Outline: Experimental Nuclear Astrophysics M. Aliotta

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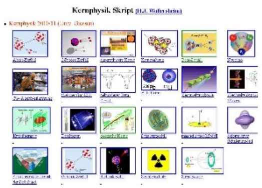
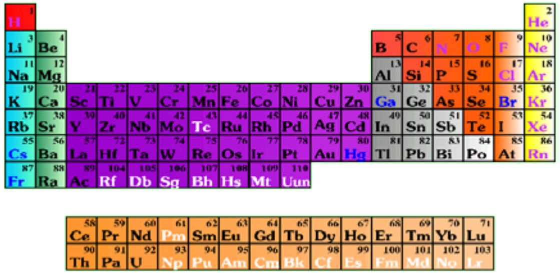


1. thermonuclear reactions
2. nuclear reaction rates
3. direct & resonant reactions

- study energy generation processes in stars
- study nucleosynthesis of the elements



- What is the origin of the elements?
- How do stars/galaxies form and evolve?
- What powers the stars?
- How old is the universe?
- …

NUCLEAR PHYSICS

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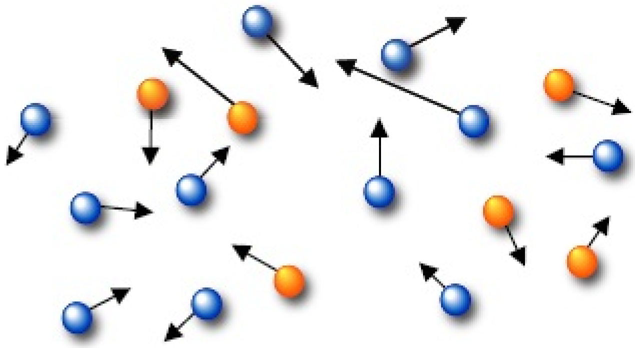
KEY for understanding

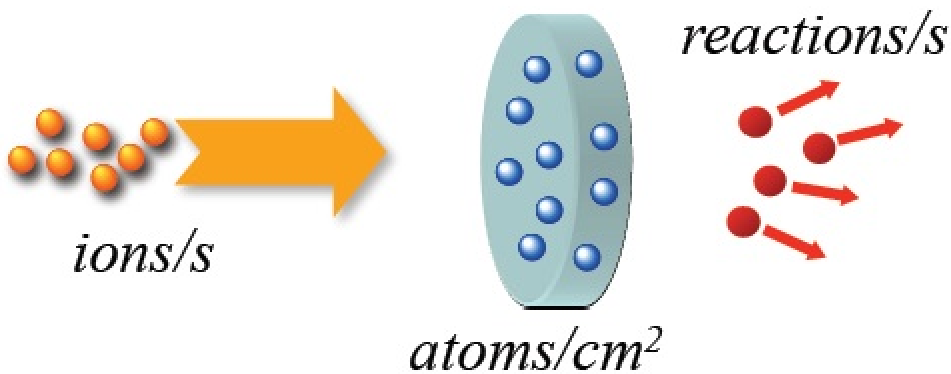
MACRO-COSMOS intimately related to MICRO-COSMOS

Nuclear reactions in the lab & in space

In astrophysical events:



In the lab:



$$\frac{reactions}{s} = \frac{ions}{s} \cdot \frac{atoms}{cm^2} \cdot \sigma(cm^2)$$

Thermonuclear reactions in stars

<u>Aston:</u>

measurements of atomic masses

$M_{nucl} < \Sigma m_p + \Sigma m_n \Rightarrow \Delta E = \Delta M_n c^2$

enormous energy stored in nuclei!

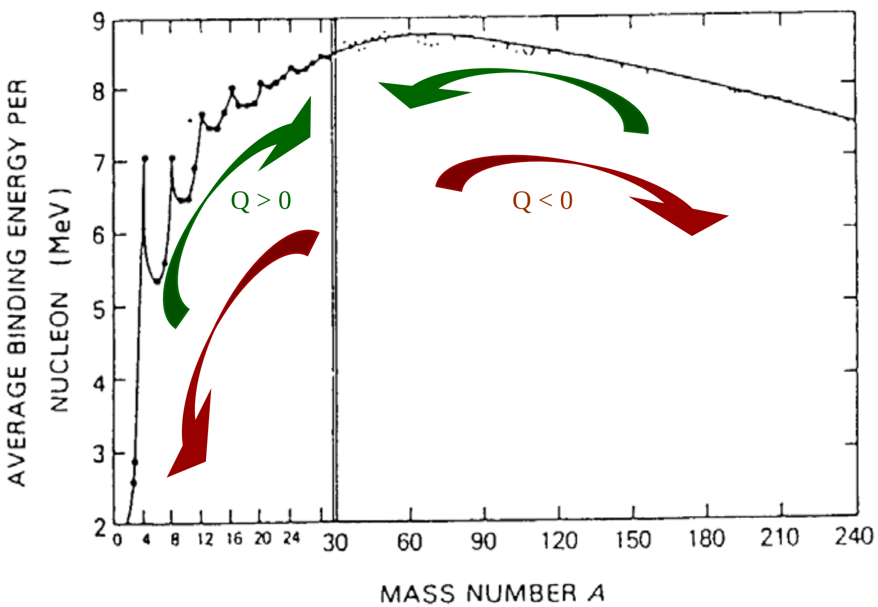
<u>Rutherford (1919):</u>

discovery of nuclear reactions

- liberate nuclear energy source
- complex nuclides formed through reactions

amount of energy liberated in nuclear reaction: $Q = [(m_1+m_2)-(m_3+m_4)]c^2 > 0$

Binding energy curve



spontaneous nuclear processes:

$Q > 0$

<u>fusion</u> up to Fe region

<u>fission</u> of heavy nuclei

H most abundant element in the Universe

⇓

FUSION reactions most effective in stars

REVIEWS OF

MODERN PHYSICS

VOLUME 29, NUMBER 4 OCTOBER, 1957

Rev. Mod. Phys. 29 (1957) 547

Synthesis of the Elements in Stars*

E. MARGARET BURBIDGE, G. R. BURBIDGE, WILLIAM A. FOWLER, AND F. HOYLE

Kellogg Radiation Laboratory, California Institute of Technology, and Mount Wilson and Palomar Observatories, Carnegie Institution of Washington, California Institute of Technology, Pasadena, California

"It is the stars, The stars above us, govern our conditions";
(*King Lear*, Act IV, Scene 3)

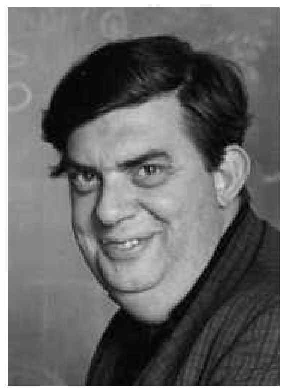
but perhaps

"The fault, dear Brutus, is not in our stars, But in ourselves,"
(*Julius Caesar*, Act I, Scene 2)

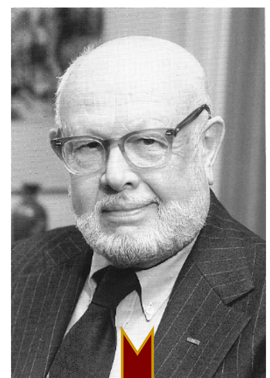
(B^2FH, 1957)

Burbidge

Burbidge



Fowler



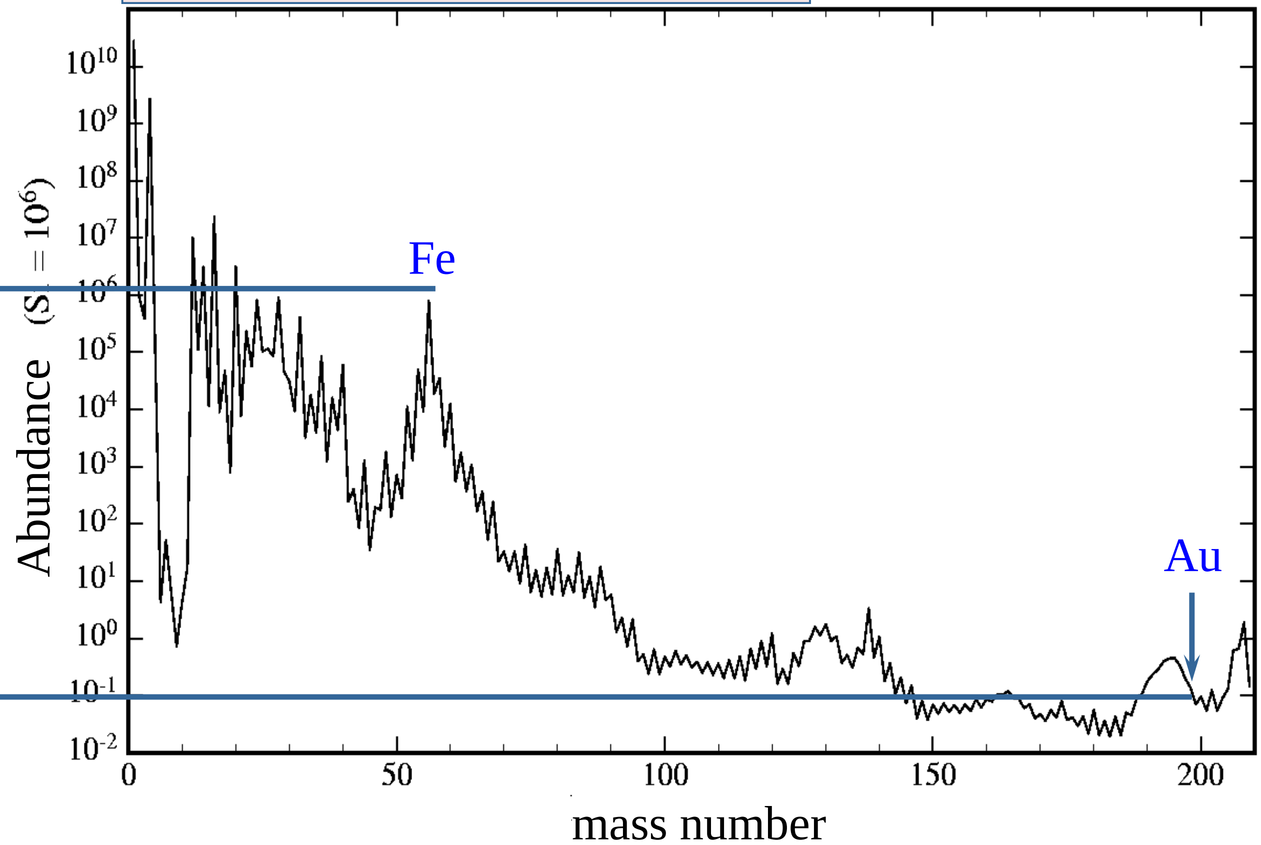
Hoyle

1983
Nobel Prize

"for his theoretical and experimental studies of the nuclear reactions of importance in the formation of the chemical elements in the universe"

Why does one kilogram of gold costs so much more than one kilogram of iron?

Abundance curve of the elements



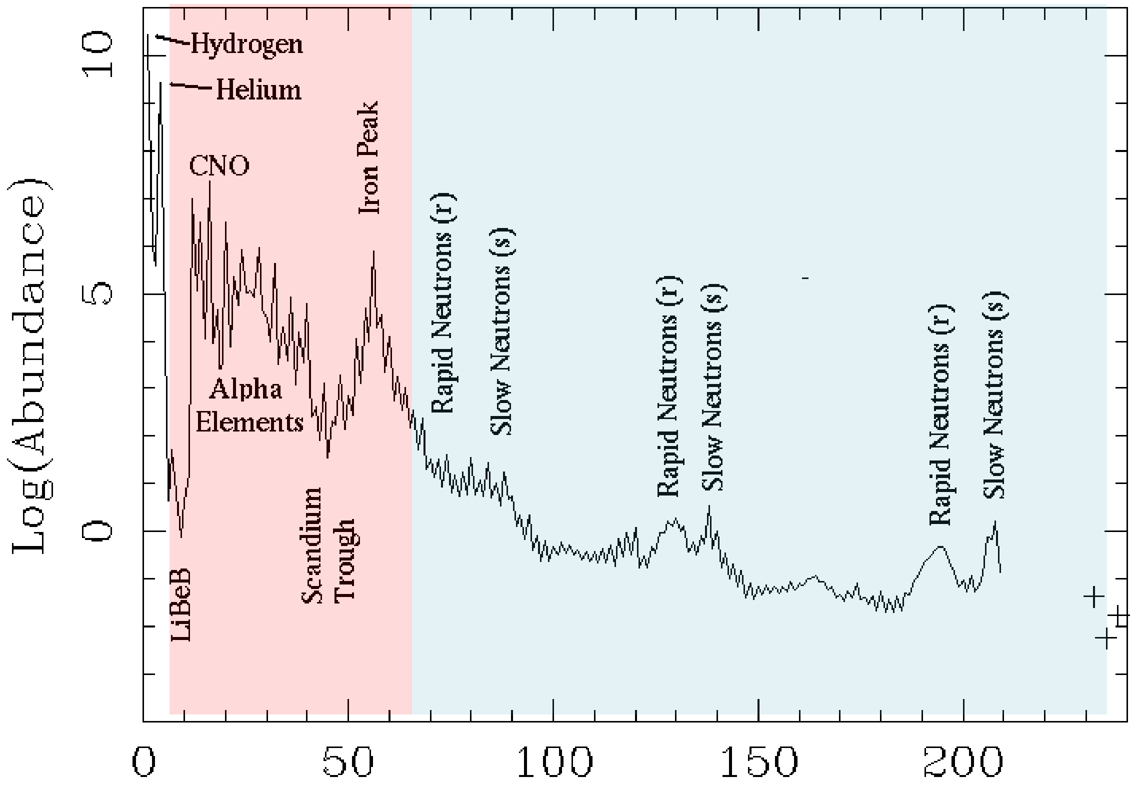
7 orders of magnitude less abundant!

WHY?

Question 3
How were the elements from iron to uranium made?

