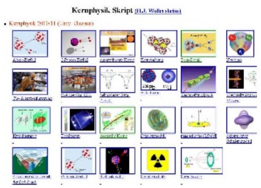
Outline: γ-decay

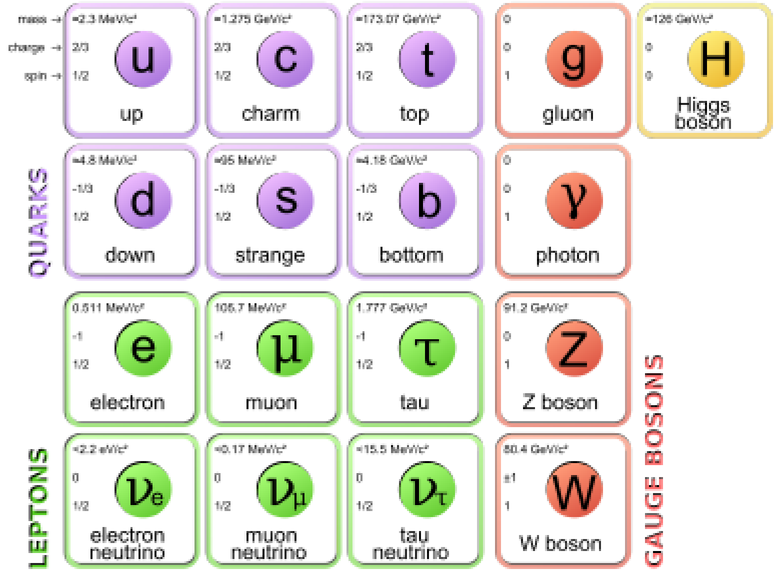
Lecturer: Hans-Jürgen Wollersheim

e-mail: h.j.wollersheim@gsi.de

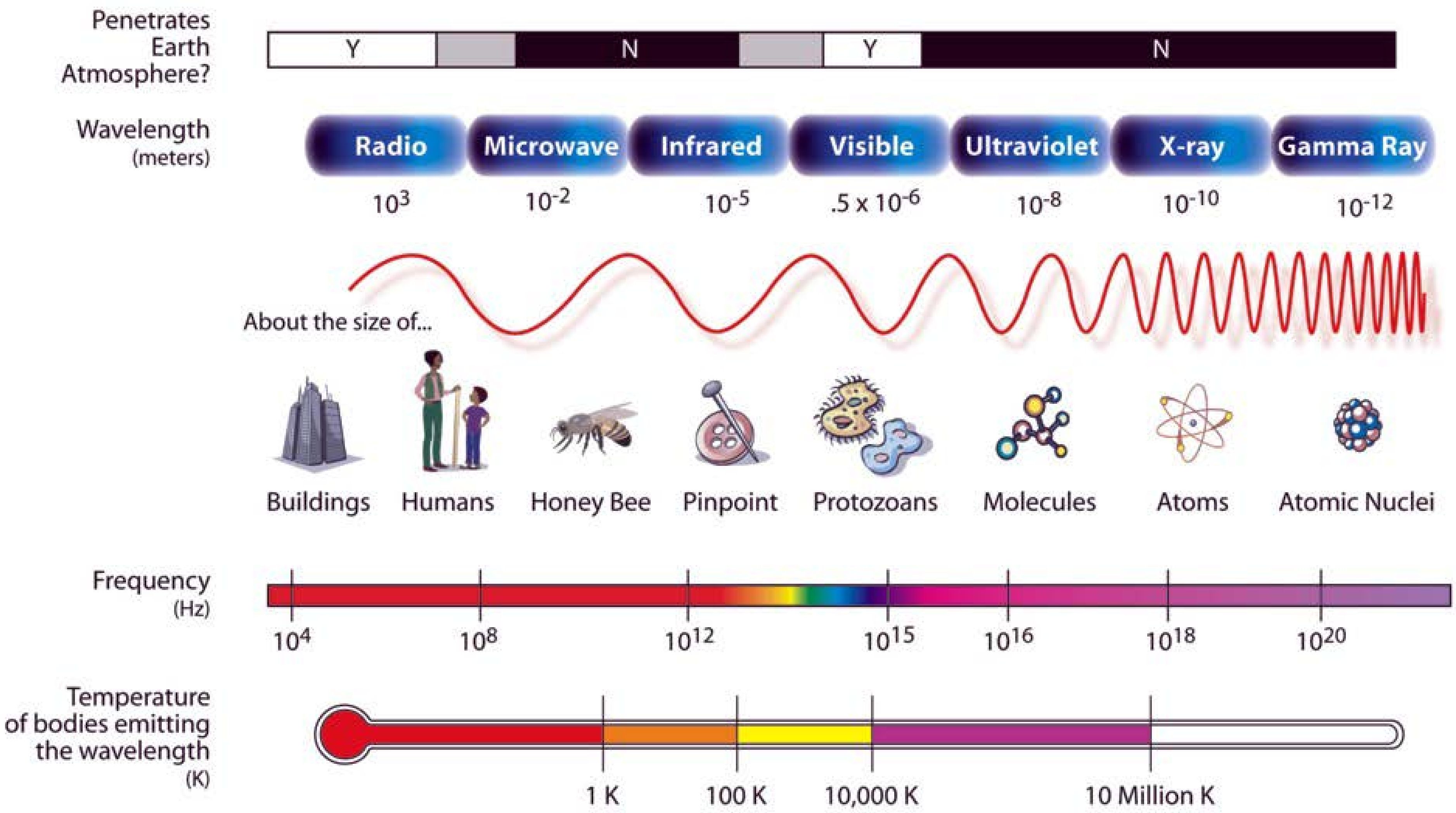
web-page: https://web-docs.gsi.de/~wolle/ and click on 

1. electromagnetic spectrum
2. angular momentum in γ-decay
3. emission of electromagnetic radiation
4. single particle transition
5. conversion electrons

- γ-decay is an *electromagnetic process* where the nucleus decreases in excitation energy, but does not change proton or neutron numbers
- This decay process only involves the emission of photons (γ-rays carry spin 1)



