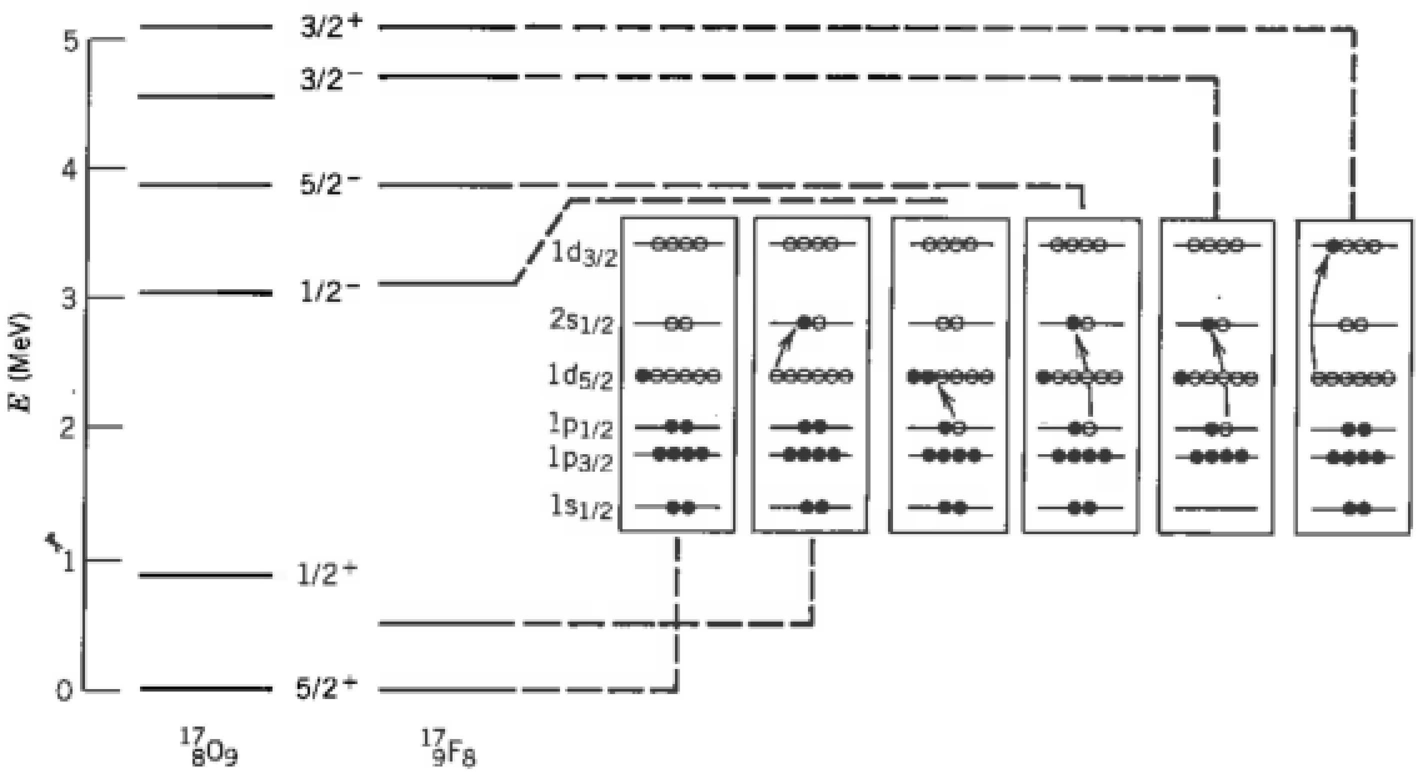




$$\boxed{{}^{A}_{Z}X^{*}_{N} \rightarrow {}^{A}_{Z}X^{(*)}_{N}}$$

γ-decay in a Nutshell

- The photon emission of the nucleus essentially results from a re-ordering of nucleons within the shells.
- This re-ordering often follows α or β decay, and moves the system into a more energetically favorable state.



Source: Krane, Fig. 5.11

γ-decay

γ-spectroscopy yields some of the most precise knowledge of nuclear structure, as spin, parity and ΔE are all measurable.

Transition rates between initial Ψ_N^* and final $\Psi_N^{\prime}$ nuclear states, resulting from electromagnetic decay producing a photon with energy E_γ can be described by Fermi´s Golden rule:

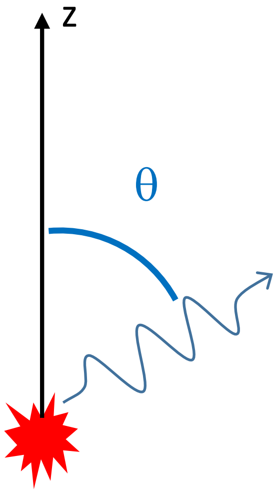
$$\lambda = \frac{2\pi}{\hbar} \left| \langle \Psi_N^{\prime} \psi_\gamma | \mathcal{M}_{em} | \Psi_N^* \rangle \right|^2 \frac{dn_\gamma}{dE_\gamma}$$

where $\mathcal{M}_{em}$ is the electromagnetic transition operator and dn_γ / dE_γ is the density of final states. The photon wave function ψ_γ and $\mathcal{M}_{em}$ are well known, therefore measurements of λ provide detailed knowledge of nuclear structure.

A γ-decay lifetime is typically 10^{-12} [s] and sometimes even as short as 10^{-19} [s]. However, this time span is an eternity in the life of an excited nucleon. It takes about $4 \cdot 10^{-22}$ [s] for a nucleon to cross the nucleus.

- **Level and γ properties**
  - Measured E_γ's → level energies
  - Efficiency-corrected γ-peak areas → intensities, branching ratios, level populations
  - γ ADs/ACs, γ polarizations, internal conversion → multipolarity → level spin, parity

Intensity of EM field (emitted photon) given by Poynting vector, which depends on spherical harmonics $Y_{lm}(\theta,\phi)$, where θ is relative to quantization axis

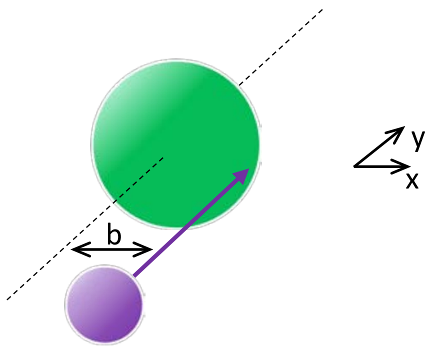


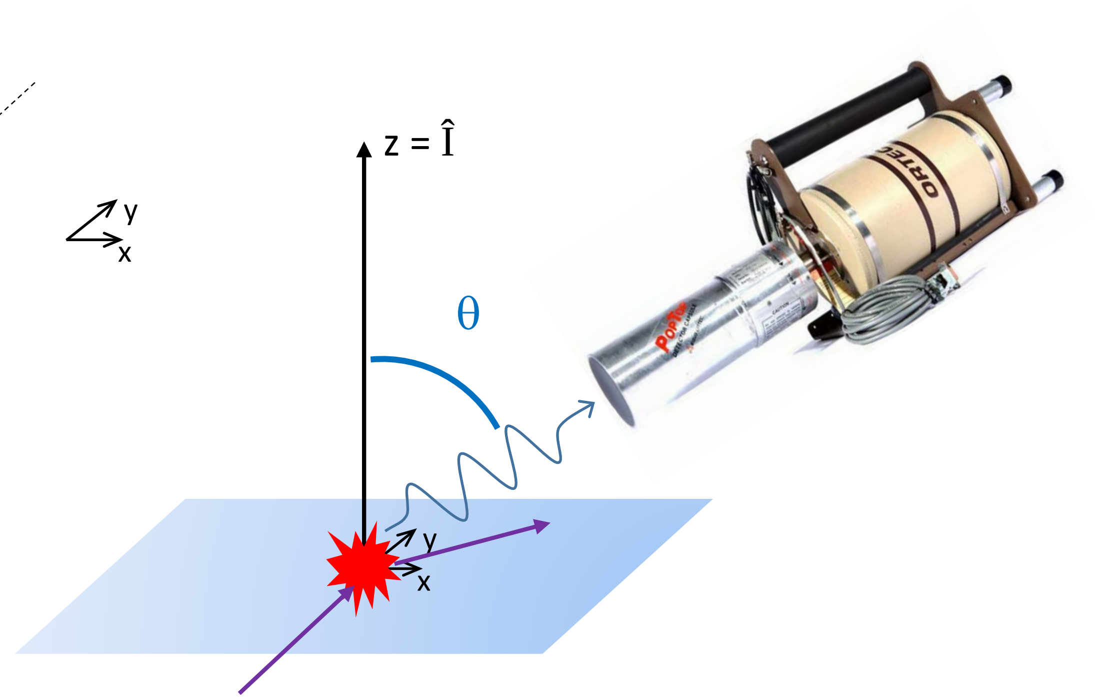
AD: angular distribution, AC: angular correlation

Building a level scheme

- **Angular distributions**
  - Quantization axis = spin direction
  - May be known event by event

Reaction plane defines spin vector





Building a level scheme

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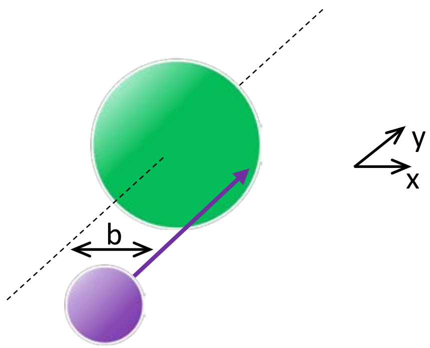
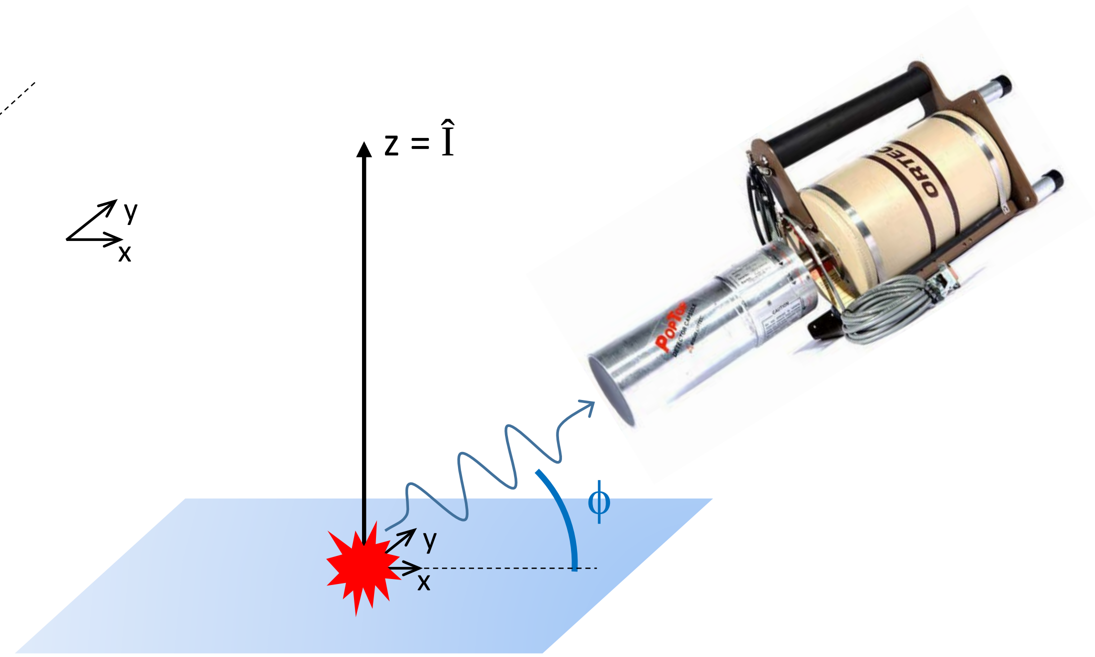


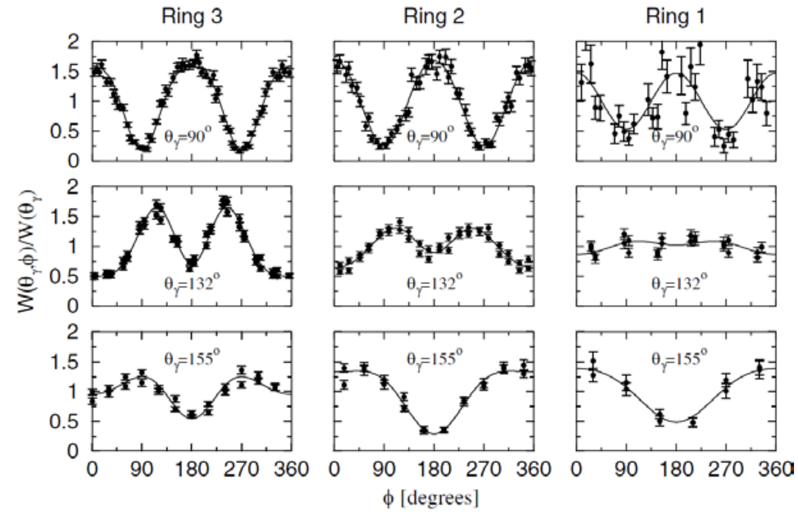


Detect outgoing particle trajectory to determine reaction plane, measure angle ϕ relative to that

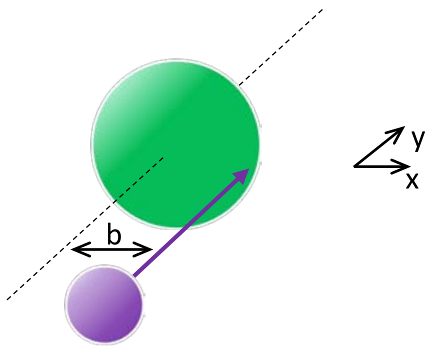
(ϕ is the difference between *lab* angles ϕ_γ and ϕ_p)

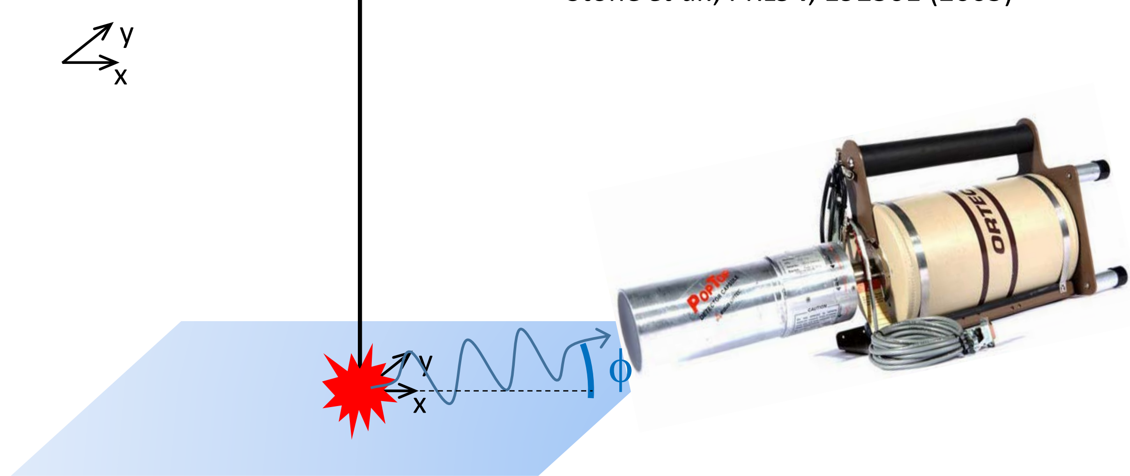
Building a level scheme

- **Angular distributions**
  - Quantization axis = spin direction
  - May be known event by event



Reaction plane defines spin vector





Stone *et al.*, PRL**94**, 192501 (2005)

Detect outgoing particle trajectory to determine reaction plane, measure angle ϕ relative to that

Measure intensity as a function of angle

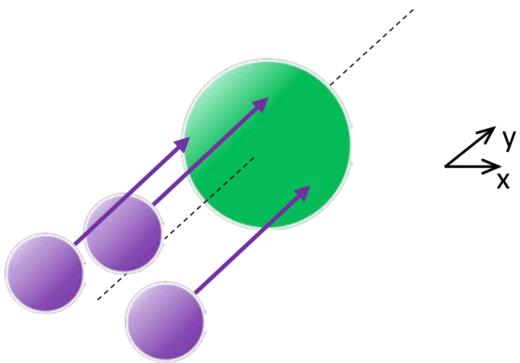
(ϕ is the difference between *lab* angles ϕ_γ and ϕ_p)

Building a level scheme

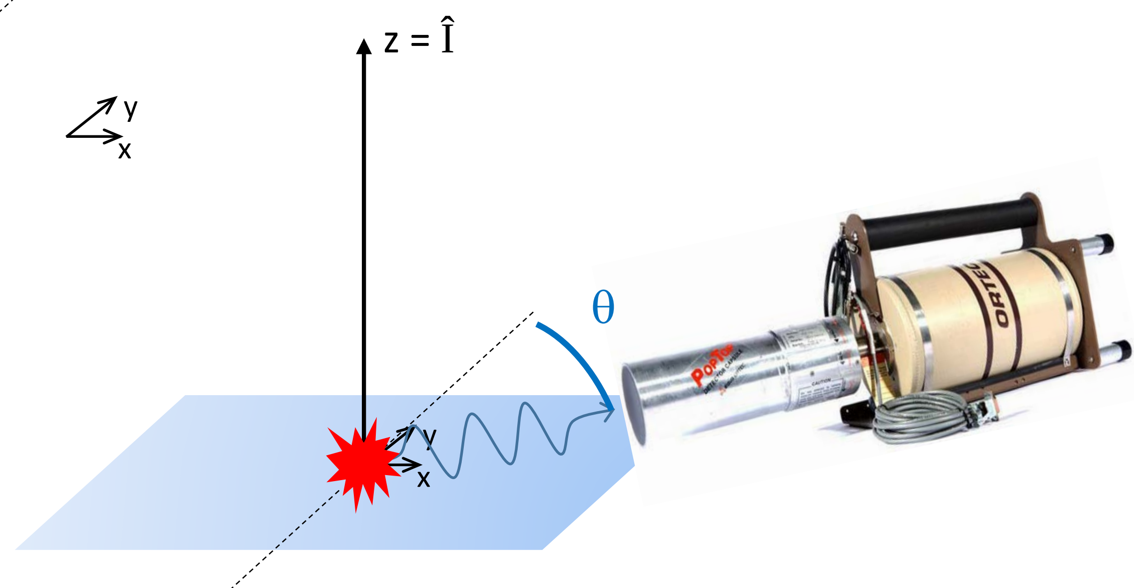
- **Angular distributions**
  - Quantization axis = spin direction
  - May be known event by event
  - …or it may not! What to do then?

$$W(\theta) = \sum_{k=0}^{L} A_{2k} P_{2k}(\cos\theta)$$

Reaction plane still defines spin vector, even if not determined experimentally



Ensemble of spins perpendicular to beam direction, symmetrically distributed ➔ use beam axis for quantization



Measure intensity as a function of angle

Building a level scheme

- **Angular distributions**
  - Quantization axis = spin direction
  - May be known event by event
  - …or it may not! What to do then?

$$W(\theta) = \sum_{k=0}^{L} A_{2k} P_{2k}(\cos\theta)$$

Pure dipole only has P_2 term, quadrupole adds P_4, octupole has P_6, etc.

With M1/E2 mixing, $A_4 \neq 0$; mixing ratio δ* can be deduced, providing insight into underlying structure

Strictly speaking, AD in singles; “gated AD”: γγ coincidence with no angle condition on gating γ (clean-up only) → if 4π coverage, same AD as in singles

AD: angular distribution, AC: angular correlation

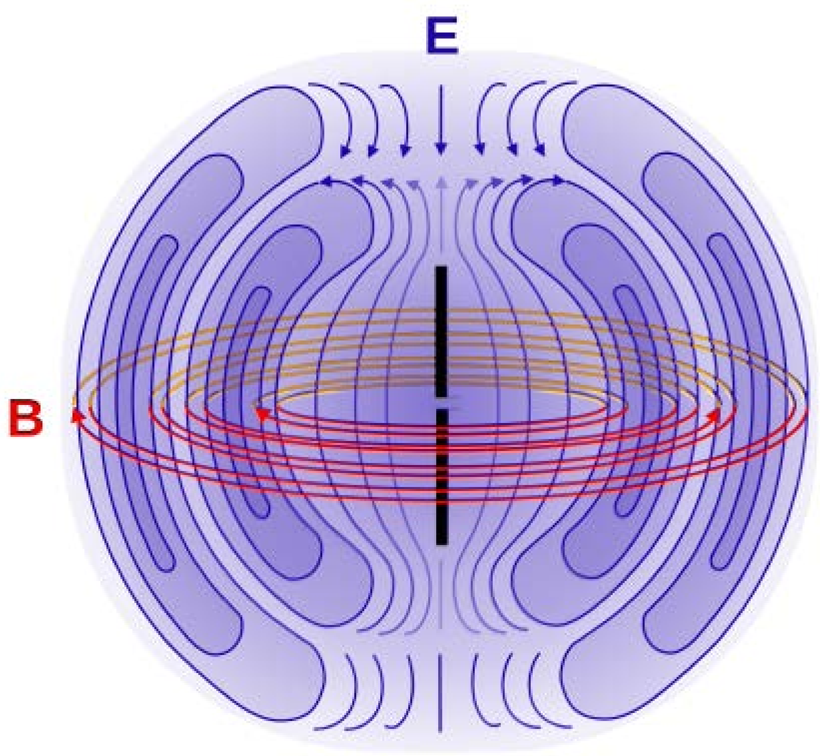
* δ^2 = (E2 intensity)/(M1 intensity)

Electric/Magnetic Dipoles

REMINDER

Electric and magnetic dipole fields have opposite parity:
Magnetic dipoles have even parity and electric dipole fields have odd parity.

$$\boxed{\Rightarrow\ \pi(M\ell) = (-1)^{\ell+1} \quad and \quad \pi(E\ell) = (-1)^{\ell}}$$



Higher Order Multipoles

It is possible to describe the angular distribution of the radiation field as a function of the *multipole order* using Legendre polynomials.

- ℓ: The index of radiation
  2^ℓ: The multipole order of the radiation

- $\ell = 1 \quad \rightarrow \quad Dipole$
  $\ell = 2 \quad \rightarrow \quad Quadrupole$
  $\ell = 3 \quad \rightarrow \quad Octupole$

- The associated Legendre polynomials $P_{2\ell}\big(cos(\theta)\big)$ are:
  For $\ell = 1$: $P_2 = \frac{1}{2}(3 \cdot cos^2(\theta) - 1)$
  For $\ell = 2$: $P_4 = \frac{1}{8}(35cos^4(\theta) - 30cos^2(\theta) + 3)$

REMINDER

Angular Momentum in γ-Decay

- *The photon is a spin-1 boson*

- Like α-decay and β-decay the emitted γ-ray can carry away units of *angular momentum* ℓ, which has given us different multipolarities for transitions.

- For orbital angular momentum, we can have values $\ell = 0,1,2,3,\cdots$ that correspond to our multipolarity.