"The 11 Greatest Unanswered Questions of Physics"
based on National Academy of Science Report, 2002
[Committee for the Physics of the Universe (CPU)]

Nuclear processes



charged-particle induced reaction

during quiescent stages of stellar evolution

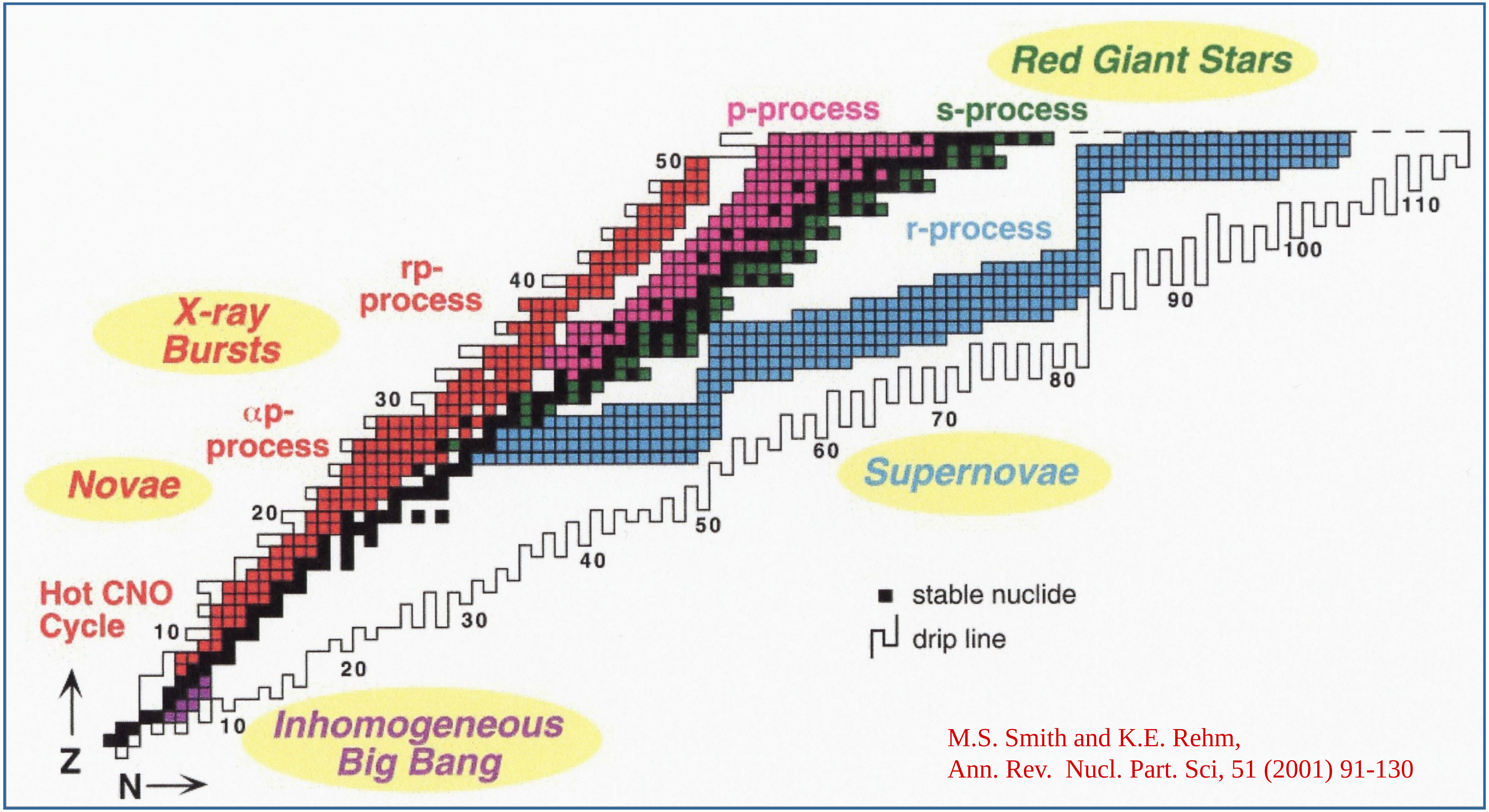
involve mainly STABLE NUCLEI

mainly neutron capture reaction

mainly during explosive stages of stellar evolution

involve mainly UNSTABLE NUCLEI

Overview of main astrophysical processes



the vast majority of reactions encountered in these processes involve UNSTABLE species hence the need for **Radioactive Ion Beams**

Nuclear reaction rates

nuclear reactions in stars: a) produce energy

b) synthesise elements

stars = cooking pots of the Universe

for reaction: T(p,e)R

(T = target, p = projectile, e = ejectile, R = recoil)

reaction Q-value: $Q=[(m_p+m_T)-(m_e+m_R)]c^2$ *(energy per single reaction)*

if $Q > 0 \Rightarrow$ net production of energy

- reaction rate per volume **r**

  (number of reactions per unit time and volume)

$$r = \frac{1}{1+\delta_{pT}} N_p N_T \langle \sigma v \rangle$$

N_i = number density of interacting species

v = relative velocity

f(v) = velocity distribution in plasma

σ(v) = reaction cross section

$$\langle \sigma v \rangle = \int \sigma(v)\phi(v) v dv$$

KEY QUANTITY

- energy production rate ε

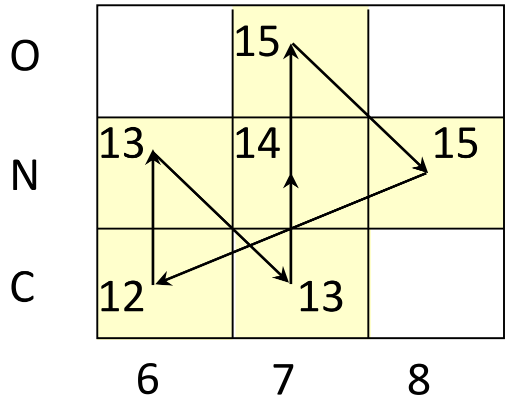
$$\varepsilon = r \cdot Q / \rho$$

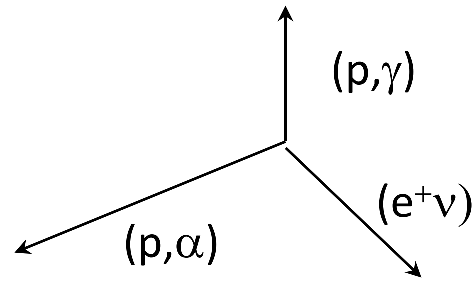
typical units: MeV g^{-1} s^{-1}

Kronecker **δ** applies for identical particles to avoid double counting

Example of nuclear reaction rates in stars

CNO cycle





$^{12}C(p,\gamma)^{13}N(e^+\nu)^{13}C(p,\gamma)^{14}N(p,\gamma)^{15}O(e^+\nu)^{15}N(p,\alpha)^{12}C$

cycle limited by β-decay of ^{13}N (t ~ 10 min) and ^{15}O (t ~ 2 min)

CNO isotopes act as catalysts

net result: $4p \rightarrow {}^4He + 2e^+ + 2\nu + Q_{eff}$ $Q_{eff} = 26.73$ MeV

nucleosynthesis **energy production**

changes in stellar conditions ⇒ changes in energy production and nucleosynthesis

need to know REACTION RATE at all temperatures to determine ENERGY PRODUCTION

Abundance changes and lifetimes

Consider reaction $1 + 2 \rightarrow 3$

Nuclei of type **1** are destroyed via capture reaction with type **2** nuclei;
type **3** nuclei are produced
Average live time of type **1** nuclei in stellar environment is given by the differential equation:

$$\left(\frac{dN_1}{dt}\right)_2 = -\lambda \cdot N_1 = -\frac{1}{\tau} \cdot N_1$$

mean lifetime
decay constant

or

$$\left(\frac{dN_1}{dt}\right)_2 = -(1+\delta_{12}) \cdot r = -N_1 N_2 \langle\sigma v\rangle$$

Kronecker symbol disappears often
the following equations are given:

$$= -N_1 \cdot \rho \cdot N_A \frac{X_2}{A_2} \langle\sigma v\rangle = -N_1 \cdot \rho \cdot N_A \cdot Y_2 \langle\sigma v\rangle$$

$$r = \frac{1}{1+\delta_{12}} N_1 \cdot N_2 \cdot \langle\sigma v\rangle$$

$$N_i = \rho \cdot N_A \frac{X_i}{A_i} = \rho \cdot N_A \cdot Y_i$$

number density	N_i
matter density	ρ
Avogadro‘s number	N_A
mass fraction	X_i
atomic mass	A_i
mol fraction	Y_i

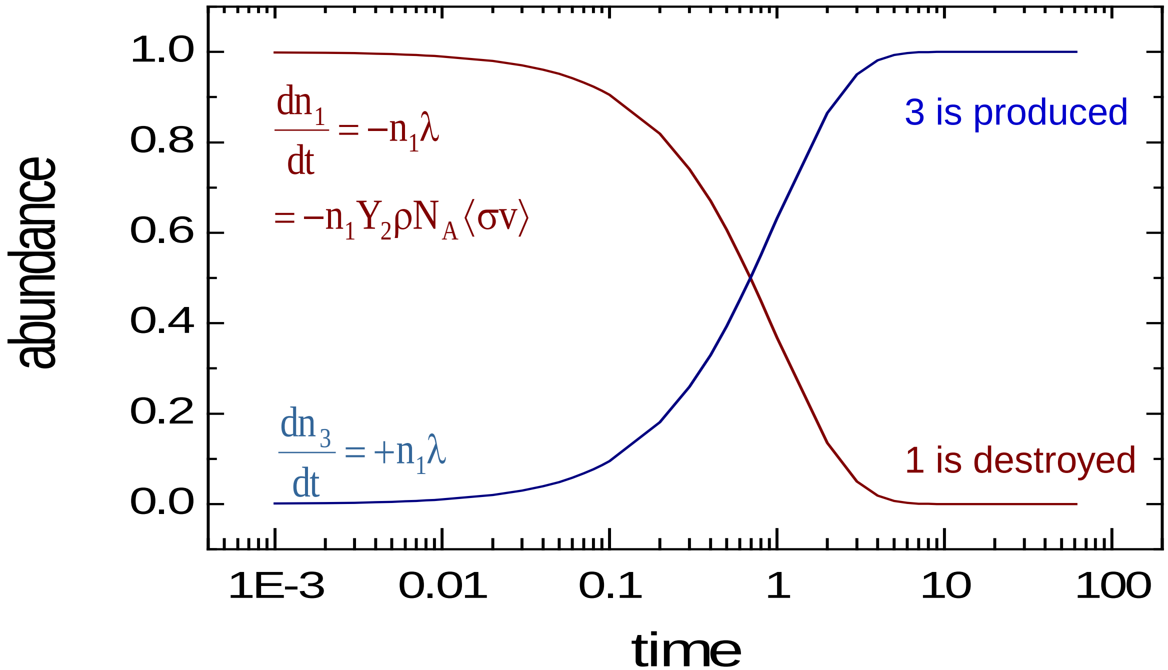
Abundance changes and lifetimes

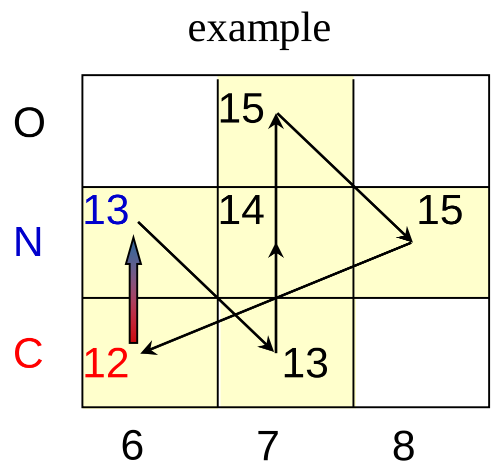
reactions are random processes with constant probability (cross section) for given conditions

⇒ abundance change is governed by same laws of radioactive decay

consider reaction **1+2 → 3**

where **1** is destroyed through capture of **2** and **3** is produced





define:

lifetime of 1 against destruction by with 2:

$$\tau = \frac{1}{\lambda} = \frac{1}{Y_2 \cdot \rho \cdot N_A \langle\sigma v\rangle}$$

need to know REACTION RATE at all temperatures to determine NUCLEOSYNTHESIS

Key quantity

stellar reaction rate $\langle \sigma v \rangle = \int \sigma(v)\phi(v) v\,dv$

need: a) velocity distribution

b) cross section

a) velocity distribution

interacting nuclei in plasma are in thermal equilibrium at temperature T

also assume non-degenerate and non-relativistic plasma

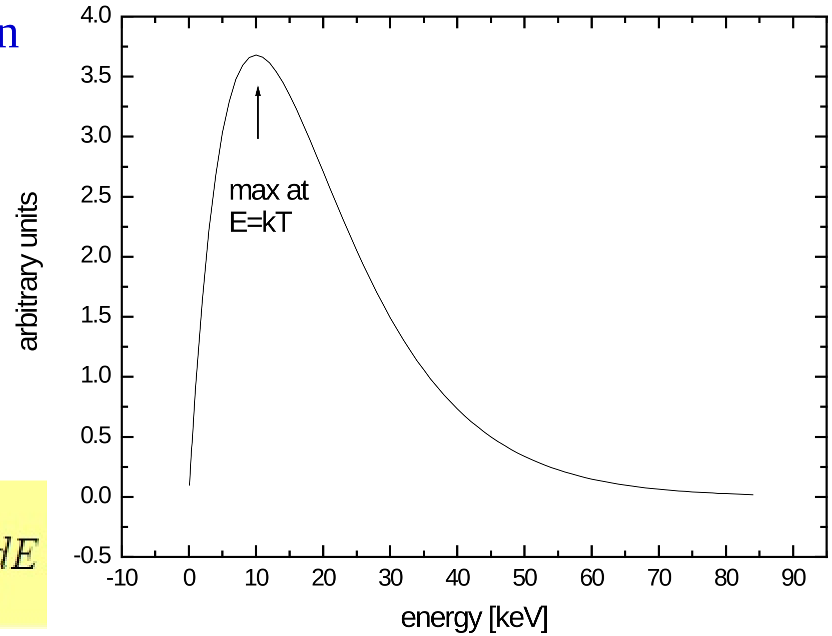
$\Rightarrow$ Maxwell-Boltzmann velocity distribution

$$\phi(v) = 4\pi \left(\frac{\mu}{2\pi kT}\right)^{3/2} v^2 \exp\left(-\frac{\mu v^2}{2kT}\right)$$

with $\mu = \frac{m_p m_T}{m_p + m_T}$ reduced mass

v = relative velocity, $E = 1/2\ \mu v^2$

$$\langle \sigma v \rangle = \left(\frac{8}{\pi \mu}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty \sigma(E) \cdot E \cdot \exp\left(-\frac{E}{kT}\right) dE$$



$kT \sim 8.6 \times 10^{-8}\ T[K]$ keV

example: Sun $T \sim 15 \times 10^6$ K $\Rightarrow$ $kT \sim 1.3$ keV

Reaction cross sections

b) cross section

no nuclear theory available to determine reaction cross section a priori

depends sensitively on:

- the properties of the nuclei involved
- the reaction mechanism

and can vary by orders of magnitude, depending on **nature of interaction**

examples:

Reaction	Force	σ (barn)	E_{proj} (MeV)
$^{15}N(p,\alpha)^{12}C$	strong	0.5	2.0
$^{3}He(\alpha,\gamma)^{7}Be$	electromagnetic	10^{-6}	2.0
$p(p,e^{+}\nu)d$	weak	10^{-20}	2.0