THE ELECTROMAGNETIC SPECTRUM



Electromagnetic decay modes

- Pair conversion coefficients
- E0 transitions: ΔL=0

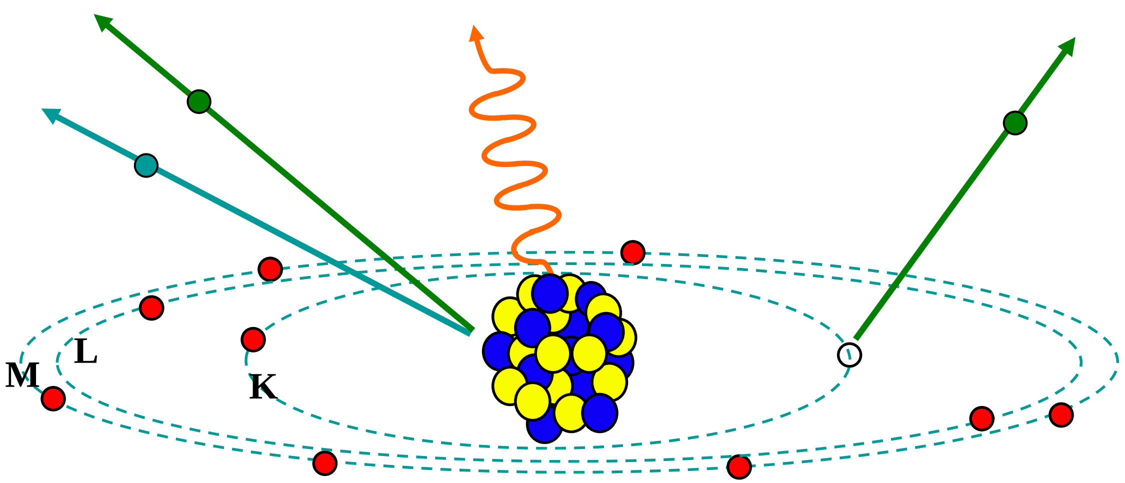
e^+ - e^- pair

- Angular distribution with spin oriented
- Angular correlations
- Polarization effects

γ-rays

- Electron conversion coefficients
- E0 transitions: ΔL=0

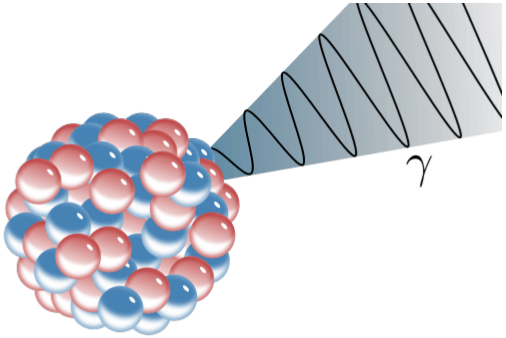
electron conversion



higher order effects: for example 2 photon emission is very weak

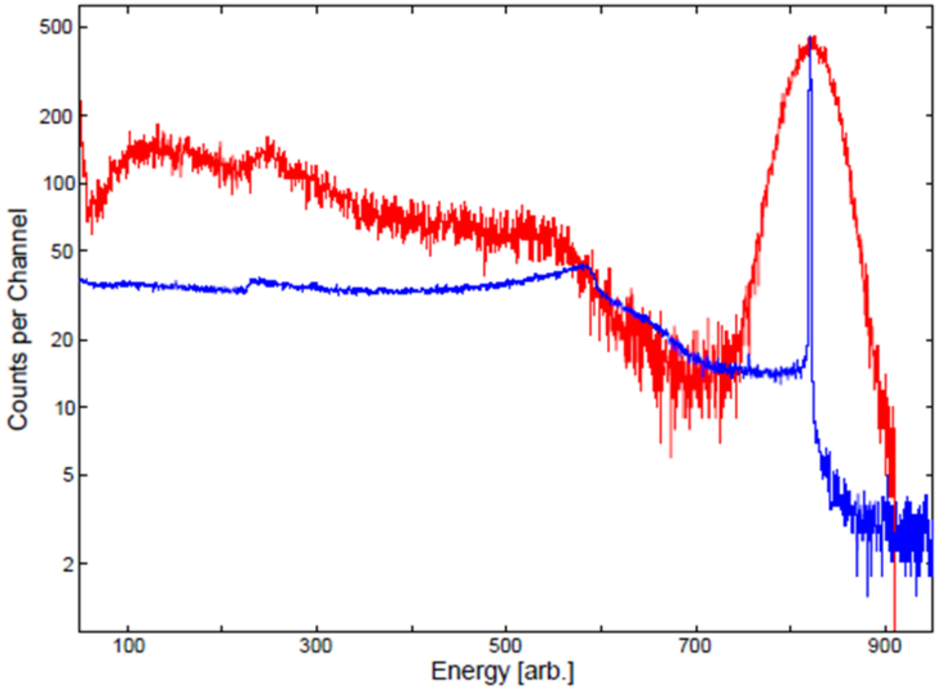
γ-decay

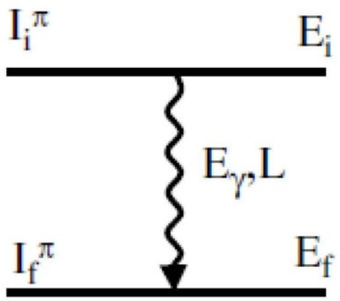
❖ Gamma-ray emission is usually the dominant decay mode



Measurements of γ-rays let us deduce:
energy, spin (angular distr. / correl.), parity (polarization), magnetic moment, lifetime (recoil distance, Doppler shift), …
of the involved nuclear levels.

^{137}Cs detected in red: NaI scintillator
blue: HPGe (high purity Ge semiconductor)