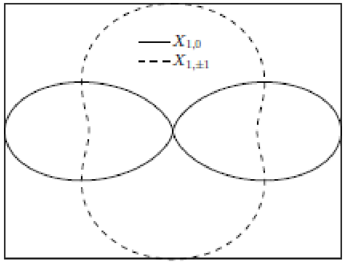
- Therefore, our selection rule is:

$$\boxed{|J_i - J_f| \leq \ell \leq |J_i + J_f|}$$

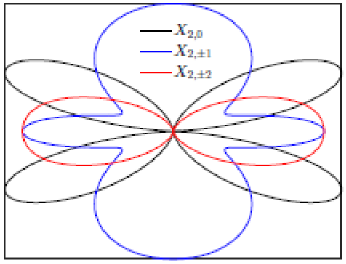
Characteristics of multipolarity

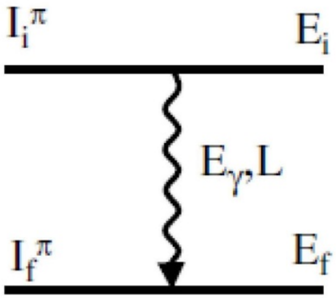
L	multipolarity	π(Eℓ) / π(Mℓ)	angular distribution
1	dipole	-1 / +1	
2	quadrupole	+1 / -1	
3	octupole	-1 / +1	
4	hexadecapole	+1 / -1	
⋮			

$\ell = 1$



$\ell = 2$





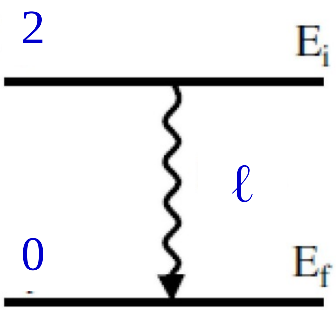
$$E_\gamma = E_i - E_f$$

$$|I_i - I_f| \leq \ell \leq I_i + I_f$$

$$\Delta\pi(E\ell) = (-1)^\ell$$

$$\Delta\pi(M\ell) = (-1)^{\ell+1}$$

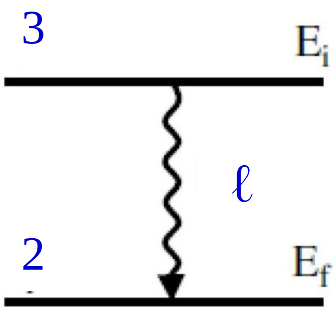
The basics of the situation



$$|2-0| \leq \ell \leq 2+0$$

Here $\Delta I = 2$ and $\ell = 2$
this is a stretched transition

The basics of the situation

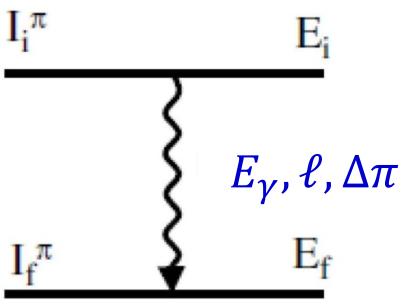


$$|3-2| \leq \ell \leq 3+2$$

Here $\Delta I = 1$ but $\ell = 1,2,3,4,5$
and the transition can be a mix of 5 multipolarities

REMINDER

The basics of the situation



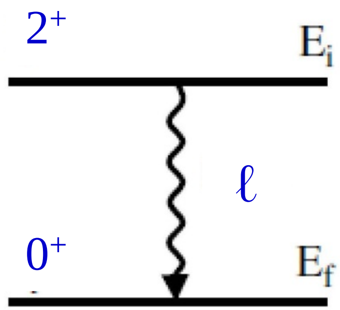
Electromagnetic transitions:

$$\Delta\pi\ (electric) = (-1)^{\ell}$$

$$\Delta\pi\ (magnetic) = (-1)^{\ell+1}$$

$\Delta\pi$	*yes*	E1	M2	E3	M4
	no	M1	E2	M3	E4

The basics of the situation

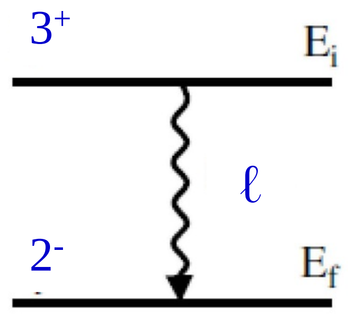


$$|2-0| \leq \ell \leq 2+0$$

$\ell = 2$ and no change in parity

$\Delta\pi$					
	no	M1	E2	M3	E4

The basics of the situation



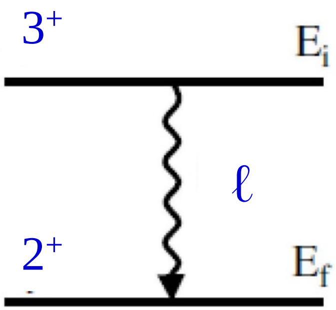
$$|3-2| \leq \ell \leq 3+2$$

Here $\Delta I = 1$ but $\ell = 1,2,3,4,5$

$\Delta\pi$	*yes*	E1	M2	E3	M4

mixed E1,M2,E3,M4,E5

The basics of the situation



$$|3-2| \leq \ell \leq 3+2$$

Here $\Delta I = 1$ but $\ell = 1,2,3,4,5$

$\Delta\pi$					
	no	M1	E2	M3	E4

mixed M1,E2,M3,E4,M5

The basics of the situation

$3^+ \rightarrow 2^+$: mixed M1,E2,M3,E4,M5

$3^+ \rightarrow 2^-$: mixed E1,M2,E3,M4,E5

In general only the lowest 2 multipoles compete

and (for reasons we will see later)

$\ell + 1$ multipole generally only competes if it is electric:

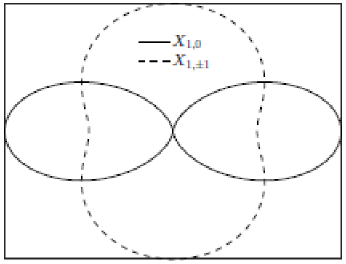
$3^+ \rightarrow 2^+$: mixed M1/E2

$3^+ \rightarrow 2^-$: almost pure E1 (very little M2 admixture)

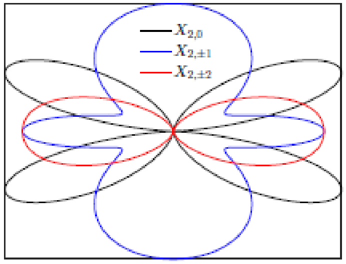
Characteristics of multipolarity

L	multipolarity	π(Eℓ) / π(Mℓ)	angular distribution
1	dipole	-1 / +1	
2	quadrupole	+1 / -1	
3	octupole	-1 / +1	
4	hexadecapole	+1 / -1	
⋮			

ℓ = 1



ℓ =2



parity: electric multipoles $\pi(E\ell) = (-1)^{\ell}$, magnetic multipoles $\pi(M\ell) = (-1)^{\ell+1}$

The power radiated is proportional to:

$$P(\sigma\ell) \propto \frac{2(\ell+1)\cdot c}{\varepsilon_0 \cdot \ell \cdot [(2\ell+1)!!]^2} \left(\frac{\omega}{c}\right)^{2\ell+2} |\mathcal{M}(\sigma\ell)|^2$$

where σ means either E or M and $\mathcal{M}(\sigma\ell)$ is the E or M multipole moment of the appropriate kind.

Emission of electromagnetic radiation

REMINDER

$$T(E1; I_i \rightarrow I_f) = 1.590\ 10^{17}\ E_\gamma^3\ B(E1; I_i \rightarrow I_f)$$

$$T(E2; I_i \rightarrow I_f) = 1.225\ 10^{13}\ E_\gamma^5\ B(E2; I_i \rightarrow I_f)$$

$$T(E3; I_i \rightarrow I_f) = 5.709\ 10^{8}\ E_\gamma^7\ B(E3; I_i \rightarrow I_f)$$

$$T(E4; I_i \rightarrow I_f) = 1.697\ 10^{4}\ E_\gamma^9\ B(E4; I_i \rightarrow I_f)$$

$$T(M1; I_i \rightarrow I_f) = 1.758\ 10^{13}\ E_\gamma^3\ B(M1; I_i \rightarrow I_f)$$

$$T(M2; I_i \rightarrow I_f) = 1.355\ 10^{7}\ E_\gamma^5\ B(M2; I_i \rightarrow I_f)$$

$$T(M3; I_i \rightarrow I_f) = 6.313\ 10^{0}\ E_\gamma^7\ B(M3; I_i \rightarrow I_f)$$

$$T(M4; I_i \rightarrow I_f) = 1.877\ 10^{-6}\ E_\gamma^9\ B(M4; I_i \rightarrow I_f)$$

where $E_\gamma = E_i - E_f$ is the energy of the emitted γ quantum in MeV (E_i, E_f are the nuclear level energies, respectively), and the reduced transition probabilities B(Eℓ) in units of $e^2(\mathrm{barn})^\ell$ and B(Mℓ) in units of $\mu_N^2 = (e\hbar/2m_N c)^2\ \ (fm)^{2\ell-2}$

Single particle transition (Weisskopf estimate)

REMINDER

$$B(E\lambda; I_i \rightarrow I_{gs}) = \frac{(1.2)^{2\lambda}}{4\pi}\left(\frac{3}{\lambda+3}\right)^2 A^{2\lambda/3} \quad e^2 (fm)^{2\lambda}$$

$$B(M\lambda; I_i \rightarrow I_{gs}) = \frac{10}{\pi}(1.2)^{2\lambda-2}\left(\frac{3}{\lambda+3}\right)^2 A^{(2\lambda-2)/3} \quad \mu_N^2 (fm)^{2\lambda-2}$$

For the first few values of λ, the Weisskopf estimates are

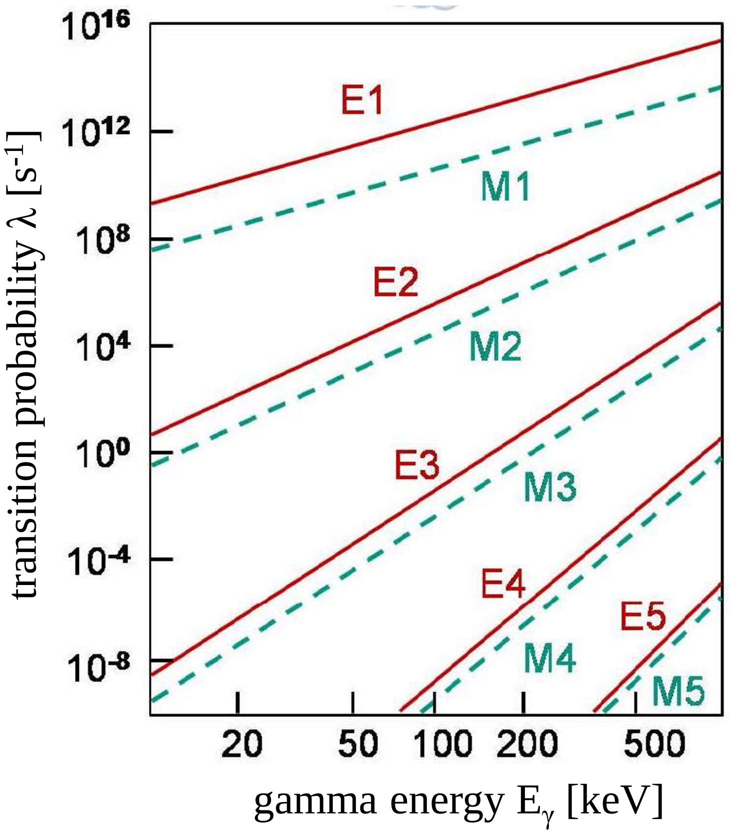
$$B(E1; I_i \rightarrow I_{gs}) = 6.446\ 10^{-4}\ A^{2/3} \quad e^2(barn)$$

$$B(E2; I_i \rightarrow I_{gs}) = 5.940\ 10^{-6}\ A^{4/3} \quad e^2(barn)^2$$

$$B(E3; I_i \rightarrow I_{gs}) = 5.940\ 10^{-8}\ A^{2} \quad e^2(barn)^3$$

$$B(E4; I_i \rightarrow I_{gs}) = 6.285\ 10^{-10}\ A^{8/3} \quad e^2(barn)^4$$

$$B(M1; I_i \rightarrow I_{gs}) = 1.790 \quad \left(\frac{e\hbar}{2Mc}\right)^2$$



Measuring g-factors

Angular correlations → *g*-factors ← Half-lives

Stuchbery *et al.*, PRC**88**, 051304 (2013)

Recoil-in-vacuum (RIV) technique

- Nucleus produced in reaction in oriented state
- Recoil exits target into vacuum → spin is deoriented by hyperfine interactions
- Deorientation dependent on *g*-factor

$$W\underbrace{\left(\theta_p,\theta_\gamma,\Delta\phi\right)} = 1 + \sum_{kq} B_{kq}(\theta_p)Q_k G_k F_k D^{k*}_{q0}(\Delta\phi,\theta_\gamma,0)$$

angles of p(articles) and γ's

statistical tensor defining initial spin alignment

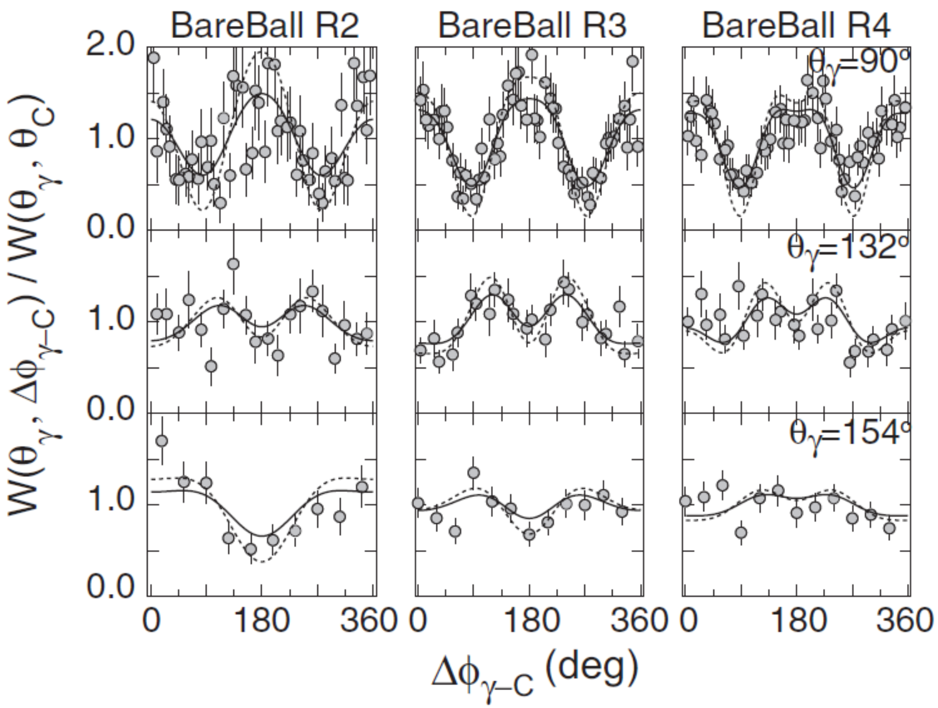
attenuation factor for finite γ detector size

"usual *F* coefficients" for ACs/ADs

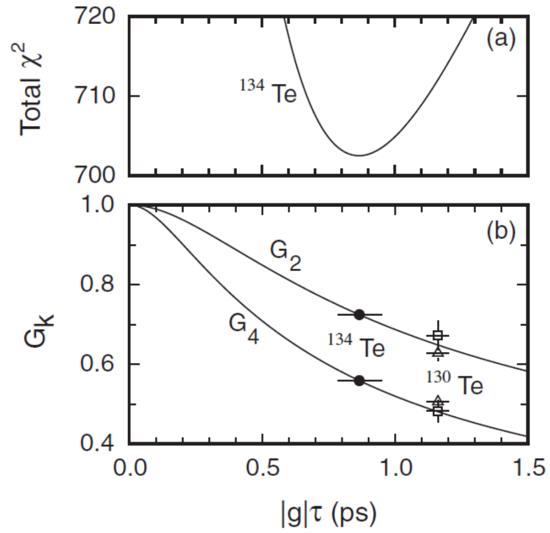
rotation matrix giving angle dependence of correlation → $P(\cos\theta_\gamma)$

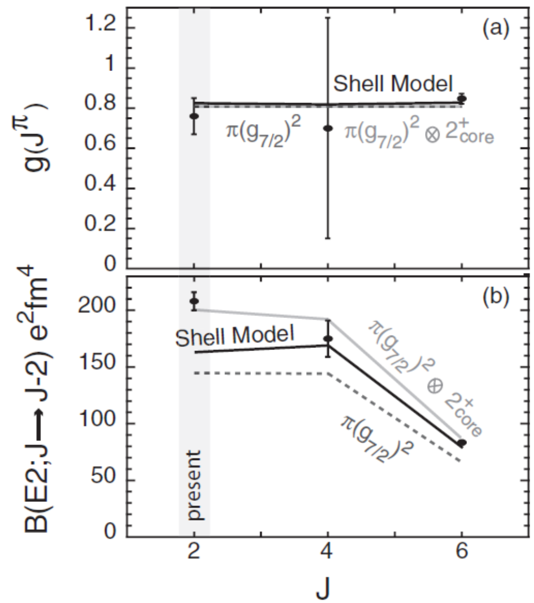
Measuring g-factors

- **R**ecoil **I**n **V**acuum in ^{134}Te
  - Coulex of ^{134}Te RIB (and also ^{130}Te SIB)
  - Simultaneously measure $B(E2)$ and g-factor for 2^+
  - Observe attenuated ACs → relate to G_k → g-factor



Stuchbery *et al.*, PRC**88**, 051304 (2013)

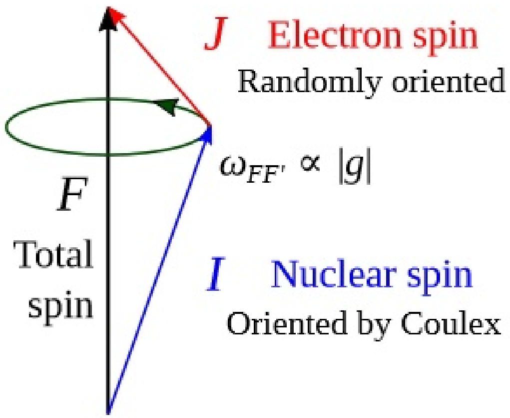




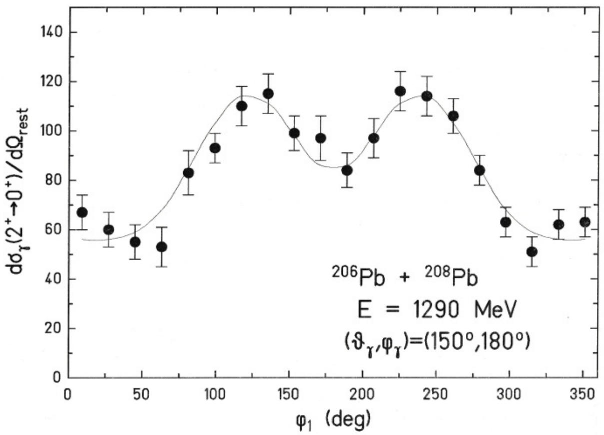
Measuring g-factors

During the collision, the projectile can be firstly excited reaching a certain excited state (I_f).

Before a γ-decay, the scattered projectile and the target recoil nucleus exit the target into the vacuum highly excited and ionized.



particle-γ angular correlation



❖ $g(2^+) = 0$ → unperturbed p-γ angular correlation

At that time, the scattered projectile decays rapidly by gamma emissions. However, the strong fluctuations of the hyperfine fields can lead to a de-orientation of the nuclear state.

$$G_k = [1 + k \cdot (k+1) \cdot \lambda \cdot \tau(I)]^{-1}$$

$$\lambda = 1/3 \cdot \tau_c \cdot \omega^2$$

$$\omega = g \cdot \frac{H \cdot \mu_N}{\hbar}$$

A. Abragam & R.V. Pound, Phys. Rev. 92 (1953) 943 $G_4 = {}^{3 \cdot G_2}/_{(10 - 7 \cdot G_2)}$

Building a level scheme

- **Angular distributions**
  - Quantization axis = spin direction
  - May be known event by event
  - …or it may not! What to do then?
  - Spins not aligned indefinitely → AD attenuated through hyperfine interactions

If there is no spin orientation, all substates contribute equally and $W(\theta)$ = constant

What to do if alignment is lost (or was never there), e.g. for γ's below **isomers**?

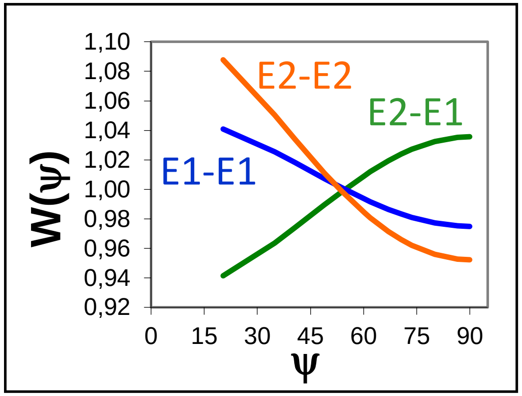
AD: angular distribution, AC: angular correlation

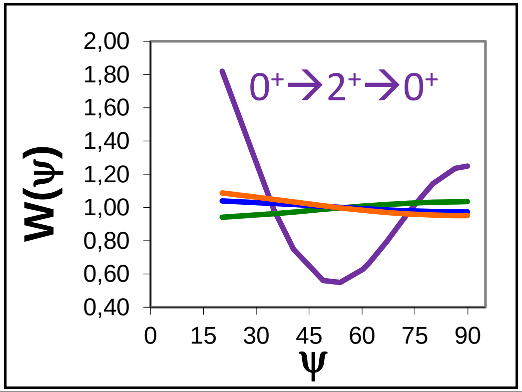
Building a level scheme

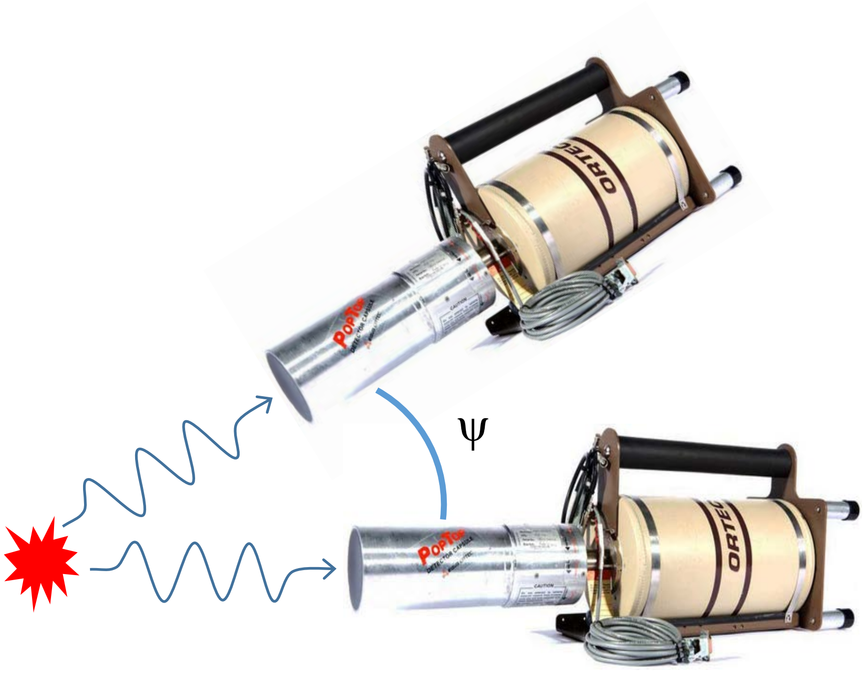
- **Angular correlations**
  - Coincidence technique
  - Detect first γ → defines quantization axis
  - Detect second γ → determine relative angle

$$W(\psi) = \sum_{k=0}^{L} A_{2k} P_{2k}(\cos\psi)$$

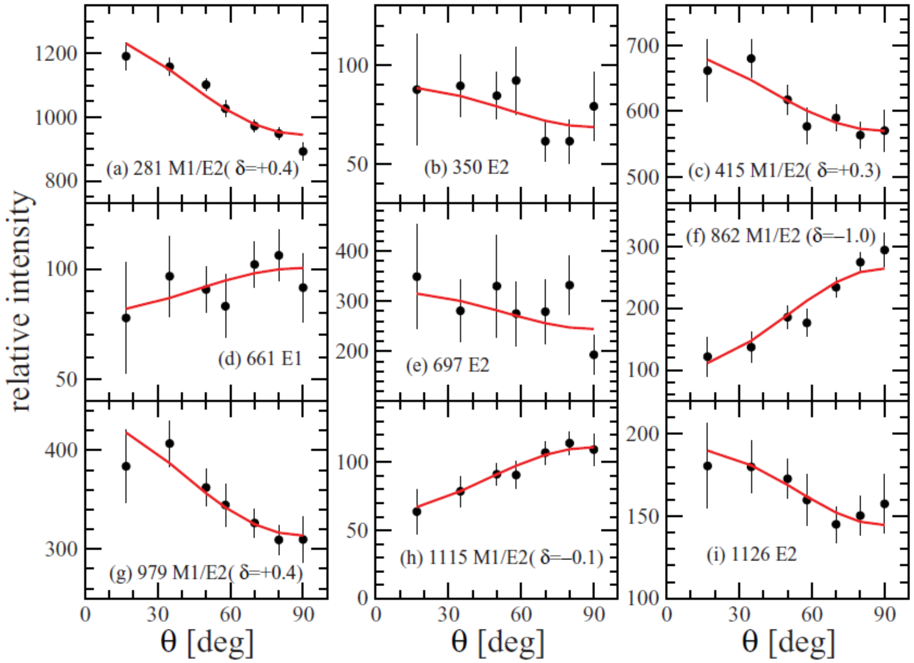
Same form as for AD, different coefficients.
L limit determined by lowest multipole.