1 barn = 10^{-24} cm^2 = 100 fm^2

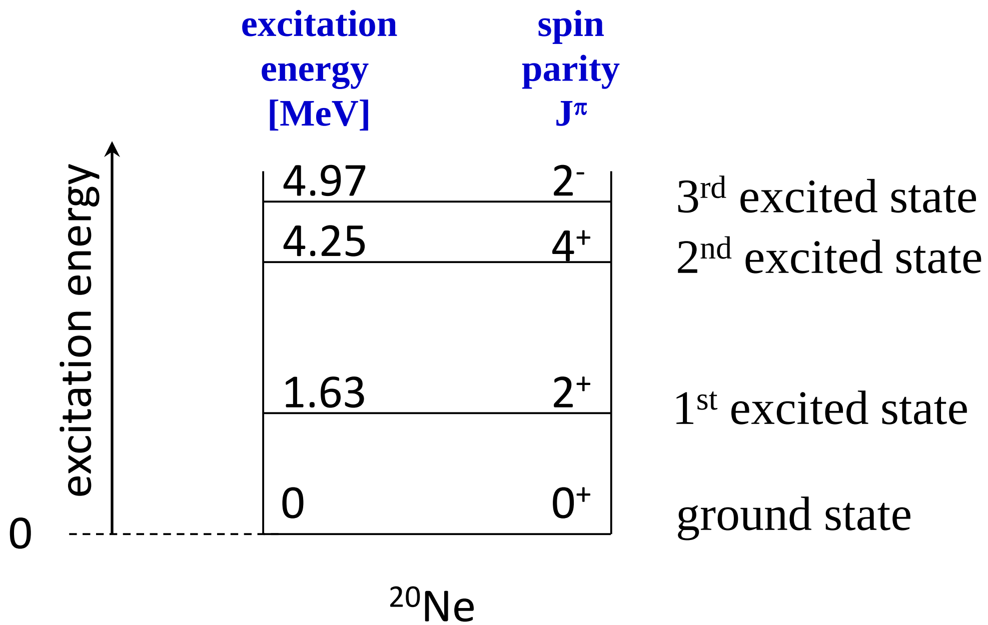
in practice, need **experiments** AND **theory** to determine stellar reaction rates

Nuclear properties relevant to reaction rates

recall: nucleons in nuclei arranged in quantised shells of given energy

$\Rightarrow$ nucleus's configuration as a whole corresponds to discrete energy levels

example



any nucleus in an excited state will eventually decay either by **γ, p, n** or **α-emission** with a characteristic **lifetime τ** which corresponds to a **width Γ** in the excitation energy of the state

$$\Gamma = \frac{\hbar}{\tau}$$ *Heisenberg's relationship*

the lifetime for each individual exit channel is usually given in terms of **partial widths**

Γ_γ, Γ_p, Γ_n and Γ_α with $\Gamma = \sum \Gamma_i$

reaction mechanisms:

I. direct reactions

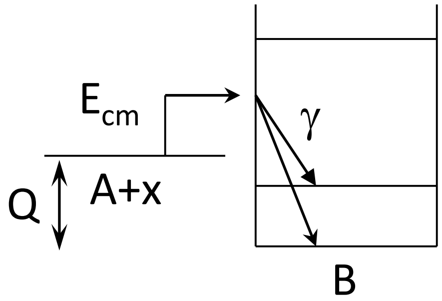
II. resonant reactions

I. direct process

one-step process

direct transition into a bound state

example: radiative capture $A(x,\gamma)B$



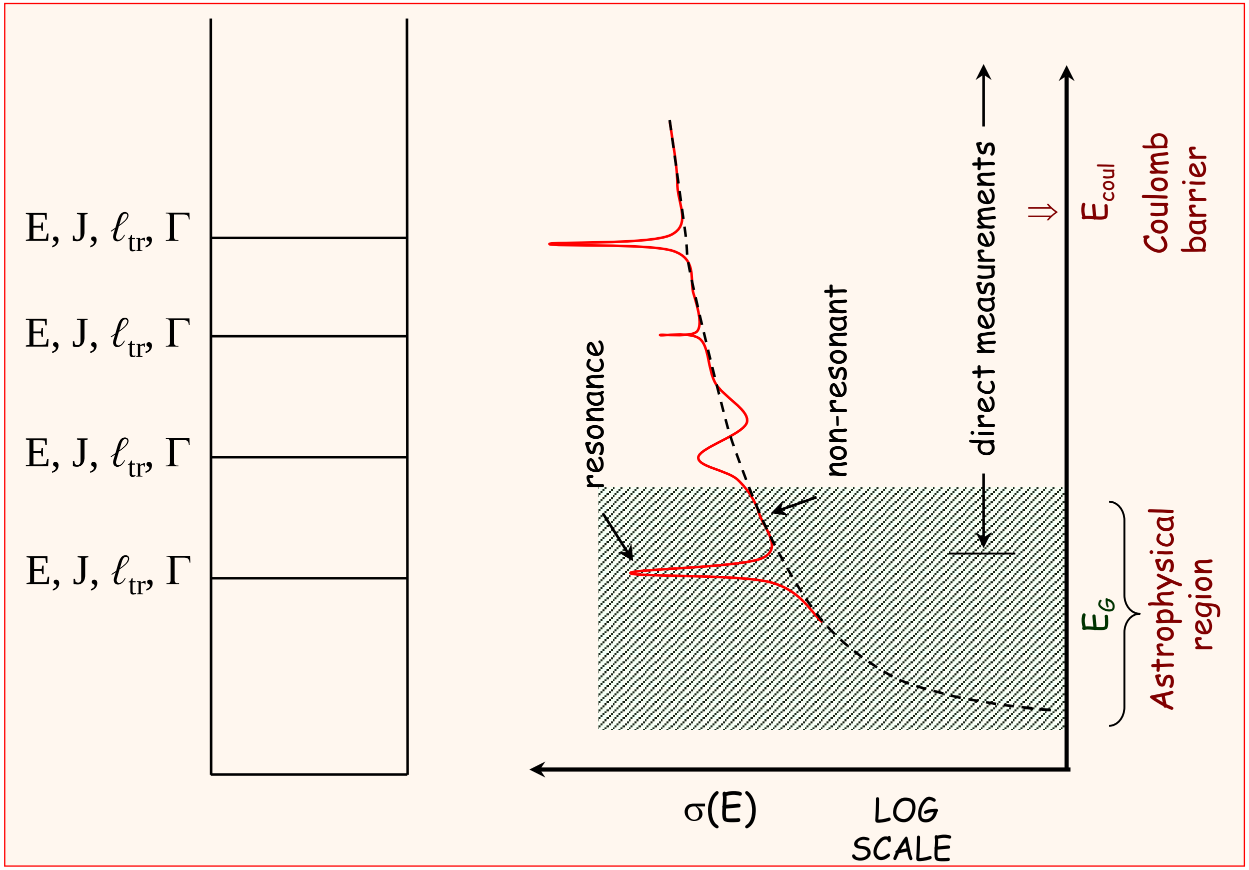
$$\sigma_\gamma \propto \left| \langle B | H_\gamma | A + x \rangle \right|^2$$

H_γ = electromagnetic operator describing the transition

- reaction cross section proportional to single matrix element
- can occur at all projectile energies
- smooth energy dependence of cross section

other direct processes: stripping, pickup, charge exchange, Coulomb excitation

II. resonant process



Reaction mechanism

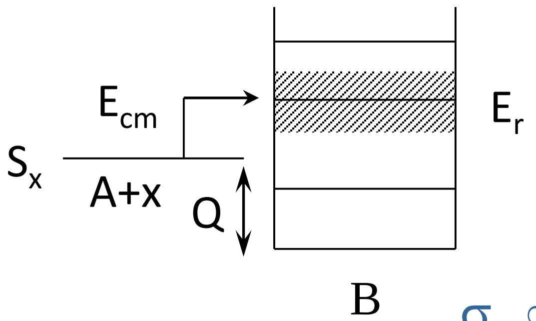
II. resonant process

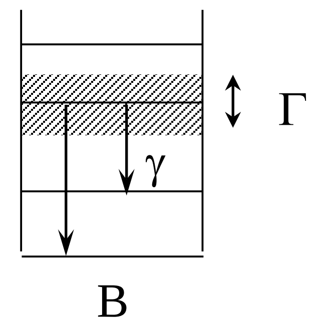
two-step process

example: **resonant radiative capture A(x,γ)B**

1. Compound nucleus formation
   (in an unbound state)

2. Compound nucleus decay
   (to lower excited states)





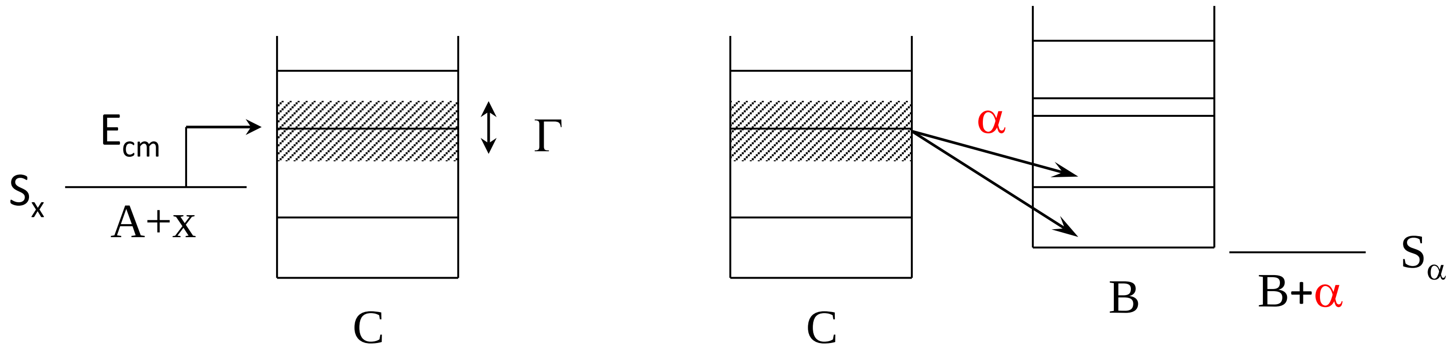
$$\sigma_\gamma \propto \underbrace{\left|\langle E_f | H_\gamma | E_r \rangle\right|^2}_{\text{compound decay}} \; \underbrace{\left|\langle E_r | H_B | A + x \rangle\right|^2}_{\text{compound formation}}$$

compound decay probability $\propto \Gamma_\gamma$ compound formation probability $\propto \Gamma_x$

- reaction cross section proportional to two matrix elements
- only occurs at energies $E_{cm} \sim E_r - Q$
- strong energy dependence of cross section

N. B. energy in entrance channel ($Q+E_{cm}$) has to match excitation energy E_r of resonant state, however all excited states have a width ⇒ there is always some cross section through tails example: **resonant reaction A(x,α)B**

1. Compound nucleus formation (in an unbound state)
2. Compound nucleus decay (by particle emission)



$$\sigma_\gamma \propto \left| \langle B+\alpha | H_\alpha | E_r \rangle \right|^2 \left| \langle E_r | H_x | A+x \rangle \right|^2$$

compound decay probability $\propto \Gamma_\alpha$

compound formation probability $\propto \Gamma_x$

N. B. energy in entrance channel (S_x+E_{cm}) has to match excitation energy E_r of resonant state, however all excited states have a width ⇒ there is always some cross section through tails

Cross section

cross section expressions

for direct reactions and resonant reactions

- with charged particles
- with neutrons

Cross sections for direct reactions

example: direct capture $A + x \rightarrow B + \gamma$

$$\sigma = \pi \lambda_x^2 \left| \langle B | H | x + A \rangle \right|^2 P_l(E)$$

"geometrical factor"
de Broglie wavelength of projectile

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

matrix element
contains nuclear properties of interaction

penetrability/transmission probability for projectile to reach target for interaction depends on projectile's angular momentum λ and energy E

$$\sigma = \frac{1}{E} \cdot P_l(E) \cdot S(E)$$

σ = (strong energy dependence) · (weak energy dependence)

S(E) = astrophysical factor contains nuclear physics of reaction

\+ can be easily: graphed, fitted, extrapolated (if needed)

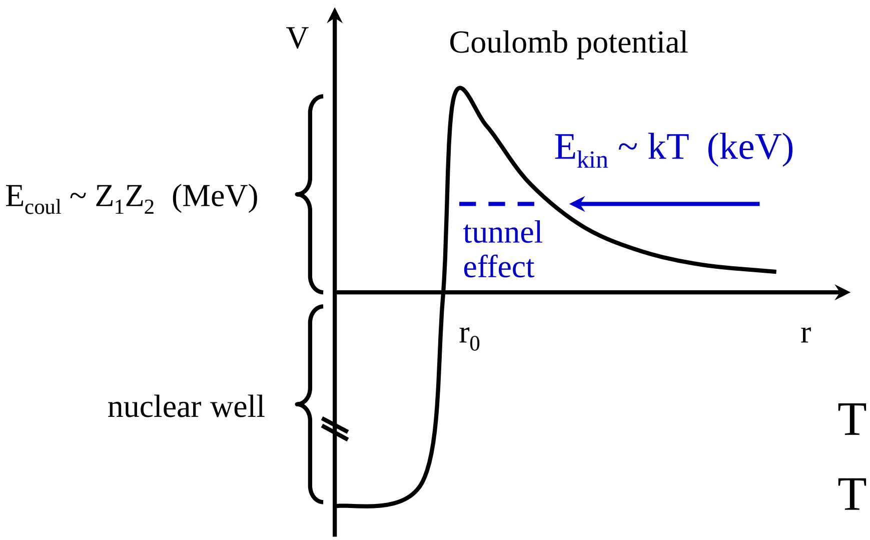
need expression for $P_\lambda(E)$

factors affecting transmission probability:

- Coulomb barrier (for charged particles only)
- centrifugal barrier (both for neutrons and charged particles)

Reactions with charged particles

charged particles ⇒ Coulomb barrier



energy available: from thermal motion

$$kT \sim 8.6 \times 10^{-8}\, T[K] \;\; keV$$

$T \sim 15 \times 10^6$ K (Sun) $\Rightarrow$ $kT \sim 1$ keV

$T \sim 10^{10}$ K (Big Bang) $\Rightarrow$ $kT \sim 2$ MeV

during quiescent burnings: $kT \ll E_c$ ⇒ reactions occur through TUNNEL EFFECT

tunneling probability $P \propto \exp(-2\pi\eta)$

$2\pi\eta$ = Gamow factor

in numerical units:

$$2\pi\eta = 31.29\, Z_1 Z_2 (\mu/E)^{1/2}$$

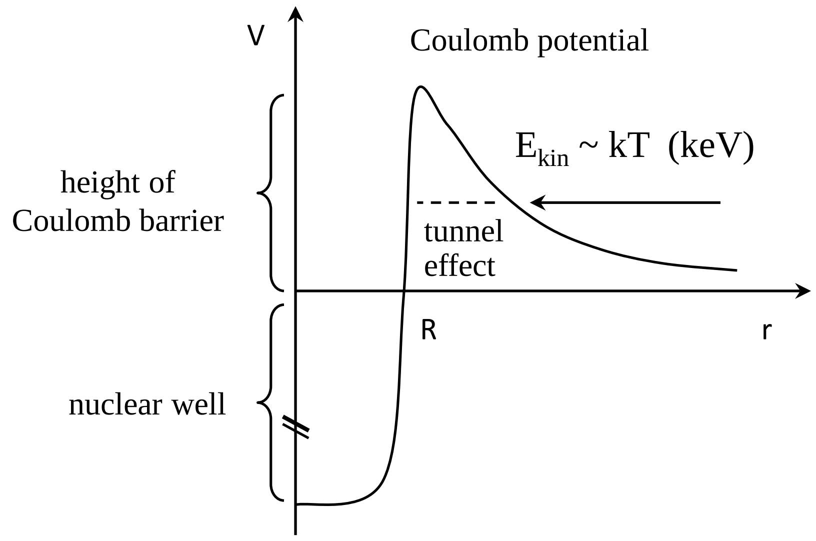
μ in amu and E_{cm} in keV

determines exponential drop in abundance curve!