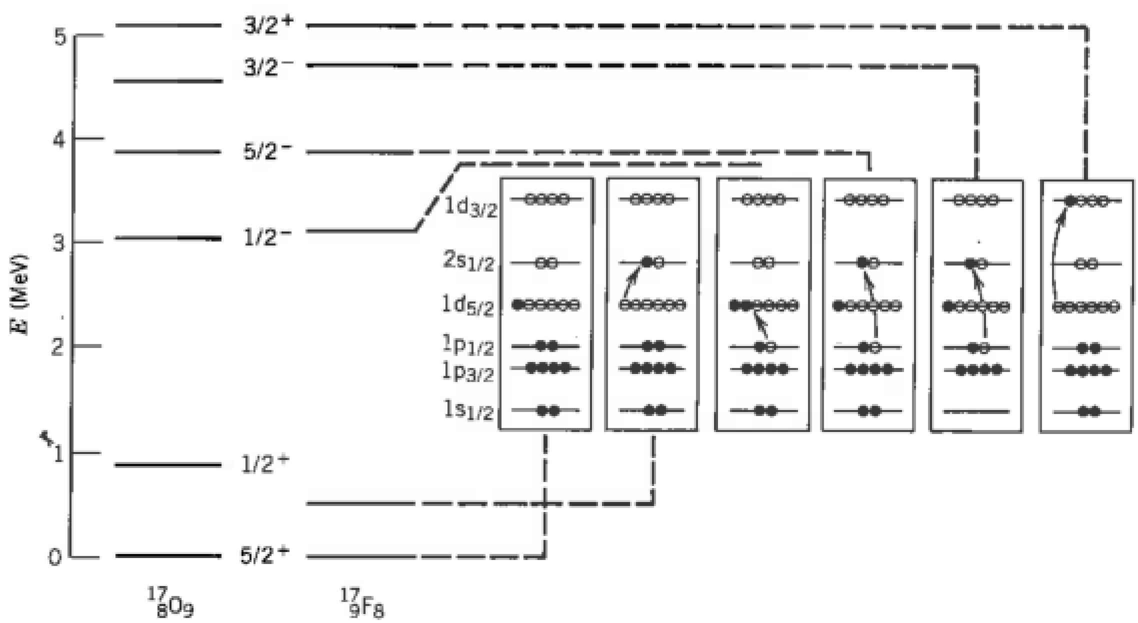




$$^A_Z X^*_N \rightarrow {}^A_Z X^{(*)}_N$$

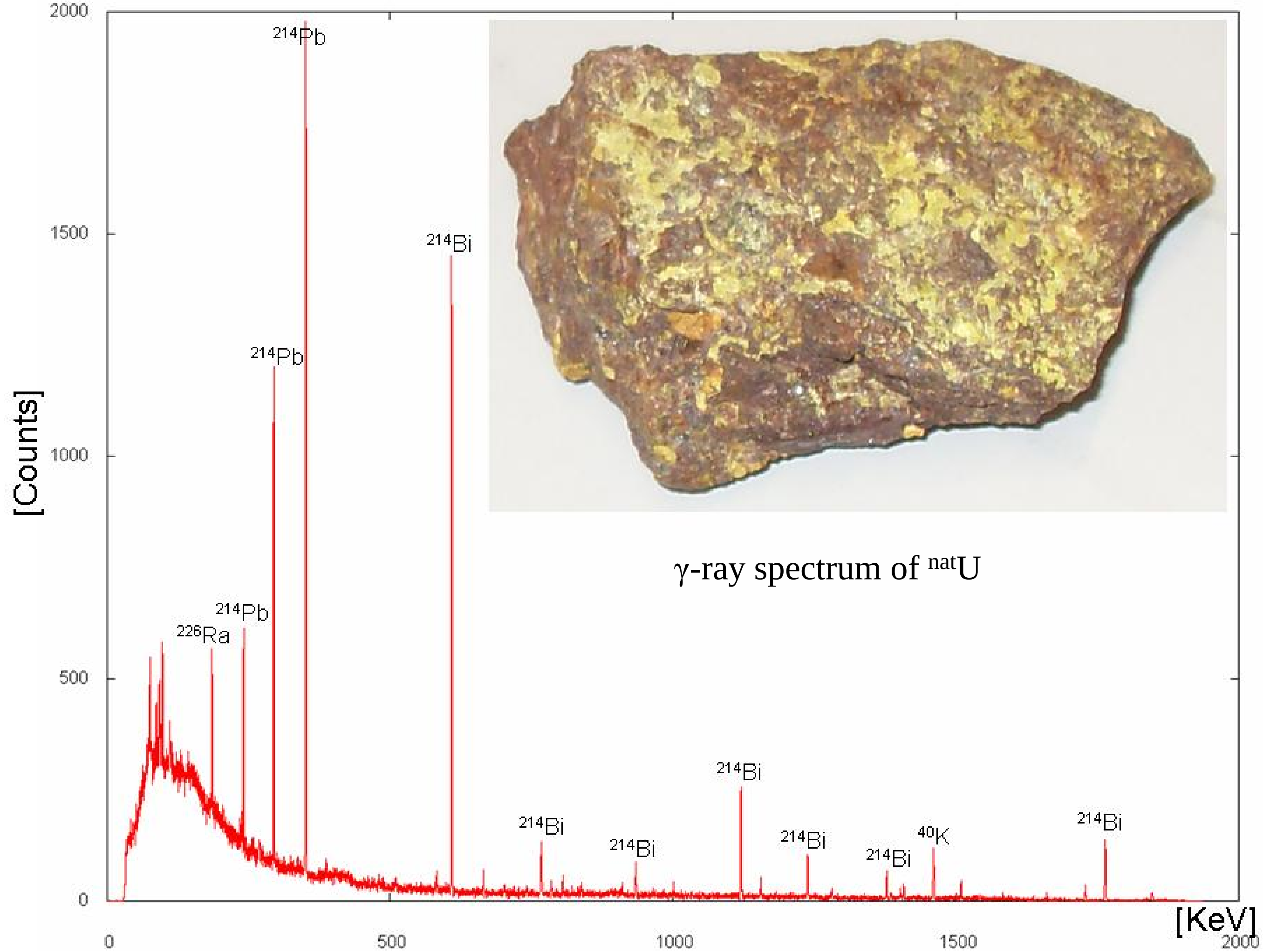
γ-decay in a nutshell

- The photon emission of the nucleus essentially results from a re-ordering of nucleons within the shells.
- This re-ordering often follows α or β decay, and moves the system into a more energetically favorable state.