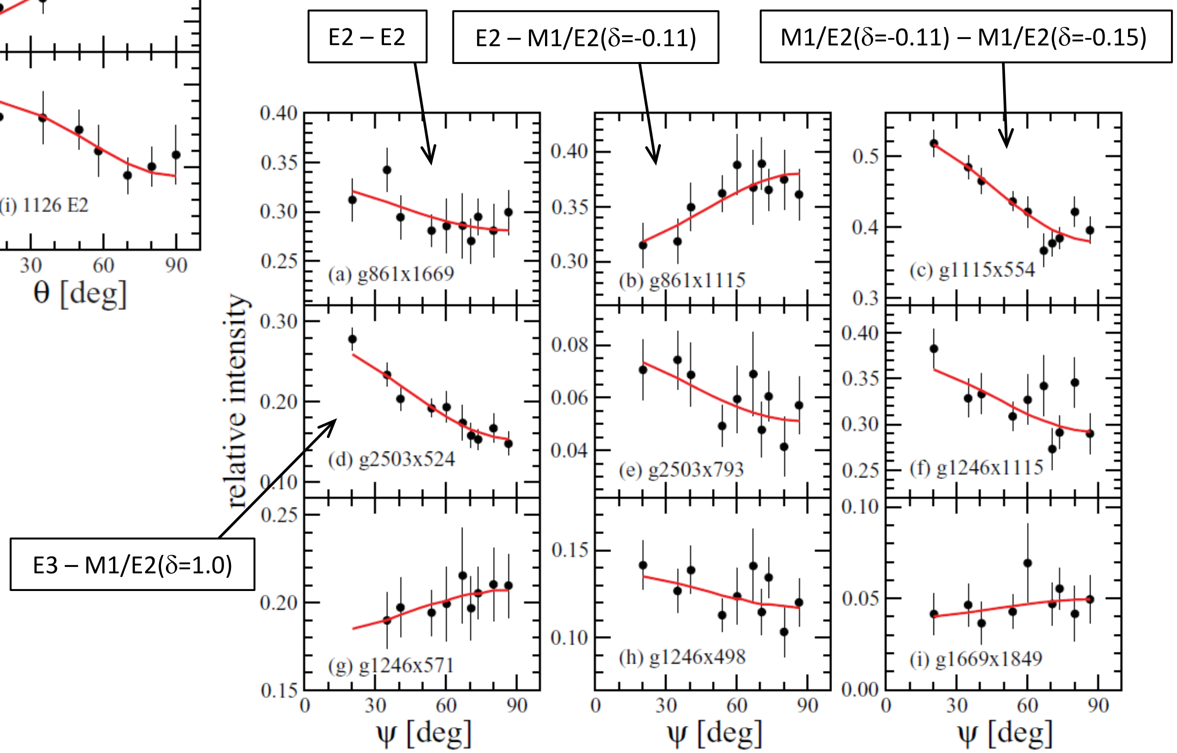




Building a level scheme



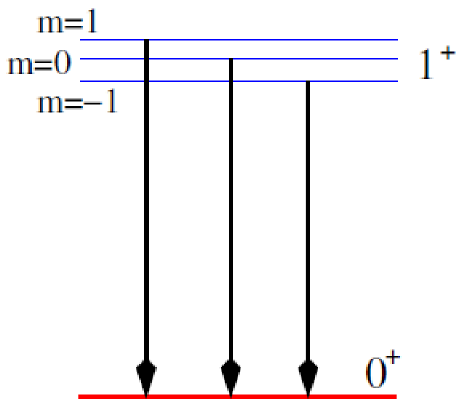
^{65}Cu ADs – ^{48}Ca+^{26}Mg



Chiara *et al.*, PRC**85**, 024309 (2012)

^{67}Cu ACs – ^{64}Ni+^{238}U

The basics of the situation



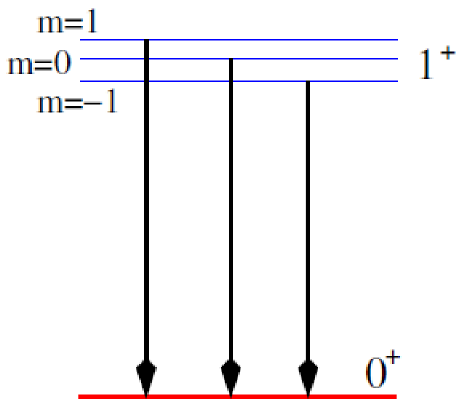
Imagine the situation of an M1 decay between two states, the initial one has J^π value of 1^+ and the final one a J^π of 0^+

The initial $J^\pi=1^+$ state has 3 degenerate magnetic substates which differ by the magnetic quantum numbers m of ± 1 and 0.

The final $J^\pi=0^+$ state has a single magnetic substate with m=0.

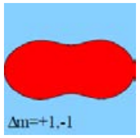
When the substates of $J^\pi=1^+$ state decay, the γ-rays emitted have different angular patterns.

The basics of the situation

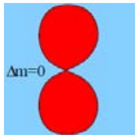


For the M1 case the angular distributions W(θ) are:

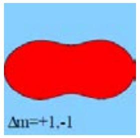
$$W_{M1,\Delta m=1}(\theta) = \frac{3}{16\pi}(1 + cos^2\theta)$$



$$W_{M1,\Delta m=0}(\theta) = \frac{3}{8\pi} sin^2\theta$$

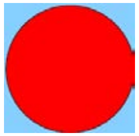


$$W_{M1,\Delta m=-1}(\theta) = \frac{3}{16\pi}(1 + cos^2\theta)$$



So the total distribution is $W_{M1} = \frac{1}{3}W_{M1,\Delta m=1} + \frac{1}{3}W_{M1,\Delta m=0} + \frac{1}{3}W_{M1,\Delta m=-1}$

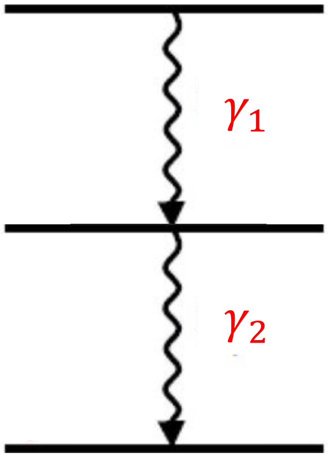
$$= \frac{1}{8\pi}(1 + cos^2\theta + sin^2\theta) = \frac{1}{4\pi}$$



no angular dependence

Angular correlation – non-oriented source

REMINDER



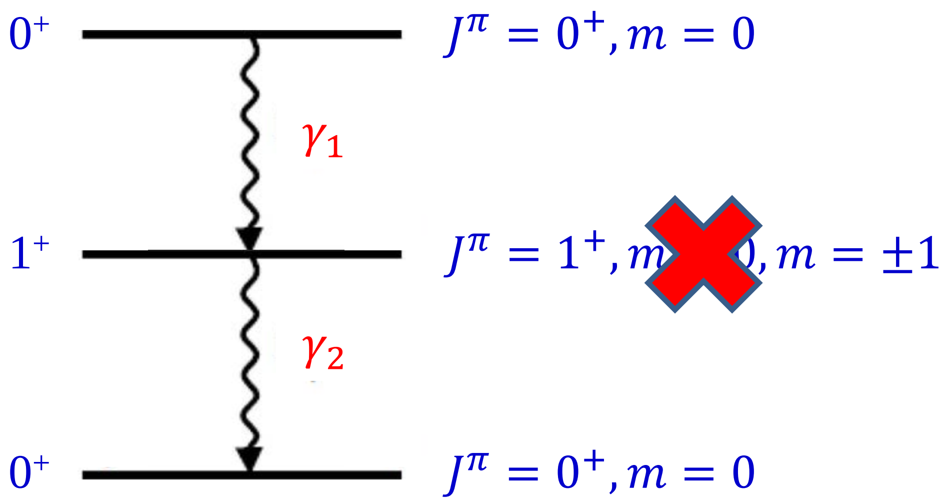
Let's imagine we have two γ-rays which follow immediately after each other in the level scheme.

If we measure γ_1 or γ_2 in singles, then the distribution will be isotropic (same intensity at all angles) ... there is no preferred direction of emission

Now imagine that we measure γ_1 and γ_2 in coincidence. We say that measuring γ_1 causes the intermediate state to be aligned. We define the z-direction as the direction of γ_1

The angular distribution of the emission of γ_2 then depends on the spin/parities of the states involved and on the multipolarity of the transition.

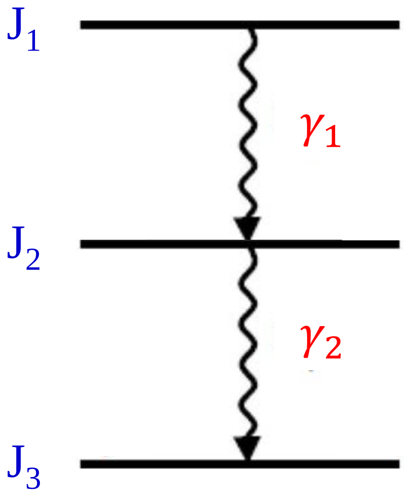
A simple example:



Hence, for γ_2 we only see the m=±1 to m=0 part of the distribution i.e. we see that the intensity measured as a function of angle (relative to γ_1) follows a $1 + cos^2\theta$ distribution.

REMINDER

General formula



$I_1(\ell_1)$	$I_2(\ell_2)$	I_3	a_2	a_4
0 (1)	1 (1)	0	1	0
1 (1)	1 (1)	0	-1/3	0
1 (2)	1 (1)	0	-1/3	0
2 (1)	1 (1)	0	1/13	0
3 (2)	1 (1)	0	-3/29	0
0 (2)	2 (2)	0	-3	4
1 (1)	2 (2)	0	-1/3	0
2 (1)	2 (2)	0	3/7	0
2 (2)	2 (2)	0	-15/13	16/13
3 (2)	2 (2)	0	-3/29	0
4 (2)	2 (2)	0	1/8	1/24

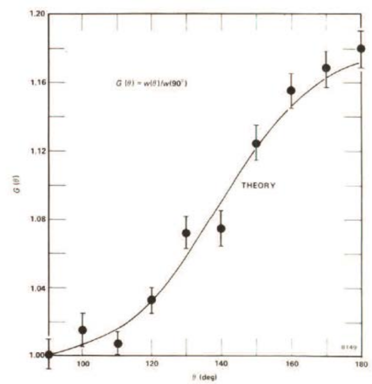
In general, the γ-ray intensity varies as:

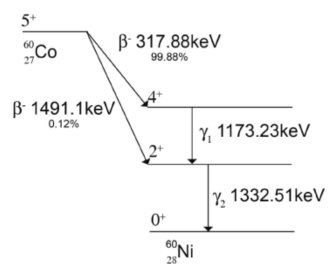
$$\boxed{W(\theta) = \sum_{k_{even}} A_k(\gamma_1)A_k(\gamma_2)Q_k(\gamma_1)Q_k(\gamma_2)P_k(cos\theta)}$$

where
θ is the relative angle between the two γ-rays
Q_k accounts for the fact that we do not have point detectors
A_k depends on the details of the transition and the spins of the level

$$P_0 = 1 \quad P_2 = \frac{1}{2}(3 \cdot cos^2(\theta) - 1) \quad P_4 = \frac{1}{8}(35cos^4(\theta) - 30cos^2(\theta) + 3)$$

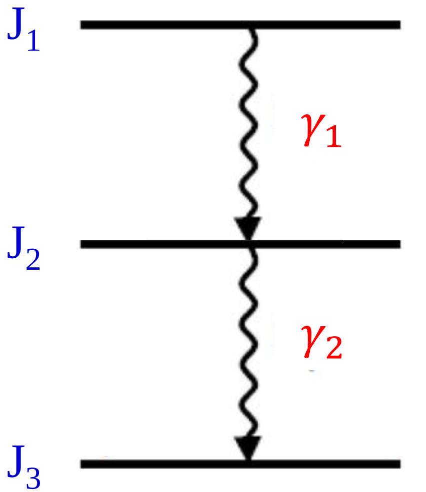
$$\boxed{W(\theta) = 1 + a_2 cos^2\theta + a_4 cos^4\theta}$$





R.D. Evans, The Atomic Nucleus

General formula



In general, the γ-ray intensity varies as:

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$$A_k(\gamma_1) = \frac{F_k(J_2J_1\ell,\ell) - 2\cdot\delta\cdot F_k\ (J_2J_1\ell,\ell+1) + \delta^2\cdot F_k\ (J_2J_1\ell+1,\ell+1)}{1+\delta^2}$$

$$A_k(\gamma_2) = \frac{F_k(J_2J_3L,L) - 2\cdot\delta\cdot F_k\ (J_2J_3L,L+1) + \delta^2\cdot F_k\ (J_2J_3L+1,L+1)}{1+\delta^2}$$

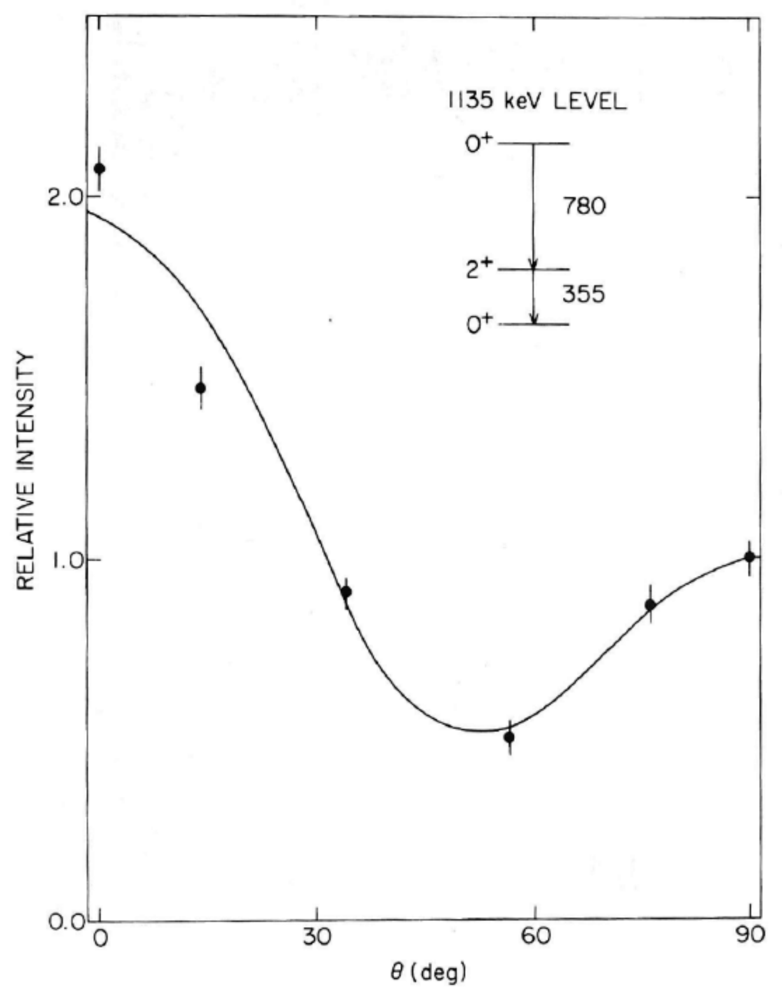
Ferentz-Rosenzweig coefficients

$$F_k(LL'I_1I_2) = (-1)^{I_1+I_2+1}\sqrt{2k+1}\sqrt{2L+1}\sqrt{2L'+1}\sqrt{2I_2+1}\begin{pmatrix} L & L' & k \\ 1 & -1 & 0 \end{pmatrix}\begin{Bmatrix} L & L' & k \\ I_1 & I_1 & I_2 \end{Bmatrix}$$

https://griffincollaboration.github.io/AngularCorrelationUtility/

A special case:

$^{195}_{78}Pt(n,\gamma)^{196}_{78}Pt$



Angular correlations with arrays

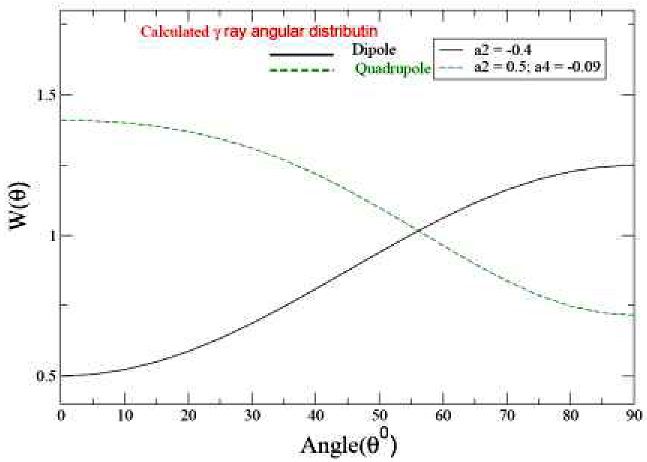
Many arrays are designed symmetrically, so the range of possible angles is reduced.

Therefore one measures a Directional Correlation from Oriented Nuclei (DCO ratio)

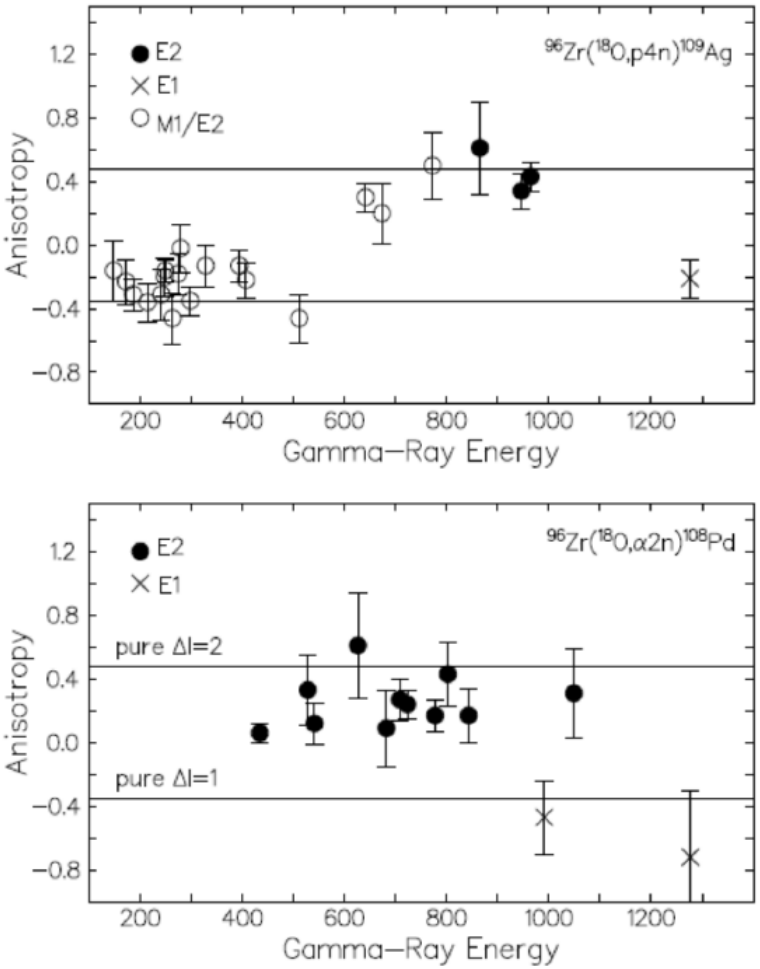
In the simplest case, if you have an array with detectors at 35^0 and 90^0.
Gate on 90^0 detector, measure coincident intensities in

- other 90^0 detectors
- 35^0 detectors

Take the ratio and compare with calculations ... can usually separate quadrupoles from dipoles but cannot measure mixing ratios



Angular correlations with arrays

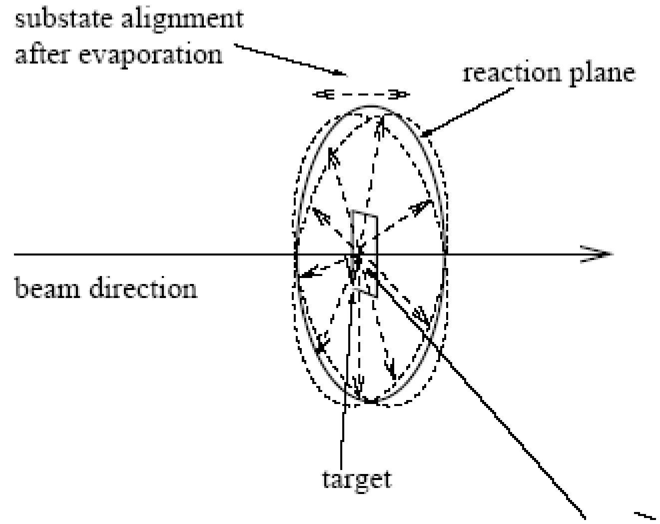


K.R.Pohl et al., Phys Rev C53 (1996) 2682

Angular distribution

In heavy-ion fusion-evaporation reactions, the compound nuclei have their spin aligned in a plane perpendicular to the beam axis:

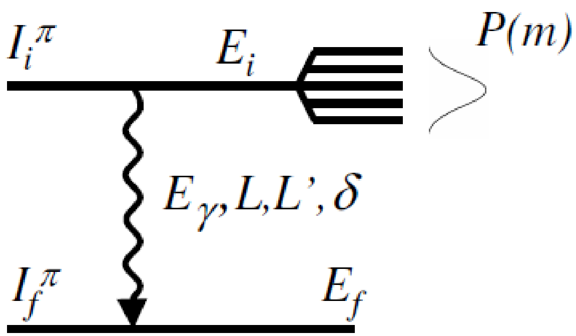
$$\boxed{\vec{\ell} = \vec{r} \times \vec{p}}$$