Coulomb barrier

for projectile and target charges Z_1 and Z_2

$$V_C = \frac{1}{4\pi\varepsilon_0}\frac{Z_1 Z_2 e^2}{R}$$



in numerical units:

$$V_C[\text{MeV}] = 1.44\frac{Z_1 Z_2}{R[\text{fm}]} \approx 1.2\frac{Z_1 Z_2}{\left(A_1^{1/3} + A_2^{1/3}\right)}$$

example: $^{12}C(p,\gamma)$ $V_C = 3$ MeV

average kinetic energies in stellar plasmas: kT ~ 1-100 keV!

⇒ fusion reactions between charged particles take place well below Coulomb barrier

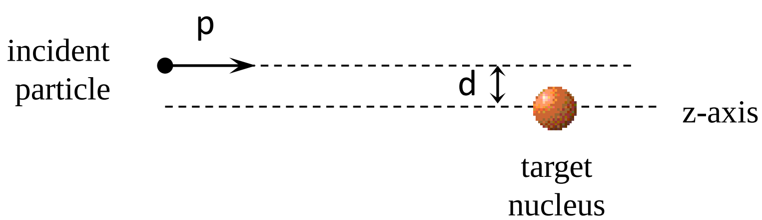
⇒ transmission probability governed by tunnel effect

for E << V_C and zero angular momentum, tunnelling probability given by:

$$P_l(E) \propto \exp(-2\pi\eta)$$ with $$2\pi\eta = 31.29 Z_1 Z_2 \left(\frac{\mu}{E}\right)^{1/2}$$ Gamow factor

Angular momentum barrier

classical treatment:



p = projectile linear momentum
d = impact parameter

incident particle can have orbital angular momentum L = p·d

angular momentum is conserved in central potential

⇒ linear momentum p (and hence energy) must increase as distance d decreases

quantum-mechanical treatment:

(discrete values only) $L = \sqrt{\lambda \cdot (\lambda + 1)}\hbar$

with parity of wave-function: $\pi = (-1)^{\lambda}$

$\lambda = 0$ s-wave
$\lambda = 1$ p-wave
$\lambda = 2$ d-wave
...

angular momentum is conserved in central potential

⇒ non-zero angular momentum implies "angular momentum energy barrier" V_{λ}

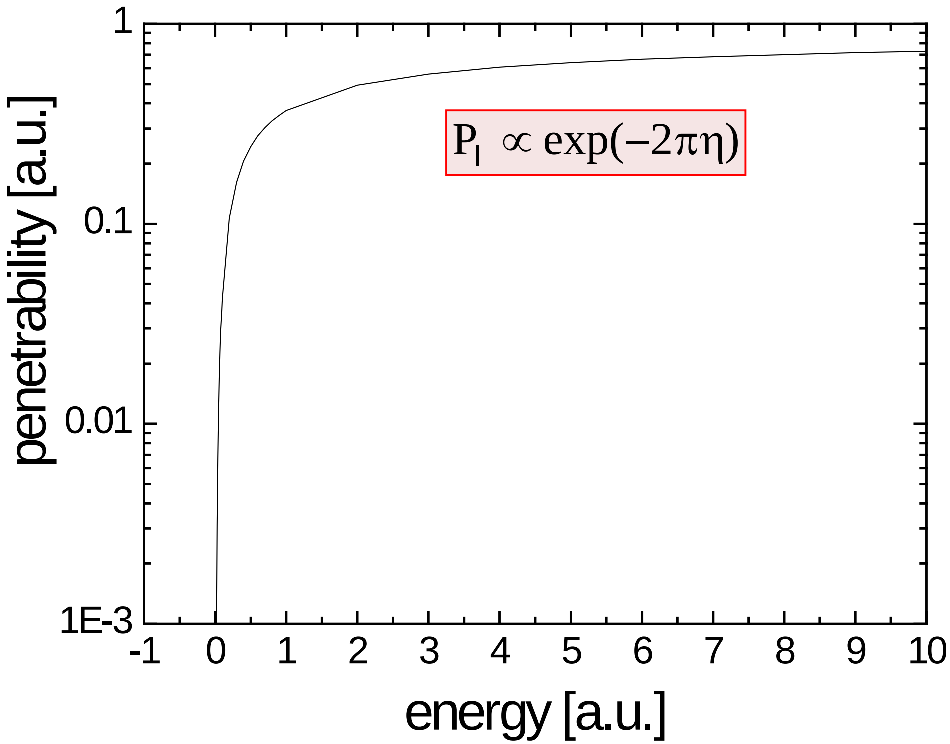
$$V_{\lambda} = \frac{\lambda \cdot (\lambda + 1)\hbar^2}{2\mu r^2}$$

μ = reduced mass of projectile-target system
r = radial distance from centre of target nucleus

Charged-particle capture

probability of tunnelling through Coulomb barrier for charged particle reactions at energies $E \ll V_{coul}$

$$P_l \propto \exp(-2\pi\eta) = \exp\left(-\frac{b}{\sqrt{E}}\right)$$



$$\eta = a \cdot k_\infty = \frac{Z_1 \cdot Z_2 \cdot e^2}{\hbar \cdot v_\infty}$$ Sommerfeld parameter

$$b = \sqrt{2\mu\pi}\frac{Z_1 Z_2 e^2}{\hbar}$$ $b^2 \equiv$ Gamow energy

assumes:

- full ion charges
- zero orbital angular momentum

$$\sigma = \frac{1}{E}\exp(-2\pi\eta)S(E)$$

units of S(E): keV barn, MeV barn …

additional angular momentum barrier leads to a roughly constant addition to the **S-factor** that strongly decreases with λ $\Rightarrow$ S-factor definition for charged particle reactions is **independent** of orbital angular momentum (unlike neutron capture processes!)

Non-resonant reaction

Non-resonant reactions

$$\sigma \propto \pi\lambda^2 \cdot P_\lambda(E) \cdot |\langle e+R|H|p+T\rangle|^2$$

geometrical factor (particle's de Broglie wavelength)

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

penetrability probability depends on projectile's angular momentum λ and energy E

interaction matrix element

$$\sigma(E) = \frac{1}{E} \cdot exp(-2\pi\eta) \cdot S(E)$$

(for s-waves only!)

non-nuclear origin
STRONG
energy dependence

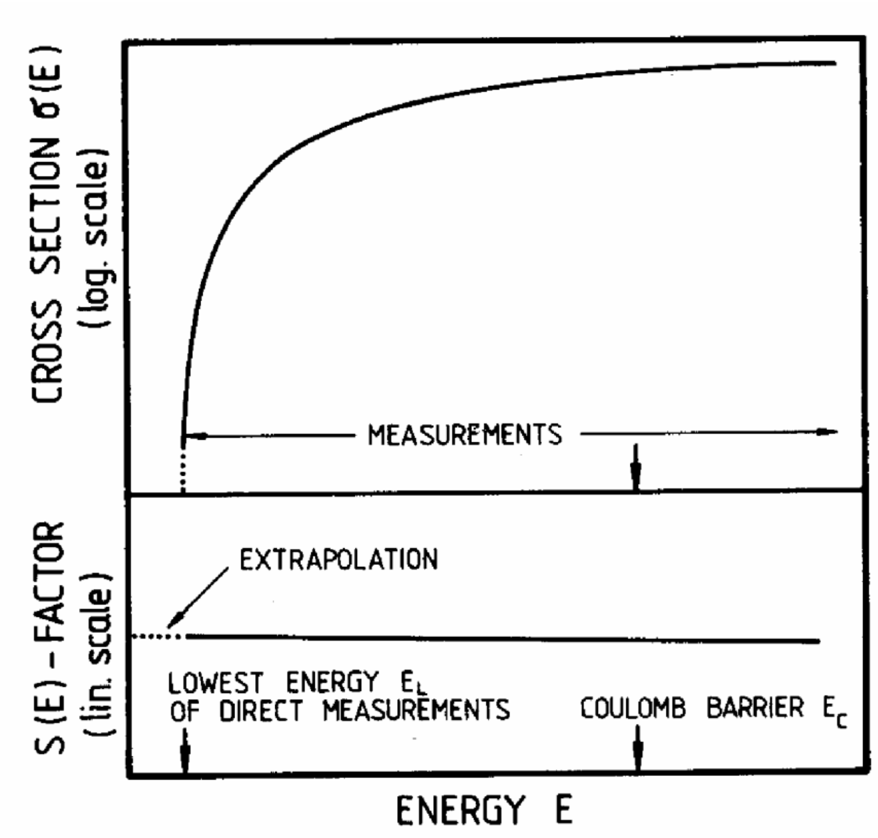
nuclear origin
WEAK
energy dependence

Above relation defines ASTROPHYSICAL S(E)-FACTOR

N.B.

If angular momentum is non zero ⇒ centrifugal barrier $V_\lambda = \frac{\lambda(\lambda+1)\hbar^2}{2\mu r^2}$ must also be taken into account

Astrophysical S-factor



"astro physical S factor" contains detailed information on nuclear structure

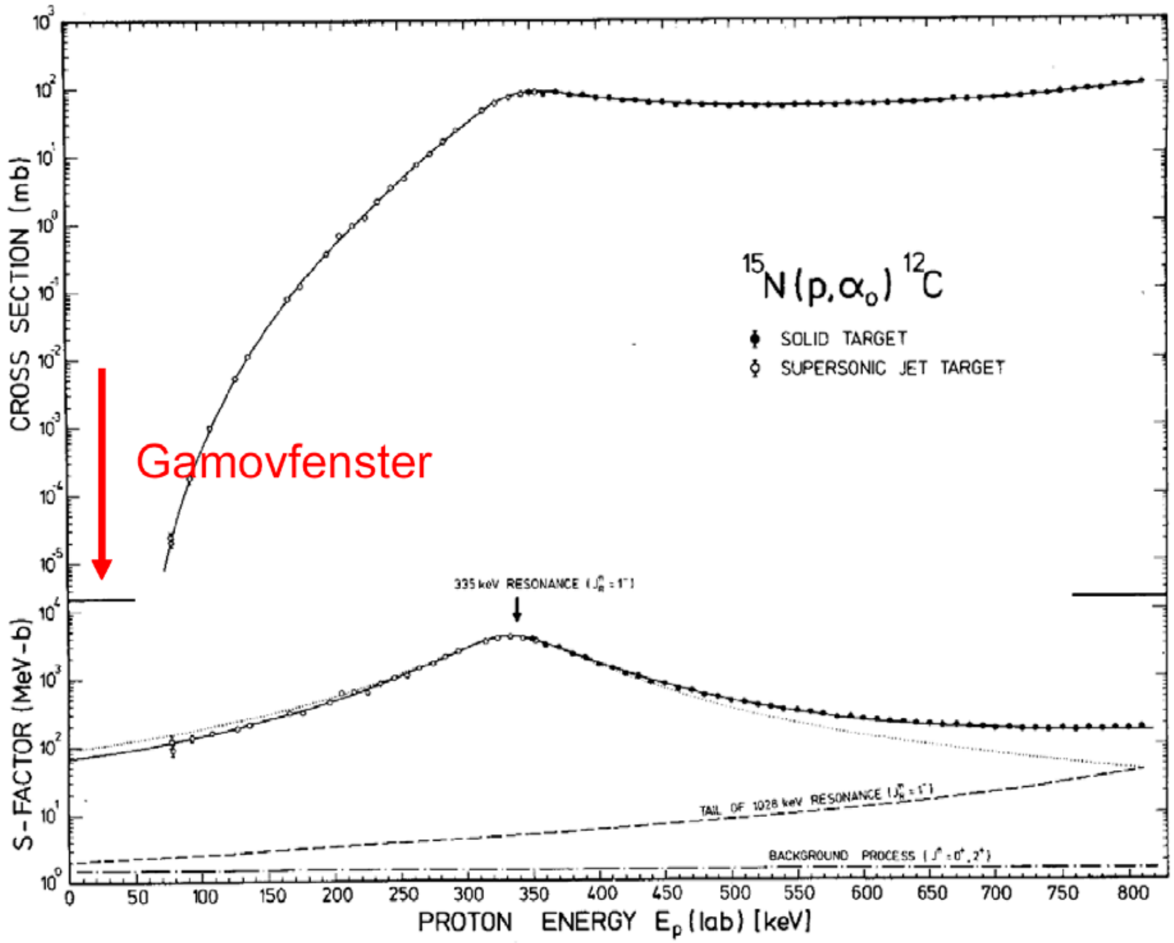
$$\sigma(E) = \frac{S(E)}{E} \cdot e^{-b/\sqrt{E}}$$

Relevant energy region for astro physics is very low, typical at the limit and below measured energy for nuclear reaction.

