Outline: The Sun

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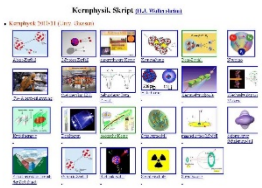
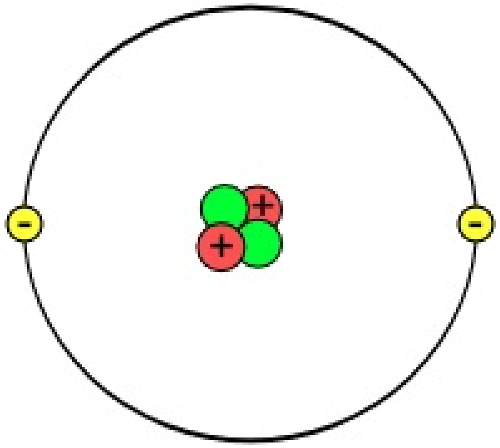
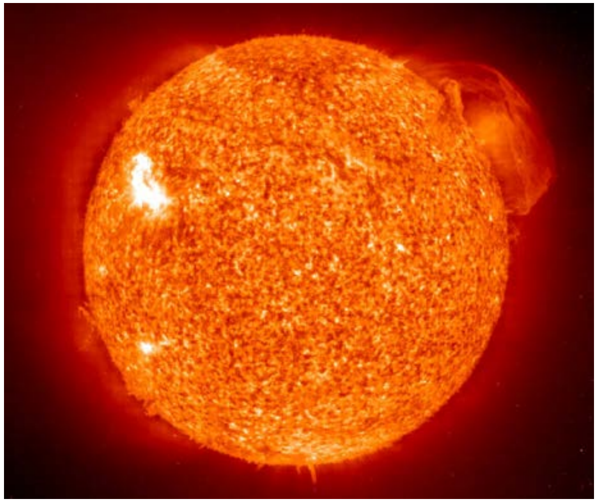
web-page: https://web-docs.gsi.de/~wolle/ and click on 

1. Hertzsprung-Russel diagram
2. black body radiation
3. evolution of the sun
4. heavier stars

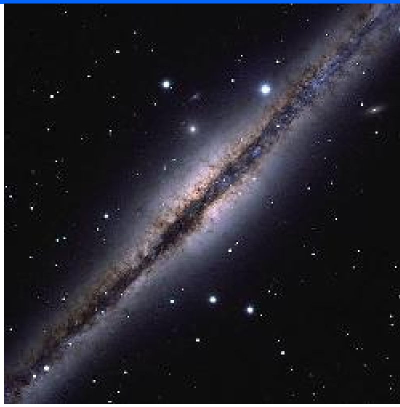




atomic nucleus $1\cdot10^{-15}$ m

The every day star $\sim1\cdot10^{9}$ m

Basic properties of stars



- Distance to stars
  - parallax method for determining distance
  - definition of the "parsec"