Depending on the number and type of particles 'boiled off' before a γ-ray is emitted, transitions are emitted from oriented nuclei and therefore their intensity shows an angular dependence.

$$W(\theta) = A_0\left(1 + \frac{A_2}{A_0} \cdot B_2 \cdot Q_2 \cdot P_2\,(cos\theta) + \frac{A_4}{A_0} \cdot B_4 \cdot Q_4 \cdot P_4(cos\theta)\right)$$

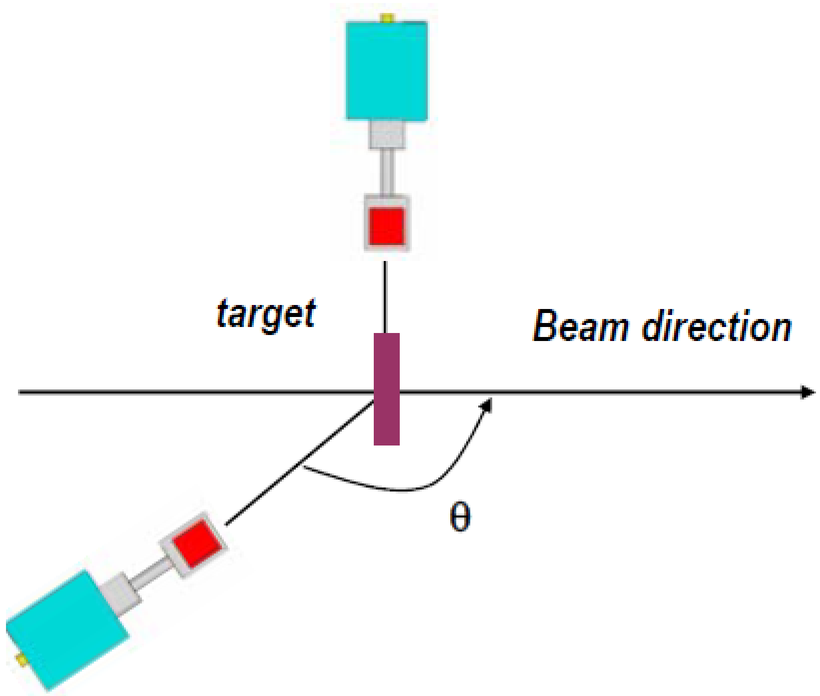
where A_k, Q_k and P_k are as before and B_k contains information about the alignment of the state

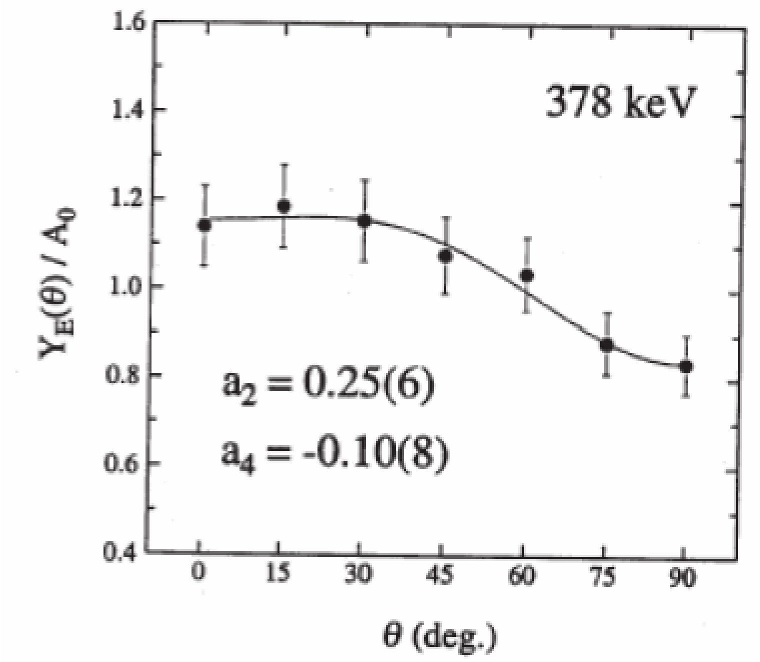


$$B_k(I_i) = \sqrt{2I_i + 1}\sum_{m=-I}^{+I}(-1)^{I_i - m}\langle I_i m I_i - m | k0\rangle P(m)$$

$$P(m) = \frac{exp\left(-\frac{m^2}{2\sigma^2}\right)}{\sum_{m'=-I}^{+I} exp\left(-\frac{m'^2}{2\sigma^2}\right)}$$

Angular distribution





Measure: the γ-ray yield as a function of θ

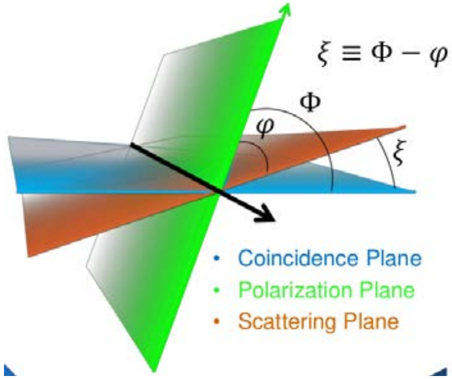
Building a level scheme

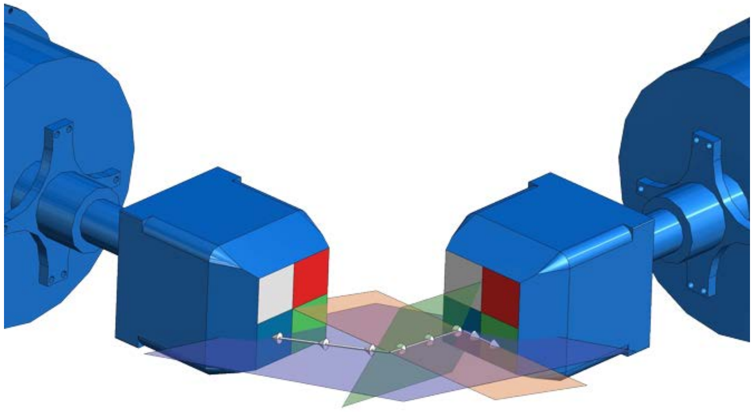
- **ADs/ACs → multipole order L, but do not directly distinguish E vs M**
  - Pure ΔI=1 E1 looks the same as pure ΔI=1 M1, e.g.
  - Nonzero mixing ratio may change that, as M1/E2 far more likely than E1/M2
  - For more direct determination of E or M: measure **polarization**

Measure Compton scattering within and perpendicular to reaction plane

$$P \propto \frac{aN_{\perp} - N_{\parallel}}{aN_{\perp} + N_{\parallel}}$$

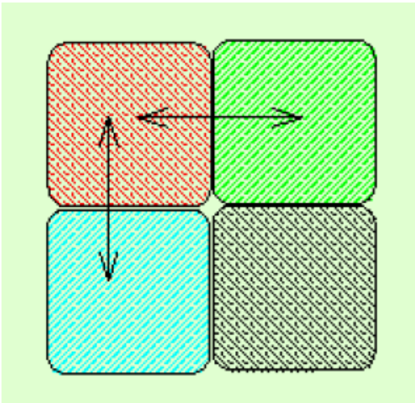
(a corrects for different intrinsic count rates)



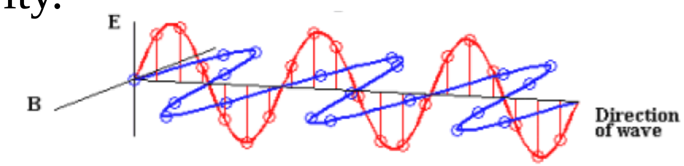


AD: angular distribution, AC: angular correlation

Linear polarization



A segmented detector can be used to measure the linear polarization which can be used to distinguish between magnetic (M) and electric (E) character of radiation of the same multipolarity.



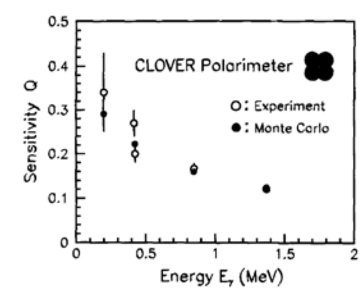
The Compton scattering cross section is larger in the direction perpendicular to the electrical field vector of the radiation.

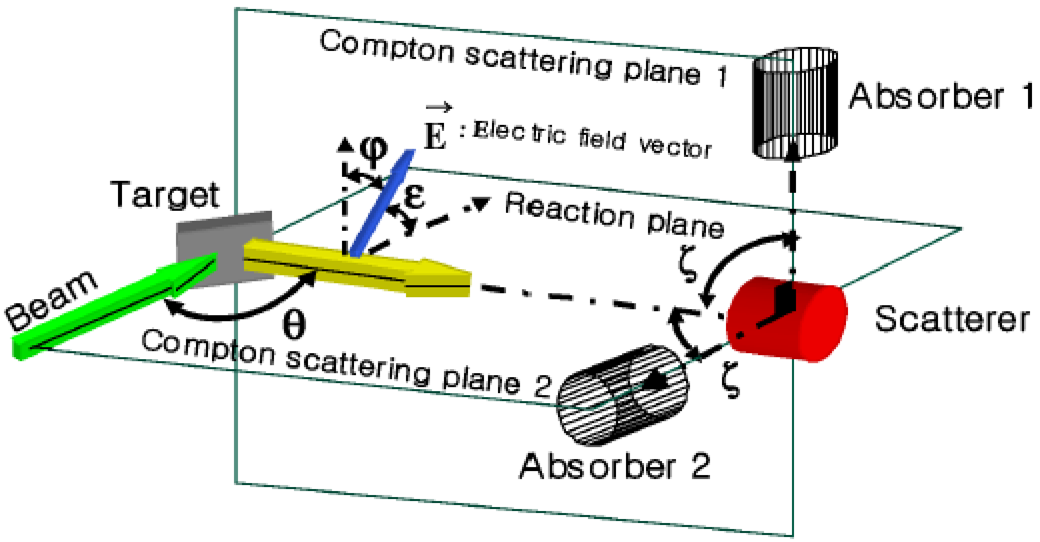
Define experimental asymmetry as: $A = \frac{N_{90} - N_0}{N_{90} + N_0}$

where N_{90} and N_0 are the intensities of scattered photons perpendicular and parallel to the reaction plane.

The experimental linear polarization P=A/Q where Q is the polarization sensitivity of the detector

Q~13% at 1 MeV



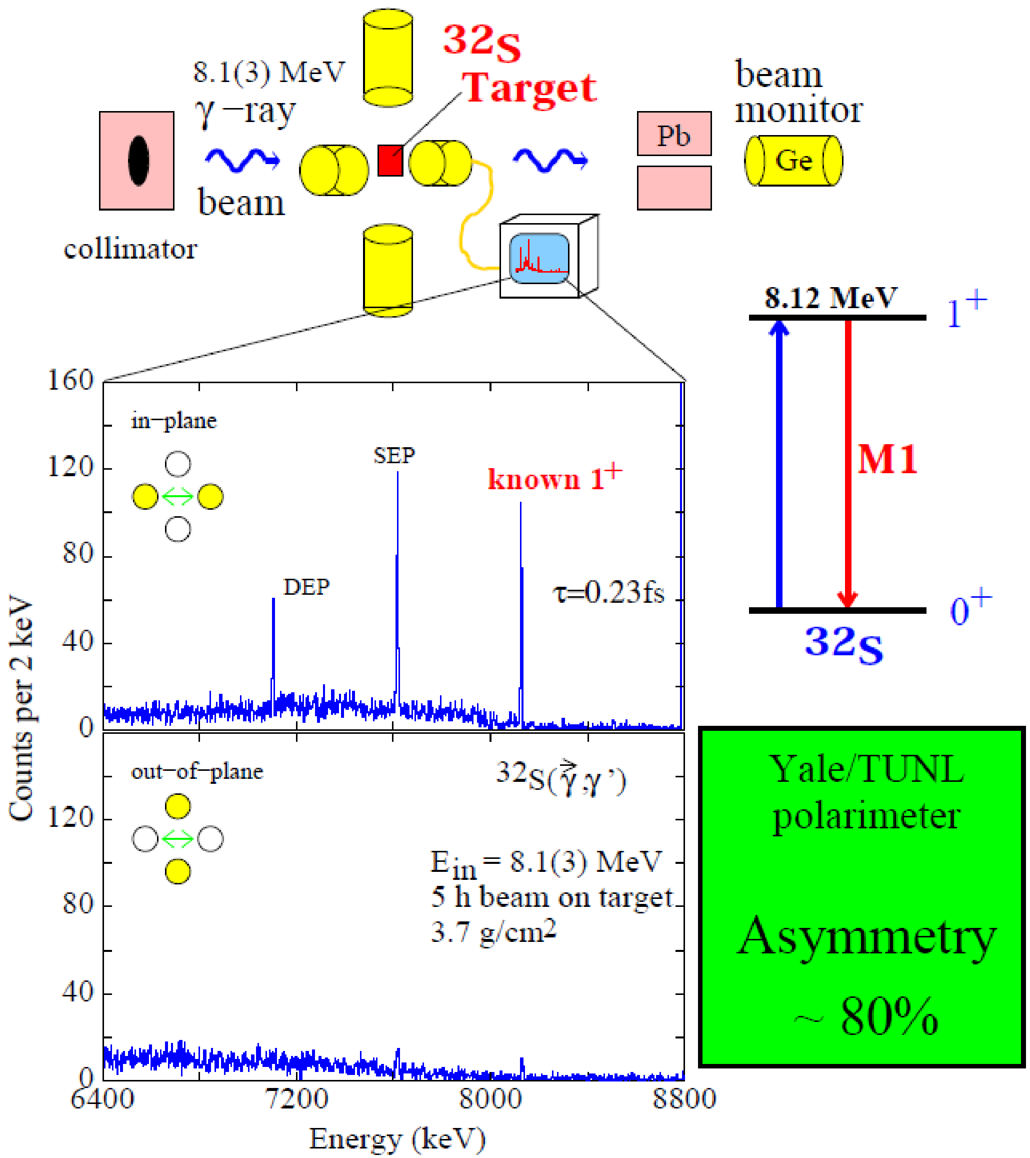


Klein-Nishina formula:

$$\frac{d\sigma_c}{d\Omega} = \frac{r_0^2}{2}\left(\frac{E_{\gamma'}}{E_\gamma}\right)^2 \cdot \left\{\frac{E_\gamma}{E_{\gamma'}} + \frac{E_{\gamma'}}{E_\gamma} - 2sin^2\theta \cdot cos^2\varphi\right\}$$

Maximum polarization at $\theta=90^0$

Proof of Principle



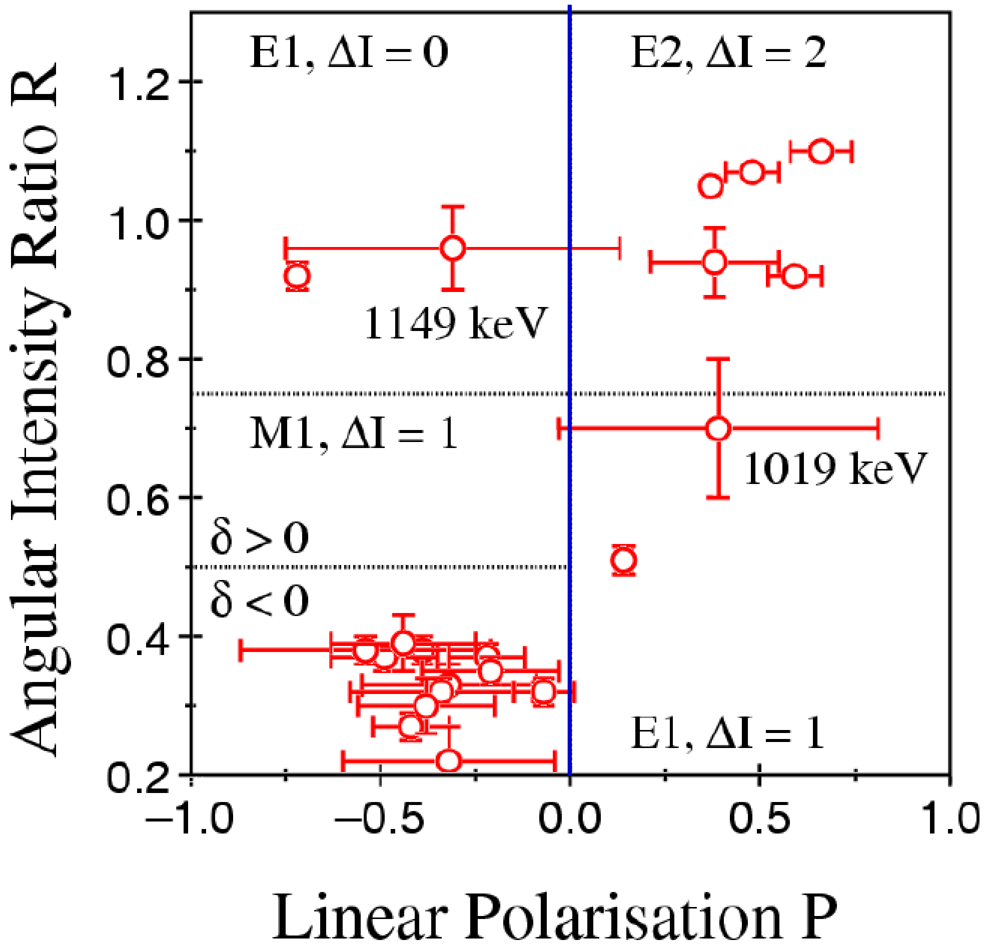
Yale/TUNL polarimeter

Asymmetry

~ 80%

N. Pietralla, Nucl Instr Meth A483, 556 (2002)

Linear polarization

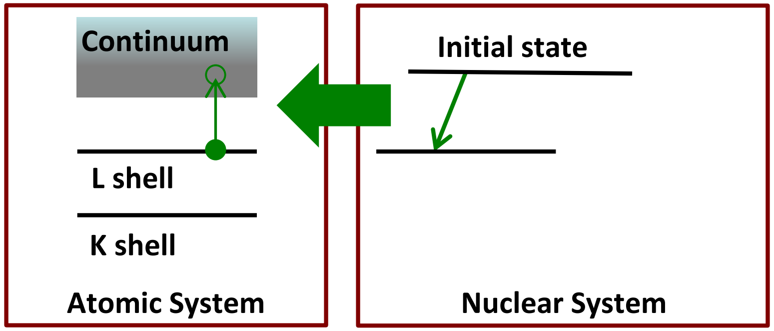


Plot P against the angular distribution information to uniquely define the multipolarity.

Data from Eurogam

Building a level scheme

- **ADs/ACs → multipole order L, but do not directly distinguish E vs M**
  - Pure ΔI=1 E1, e.g., looks the same as pure ΔI=1 M1
  - Nonzero mixing ratio may change that, as M1/E2 far more likely than E1/M2
  - For more direct determination of E or M: measure polarization (e.g. Clover detectors)
  - …or **internal-conversion electrons**



Nuclear level decay via **internal conversion** (IC)

- first observed over 100 y ago
  [van Baeyer and Hahn, Physik. Z. **11**, 488 (1 910)] *
- ubiquitous process
- usable as analysis tool

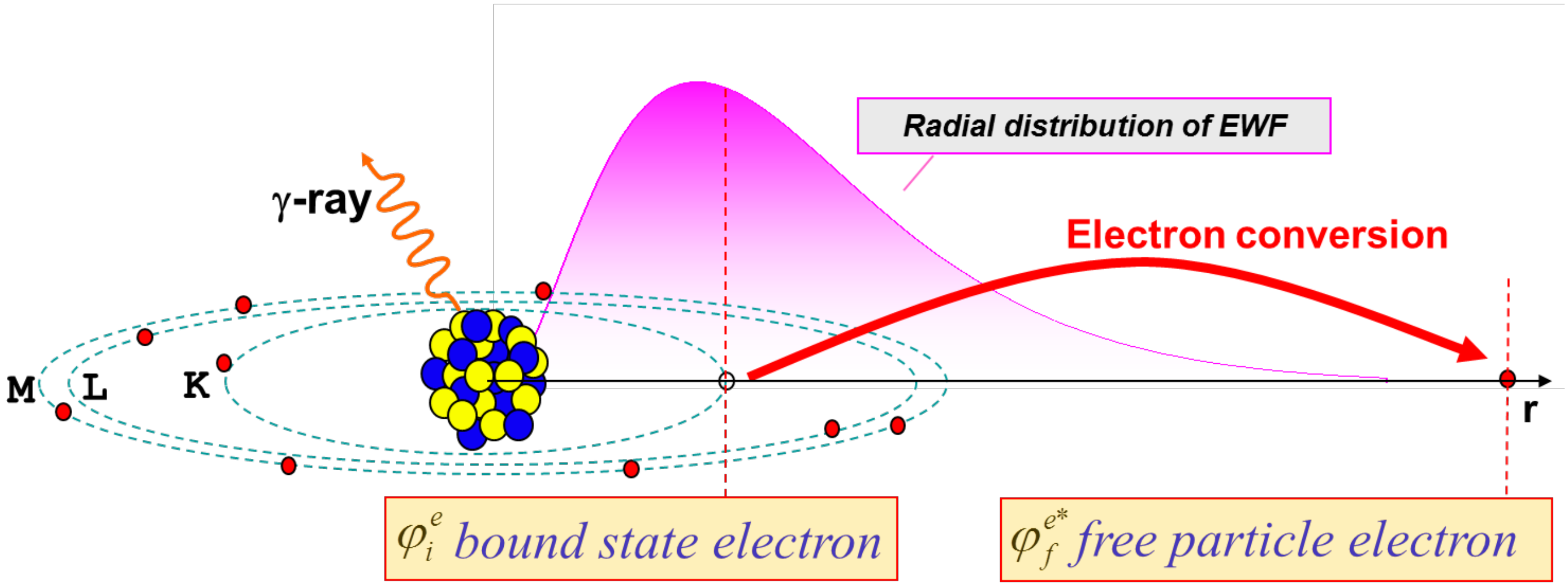
(cf. BrIcc, http://bricc.anu.edu.au/)

Instead of γ emission, energy transferred to atomic electron, which is ejected

* nucleus discovered in 1911!

AD: angular distribution, AC: angular correlation

Internal conversion



Energetics of CE-decay (i=K, L, M,….)