In some cases S€ can be extrapolated

Rolfs & Rodney p. 156

Astrophysical S-factor



Rolfs & Rodney p. 156

Typically data for astrophysical processes are given by S(E)
In some cases S€ varies strongly with energy

Gamow peak

With above definition of cross section:

$$\langle\sigma v\rangle_{12} = \left(\frac{8}{\pi\mu_{12}}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \cdot exp\left[-\frac{E}{kT} - \frac{b}{E^{1/2}}\right] dE \qquad b = \sqrt{2\mu}\frac{\pi Z_1 Z_2 e^2}{\hbar}$$

$S(E)$: varies smoothly with energy

$exp[\ldots] = f(E)$: governs energy dependence

MAXIMUM reaction rate:

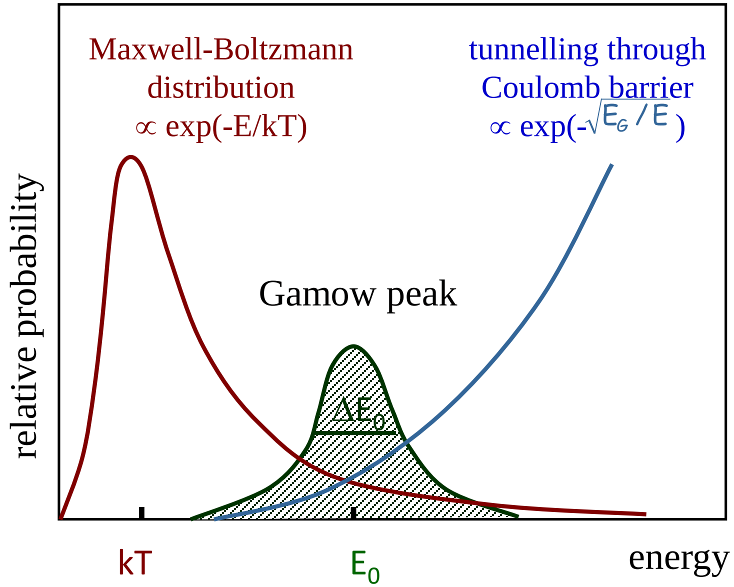
$$\frac{df}{dE} = 0 \quad \Rightarrow \quad E_0 = \left(\frac{b \cdot kT}{2}\right)^{2/3}$$

$\Delta E_0 < E_0$

only small energy range contributes to reaction rate

⇓

OK to set $S(E) \sim S(E_0)$ = const.



Gamow peak

$$\langle\sigma v\rangle = \int \sigma(v)\phi(v) v\,dv = \int \sigma(E)\exp(-E/kT) E\,dE$$

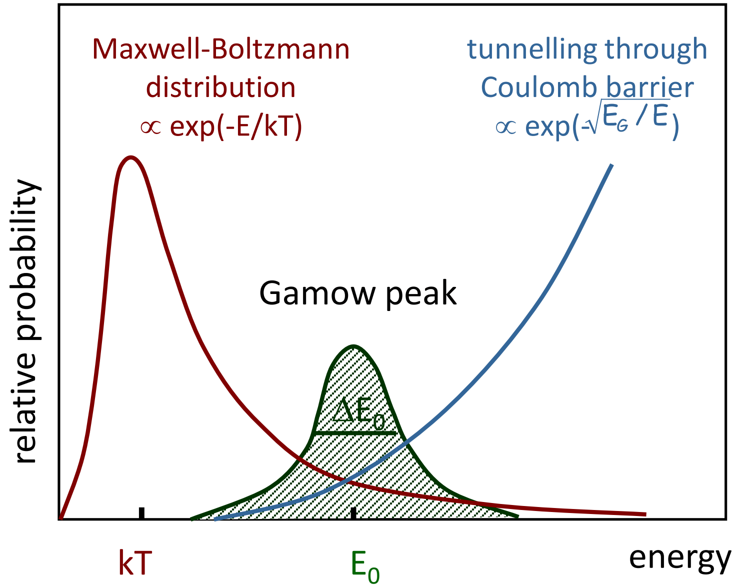
and substituting for σ: $\langle\sigma v\rangle \propto \int S(E)\exp\left(-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right)dE$

maximum reaction rate at E_0: $\frac{d}{dE}\left[\exp\left(-\frac{E}{kT} - \frac{b}{\sqrt{E}}\right)\right] = 0$

Gamow peak

$$E_0 = \left(\frac{bkT}{2}\right)^{3/2} = 0.122\left(Z_1^2 Z_2^2 A\right)^{1/3} T_9^{2/3} \ \text{MeV}$$

$$\Delta E = \frac{4}{\sqrt{3}}\sqrt{E_0 kT} = 0.237\left(Z_1^2 Z_2^2 A\right)^{1/6} T_9^{5/6} \ \text{MeV}$$



E_0 = relevant energy for astrophysics >> kT

N.B. Gamow energy depends on **reaction** and **temperature**

Gamow peak

Gamow peak: most effective energy region for thermonuclear reactions

$E_0 \pm \Delta E_0/2$ energy window of astrophysical interest

$$E_0 = f(Z_1, Z_2, T)$$

⇓

varies depending on reaction and/or temperature

Examples: $T \sim 15 \times 10^6$ K ($T_6 = 15$)

reaction	Coulomb barrier (MeV)	E_0 (keV)	$\exp(-3E_0/kT)\ \Delta E_0$
p + p	0.55	5.9	7.0×10^{-6}
$\alpha + {}^{12}C$	3.43	56	5.9×10^{-56}
${}^{16}O + {}^{16}O$	14.07	237	2.5×10^{-237}

⇒ area of Gamow peak (height x width) ~ $\langle \sigma v \rangle$

STRONG sensitivity to Coulomb barrier

separate stages: H-burning
He-burning
C/O-burning
...

Resonance curve

Wave function for a decaying intermediate state Z* with energy E_0 and live time τ

$$\psi(t) = \psi(0) \cdot e^{-i\omega_0 t} \cdot e^{-t/2\tau} \qquad E = \hbar \cdot \omega \qquad \hbar = 1$$

$$|\psi(t)|^2 = |\psi(0)|^2 \cdot e^{-t/\tau}$$

Exponential decay
τ decay constant

Frequency- or energy dependence is obtained from wave function via Fourier transformation of $\psi(t)$

$$f(\omega) = \int_0^\infty \psi(t)\, e^{i\omega t} dt$$

Resonance curve

Fourier transformation:

$$f(\omega) = \int_0^\infty \psi(t) e^{i\omega t}\, dt$$

$$f(E) = \int_0^\infty \psi(t)\, e^{iEt} dt$$

$$= \int_0^\infty \psi(0) \cdot e^{-iE_0 t} \cdot e^{-t/2\tau} \cdot e^{iEt} dt$$

$$= \int_0^\infty \psi(0) \cdot e^{-t\cdot\left(i(E_0 - E) + \frac{1}{2\tau}\right)} dt$$

$$= \frac{\psi(0)}{(E_0 - E) - {}^{i}\!/_{2\tau}}$$

Resonance curve

Probability to find a state with energy E

$$E = f^* \cdot f$$

$$P(E) = f^*(E) \cdot f(E) = \frac{\psi(0)}{(E_0 - E) - {}^{i}/_{2\tau}} \cdot \frac{\psi(0)}{(E_0 - E) + {}^{i}/_{2\tau}}$$

$$= \frac{|\psi(0)|^2}{(E_0 - E)^2 + {}^{1}/_{4\tau^2}}$$

REMINDER

Energy dependence – line shape

probability for Z* to heave energy E

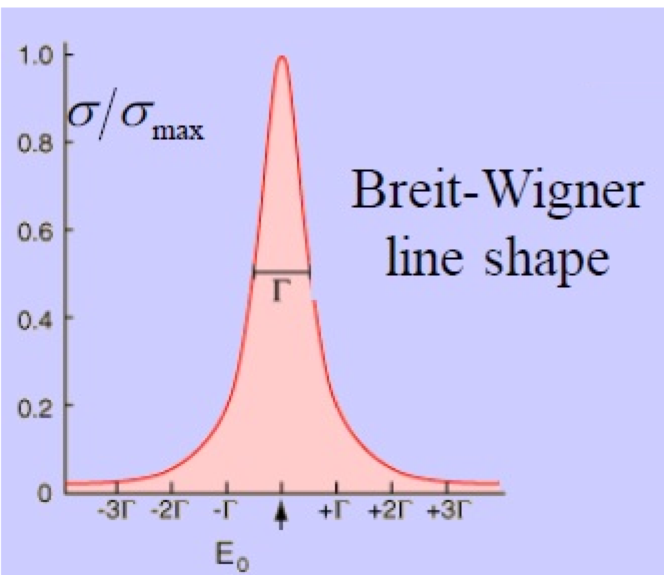
Full Width at Half Maximum FWHM = Γ

$$\frac{1}{\left(E_0 - E_0 + {}^{\Gamma}/_{2}\right)^2 + {}^{1}/_{4\tau^2}} = \frac{1}{2} \cdot 4 \cdot \tau^2$$

$$\frac{1}{\frac{\Gamma^2}{4} + \frac{1}{4\tau^2}} = 2\tau^2$$

$$\Rightarrow \quad \frac{1}{\frac{\Gamma^2\tau^2 + 1}{4\tau^2}} = 2\tau^2$$

$$\Rightarrow \quad \frac{2}{\Gamma^2\tau^2 + 1} = 1$$



$\tau \cdot \Gamma = 1$ Heisenberg uncertainty principle ($\hbar$=1)

limited live time ↔ uncertainty in energy

GSI

Cross section for resonant reactions

for a **single** isolated resonance:

resonant cross section given by **Breit-Wigner expression**

$$E_r = E_0 - i\frac{\Gamma}{2} \qquad |E_r| = \sqrt{E_0^2 + \frac{\Gamma^2}{4}}$$

$$\sigma(E) = \pi ƛ^2 \frac{2J+1}{(2J_1+1)(2J_T+1)} \frac{\Gamma_1\Gamma_2}{(E-E_r)^2 + (\Gamma/2)^2}$$

for reaction: 1 + T → C → F + 2

geometrical factor

$\propto 1/E$

spin factor ω

J = spin of CN's state

J_1 = spin of projectile

J_T = spin of target

strongly energy-dependent term

Γ_1 = partial width for decay via emission of particle 1

= probability of compound formation via entrance channel

Γ_2 = partial width for decay via emission of particle 2

= probability of compound decay via exit channel

Γ = total width of compound's excited state

$= \Gamma_1 + \Gamma_2 + \Gamma_\gamma + \ldots$

E_r = resonance energy

what about penetrability considerations? $\Rightarrow$ look for energy dependence in partial widths!

partial widths are NOT constant but energy dependent!

Energy dependence of partial widths

particle widths

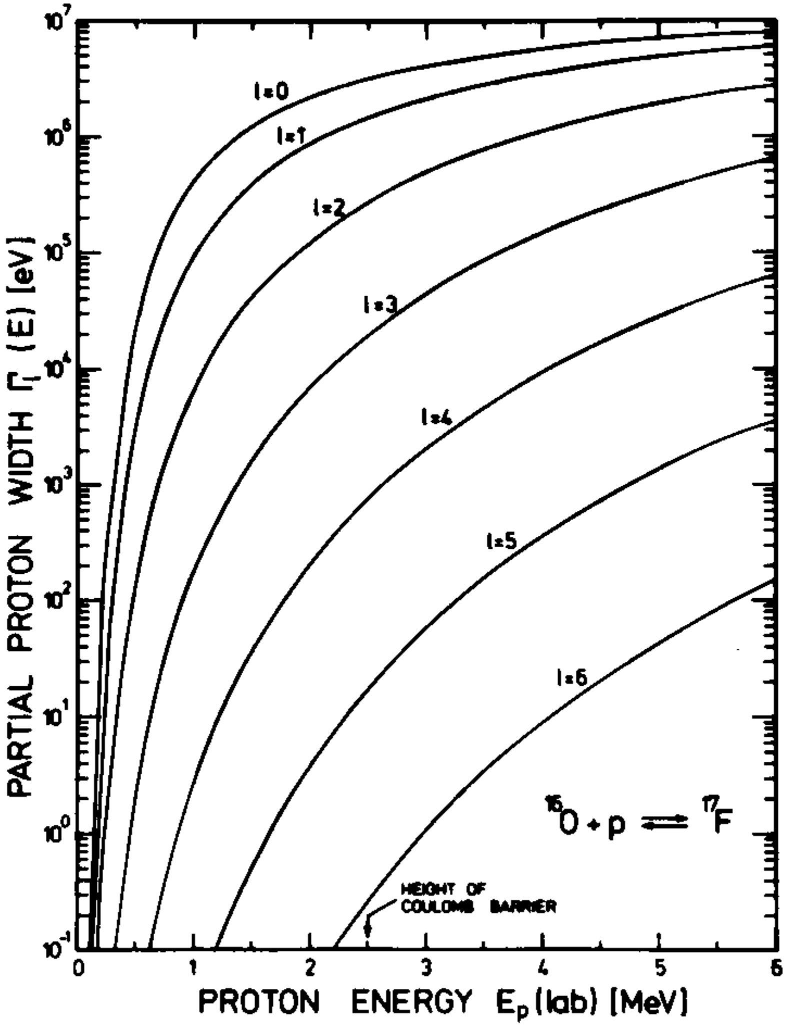
$$\Gamma_l = \frac{2\hbar}{R} P_l(E_l)\,\theta_l^2$$

θ_λ = "reduced width" (contains nuclear physics info)

P_λ gives strong energy dependence

example: $^{16}O(p,\gamma)^{17}F$

energy dependence of proton partial width Γ_p as function of λ



particle partial widths have approximately same energy dependence as penetrability function seen in direct reaction processes

Reaction rates for resonant processes

$$\langle \sigma v \rangle = \int \sigma(v)\phi(v)v\,dv = \int \sigma(E)\exp(-E/kT)E\,dE$$

here Breit-Wigner cross section

$$\sigma(E) = \pi D^2 \frac{2J+1}{(2J_1+1)(2J_T+1)} \frac{\Gamma_1\Gamma_2}{(E-E_r)^2+(\Gamma/2)^2}$$

integrate over appropriate energy region

E ~ kT	for neutron induced reactions
E ~ Gamow window	for charged particle reactions

if compound nucleus has an exited state (or its wing) in this energy range

$\Rightarrow$ RESONANT contribution to reaction rate (if allowed by selection rules)

typically:

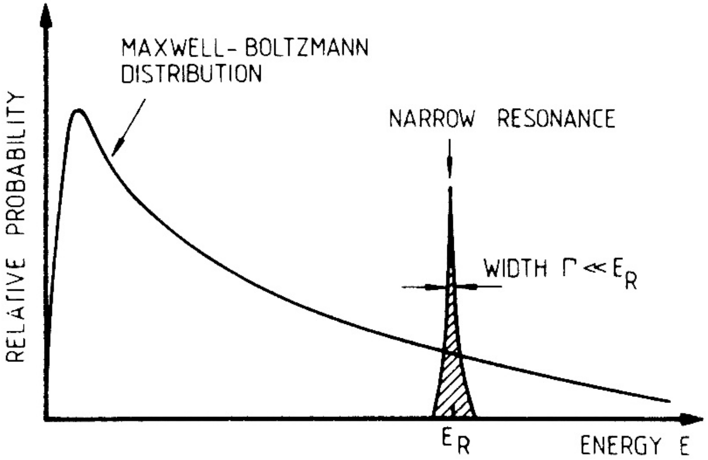
- resonant contribution dominates reaction rate
- reaction rate critically depends on resonant state properties

two simplifying cases:

- narrow (isolated) resonances
- broad resonances

Resonant reactions

1. Narrow resonances $\Gamma \ll E_R$



Breit-Wigner formula

$$\sigma(E)_{BW} = \pi \lambda^2 (1+\delta_{12}) \frac{2J+1}{(2J_1+1)(2J_2+1)} \frac{\Gamma_a \Gamma_b}{(E-E_r)^2 + (\Gamma/2)^2}$$

insert in expression for reaction rate, integrate and get:

$$\langle \sigma v \rangle_{12} = \left(\frac{2\pi}{\mu_{12} kT} \right)^{3/2} \eta^2 (\omega\gamma)_R \cdot exp\left(-\frac{E_R}{kT} \right)$$ (for single resonance)

resonance strength
(integrated cross section over resonant region)

low-energy resonances ($E_R \rightarrow kT$) dominate reaction rate

Experiment: determine $(\omega\gamma)_R$ and E_R

Some considerations

$$\langle\sigma v\rangle_{12} = \left(\frac{2\pi}{\mu_{12}kT}\right)^{3/2} \hbar^2 (\omega\gamma)_R \exp\left(-\frac{E_R}{kT}\right)$$

rate entirely determined by **"resonance strength"** $\omega\gamma$ and **energy of the resonance E_R**

resonance strength
(= integrated cross section over resonant region)

$$\omega\gamma = \frac{2J+1}{(2J_1+1)(2J_T+1)} \frac{\Gamma_1\Gamma_2}{\Gamma}$$

(Γ_i values at resonant energies)

often

$$\Gamma = \Gamma_1 + \Gamma_2$$

$$\Gamma_1 << \Gamma_2 \longrightarrow \Gamma \approx \Gamma_2 \longrightarrow \frac{\Gamma_1\Gamma_2}{\Gamma} \approx \Gamma_1$$

$$\Gamma_2 << \Gamma_1 \longrightarrow \Gamma \approx \Gamma_1 \longrightarrow \frac{\Gamma_1\Gamma_2}{\Gamma} \approx \Gamma_2$$

reaction rate is determined by the **smaller** width !

experimental info needed:

- partial widths Γ_i
- spin **J**
- energy E_R

note: for many unstable nuclei

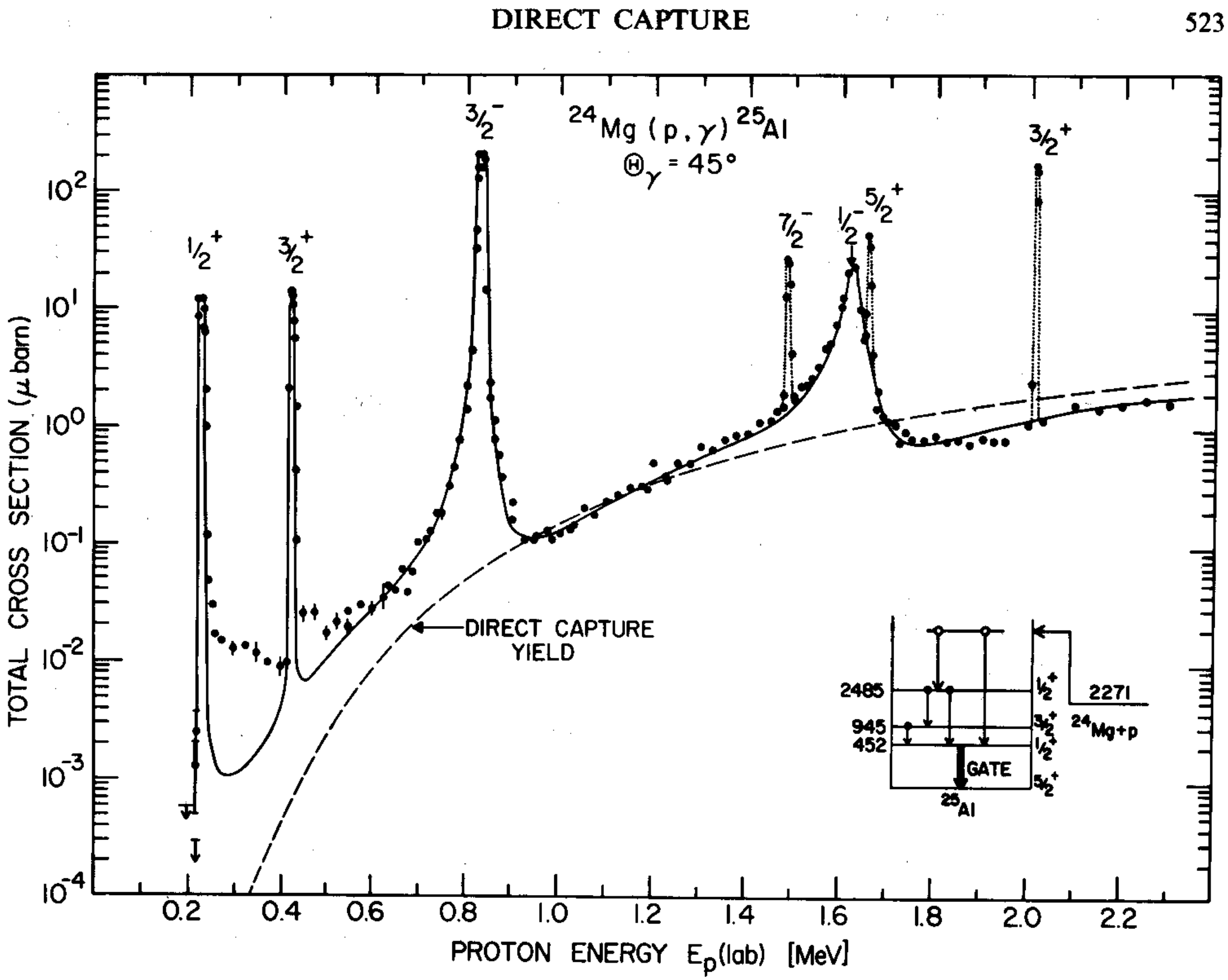
most of these parameters are

UNKNOWN!

Narrow resonant case

example: $^{24}Mg(p,\gamma)^{25}Al$

the cross section

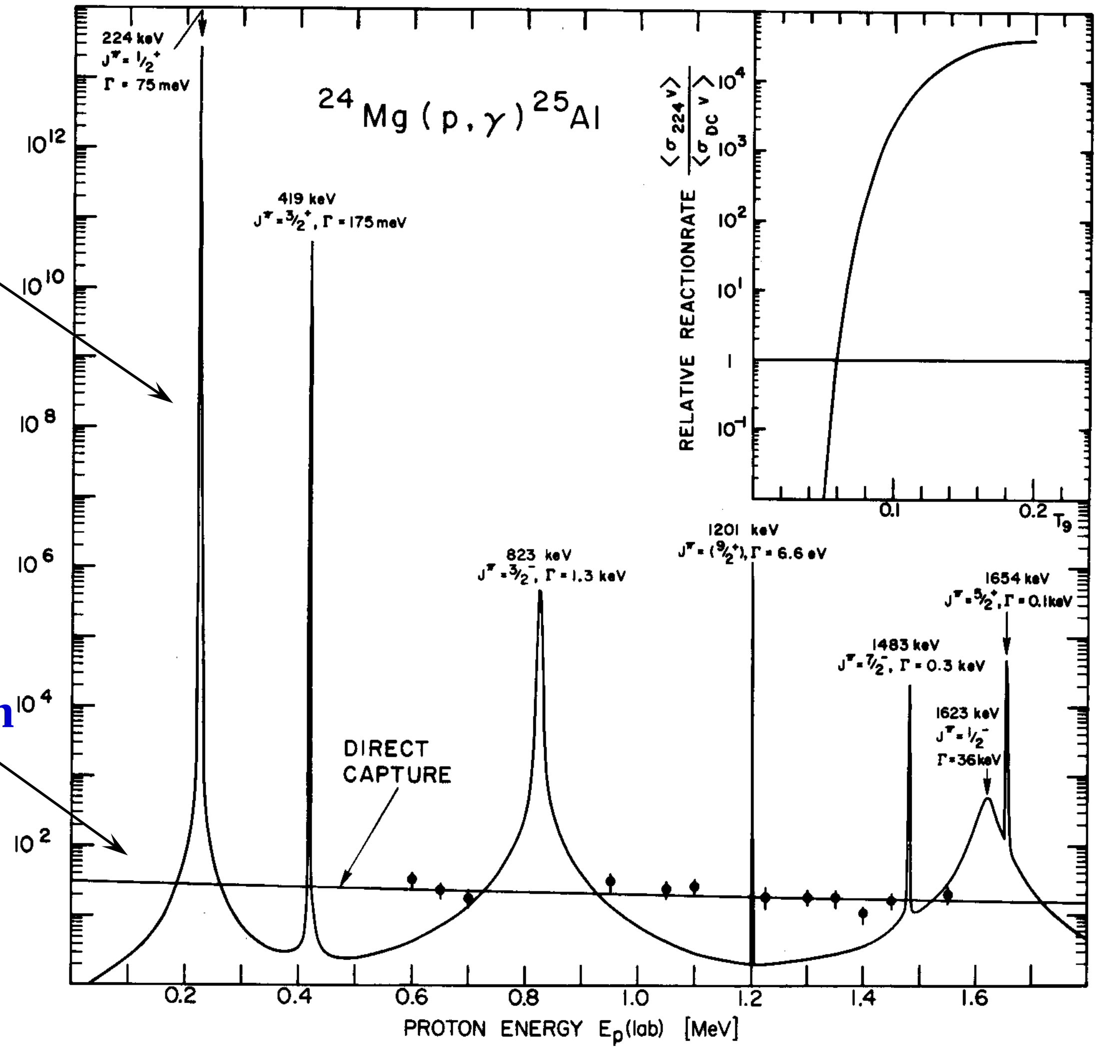


Narrow resonant case

… and the corresponding S-factor

non-constant S-factor
resonant contribution

almost constant S-factor
direct capture contribution



Note varying widths of resonant states !

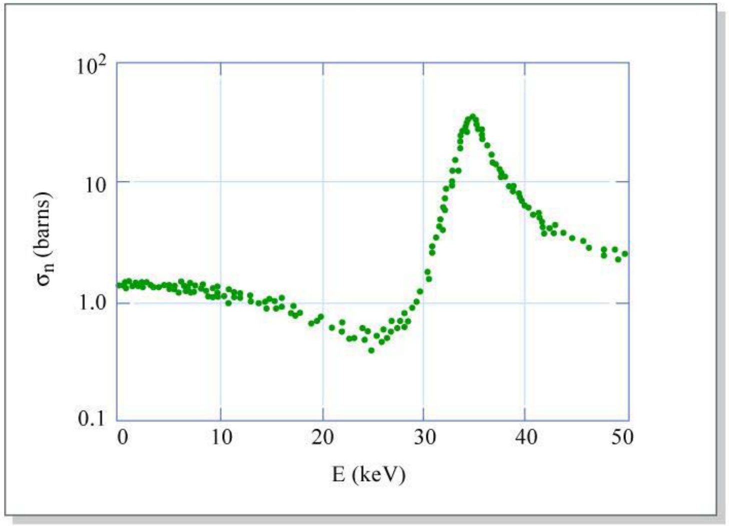
Remarks

- There was only one resonance. There can be several overlapping resonances.
- Resonance amplitude $\sim \dfrac{1}{i(E_0 - E) + \Gamma/2}$

  fast change of phase while crossing the resonance energy

Typically there are two processes:
- non resonant scattering
  e.g. Rutherford scattering
- resonant scattering

interferences changes the shape of the resonances, typically asymmetric lines respectively interferences in cross section



example: n scattering with ^{27}Al

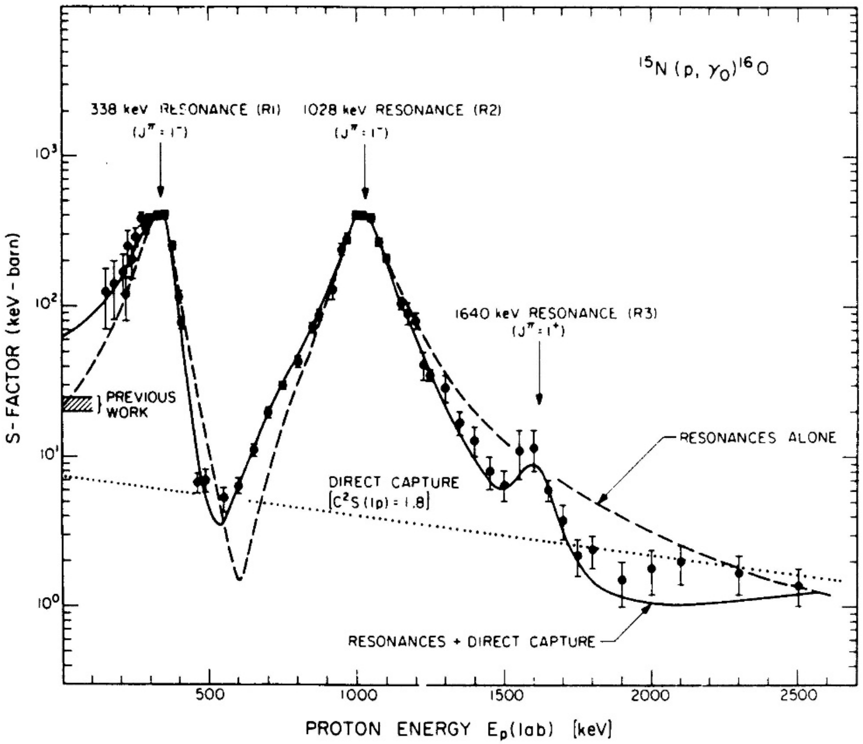
2. Broad resonances $\Gamma \sim E_R$

Breit-Wigner formula

+

energy dependence of partial $\Gamma_a(E)$, $\Gamma_b(E)$ and total $\Gamma(E)$ widths

N.B. Overlapping broad resonances of same J^π → interference effects

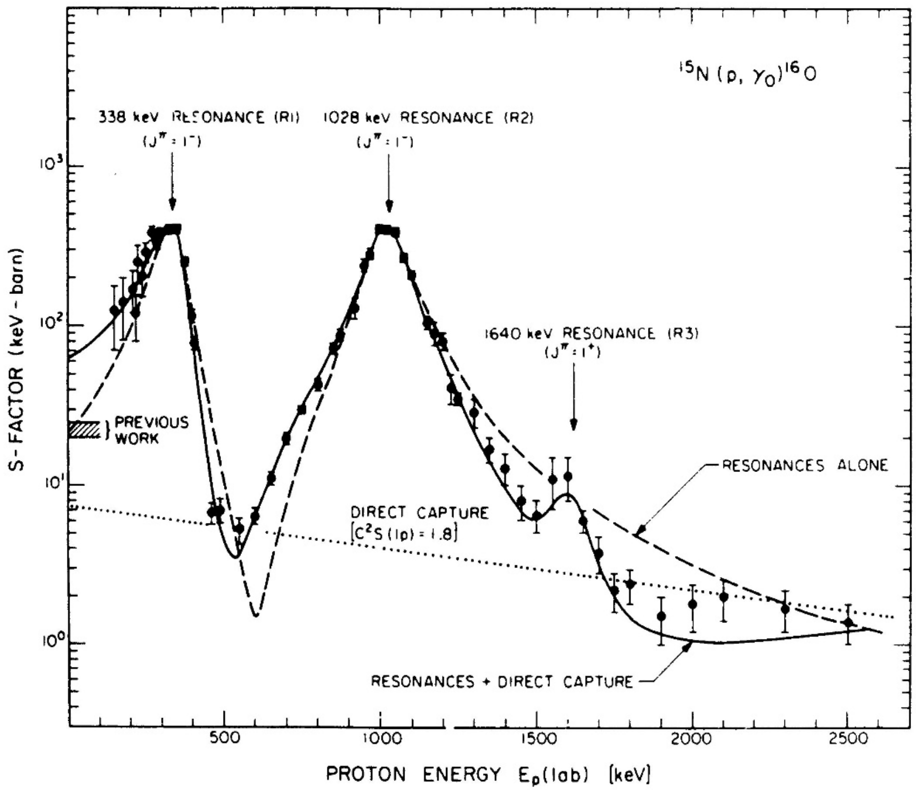


Broad resonance case

$\Gamma \sim E_R$

broader than the relevant energy window for the given temperature

resonances outside the energy range can also contribute through their wings



Breit-Wigner formula
+
energy dependence of partial and total widths

assume:

Γ_2= const, Γ = const and use simplified

$$\sigma(E) = \pi D^2 \Gamma_1(E)\omega \frac{\Gamma_2}{(E - E_R)^2 + (\Gamma/2)^2}$$

same energy dependence as in direct process

for $E \ll E_R$ very weak energy dependence

N.B. overlapping broad resonances of same J^π → **interference effects**

3. Sub-threshold resonances

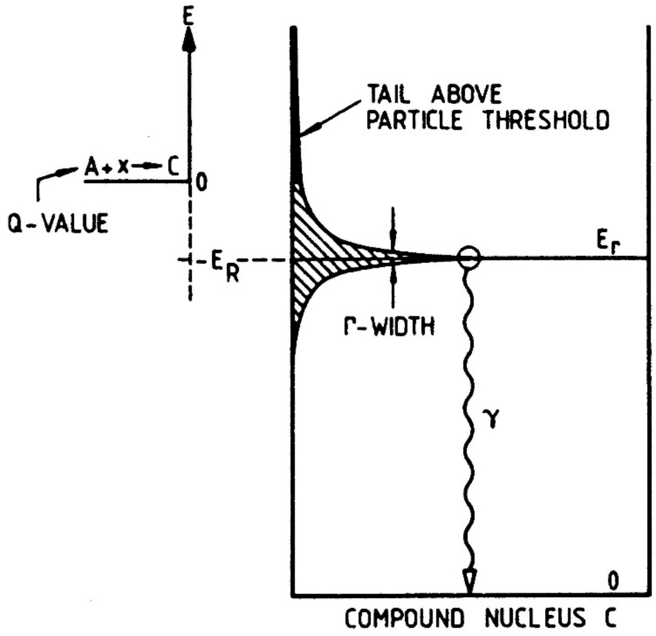
any exited state has a finite width

$\Gamma \sim h/\tau$

high energy wing can extend above particle threshold

⇓

cross section can be entirely dominated by contribution of sub-threshold state(s)



Example: $^{12}C(\alpha,\gamma)^{16}O$

TOTAL REACTION RATE

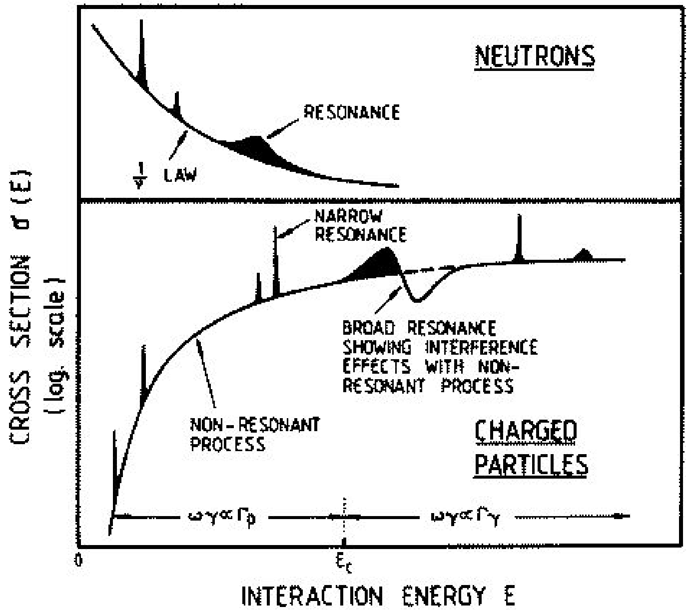
$$\langle\sigma v\rangle_{tot} = \langle\sigma v\rangle_r + \langle\sigma v\rangle_{nr}$$

Summary

stellar reaction rate of nuclear reaction determined by the sum of contributions due to

- direct transitions to the various bound states
- all narrow resonances in the relevant energy window
- broad resonances (tails) e.g. from higher lying resonances
- any interference term

total rate $$\langle\sigma v\rangle = \sum_i \langle\sigma v\rangle_{DCi} + \sum_i \langle\sigma v\rangle_{Ri} + \langle\sigma v\rangle_{tails} + \langle\sigma v\rangle_{interference}$$

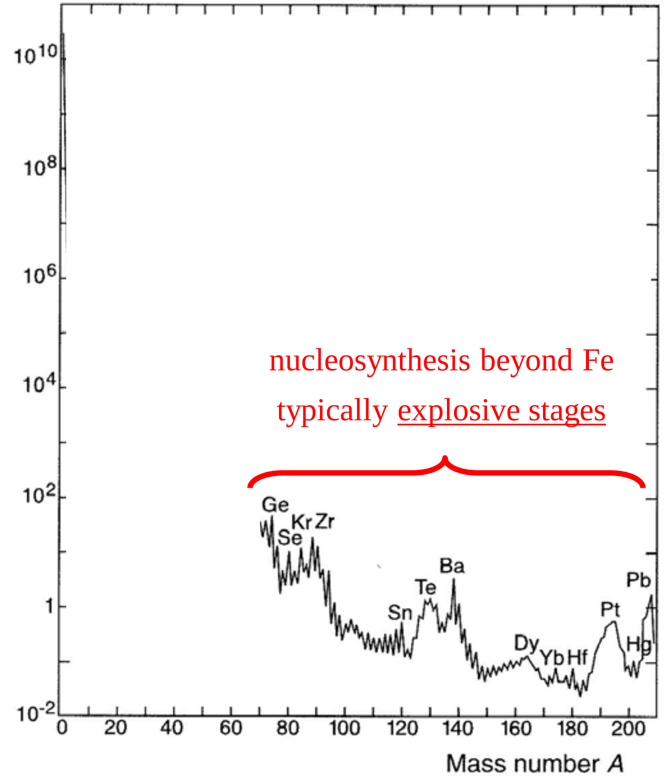


Rolfs & Rodney:
Cauldrons in the Cosmos, 1988

the Gamow window moves to higher energies with increasing temperature

$\Rightarrow$ different resonances play a role at different temperatures

Reactions with neutrons

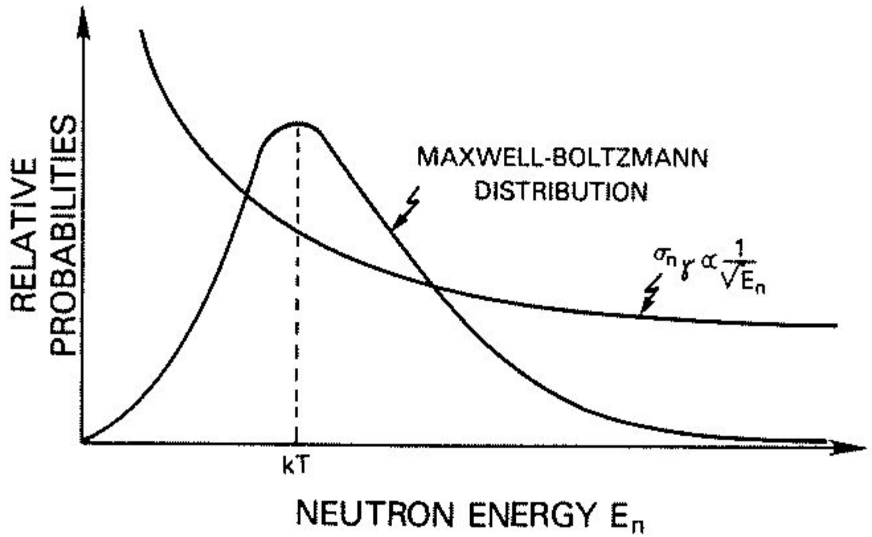


No Coulomb barrier

neutrons produced in stars are quickly thermalized

$E_0 \sim kT$ = relevant energy (e.g. $T \sim 1\text{-}6 \times 10^8$ K $\Rightarrow$ $E_0 \sim 30$ keV)

Typically: $\sigma \sim \frac{1}{v}$ ⇒ $\langle\sigma v\rangle \sim const = \langle\sigma_T v_T\rangle$ ⇒ accounts for almost flat abundance distribution beyond iron peak



neutron-capture cross sections can be measured DIRECTLY at relevant energies

s-process site(s) and conditions

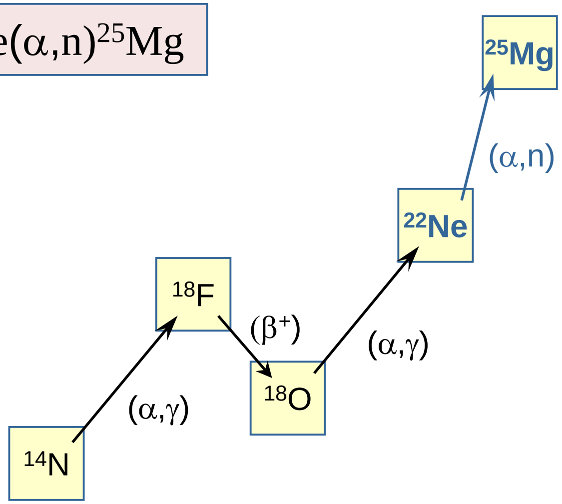
free neutrons are unstable $\Rightarrow$ they must be produced in situ

in principle many (α,n) reactions can contribute

in practice, one needs suitable reaction rate & abundant nuclear species

most likely candidates as neutron source are:

$^{22}Ne(\alpha,n)^{25}Mg$

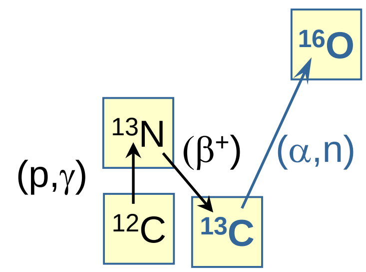


astrophysical site:

core He burning (and shell C-burning)
in massive stars (e.g. 25 solar masses)
$T_8 \sim 2.2 – 3.5$

contribution to weak s-process

$^{13}C(\alpha,n)^{16}O$



astrophysical site:

He-flashes followed by H mixing
into ^{12}C enriched zones
low-mass (1.5 - 3 M_{sun}) TP-AGB stars
$T_8 \sim 0.9 – 2.7$

contribution to main s-process

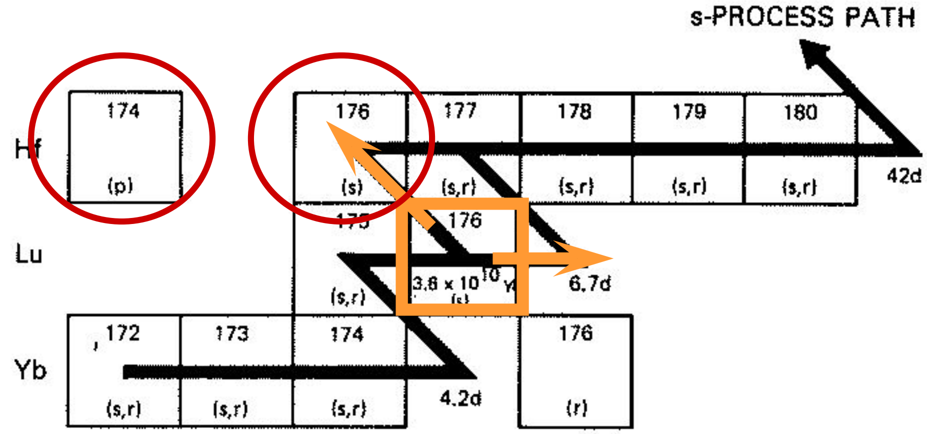
in some cases: $\tau_\beta \sim \tau_n$ $\Rightarrow$ a branching occurs in nucleosynthesis path

example:

$^{176}Lu^{gs} \rightarrow {}^{176}Hf$ $\tau_\beta \sim 5\text{x}10^{10}$ y

$^{176}Lu^{m} \rightarrow {}^{176}Hf$ $\tau_\beta \sim 5.3$ h

for $N_n \sim 10^8$ cm^{-3} $\Rightarrow$ $\tau_n \sim 1$ y



$^{176}Lu^{gs}$ essentially STABLE

$^{176}Lu^{m}$ quickly decays into ^{176}Hf

from abundance determinations:

$$\frac{^{176}Hf}{^{174}Hf} = 29$$ (note: ^{174}Hf = p-only nucleus, i.e. not affected by s-process)

$\Rightarrow$ significant amount of s-process branching from $^{176}Lu^{m}$ β-decay is required

$\Rightarrow$ need temperatures $T_8 > 1$ to guarantee that isomeric state is significantly populated