- Flux, luminosity and stellar magnitude

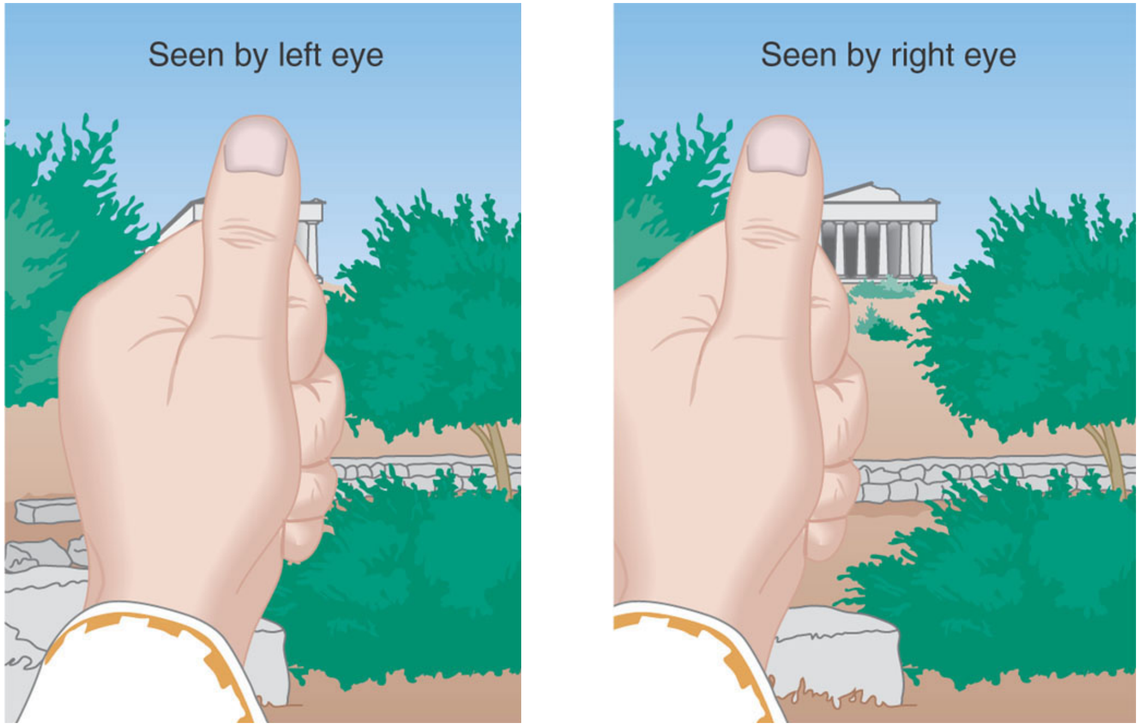
- Hertzsprung-Russell diagram

The distance to the stars

- The **distance** to any astronomical object is the most basic parameter
  - quire knowledge of distance in order to calculate just about any other property of the object
  - distance is often difficult to determine!

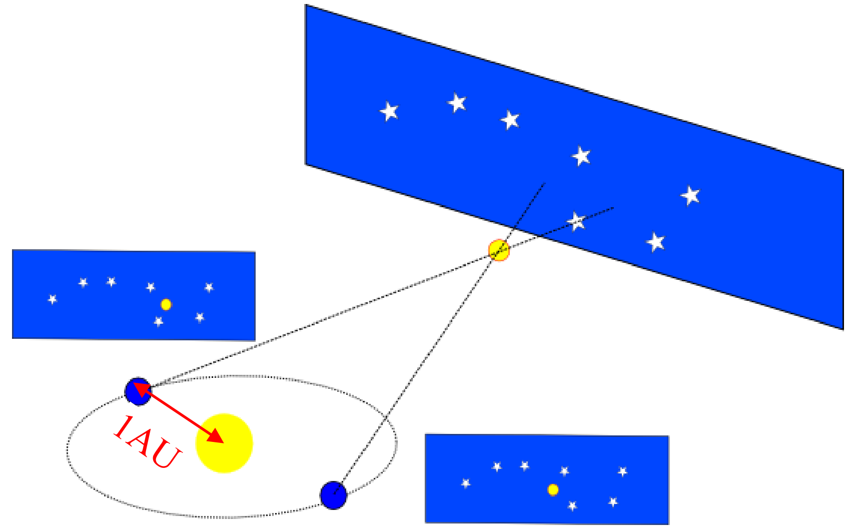
- Most direct method to measure distances to “nearby” stars uses trigonometric **parallax**
  - as Earth orbits Sun, we view a star along a slightly different line of sight
  - this causes the star to **appear** to move slightly with respect to much more distant stars
  - we can currently use this technique to measure stellar distances out to ~3000 light years from Earth