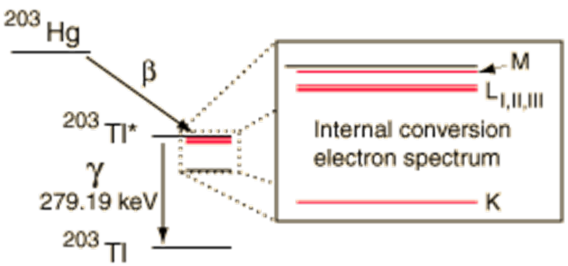
$$E_i = E_f + E_{ce,i} + E_{BE,i}$$

γ- and CE-decays are independent; transition probability (λ ~ Intensity)

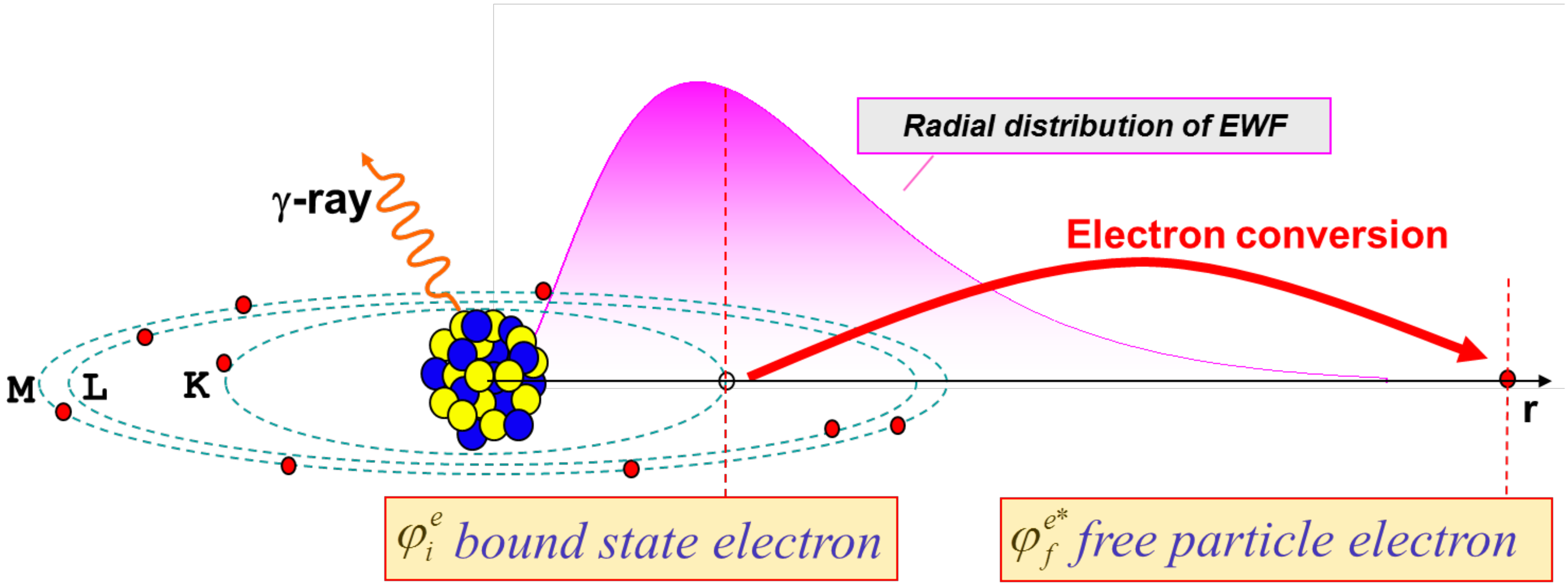
$$\lambda_T = \lambda_\gamma + \lambda_{CE} = \lambda_\gamma + \lambda_K + \lambda_L + \lambda_M \ldots\ldots$$

Conversion coefficient

$$\alpha_i = \frac{\lambda_{CE,i}}{\lambda_\gamma}$$



Internal conversion



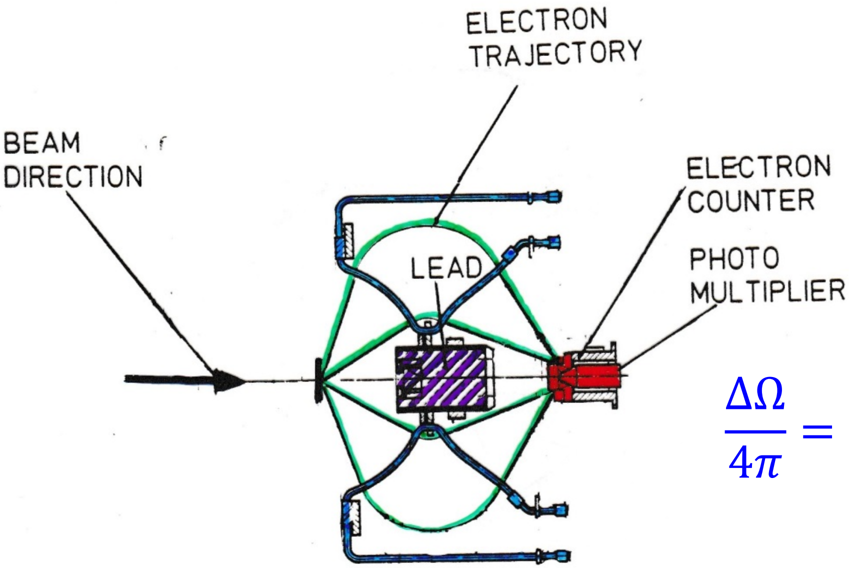
❖ For an electromagnetic transition internal conversion can occur instead of emission of gamma radiation. In this case the transition energy $Q = E_\gamma$ will be transferred to an electron of the atomic shell.

$$\boxed{T_e = E_\gamma - B_e}$$

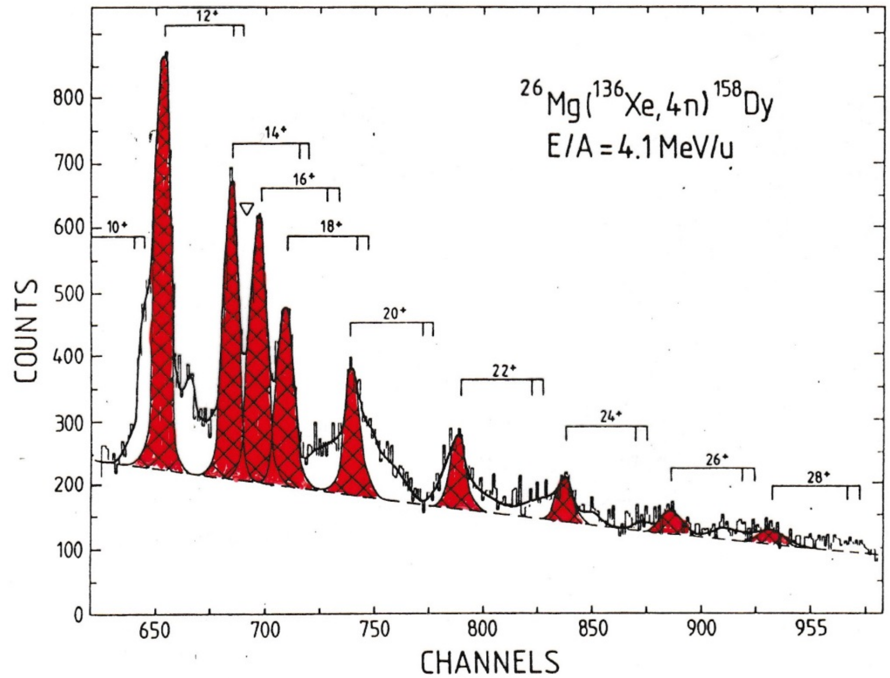
T_e: kinetic energy of the electron
B_e: binding energy of the electron

internal conversion is important for:
- heavy nuclei ~ Z^3
- high multipolarities Eℓ or Mℓ
- small transition energies

$$\alpha_k(El) \propto Z^3 \left(\frac{L}{L+1}\right)\left(\frac{2m_ec^2}{E}\right)^{L+5/2}$$



$$\frac{\Delta\Omega}{4\pi} = 26\%$$



		$\left(\frac{\Delta p}{p}\right)_e / \%$
resolution of the spectrometer including Doppler correction as calculated for a point source		0.4
scattering in the target	(i)	0.004
beam optics	(ii)	0.11
evaporation of neutrons	(iii)	0.09
energy loss in the target	(iv)	0.31
energy straggling of the projectiles	(v)	0.006
quadratic sum		0.53 %
experimental resolution		0.56 %

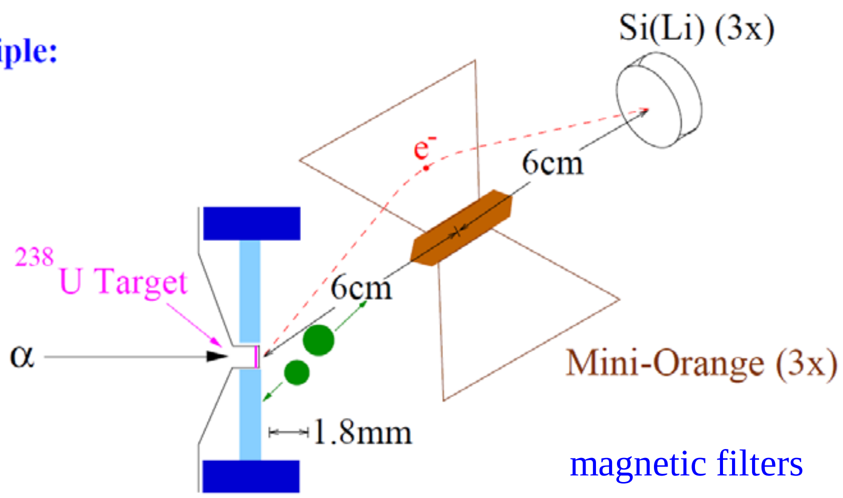
Doppler shift correction for projectile:

$$\boxed{T_e^* = \gamma \cdot T_e \cdot \left\{1 - \beta_1 \cdot \sqrt{1 + 2m_e\, c^2/T_e} \cdot cos\theta_{e1}\right\} + m_e c^2 \cdot (\gamma - 1)}$$

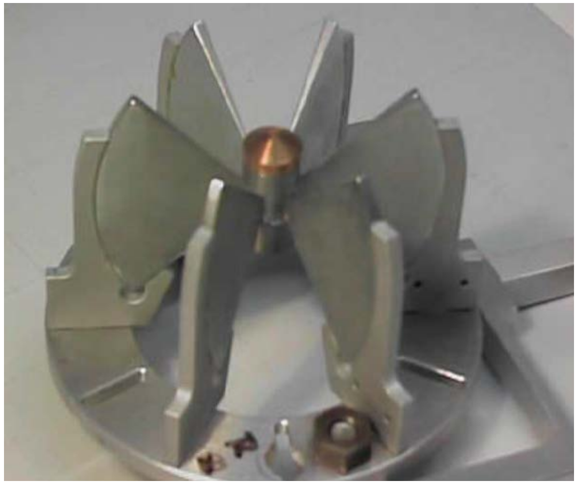
$$cos\theta_{e1} = cos\vartheta_1 cos\vartheta_e + sin\vartheta_1 sin\vartheta_e cos(\varphi_e - \varphi_1)$$

Mini Orange setup for conversion electron spectroscopy

Principle:

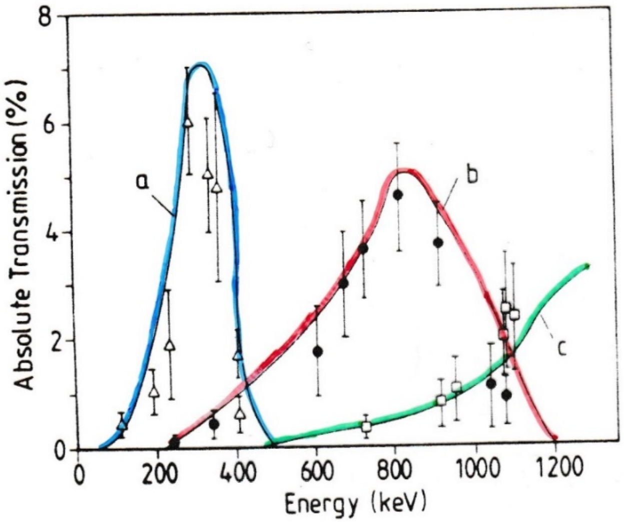


annular Si detector
(delayed fission)

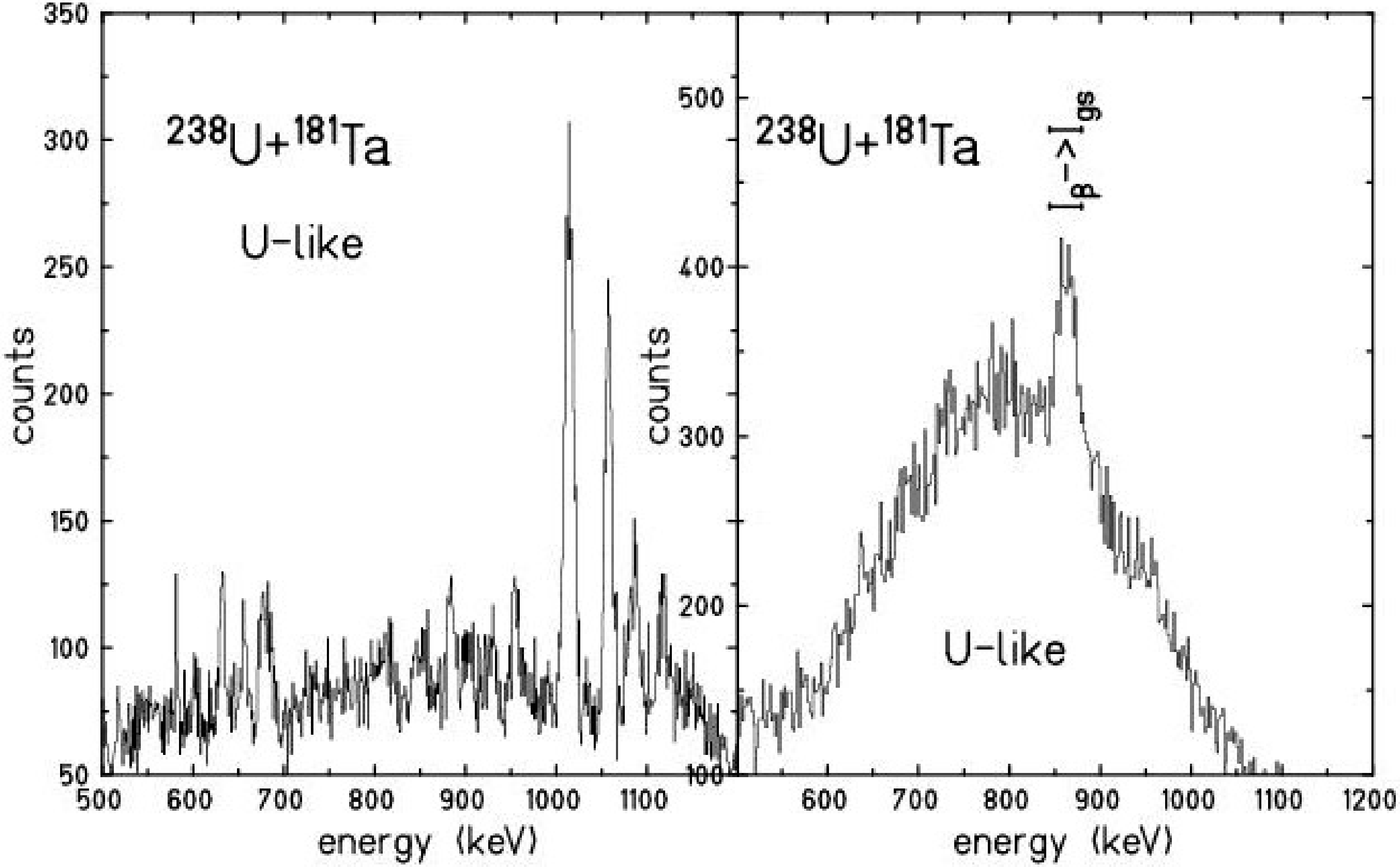


magnetic filters

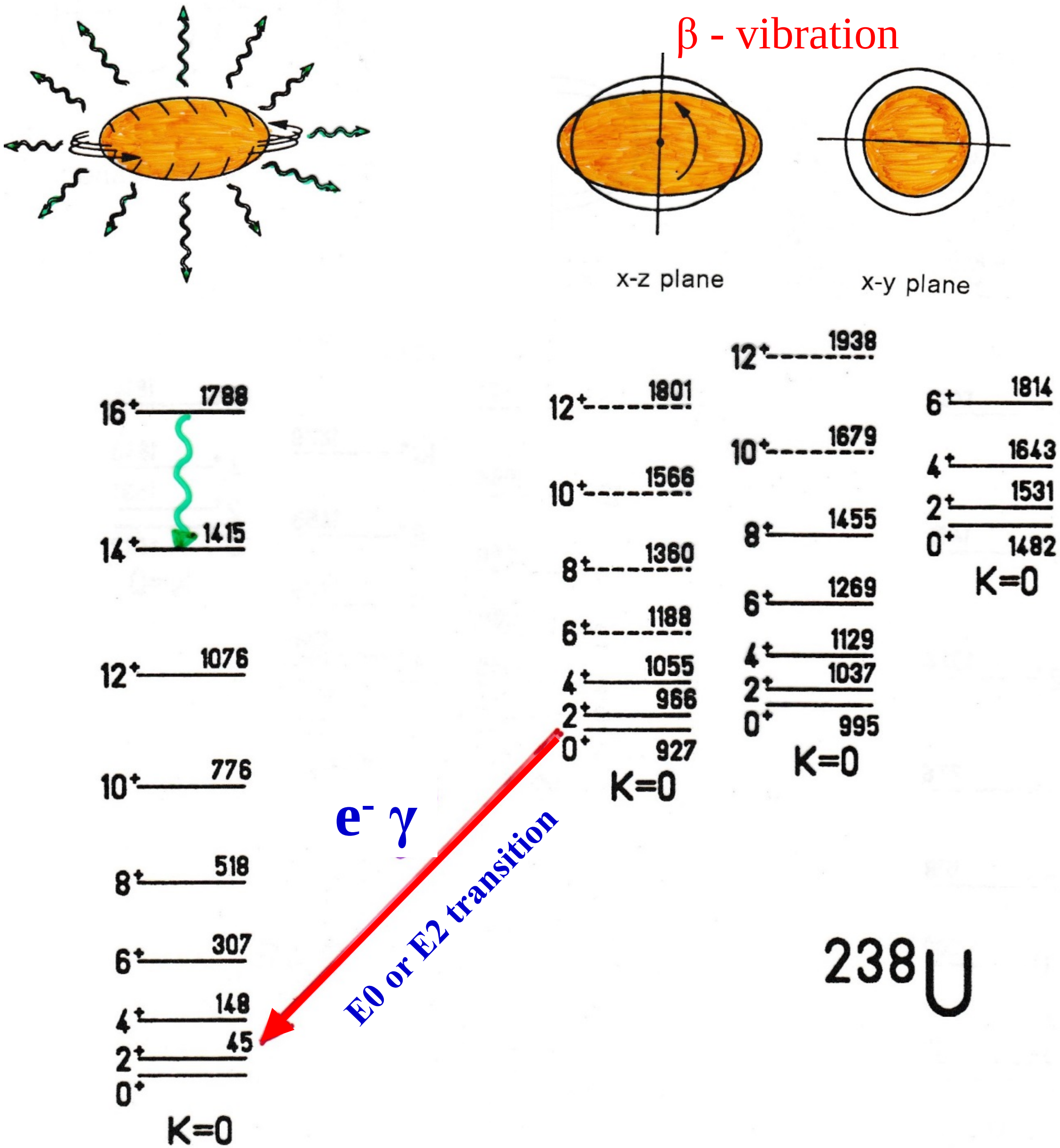
$SmCo_5$ magnets
for symmetric configuration 1 – 5 kG



Coulomb excitation experiment

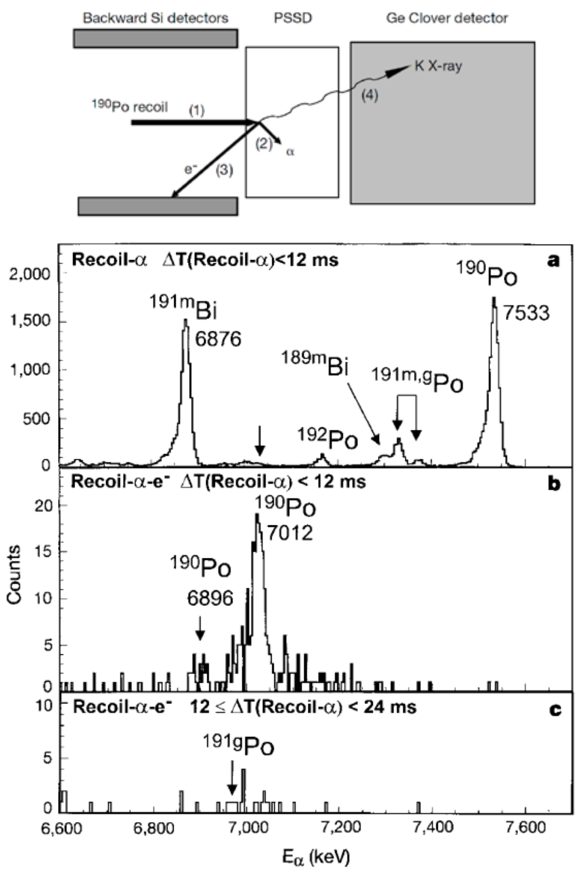


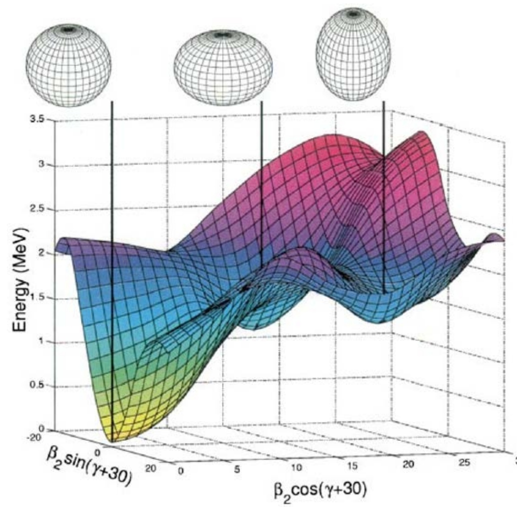
Surface oscillations in deformed nuclei



Building a level scheme

- **E0 decays**
  - Most transitions have γ vs e^- competition, but one case that "never" has a γ: $\mathbf{0^+ \rightarrow 0^+}$
  - Transition would be E0 (monopole moment = charge), which cannot radiate to points external to nucleus

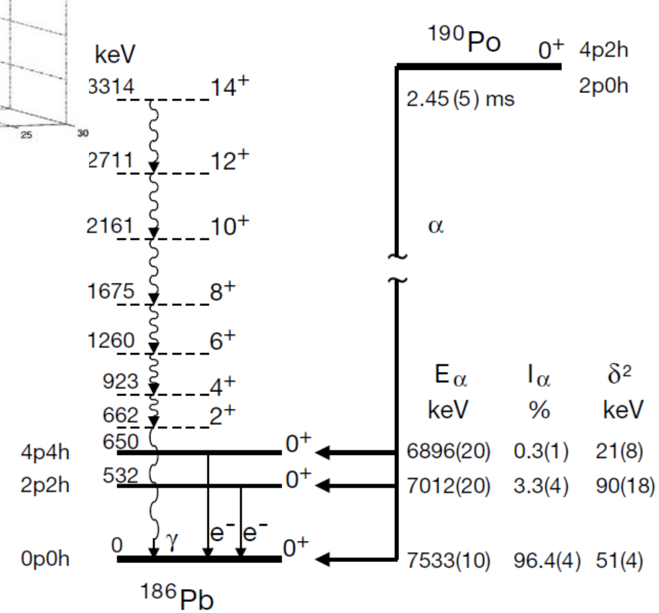




Andreyev *et al.*, Nature **405**, 430 (2000)

reduced α width:

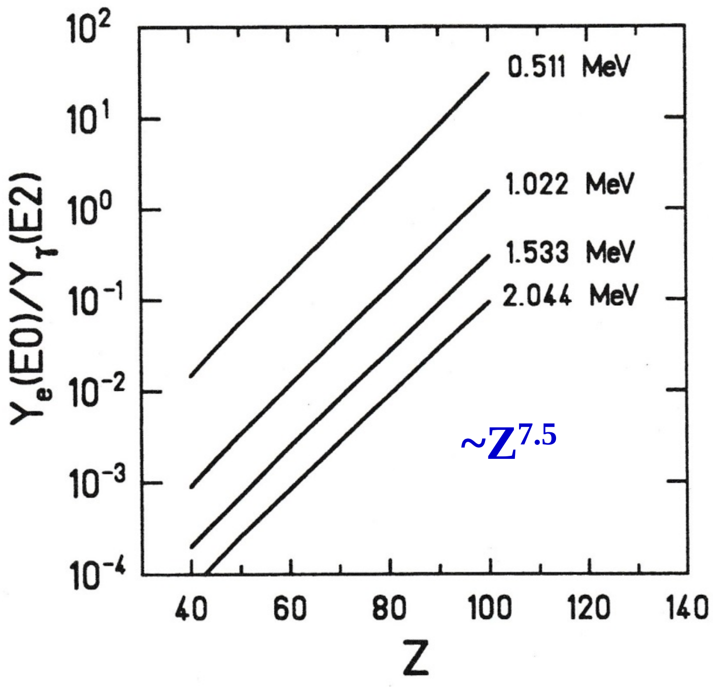
$$\delta^2 = \frac{h\,(\ln 2) b_\alpha}{T_{1/2} P}$$



E0/E2 branching ratio

$$\frac{Y_e(E0)}{Y_\gamma(E2)} = \frac{\Omega_K\ [s^{-1}]}{2.56\cdot 10^9\cdot A^{4/3}\cdot E_\gamma{}^5\ \ [MeV]}\cdot \underbrace{\frac{B(E0;I\to I')}{B(E2;I\to I')}}_{}$$

= 14 β^2 for $2_\beta \to 2$



Ω_K: conversion probability electronic factor

D.A. Bell et al.; Can. J. of Phys. 48 (1970),2542

Building a level scheme

- **E0 decays**
  - Most transitions have γ vs e^- competition, but one case that "never" has a γ: $\mathbf{0^+ \rightarrow 0^+}$
  - Transition would be E0 (monopole moment = charge), which cannot radiate to points external to nucleus

Why "never" and not *never*?