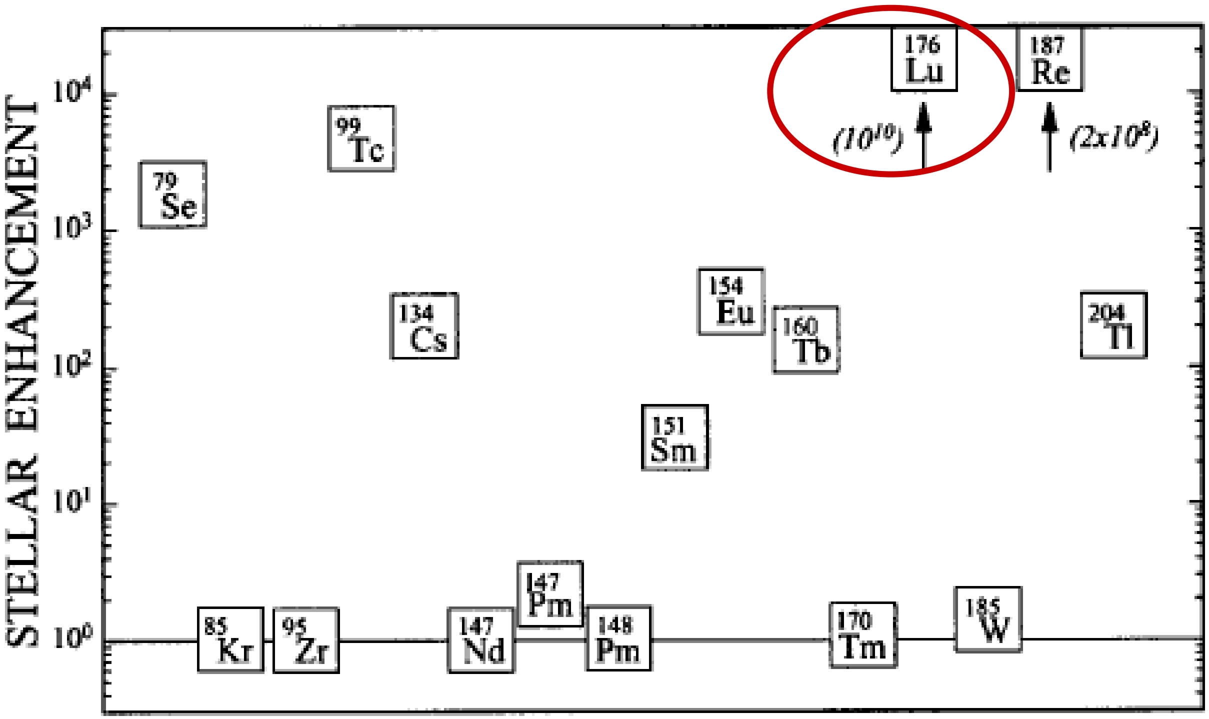
branching points can be used to determine

- neutron flux
- temperature
- density

in the star during the s process

about 15-20 branchings relevant to s process

stellar enhancement of decay (stellar decay rate/terrestrial rate)
for some important branching-point nuclei in s-process path @ kT = 30 keV

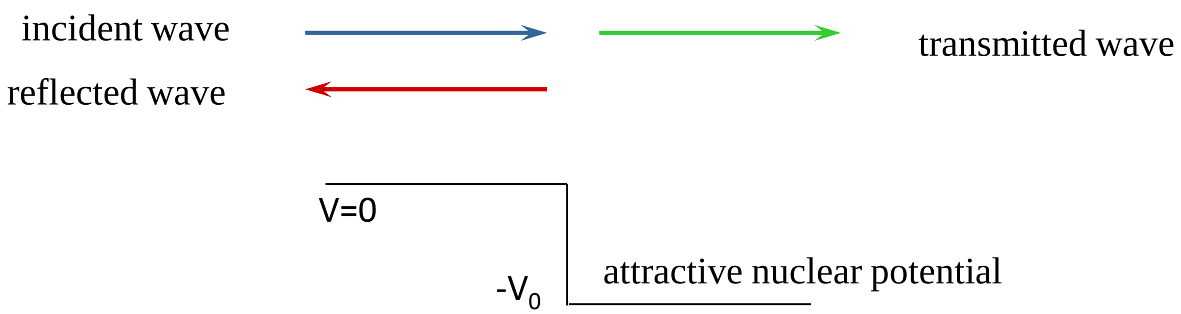


F. Kaeppeler: Prog. Part. Nucl. Phys. 43 (1999) 419 – 483

Neutron capture

simplest case: s-wave neutrons $\Rightarrow V_\lambda = 0$ and also $V_C = 0$

discontinuity in potential gives rise to partial reflection of incident wave



transmission probability:

$P_l \propto E^{1/2}$ for $\lambda = 0$ and hence: $\sigma \propto \frac{1}{\sqrt{E}} = \frac{1}{v}$

$P_l \propto E^{1/2+l}$ for $\lambda \neq 0$ and hence: $\sigma \propto E^{l-1/2}$

consequences:

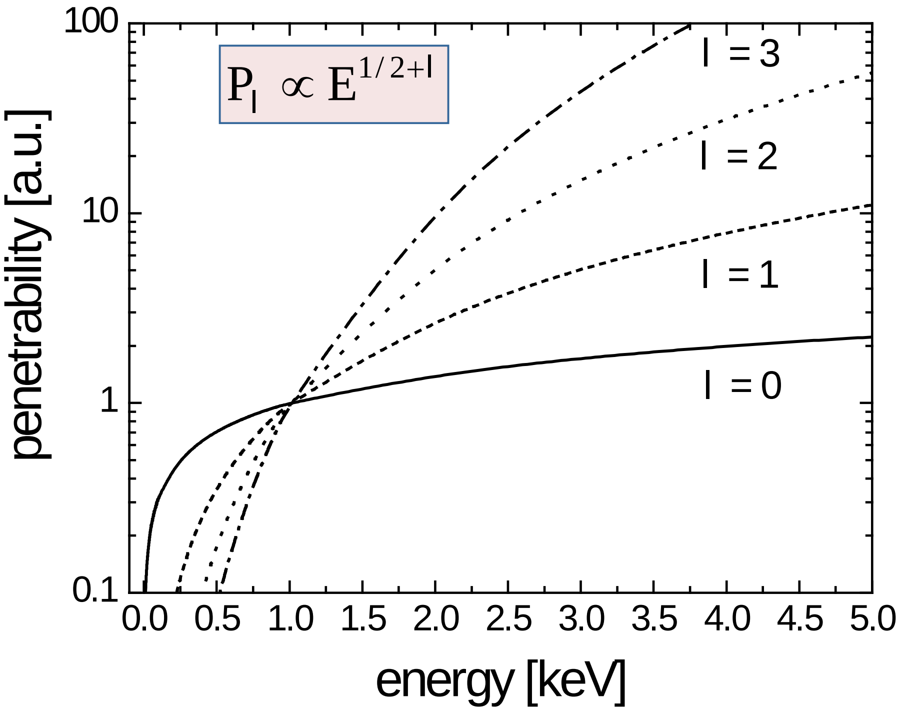
s-wave neutron capture usually dominates at **low energies** (except if hindered by selection rules)

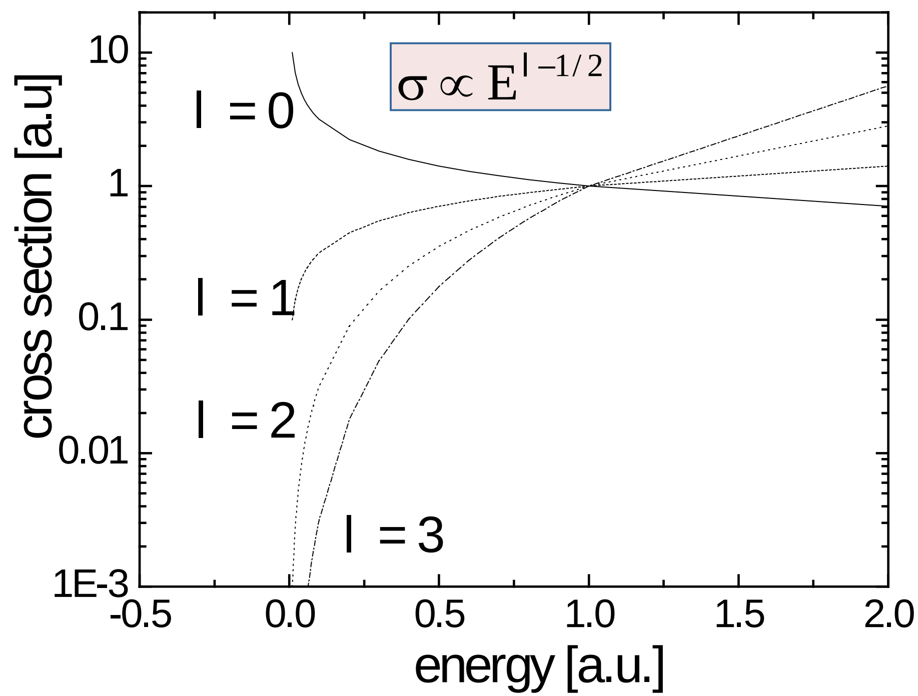
higher λ neutron capture only plays role at **higher energies** (or if λ=0 capture suppressed)

Neutron capture

λ dependence of penetrability through centrifugal barrier

λ dependence of neutron capture cross section





lower λ values dominate reaction rate

note: arbitrary scale between different λ values

cross section decreases strongly with decreasing energy (effect of barrier)

Stellar reaction rates for neutron capture

$$\langle\sigma v\rangle = \int \sigma(v)\phi(v) v dv = \int \sigma(E)\exp(-E/kT) E dE$$

energy range of interest for astrophysics depends on:

temperature and **cross section shape**

s-wave neutron capture

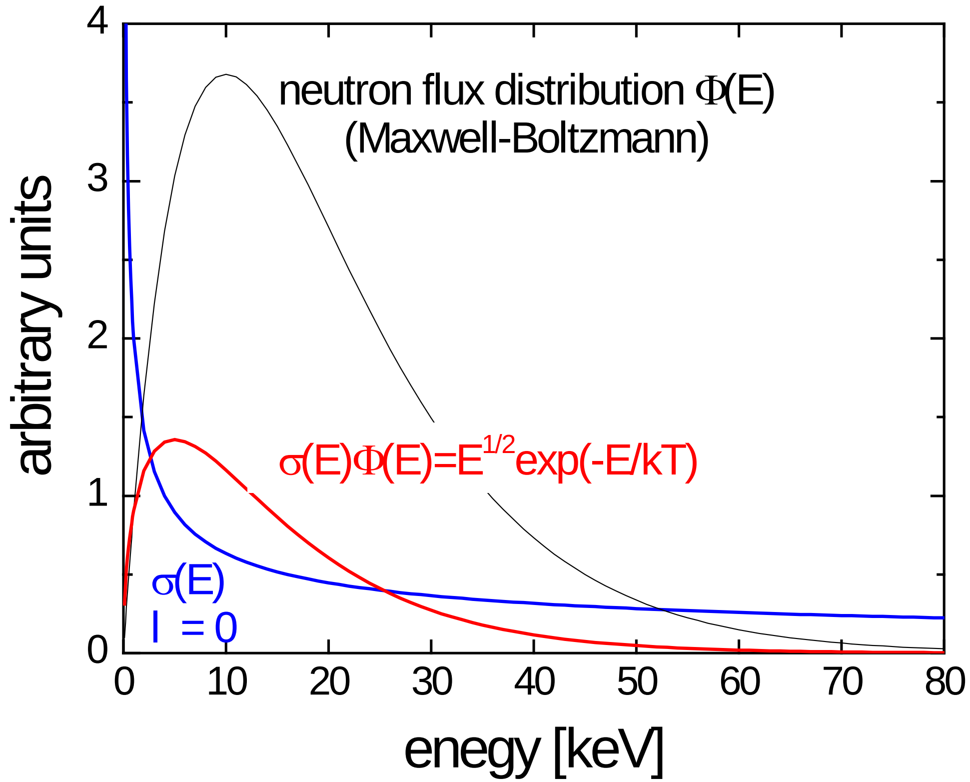
energy range of interest $E \sim kT$

$$\sigma \propto \frac{1}{v}$$

$$\sigma v = \text{const} = \langle\sigma v\rangle$$

stellar reaction rate

$$\langle\sigma v\rangle = v_T \sigma_{th}$$



σ_{th} = measured cross section for thermal neutrons

$$v_T = \sqrt{\frac{2kT}{\mu}}$$

most probable velocity, corresponding to $E_{cm} = kT$

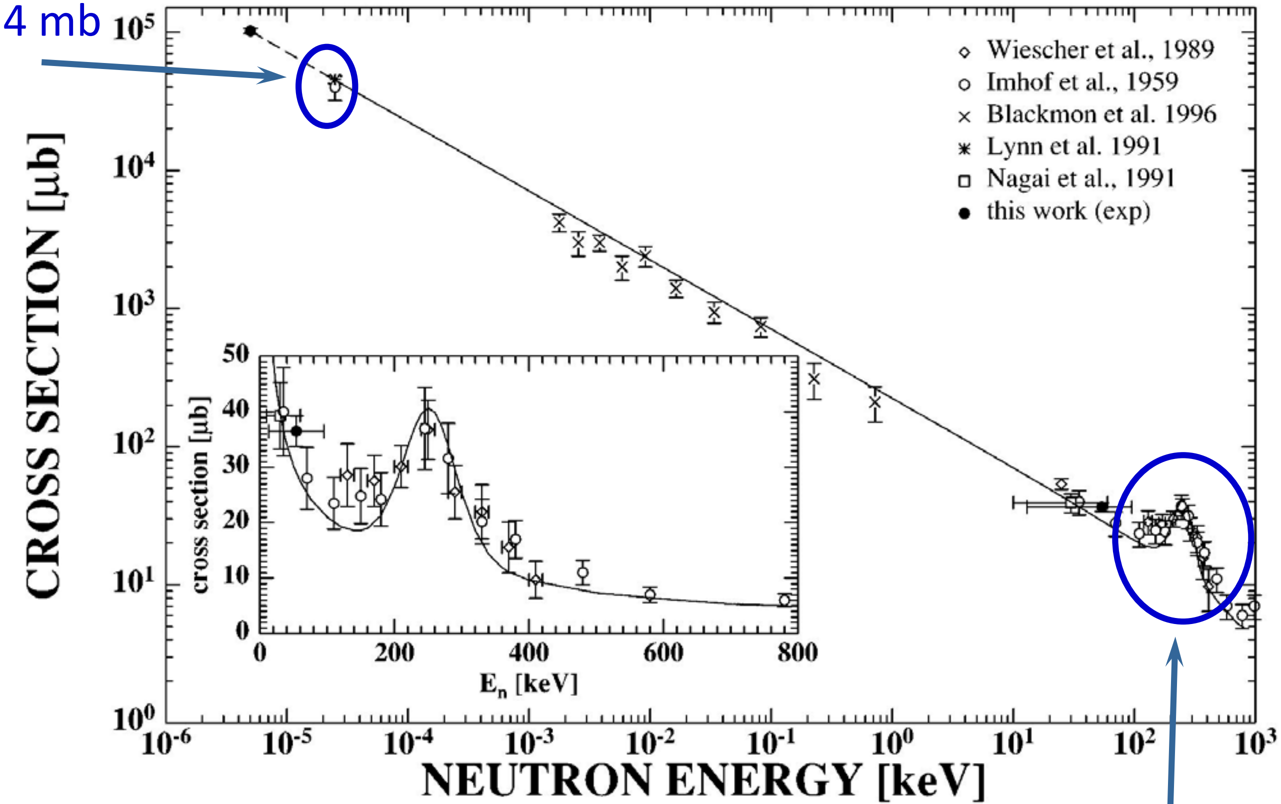
Special case: $\lambda = 0$

s-wave neutron capture

$$\sigma \propto \frac{1}{\sqrt{E}} = \frac{1}{v}$$

thermal
cross section
$<\sigma> = 45.4$ mb

example: $^7Li(n,\gamma)^8Li$



deviation from 1/v trend due to
resonant contribution (see later)