Determination of distances - parallax

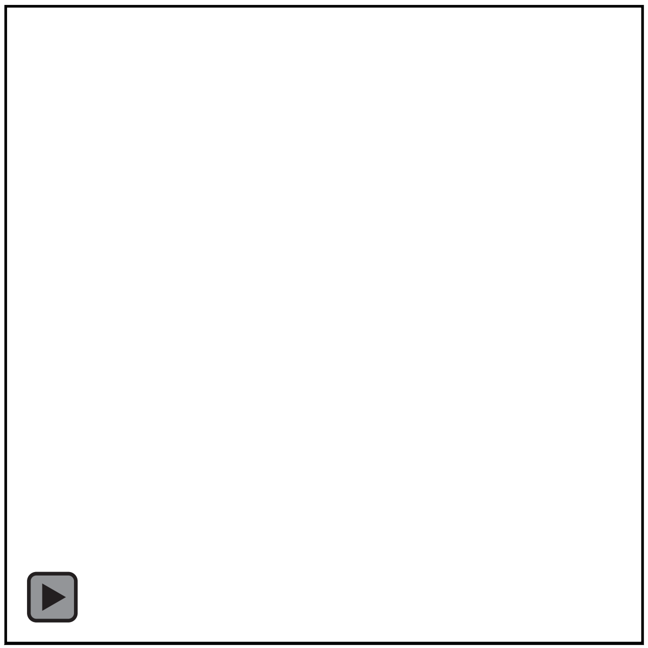


Trigonometry:

$$1'' \hat{=} 3{,}26 Ly = 1 pc$$



- Limited to stars no more than 100pc distance



$$1\,arc\sec = \frac{1^0}{3600} = \frac{1^0}{3600}\cdot\frac{\pi}{180^0} = 4.85\cdot 10^{-6}\,rad$$

$$1\,Ly = 2.998\cdot 10^8\left(\frac{m}{s}\right)\cdot 86400\left(\frac{s}{d}\right)\cdot 365\left(\frac{d}{y}\right) = 9.46\cdot 10^{15}\,m$$

$$1\,pc = \frac{1.5\cdot 10^{11}\,m}{4.85\cdot 10^{-6}} = 3.086\cdot 10^{16}\,m = 3.26\,Ly$$

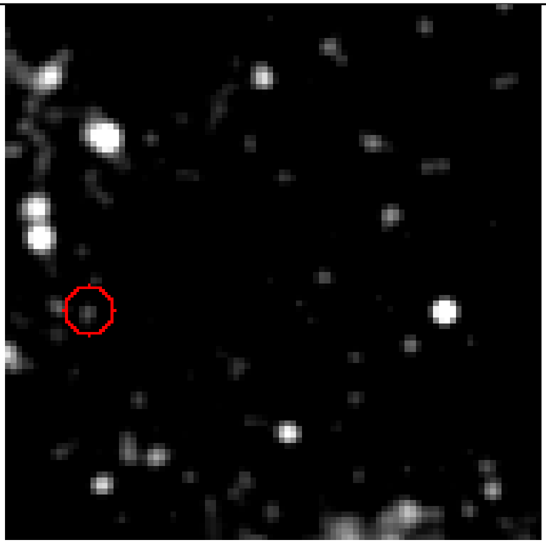
1parsec (pc) = unit of length, measures the distance of a star with a parallax of 1arc-second

mean Earth – Sun distance = astronomical unit 149 597 870 km

Cepheid – intrinsic stellar pulsation

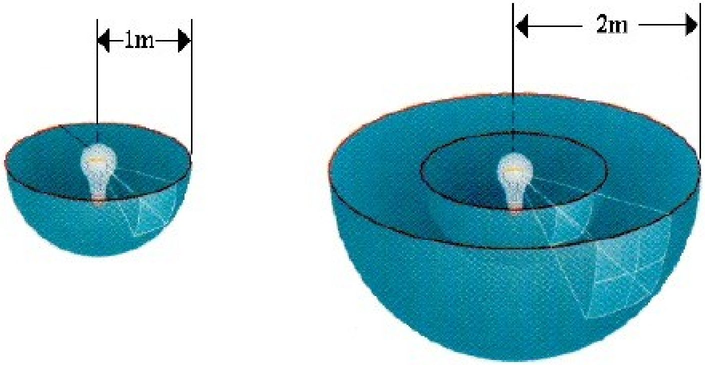
- Cepheids are stars, that undergo pulsations.
- (imbalance between ionization and gravitation)

- 1912: H. Leavitt, H.Shapley:
  - There is a linear relationship between luminosity and pulsation period.
  - This method allows distance measurements up to 50 Mpc.