If $E > 2m_ec^2 = 1.022$ MeV, internal pair (e^+e^-) production is possible.

e^+ subsequently annihilates with another e^-, yielding 511-keV γ's.

Building a level scheme

- **Level and γ properties**
  - Measured E_γ's → level energies
  - Efficiency-corrected γ peak areas → intensities, branching ratios, level populations
  - γ ADs/ACs, γ polarizations, internal conversion → multipolarity → level spin, parity
  - Relative γ times → level half-lives, B(XL)'s

Reduced transition rates $B(XL) \propto E_\gamma^{-(2L+1)} T_{1/2,\gamma}^{-1}$

can be a sensitive probe of matrix elements connecting initial and final states:

$$B(XL; I_i \to I_f) = \frac{1}{2I_i + 1} \langle \psi_f \| XL \| \psi_i \rangle^2$$

Measuring half-lives

Excited-state half-lives range from ~10^{-23} s (>10-MeV resonances) to > 4.5×10^{16} y (^{180m}Ta). That's over 47 orders of magnitude!

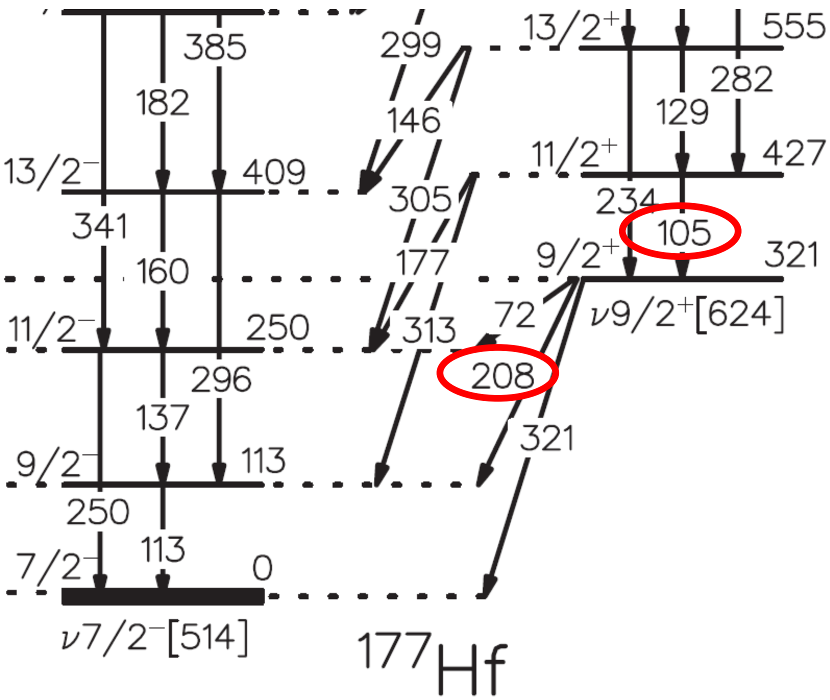
Most excited states have sub-ns half-lives; why are some exceptionally long?

- Low transition energy
- Large spin difference
- Large difference in underlying configurations
- *K*-forbiddenness (change in spin projection on symmetry axis $\Delta K > L$)

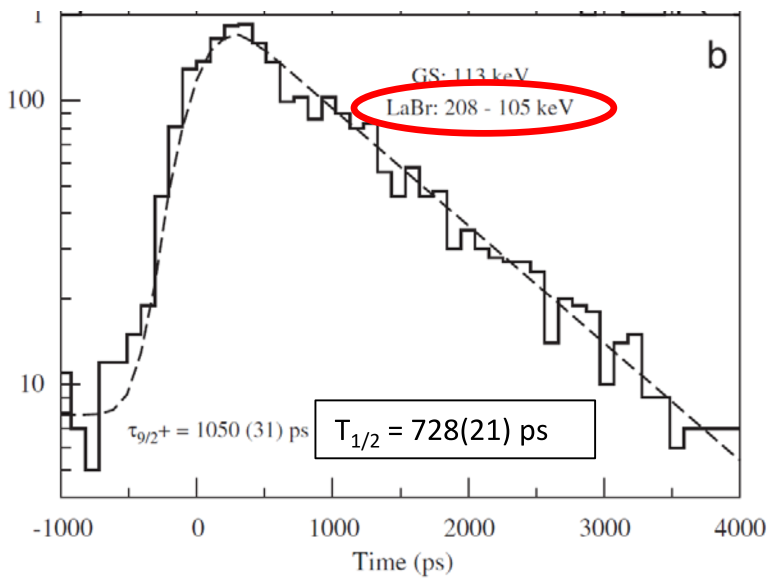
No single approach to measuring half-lives will apply to all cases.

Measuring half-lives

- **Observe exponential decay for half-lives of ~tens of ps or longer**
  - Measure γ-ray time relative to a fast START signal
  - START with preceding β decay in e.g. fast plastic
  - …or START with RF (clock tick marking reaction)
  - …or START with preceding γ in e.g. Ge ($T_{1/2}$ > ns) or $LaBr_3$ (sub-ns)
  - Get half-life from slope of exponential

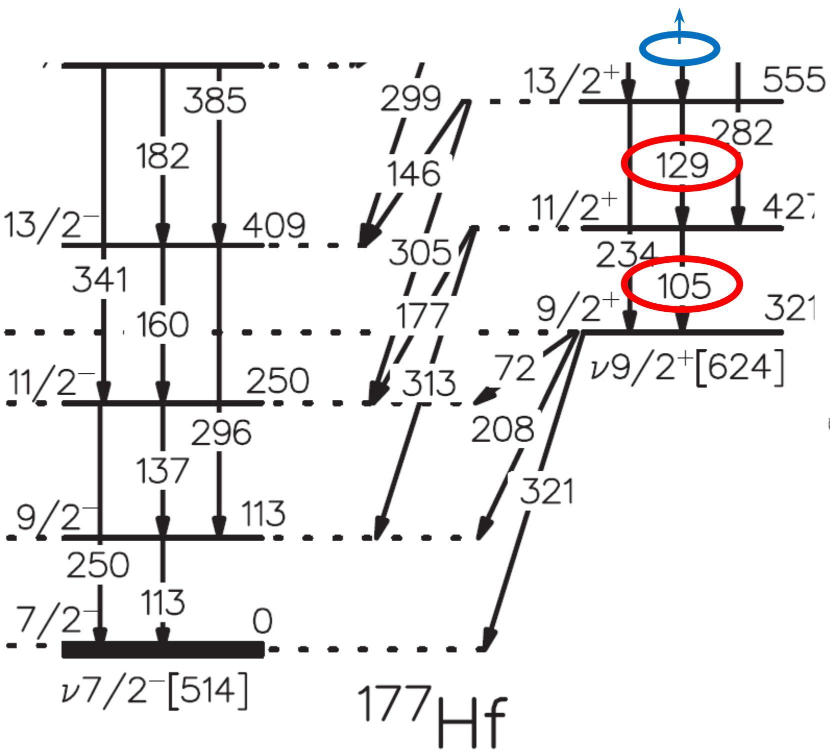


Kondev *et al.*, PRC**85**, 027304 (2012)

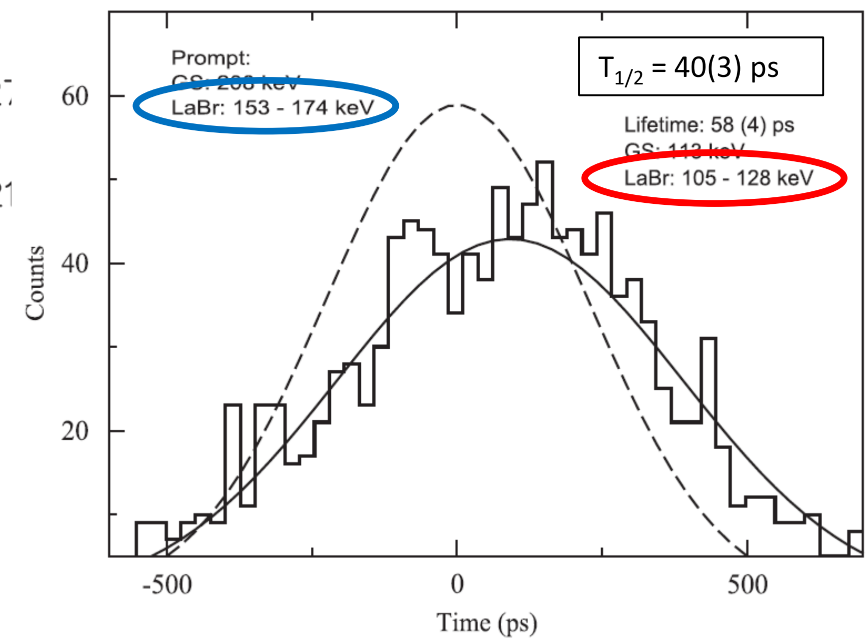


Zhu *et al.*, NIMA**652**, 231 (2011)

- **Observe exponential decay for half-lives of ~tens of ps or longer**
  - Measure γ-ray time relative to a fast START signal
  - START with preceding β decay in e.g. fast plastic
  - …or START with RF (clock tick marking reaction)
  - …or START with preceding γ in e.g. Ge ($T_{1/2}$ > ns) or $LaBr_3$ (sub-ns)
  - Get half-life from slope of exponential…or from centroid shift



Kondev *et al.*, PRC**85**, 027304 (2012)



Zhu *et al.*, NIMA**652**, 231 (2011)

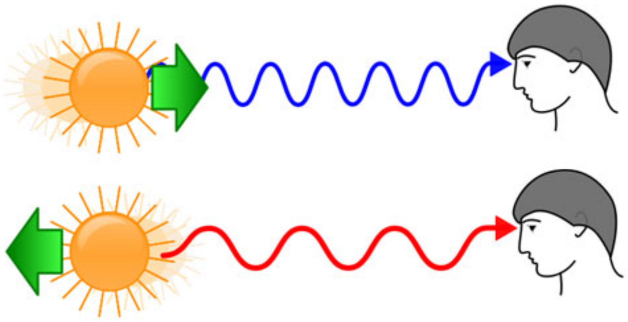
Measuring half-lives

- **For ~ps half-lives, instrumentation not fast enough**
  - Exploit physics of in-beam measurement: Doppler shifts

Brief Doppler diversion…

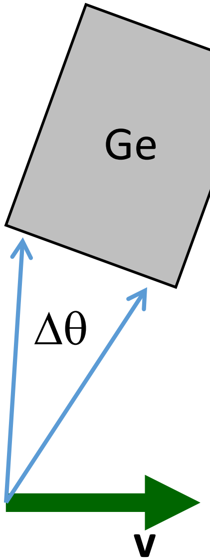
Doppler effect

Doppler shift:



$$E = E_0 \frac{\sqrt{1-(v/c)^2}}{1-(v/c)\cos\theta}$$

Doppler broadening:



$$\Delta E \propto E_0 (v/c) \sin\theta \, \Delta\theta$$

Ions lose energy while traversing a medium ➔ v/c decreases with time.

By modelling the energy loss, time-dependent Doppler shift determined ➔ Doppler-shift attenuation and (differential) plunger techniques

Measuring half-lives

- **For ~ps half-lives, instrumentation not fast enough**
  - Exploit physics of in-beam measurement: Doppler shifts
  - Recoil-distance method (plunger)

- Produce nucleus of interest with initial velocity v_0 via Coulex or KO reactions
- Observe Doppler-shifted γ's emitted between target and degrader
- Ion slows in the degrader to velocity $v_d < v_0$
- Observe Doppler-shifted γ's emitted after degrader
- Increase distance between target and degrader; change in relative peak intensities depends on $T_{1/2}$
- $T_{1/2}$ ~ 10-100s of ps can be measured

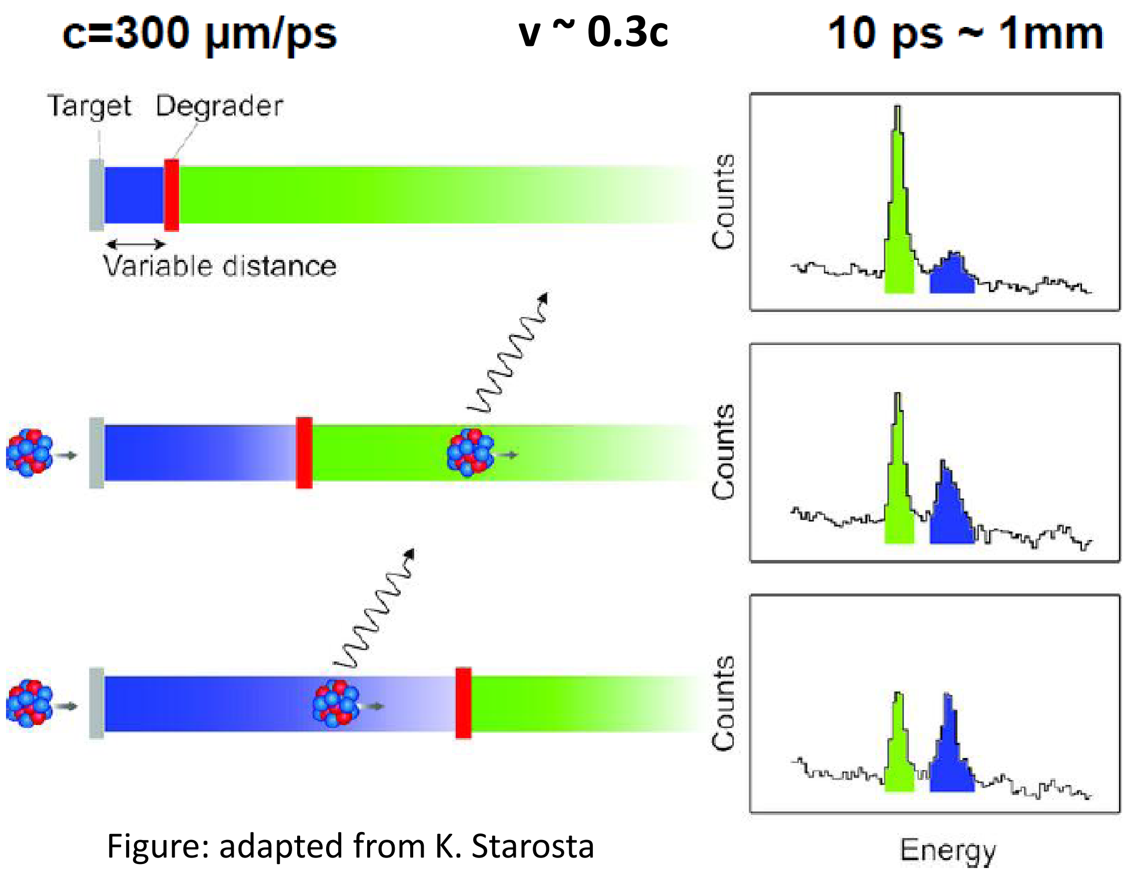


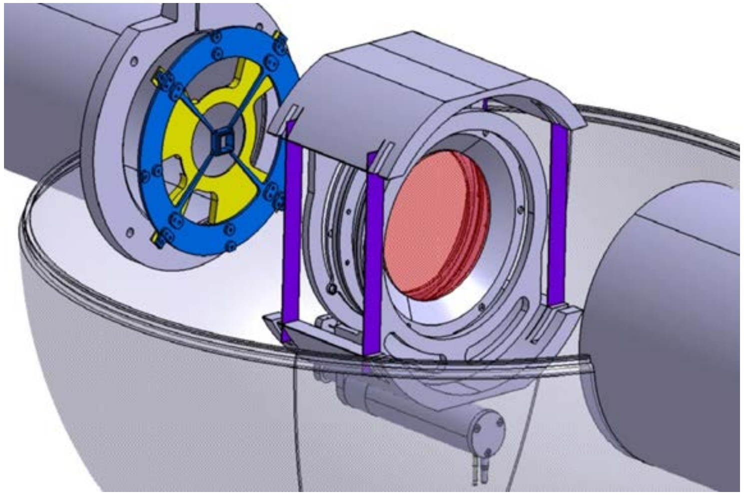
Figure: adapted from K. Starosta

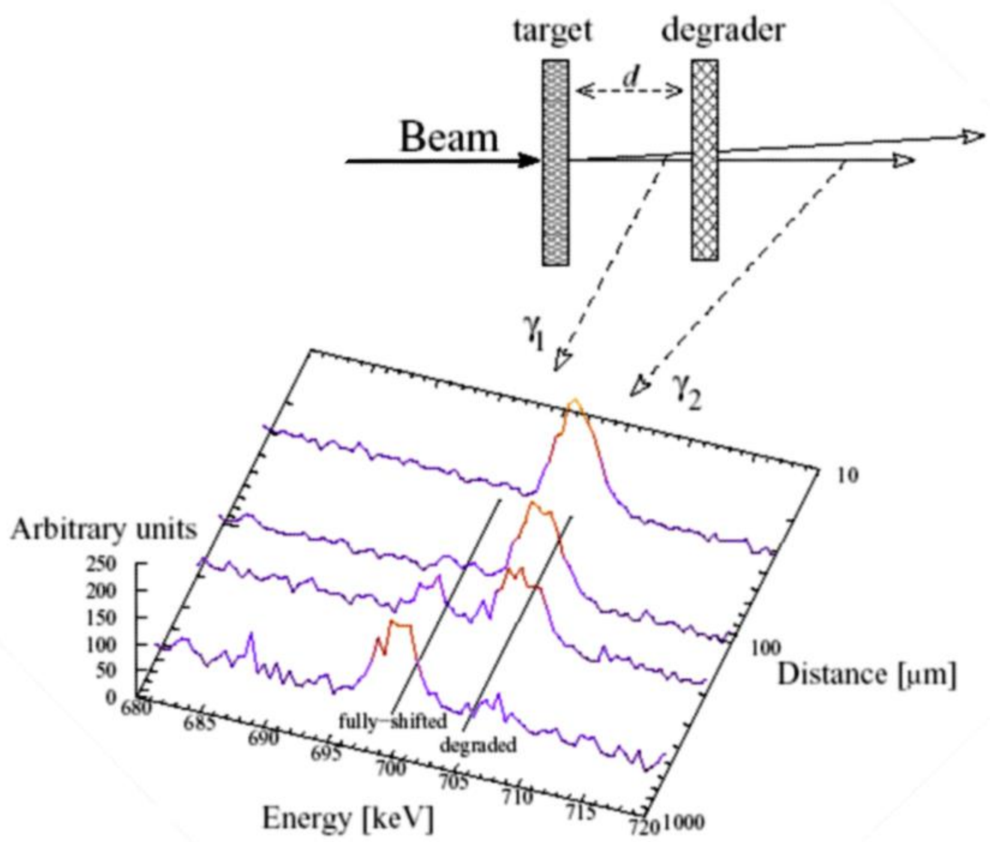
Recoil distance method

$$I_{degraded} = I \cdot e^{-d/v\tau}$$

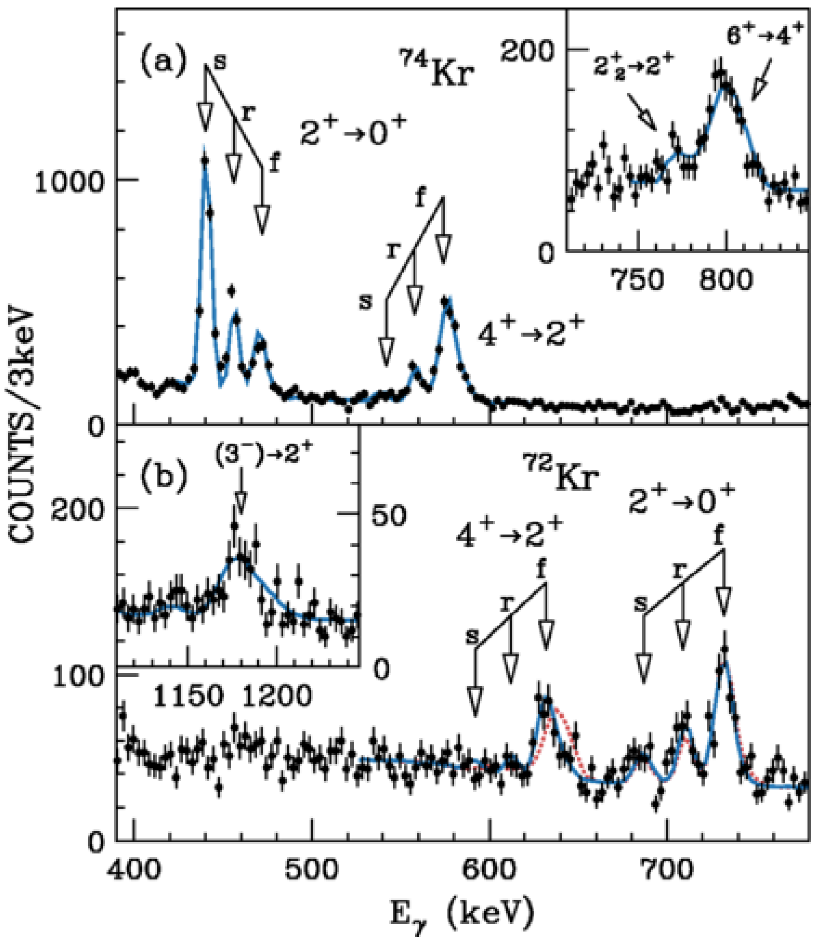
$$I_{shifted} = \left(1 - e^{-d/v\tau}\right)$$