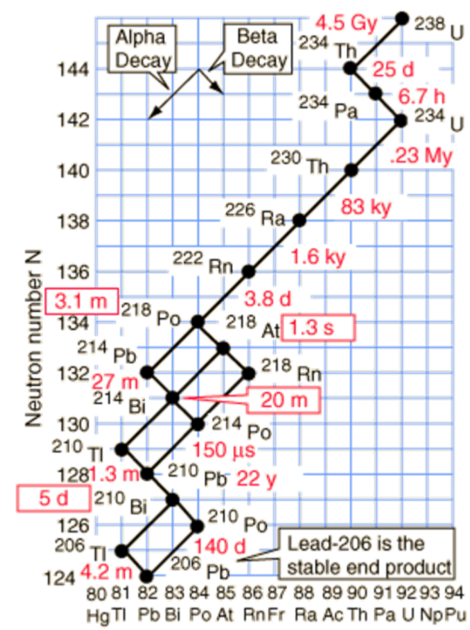


Source: Krane, Fig. 5.11

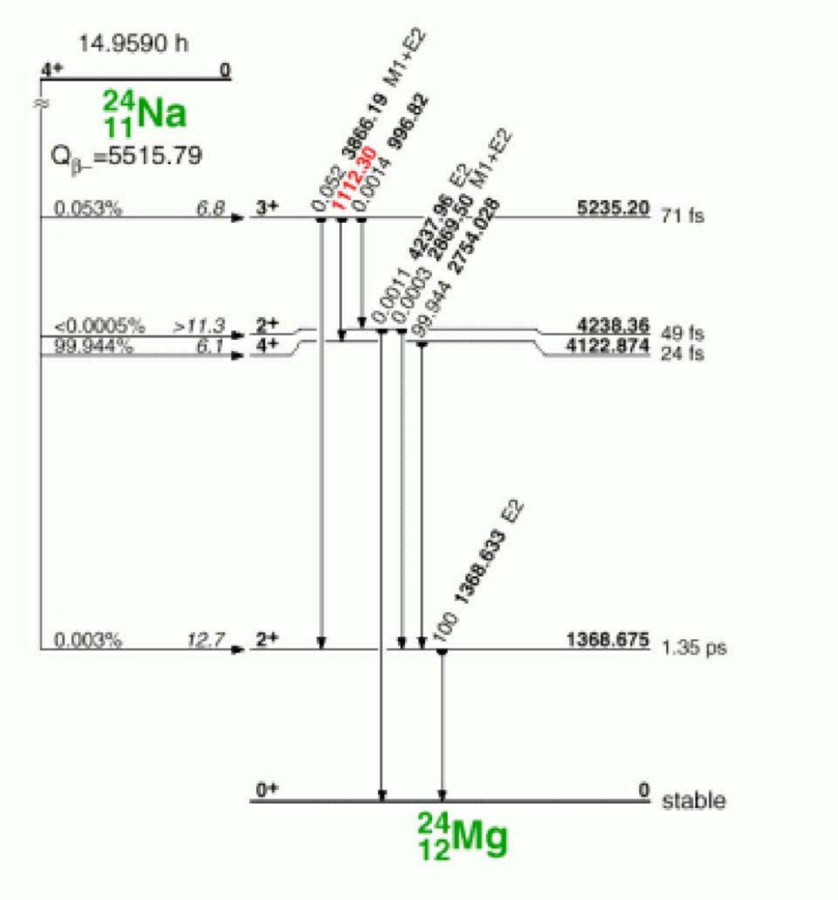
γ-decay



γ-ray spectrum of ^{nat}U



Most β-decay transitions are followed by γ-decay.



Classical electrodynamics

- The nucleus is a collection of moving charges, which can induce magnetic/electric fields
- The power radiated into a small area element is proportional to $sin^2(\theta)$
- The average power radiated for an electric dipole is:

$$P = \frac{1}{12\pi\epsilon_0}\frac{\omega^4}{c^3}d^2$$

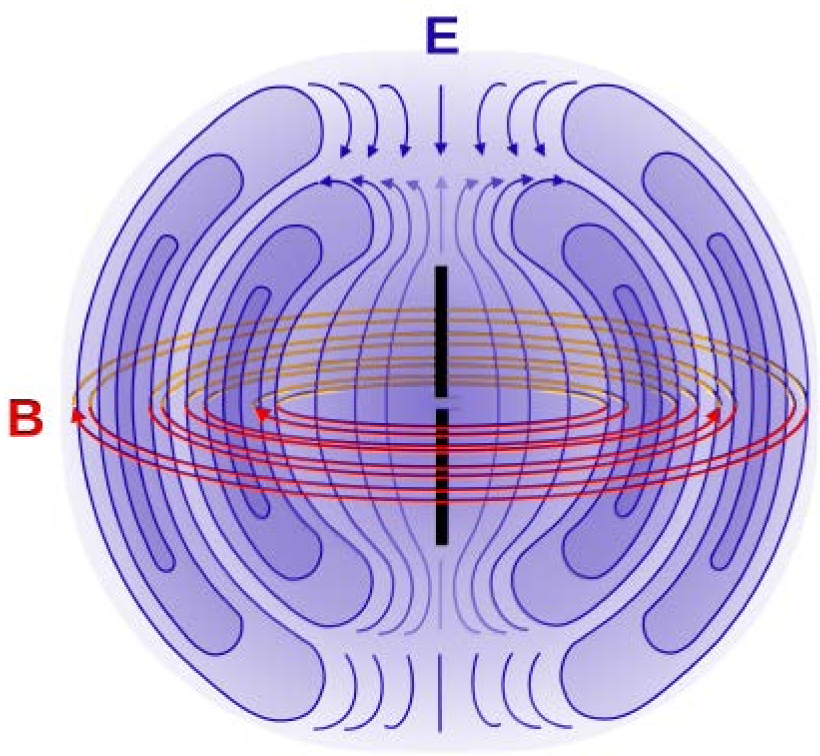
- For a magnetic dipole is

$$P = \frac{1}{12\pi\epsilon_0}\frac{\omega^4}{c^5}\mu^2$$

Electric/magnetic dipoles

Electric and magnetic dipole fields have opposite parity:
Magnetic dipoles have even parity and electric dipole fields have odd parity.

$$\Rightarrow \; \pi(M\ell) = (-1)^{\ell+1} \;\; and \;\; \pi(E\ell) = (-1)^{\ell}$$



Higher order multipoles

It is possible to describe the angular distribution of the radiation field as a function of the *multipole order* using Legendre polynomials.

- ℓ: The index of radiation
  2^{ℓ}: The multipole order of the radiation

- $\ell = 1 \rightarrow Dipole$
  $\ell = 2 \rightarrow Quadrupole$
  $\ell = 3 \rightarrow Octupole$

- The associated Legendre polynomials $P_{2\ell}\left(cos(\theta)\right)$ are:
  For $\ell = 1$: $P_2 = \frac{1}{2}(3 \cdot cos^2(\theta) - 1)$
  For $\ell = 2$: $P_4 = \frac{1}{8}(35cos^4(\theta) - 30cos^2(\theta) + 3)$