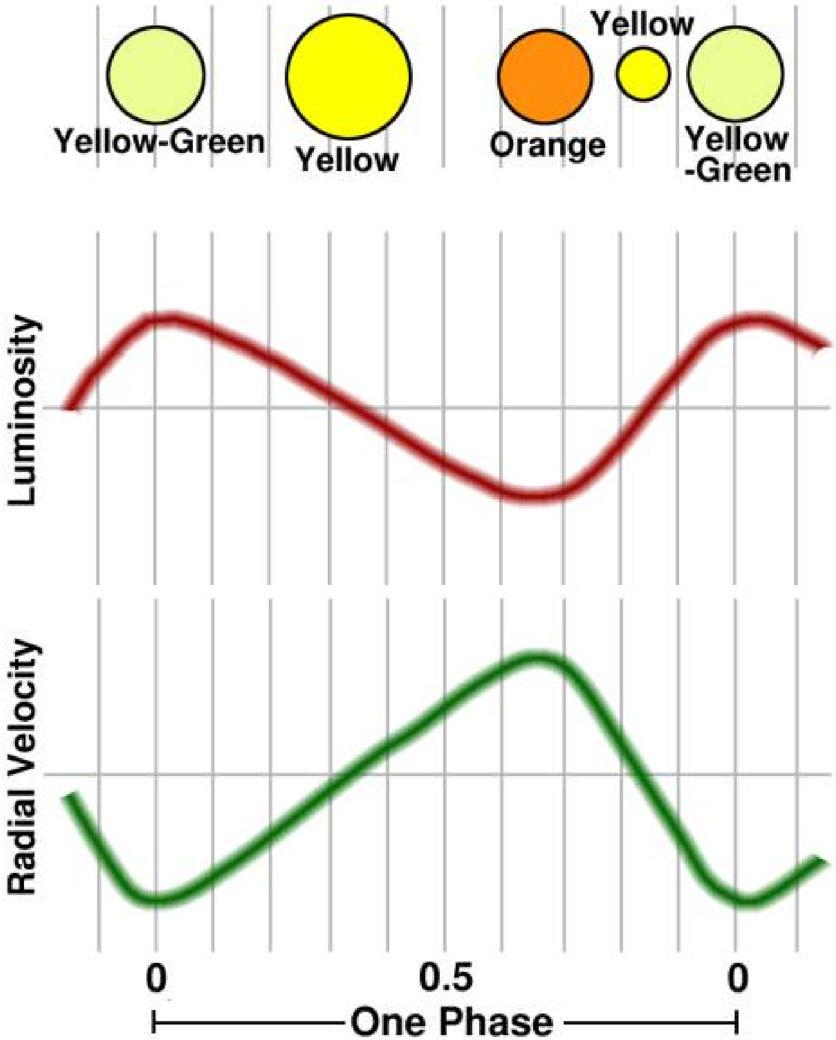


- If one measures the pulsation period of a Cepheid,
- one knows its true luminosity.
- One compares this with the observed brightness
- on Earth and obtains the cosmic distance to the star.