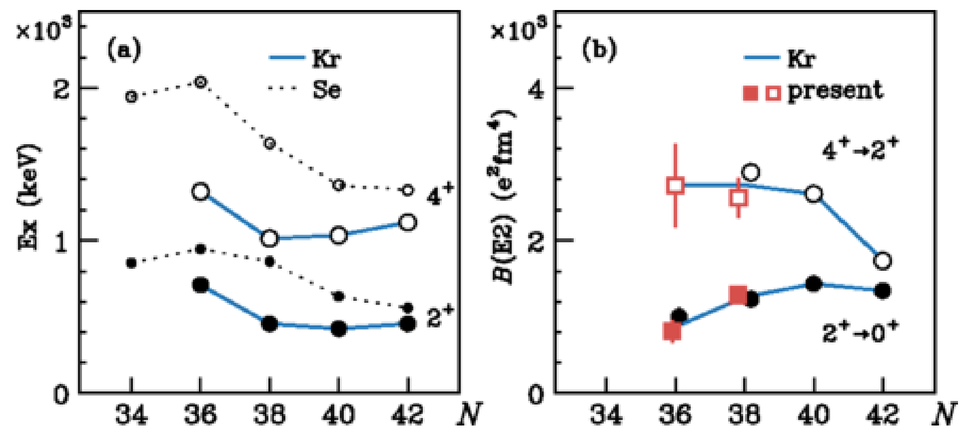
$$\boxed{\frac{I_{degraded}}{I_{degraded} + I_{shifted}} = e^{-d/v\tau}}$$





- Double-degrader plunger



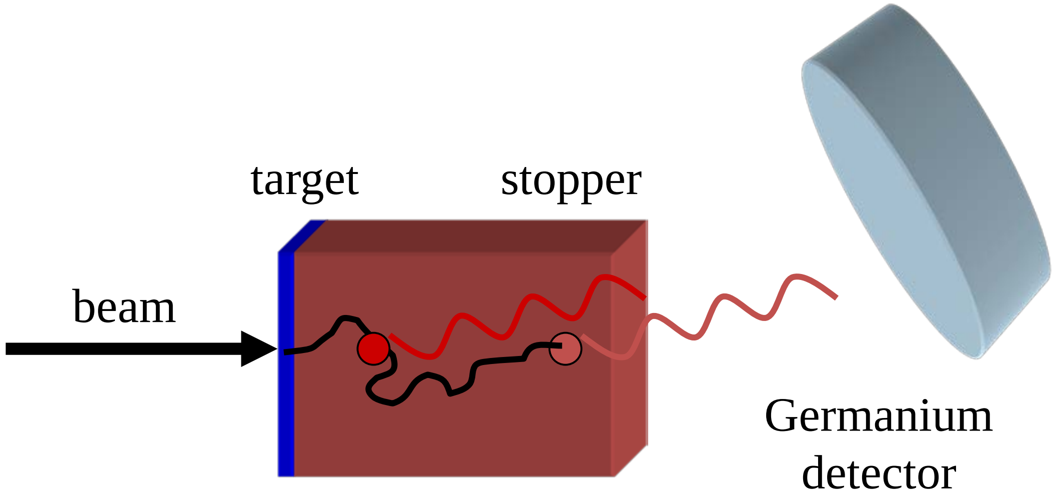


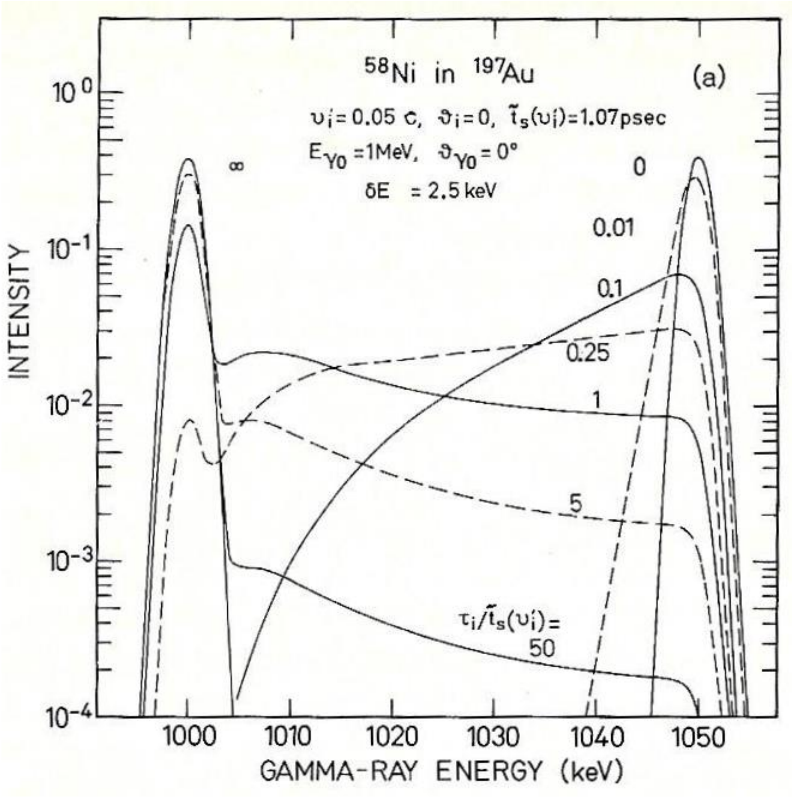
$T_{1/2}$ → B(E2) → indicator for collectivity

Behavior of the 4^+ and 2^+ states in ^{72}Kr suggest rapid shape evolution occurring

Iwasaki *et al.*, PRL**112**, 142502 (2014).

Doppler Shift Attenuation Method

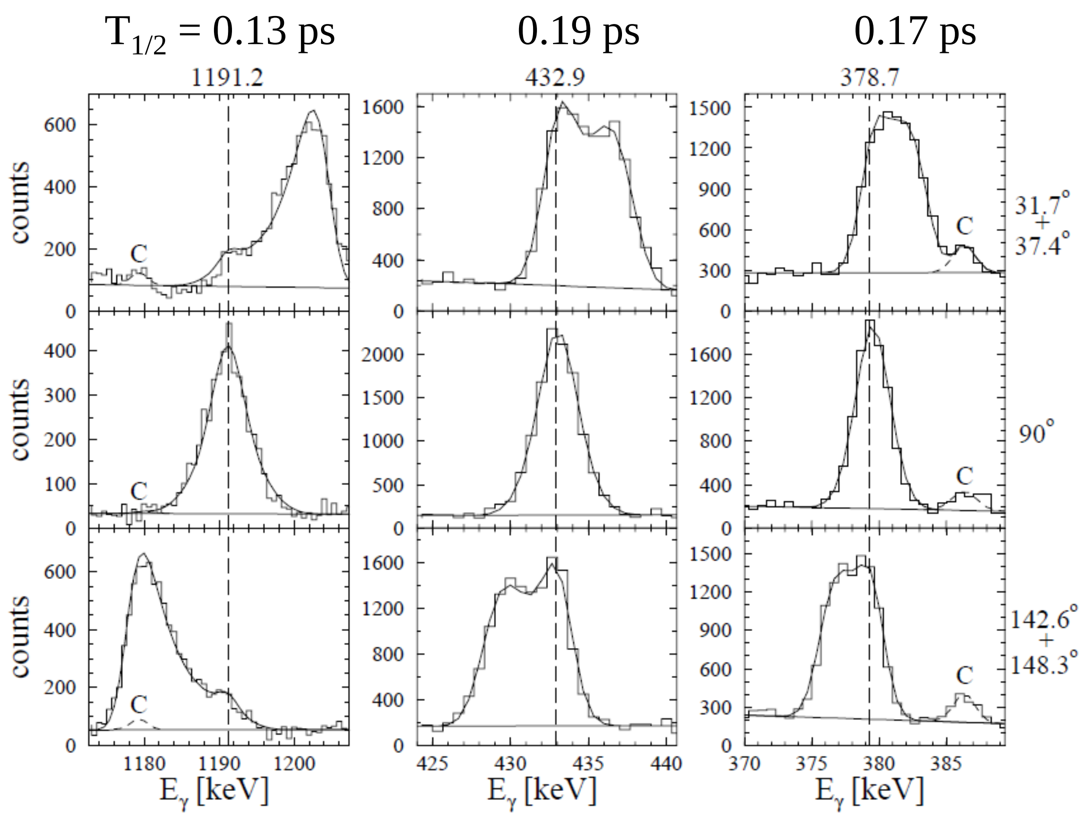




Measuring half-lives

- For ~ps half-lives, instrumentation not fast enough
  - Exploit physics of in-beam measurement: Doppler shifts
  - Recoil-distance method (plunger)
  - Doppler-broadened line shapes

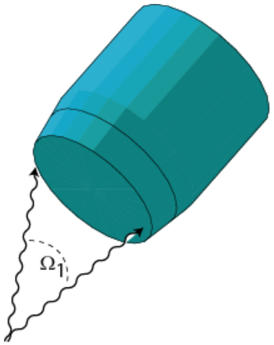
- If ion slows substantially in foil, variation in Doppler shift → line shape
- Sensitive to sub-ps $T_{1/2}$'s

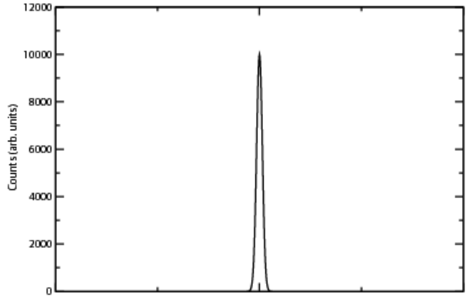


Chiara *et al.*, PRC**61**, 034318 (2000)

Lorentz transformation

Efficiency versus resolution

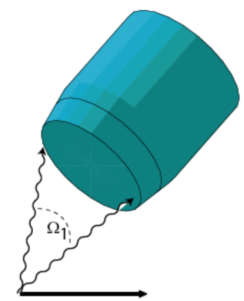


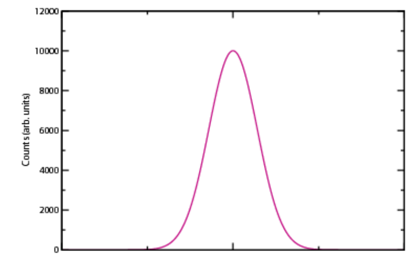


With a source at rest, the intrinsic resolution of the detector can be reached;

efficiency decreases with the increasing detector-source distance.

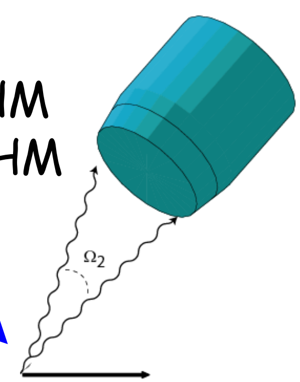
With a moving source also the effective energy resolution depends on the detector-source distance (Doppler effect)

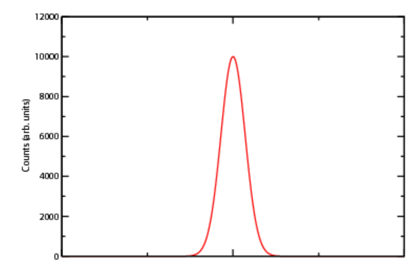




Small *d* ⟷ Large Ω ⟷ High ε ⟷ Poor FWHM

Large *d* ⟷ Small Ω ⟷ Low ε ⟷ Good FWHM

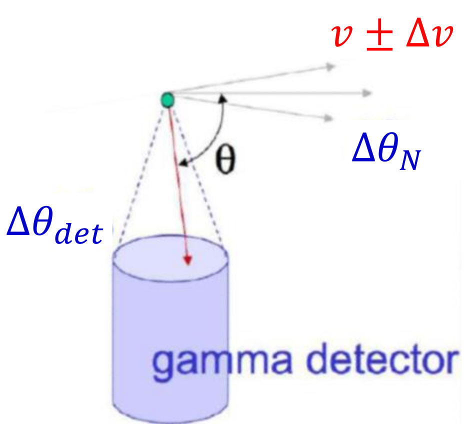




Energy resolution

The major factors affecting the final energy resolution (FWHM) at a particular energy are as follows:

$$\Delta E_\gamma^{final} = \left(\Delta E_{Int}^2 + \Delta\theta_{det}^2 + \Delta\theta_N^2 + \Delta v^2\right)^{1/2}$$



ΔE_{Int} – The intrinsic resolution of the detector system. It includes contributions from the detector itself and the electronic components used to process the signal.

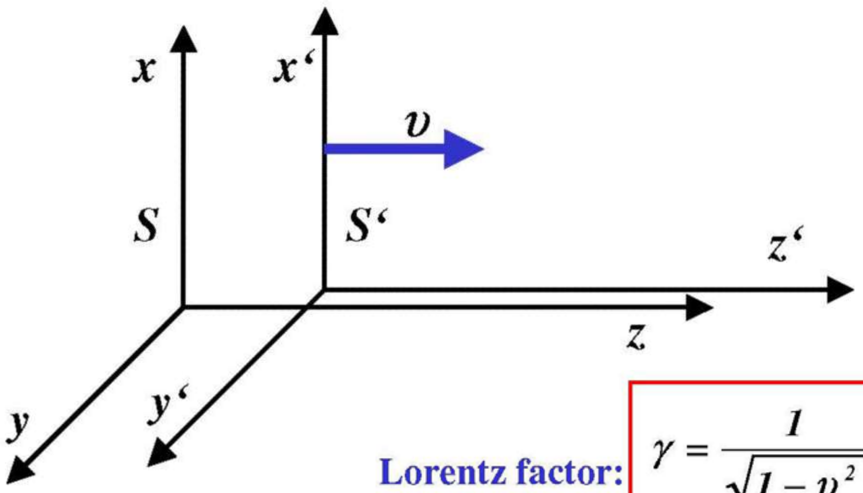
$\Delta\theta_{det}$ – The Doppler broadening arising from the opening angle of the detectors

$\Delta\theta_N$ – The Doppler broadening arising from the angular spread of the recoils in the target

Δv – The Doppler broadening arising from the velocity (energy) variation of the excited nucleus

Special relativity

Lorentz transformation:



Lorentz factor: $\boxed{\gamma = \frac{1}{\sqrt{1-v^2}}}$

❑ Consider the space-time point
- in a given frame S: (t, x, y, z)
- and in a (moving) frame S': (t', x', y', z')

1) S' moves with a constant velocity v along z-axis

Space-time Lorentz transformation S←→S':

$\underline{S \Rightarrow S'}$	$\underline{S' \Rightarrow S}$
$x' = x$	$x = x'$
$y' = y$	$y = y'$
$z' = \gamma(z - vt)$	$z = \gamma(z' + vt)$
$t' = \gamma(t - vz)$	$t = \gamma(t' + vz)$

Note: units c=1

$$t' = \gamma(t - \frac{v}{c^2}z)$$

$$t = \gamma(t' + \frac{v}{c^2}z)$$

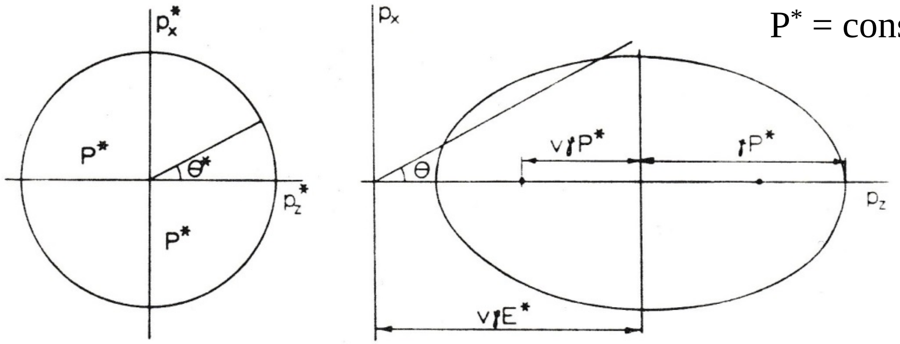
$$v_z = |\vec{v}| = v$$

❑ Consider the 4-momentum:
- in a given frame S: $p \equiv (E, p) = (E, p_x, p_y, p_z)$
- in the (moving) frame S': $p' \equiv (E', \vec{p}') = (E', p'_x, p'_y, p'_z)$

Lorentz transformation for 4-momentum S←→S':

$$\boxed{\begin{array}{l} p'_x = p_x, \quad p'_y = p_y \\ p'_z = \gamma(p_z - vE) \\ E' = \gamma(E - vp_z) \end{array}}$$

Lorentz transformation



P^* = const.

rest system   laboratory system

total energy:

$$E^* = \gamma \cdot E - \gamma \cdot v \cdot P \cdot cos\theta$$

with

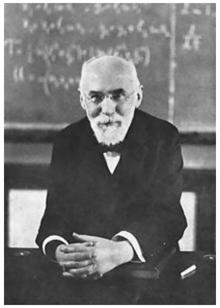
$$E = \sqrt{(mc^2)^2 + (Pc)^2}$$

E^*, P^* total energy and momentum in the rest system
E, P   total energy and momentum in the laboratory system

Doppler formula for zero-mass particle (photon):   E=Pc

$$E^* = \gamma \cdot E - \gamma \cdot \beta \cdot E \cdot cos\theta$$

$$\boxed{E^* = \gamma \cdot E(1 - \beta \cdot cos\theta)}$$



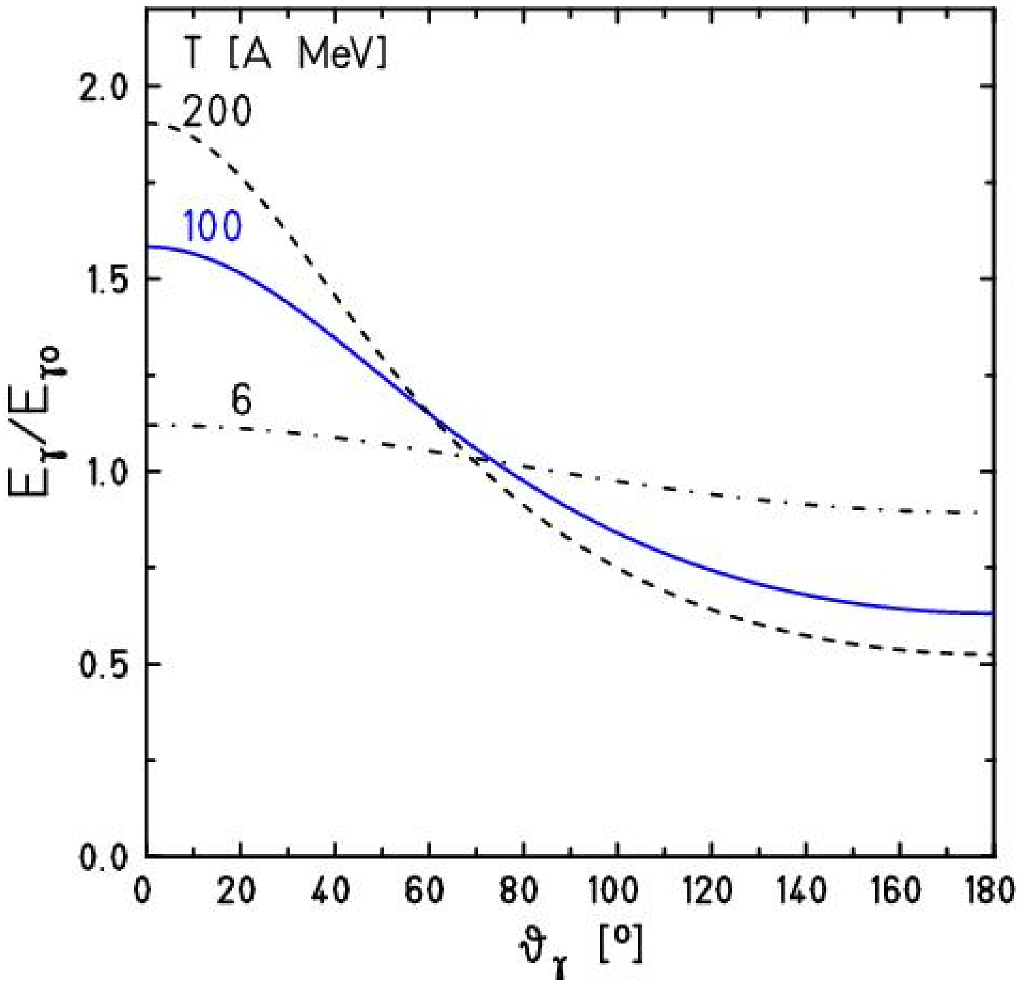
Hendrik Lorentz

E. Byckling, K. Kajantie   J. Wiley & Sons   London

$$\frac{E_{\gamma 0}}{E_\gamma} = \frac{1 - \beta \cdot \cos \vartheta_\gamma^{lab}}{\sqrt{1-\beta^2}}$$

for $\vartheta_p \cong 0^0$

$$\frac{d\Omega_{rest}}{d\Omega_{lab}} = \left(\frac{E_\gamma}{E_{\gamma 0}}\right)^2$$



Doppler broadening and position resolution

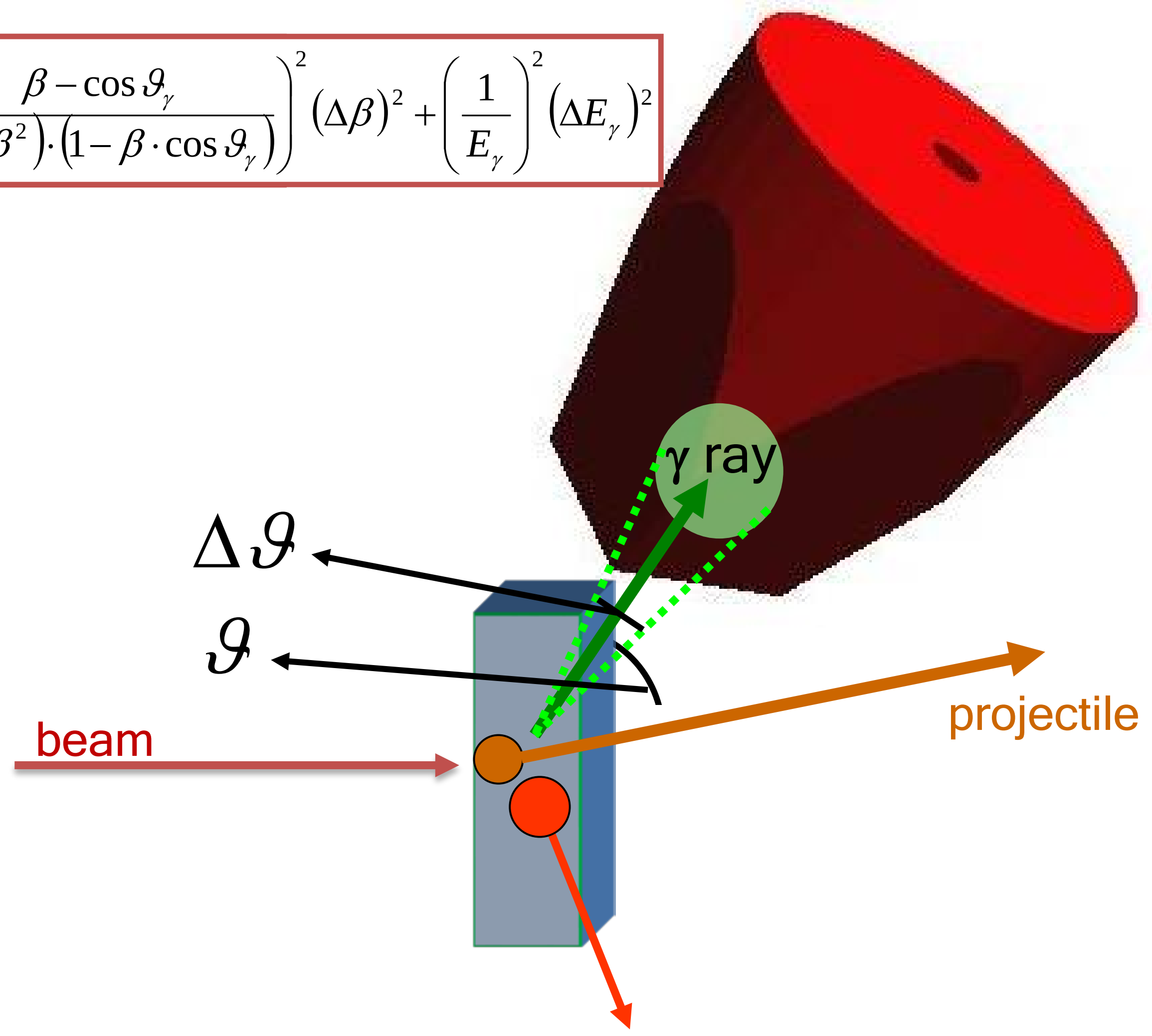
$$E_{\gamma 0} = E_\gamma \frac{1-\beta\cdot\cos\vartheta_\gamma}{\sqrt{1-\beta^2}} \qquad \left(\beta, \vartheta_p = 0^0, \vartheta_\gamma \text{ and } E_\gamma \text{ in lab}-\text{frame}\right)$$

$$\left(\frac{\Delta E_{\gamma 0}}{E_{\gamma 0}}\right)^2 = \left(\frac{\beta\cdot\sin\vartheta_\gamma}{1-\beta\cdot\cos\vartheta_\gamma}\right)^2 (\Delta\vartheta_\gamma)^2 + \left(\frac{\beta-\cos\vartheta_\gamma}{(1-\beta^2)\cdot(1-\beta\cdot\cos\vartheta_\gamma)}\right)^2 (\Delta\beta)^2 + \left(\frac{1}{E_\gamma}\right)^2 (\Delta E_\gamma)^2$$