Angular momentum in γ-decay

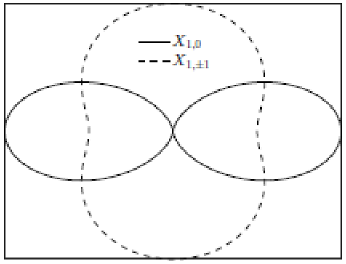
- *The photon is a spin-1 boson*
- Like α-decay and β-decay the emitted γ-ray can carry away units of *angular momentum* ℓ, which has given us different multipolarities for transitions.
- For orbital angular momentum, we can have values $\ell = 0,1,2,3,\cdots$ that correspond to our multipolarity.
- Therefore, our selection rule is:

$$|I_i - I_f| \leq \ell \leq I_i + I_f$$

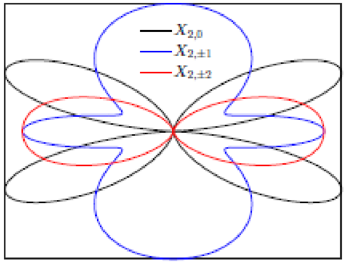
Characteristics of multipolarity

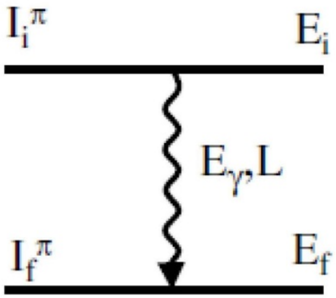
L	multipolarity	π(Eℓ) / π(Mℓ)	angular distribution
1	dipole	-1 / +1	
2	quadrupole	+1 / -1	
3	octupole	-1 / +1	
4	hexadecapole	+1 / -1	
⋮			

ℓ = 1



ℓ =2





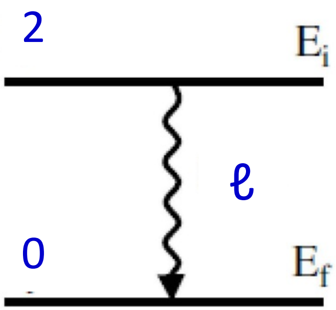
$$E_\gamma = E_i - E_f$$

$$|I_i - I_f| \leq \ell \leq I_i + I_f$$

$$\Delta\pi(E\ell) = (-1)^\ell$$

$$\Delta\pi(M\ell) = (-1)^{\ell+1}$$

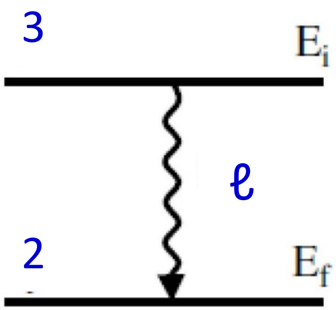
The basics of the situation



$$|2-0| \leq \ell \leq 2+0$$

Here $\Delta I = 2$ and $\ell = 2$
this is a stretched transition

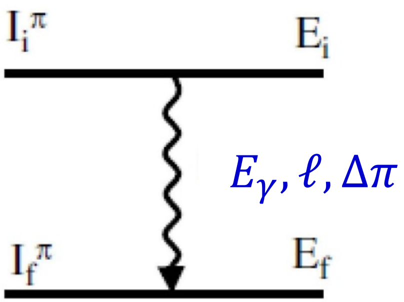
The basics of the situation



$$|3-2| \leq \ell \leq 3+2$$

Here $\Delta I = 1$ but $\ell = 1,2,3,4,5$
and the transition can be a mix of 5 multipolarities

The basics of the situation



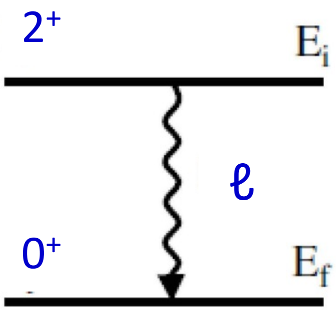
Electromagnetic transitions:

$$\Delta\pi\ (electric) = (-1)^{\ell}$$

$$\Delta\pi\ (magnetic) = (-1)^{\ell+1}$$

$\Delta\pi$	*yes*	E1	M2	E3	M4
	no	M1	E2	M3	E4

The basics of the situation

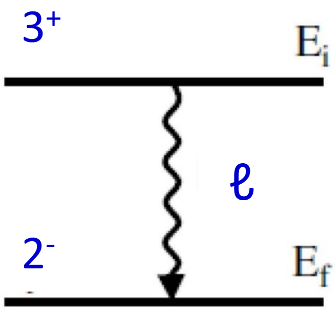


$$|2-0| \le \ell \le 2+0$$

$\ell = 2$ and no change in parity

$\Delta\pi$					
	no	~~M1~~	E2	M3	E4

The basics of the situation



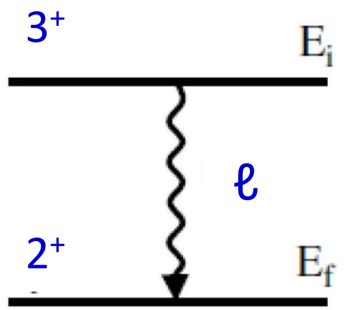
$$|3-2| \leq \ell \leq 3+2$$

Here $\Delta I = 1$ but $\ell = 1,2,3,4,5$

$\Delta\pi$	*yes*	E1	M2	E3	M4

mixed E1,M2,E3,M4,E5

The basics of the situation



$$|3-2| \leq \ell \leq 3+2$$

Here $\Delta J = 1$ but $\ell = 1,2,3,4,5$

$\Delta\pi$					
	no	M1	E2	M3	E4

mixed M1,E2,M3,E4,M5

The basics of the situation

$3^+ \rightarrow 2^+$: mixed M1,E2,M3,E4,M5

$3^+ \rightarrow 2^-$: mixed E1,M2,E3,M4,E5

In general only the lowest 2 multipoles compete

and (for reasons we will see later)

$\ell + 1$ multipole generally only competes if it is electric:

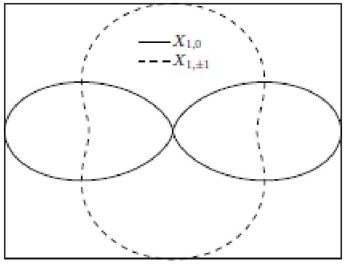
$3^+ \rightarrow 2^+$: mixed M1/E2

$3^+ \rightarrow 2^-$: almost pure E1 (very little M2 admixture)

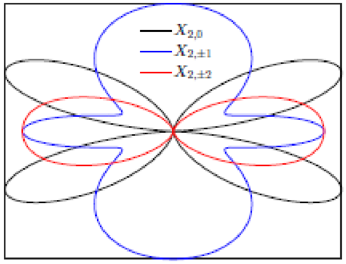
Characteristics of multipolarity

L	multipolarity	π(Eℓ) / π(Mℓ)	angular distribution
1	dipole	-1 / +1	
2	quadrupole	+1 / -1	
3	octupole	-1 / +1	
4	hexadecapole	+1 / -1	
⋮			

$\ell = 1$



$\ell = 2$



parity: electric multipoles $\pi(E\ell) = (-1)^{\ell}$, magnetic multipoles $\pi(M\ell) = (-1)^{\ell+1}$

The power radiated is proportional to:

$$P(\sigma\ell) \propto \frac{2(\ell+1)\cdot c}{\varepsilon_0 \cdot \ell \cdot [(2\ell+1)!!]^2} \left(\frac{\omega}{c}\right)^{2\ell+2} |\mathcal{M}(\sigma\ell)|^2$$

where σ means either E or M and $\mathcal{M}(\sigma\ell)$ is the E or M multipole moment of the appropriate kind.