Position resolution

Angular resolution

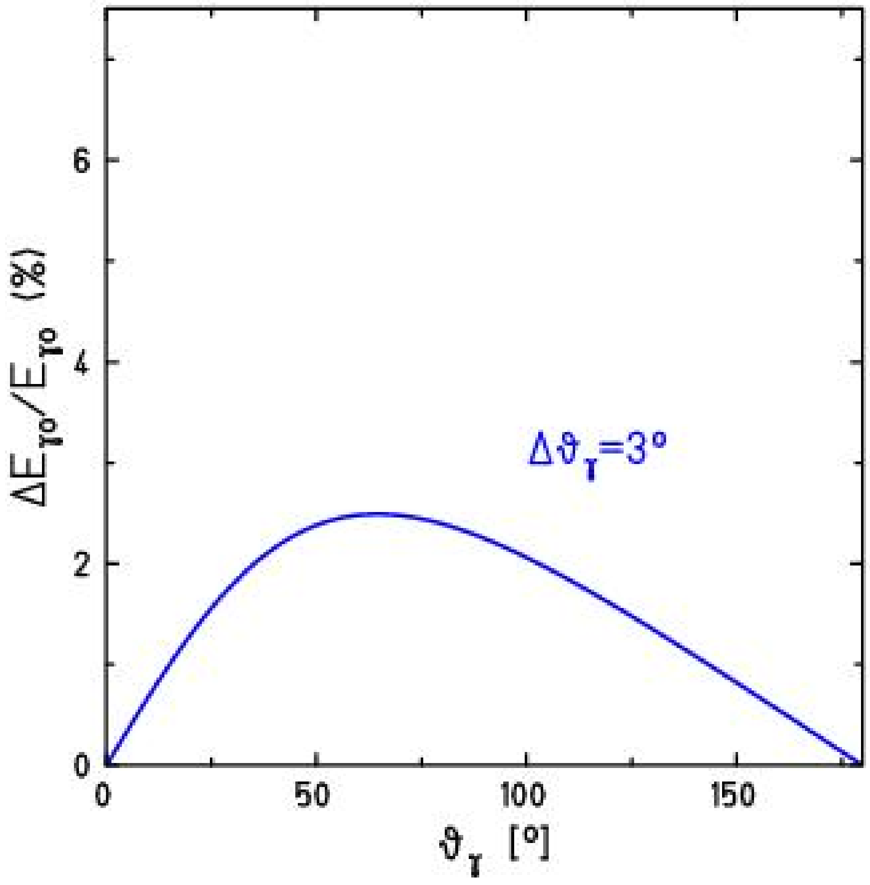
Energy resolution



Doppler broadening (opening angle of detector)

$$\frac{\Delta E_{\gamma 0}}{E_{\gamma 0}} = \frac{\beta \cdot \sin \vartheta_\gamma}{1 - \beta \cdot \cos \vartheta_\gamma} \cdot \Delta\vartheta_\gamma$$

for $\vartheta_p \cong 0^0$



with

$$\Delta\vartheta_\gamma = 0.622 \cdot \arctan \frac{d[mm]}{R[mm] + 30[mm]}$$

$$R = 700[mm]$$

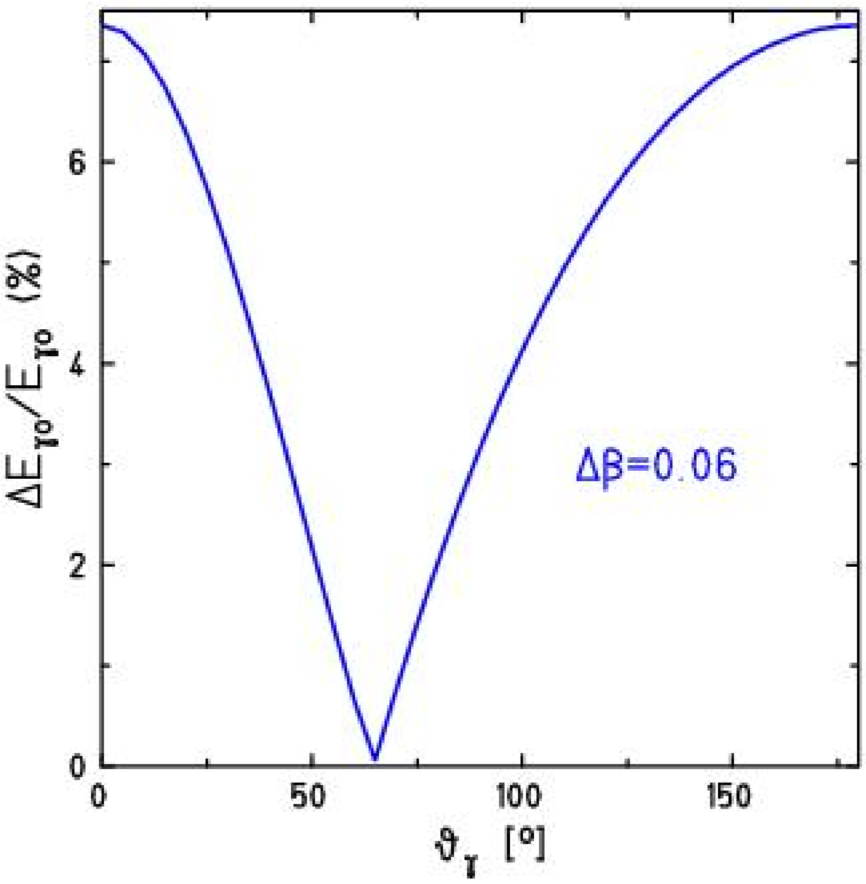
$$d = 59[mm]$$

Doppler broadening (velocity variation)

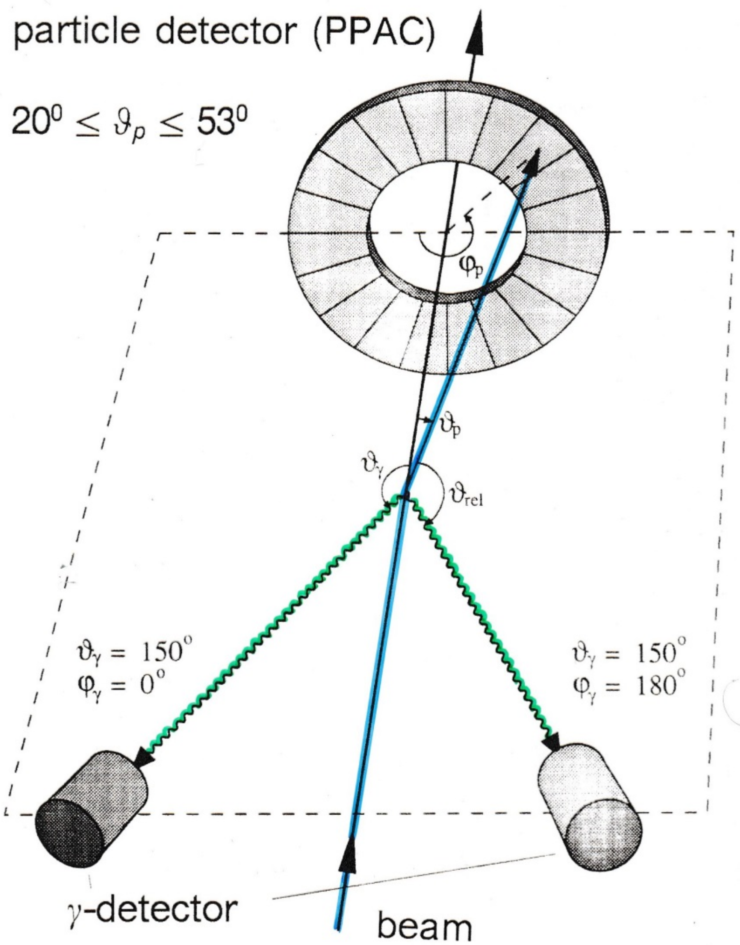
$$\frac{\Delta E_{\gamma 0}}{E_{\gamma 0}} = \frac{\beta - \cos\vartheta_\gamma}{\left(1-\beta^2\right)\cdot\left(1-\beta\cdot\cos\vartheta_\gamma\right)}\cdot\Delta\beta$$

for $\vartheta_p \cong 0^0$

with $\Delta\beta = 6\%$



Experimental arrangement



experimental problem:

Doppler broadening due to finite size of Ge-detector

$\frac{\Delta E}{E} \sim 1\%$ for $\Delta\vartheta_\gamma = 20^0 \quad \beta_1 \cong 10\%$

For projectile excitation:

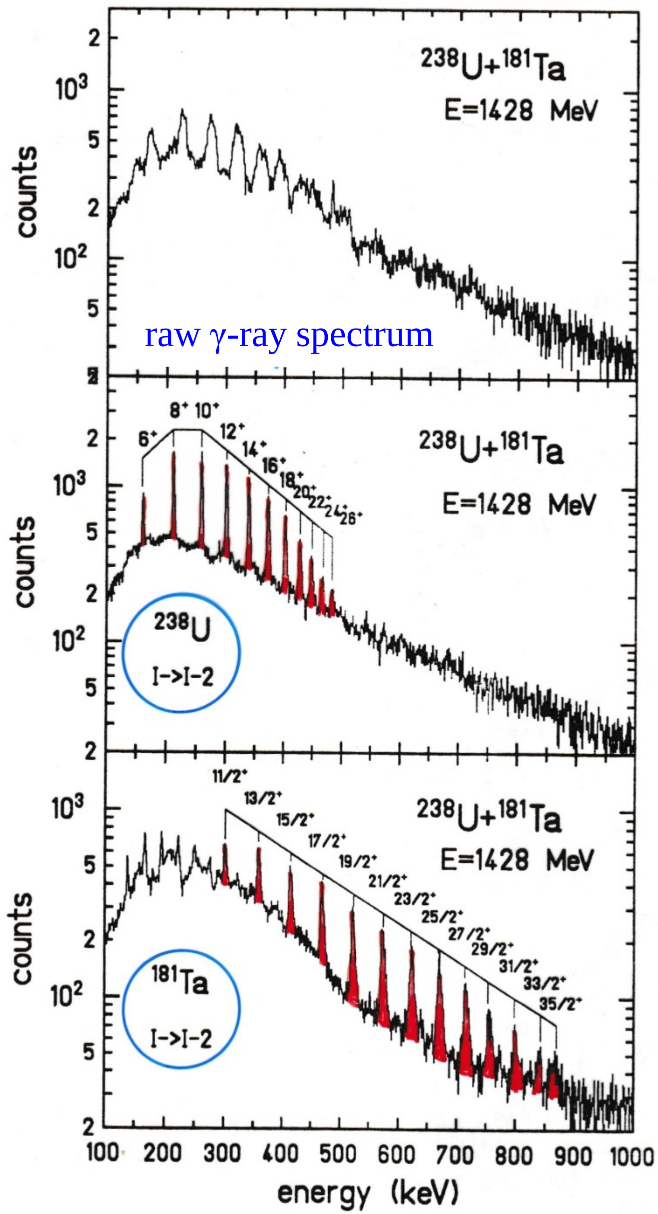
$E^* = \gamma \cdot E \cdot (1 - \beta_1 \cdot cos\theta_{\gamma 1})$ Doppler shift

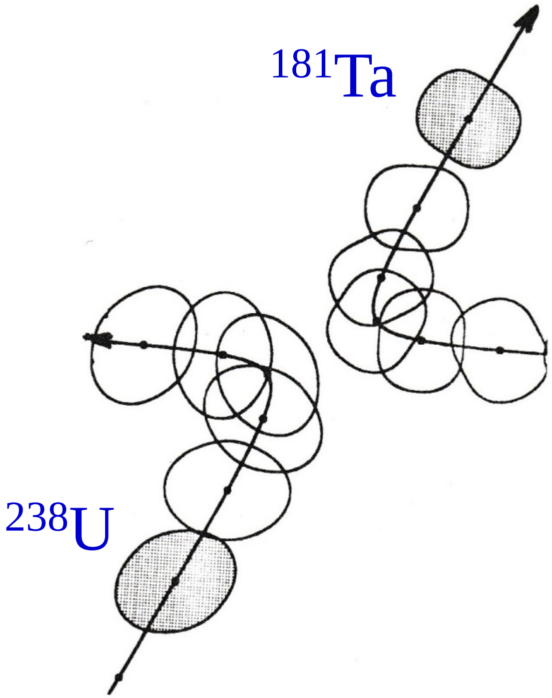
with

$cos\theta_{\gamma 1} = cos\vartheta_1 cos\vartheta_\gamma + sin\vartheta_1 sin\vartheta_\gamma cos(\varphi_\gamma - \varphi_1)$

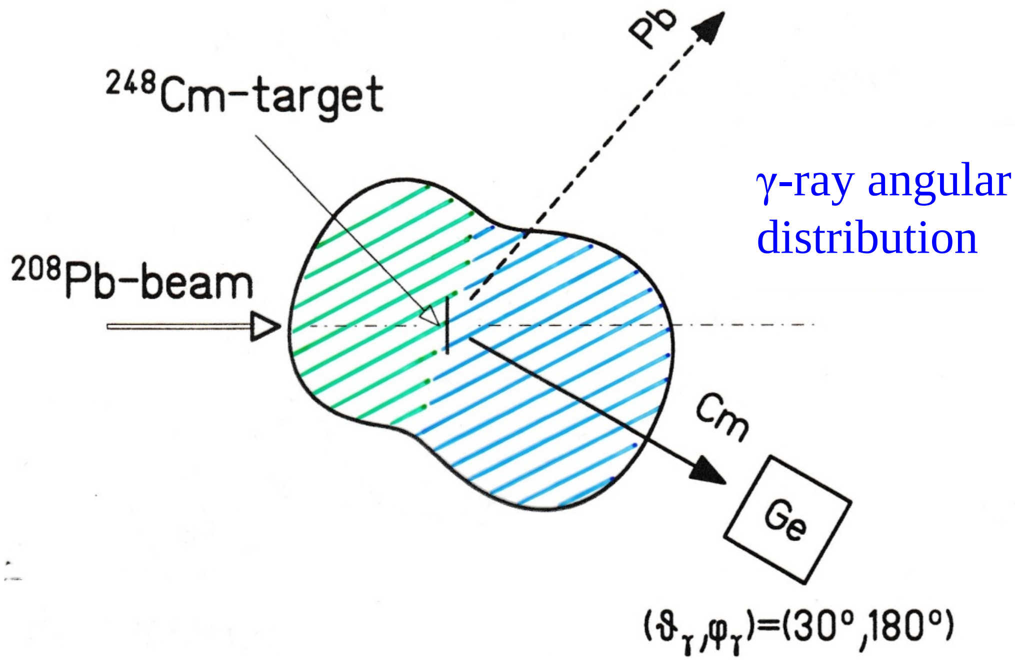
$\Delta E \cong E^* \cdot \beta_1 \cdot sin\theta_{\gamma 1} \cdot \Delta\theta_{\gamma 1}$ Doppler broadening

Inelastic heavy-ion scattering





Lorentz transformation



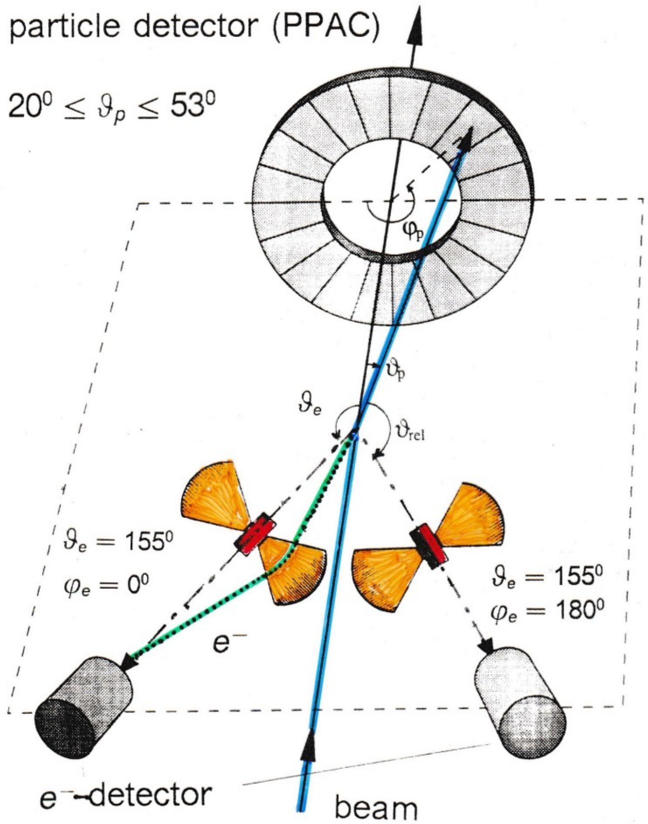
Contraction of the solid angle element in the laboratory system

$$\frac{d\Omega}{d\Omega^*} = \left\{\frac{E^*}{E}\right\}^2$$

with

$$E^* = \gamma \cdot E \cdot (1 - \beta \cdot cos\theta) \qquad \text{Doppler formula}$$

Experimental arrangement (electron detection)



Doppler broadening

$\Delta\vartheta_e = 20^0$
target – Mini-Orange: 19 cm
Mini-Orange – Si detector: 6 cm

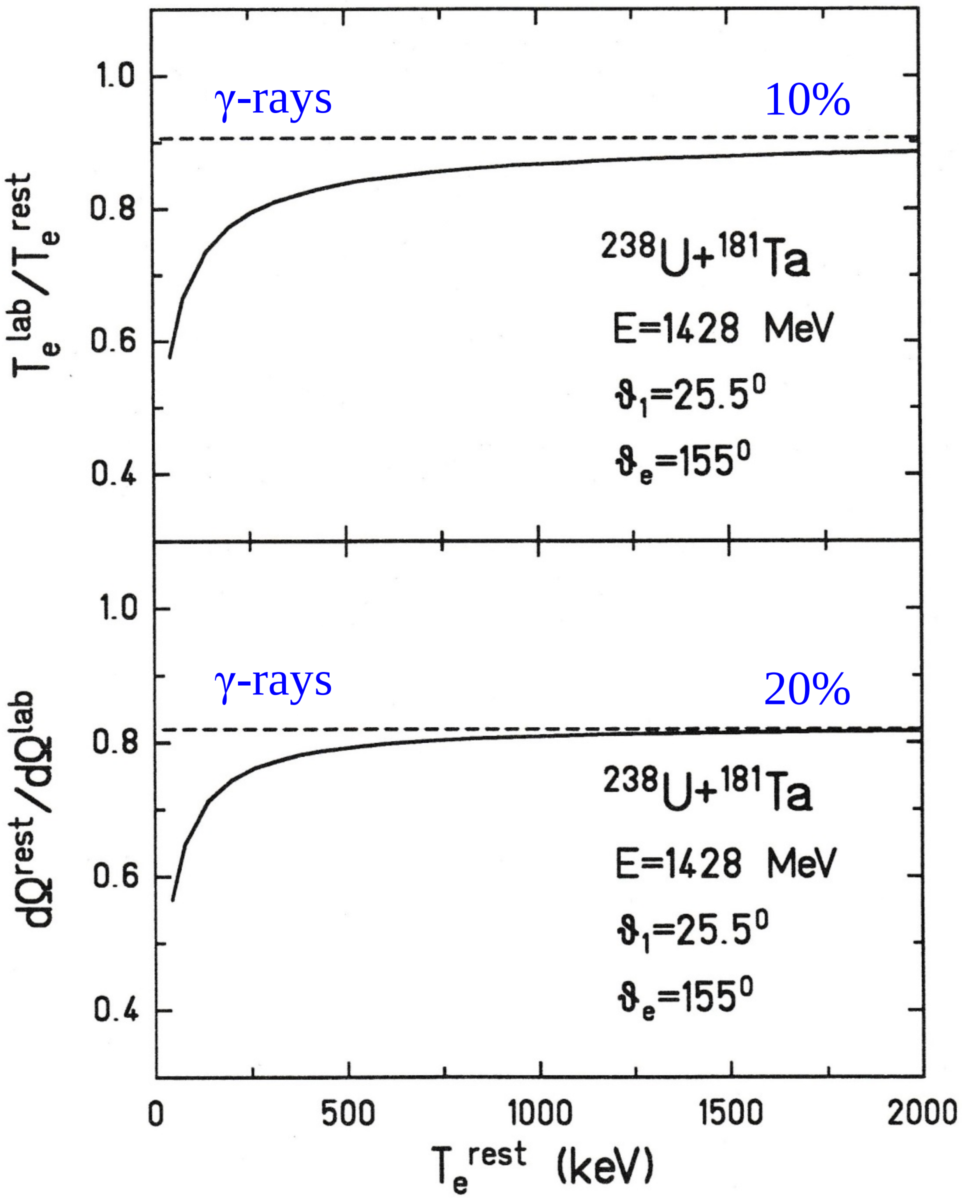
For projectile excitation:

$$\boxed{T_e^* = \gamma \cdot T_e \cdot \left\{1 - \beta_1 \cdot \sqrt{1 + 2m_e\, c^2/T_e} \cdot cos\theta_{e1}\right\} + m_e c^2 \cdot (\gamma - 1)}$$

with

$$cos\theta_{e1} = cos\vartheta_1 cos\vartheta_e + sin\vartheta_1 sin\vartheta_e cos(\varphi_e - \varphi_1)$$

Lorentz transformation



energy shift

solid angle contraction

Segmented detectors