Emission of electromagnetic radiation

$$T(E1; I_i \rightarrow I_f) = 1.590\ 10^{17}\ E_\gamma^3\ B(E1; I_i \rightarrow I_f)$$

$$T(E2; I_i \rightarrow I_f) = 1.225\ 10^{13}\ E_\gamma^5\ B(E2; I_i \rightarrow I_f)$$

$$T(E3; I_i \rightarrow I_f) = 5.709\ 10^{8}\ E_\gamma^7\ B(E3; I_i \rightarrow I_f)$$

$$T(E4; I_i \rightarrow I_f) = 1.697\ 10^{4}\ E_\gamma^9\ B(E4; I_i \rightarrow I_f)$$

$$T(M1; I_i \rightarrow I_f) = 1.758\ 10^{13}\ E_\gamma^3\ B(M1; I_i \rightarrow I_f)$$

$$T(M2; I_i \rightarrow I_f) = 1.355\ 10^{7}\ E_\gamma^5\ B(M2; I_i \rightarrow I_f)$$

$$T(M3; I_i \rightarrow I_f) = 6.313\ 10^{0}\ E_\gamma^7\ B(M3; I_i \rightarrow I_f)$$

$$T(M4; I_i \rightarrow I_f) = 1.877\ 10^{-6}\ E_\gamma^9\ B(M4; I_i \rightarrow I_f)$$

where $E_\gamma = E_i - E_f$ is the energy of the emitted γ quantum in MeV (E_i, E_f are the nuclear level energies, respectively), and the reduced transition probabilities B(Eℓ) in units of $e^2(\text{barn})^\ell$ and B(Mℓ) in units of $\mu_N^2 = (e\hbar/2m_N c)^2\ \ (fm)^{2\ell-2}$

Single particle transition (Weisskopf estimate)

$$B(E\lambda; I_i \to I_{gs}) = \frac{(1.2)^{2\lambda}}{4\pi}\left(\frac{3}{\lambda+3}\right)^2 A^{2\lambda/3} \quad e^2(fm)^{2\lambda}$$

$$B(M\lambda; I_i \to I_{gs}) = \frac{10}{\pi}(1.2)^{2\lambda-2}\left(\frac{3}{\lambda+3}\right)^2 A^{(2\lambda-2)/3} \quad \mu_N^2(fm)^{2\lambda-2}$$

For the first few values of λ, the Weisskopf estimates are

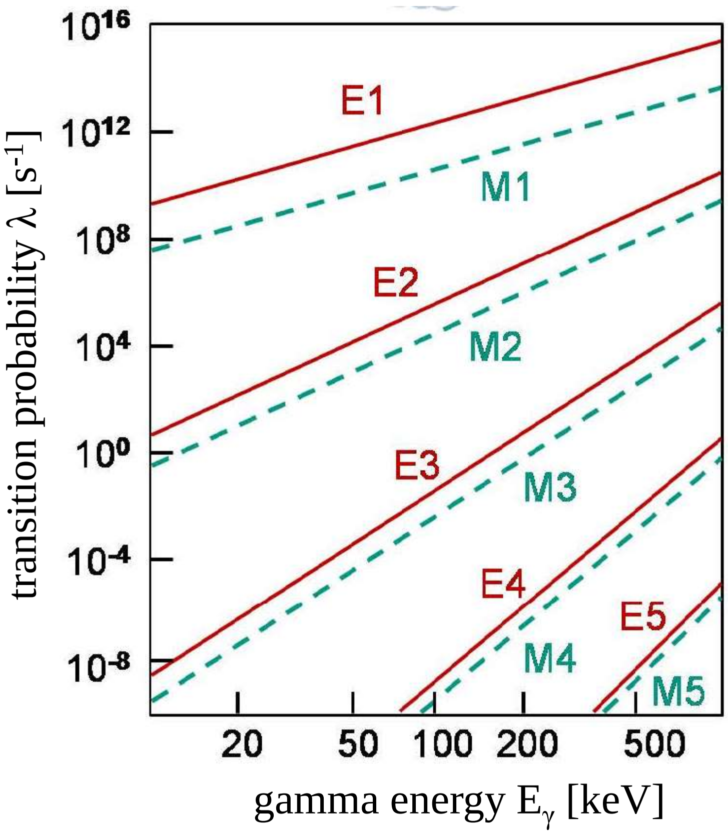
$$B(E1; I_i \to I_{gs}) = 6.446\ 10^{-4}\ A^{2/3} \quad e^2(barn)$$

$$B(E2; I_i \to I_{gs}) = 5.940\ 10^{-6}\ A^{4/3} \quad e^2(barn)^2$$

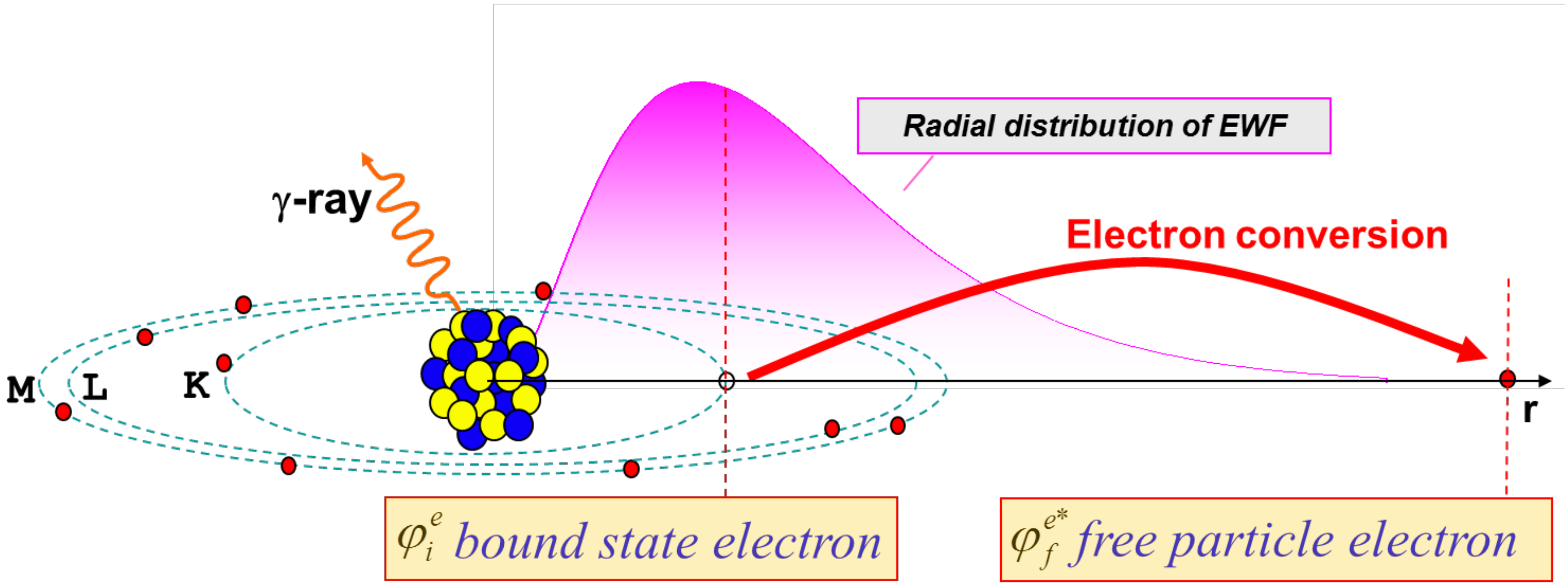
$$B(E3; I_i \to I_{gs}) = 5.940\ 10^{-8}\ A^{2} \quad e^2(barn)^3$$

$$B(E4; I_i \to I_{gs}) = 6.285\ 10^{-10}\ A^{8/3} \quad e^2(barn)^4$$

$$B(M1; I_i \to I_{gs}) = 1.790 \quad \left(\frac{e\hbar}{2Mc}\right)^2$$



Conversion electrons



Energetics of CE-decay (i=K, L, M,….)

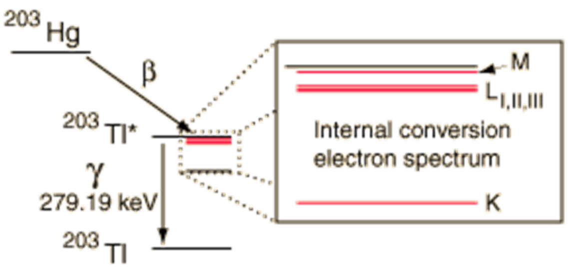
$$E_i = E_f + E_{ce,i} + E_{BE,i}$$

γ- and CE-decays are independent; transition probability (λ ~ Intensity)

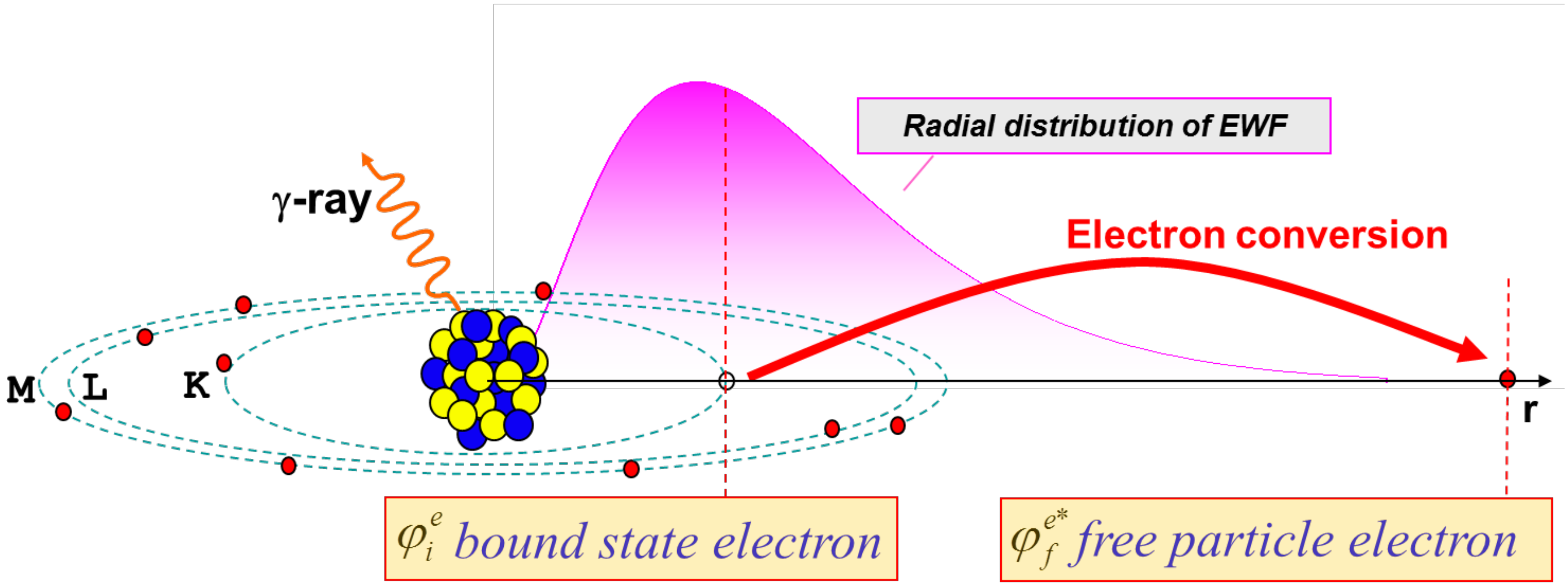
$$\lambda_T = \lambda_\gamma + \lambda_{CE} = \lambda_\gamma + \lambda_K + \lambda_L + \lambda_M \ldots\ldots$$

Conversion coefficient

$$\alpha_i = \frac{\lambda_{CE,i}}{\lambda_\gamma}$$



Internal conversion



❖ For an electromagnetic transition internal conversion can occur instead of emission of gamma radiation. In this case the transition energy $Q = E_\gamma$ will be transferred to an electron of the atomic shell.

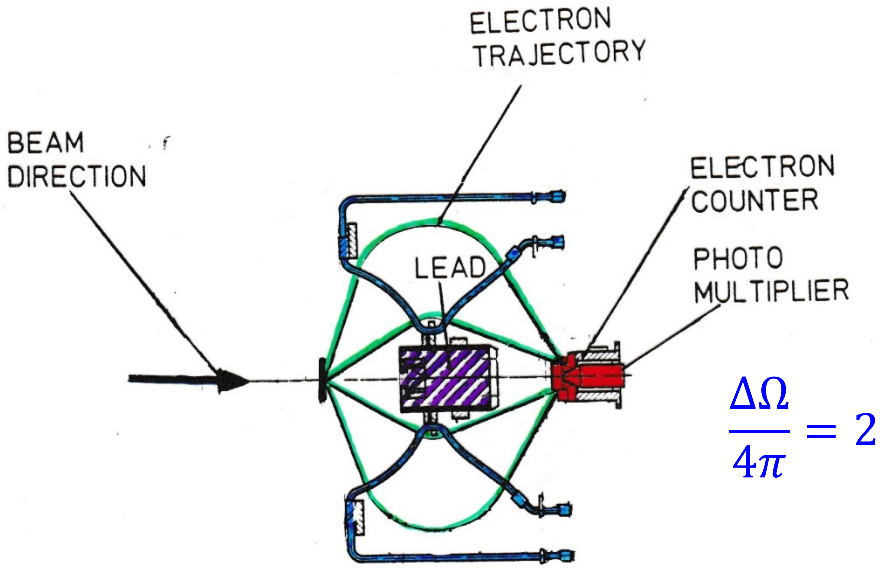
$$T_e = E_\gamma - B_e$$

T_e: kinetic energy of the electron
B_e: binding energy of the electron

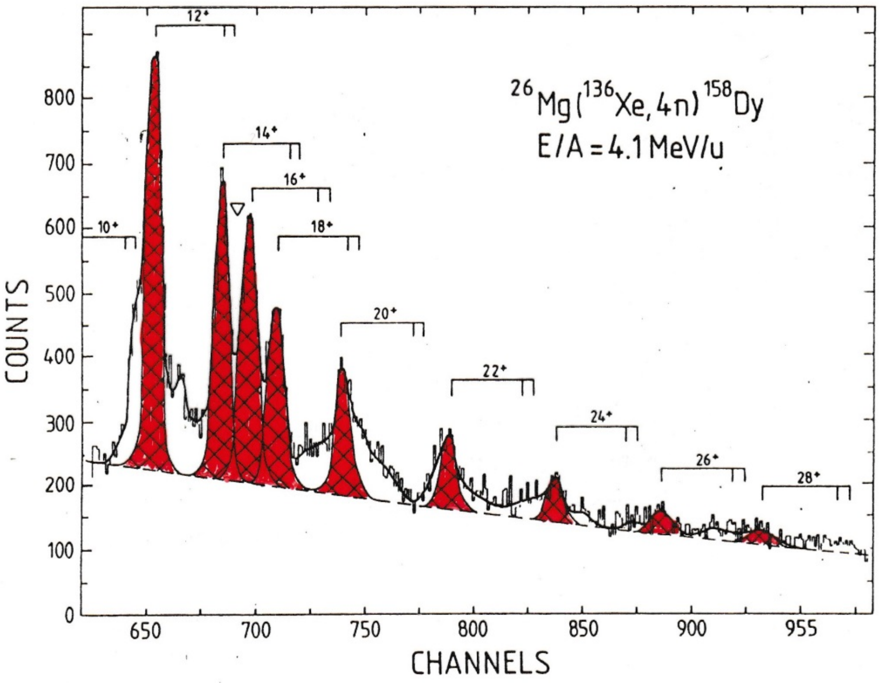
internal conversion is important for:
- heavy nuclei ~ Z^3
- high multipolarities Eℓ or Mℓ
- small transition energies

$$\alpha_k(El) \propto Z^3 \left(\frac{L}{L+1}\right)\left(\frac{2m_ec^2}{E}\right)^{L+5/2}$$

Electron spectroscopy



$$\frac{\Delta\Omega}{4\pi} = 26\%$$



Doppler shift correction for projectile:

$$T_e^* = \gamma \cdot T_e \cdot \left\{1 - \beta_1 \cdot \sqrt{1 + 2m_e\, c^2/T_e} \cdot cos\theta_{e1}\right\} + m_e c^2 \cdot (\gamma - 1)$$

$$cos\theta_{e1} = cos\vartheta_1 cos\vartheta_e + sin\vartheta_1 sin\vartheta_e cos(\varphi_e - \varphi_1)$$

		$(\frac{\Delta p}{p})_e / \%$
resolution of the spectrometer including Doppler correction as calculated for a point source		0.4
scattering in the target	(i)	0.004
beam optics	(ii)	0.11
evaporation of neutrons	(iii)	0.09
energy loss in the target	(iv)	0.31
energy straggling of the projectiles	(v)	0.006
quadratic sum		0.53 %
experimental resolution		0.56

Comparison of α-decay, β-decay and γ-decay

de Broglie wavelength: $\lambda = \frac{h}{p} = \frac{h \cdot c}{\sqrt{E_{kin} \cdot (E_{kin} + 2mc^2)}} = \frac{1239.84[MeV\ fm]}{\sqrt{E_{kin} \cdot (E_{kin} + 2mc^2)}}$

decay	Energy [MeV]	de Broglie λ [fm]
α-particle, m_α = 3727 MeV/c²	5	6.42
β-particle, m_e = 0.511 MeV/c²	1	871.92
γ-photon	1	$\lambda = {h \cdot c}/{E} = {1240}/{E}$

For α-particles this dimension is somewhat smaller than the nucleus and this is why a semi-classical treatment of α-decay is successful.
The typical β-particle has a large wavelength λ in comparison to the nuclear size and a quantum mechanical is dictated and wave analysis is called for.
For γ-decay the wavelength λ ranges from 12400 – 1240 fm (0.1 – 1 MeV). Clearly, only a quantum mechanical approach has a chance of success.

γ-decay

γ-spectroscopy yields some of the most precise knowledge of nuclear structure, as spin, parity and ΔE are all measurable.

Transition rates between initial Ψ_N^* and final $\Psi_N^{\prime}$ nuclear states, resulting from electromagnetic decay producing a photon with energy E_γ can be described by Fermi´s Golden rule:

$$\lambda = \frac{2\pi}{\hbar} \left| \left\langle \Psi_N^{\prime} \psi_\gamma \middle| \mathcal{M}_{em} \middle| \Psi_N^* \right\rangle \right|^2 \frac{dn_\gamma}{dE_\gamma}$$

where $\mathcal{M}_{em}$ is the electromagnetic transition operator and dn_γ / dE_γ is the density of final states. The photon wave function ψ_γ and $\mathcal{M}_{em}$ are well known, therefore measurements of λ provide detailed knowledge of nuclear structure.

A γ-decay lifetime is typically 10^{-12} [s] and sometimes even as short as 10^{-19} [s]. However, this time span is an eternity in the life of an excited nucleon. It takes about $4 \cdot 10^{-22}$ [s] for a nucleon to cross the nucleus.