$$L_d = \frac{L_0}{4 \cdot \pi \cdot d^2}$$



Cepheids – intrinsic stellar pulsation

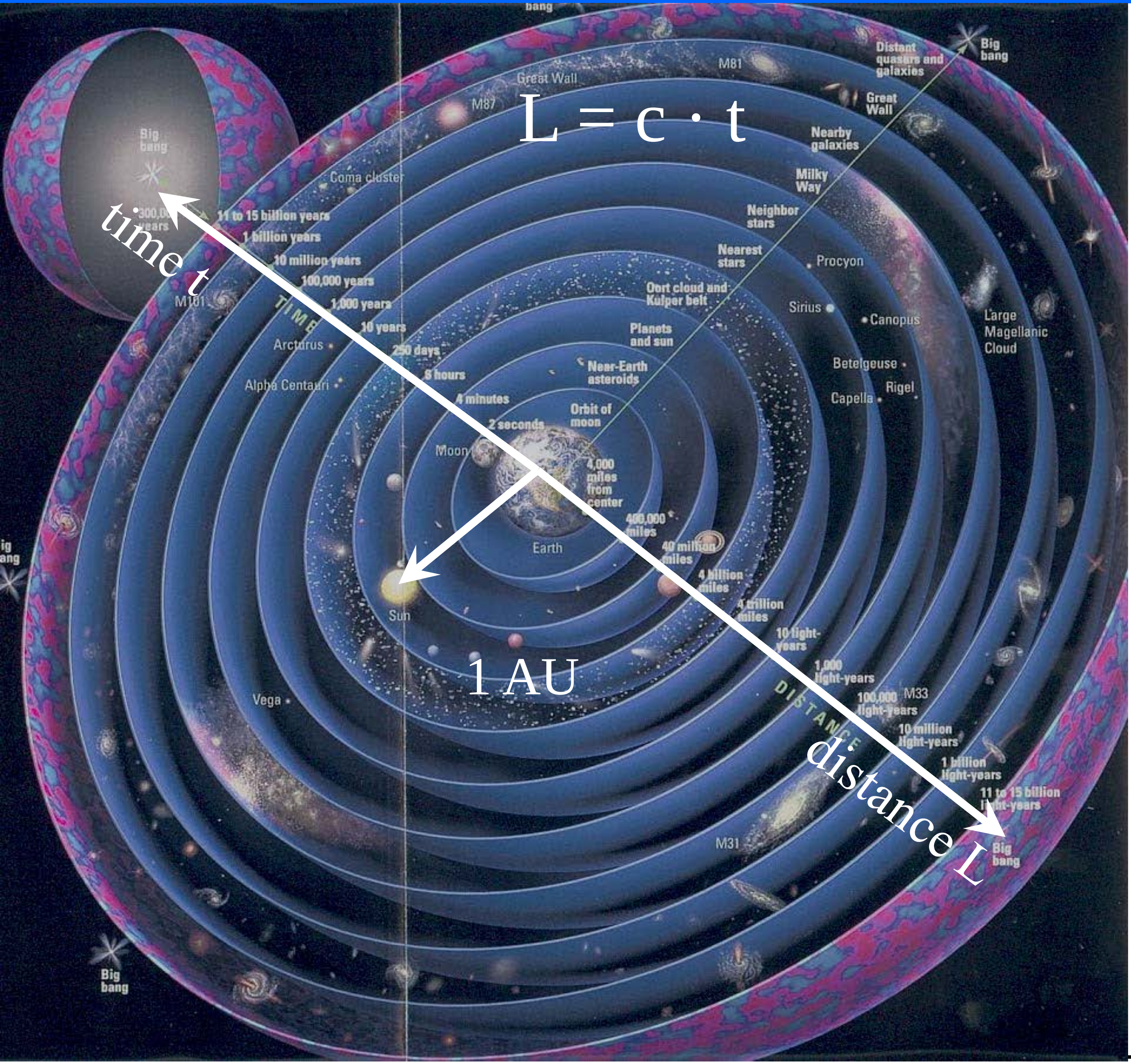


Luminosity:

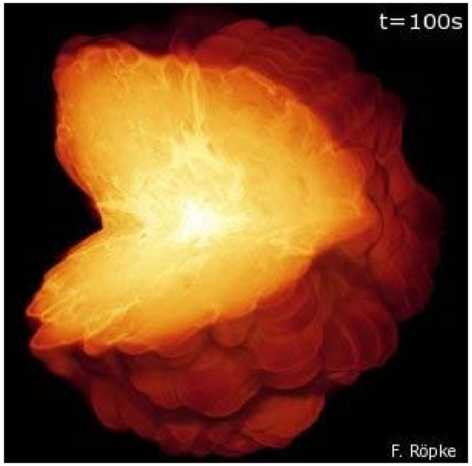
small but hot ***bright***

big but cool ***dim***

Cepheids – intrinsic stellar pulsation



50 Mpc – 3 Gpc



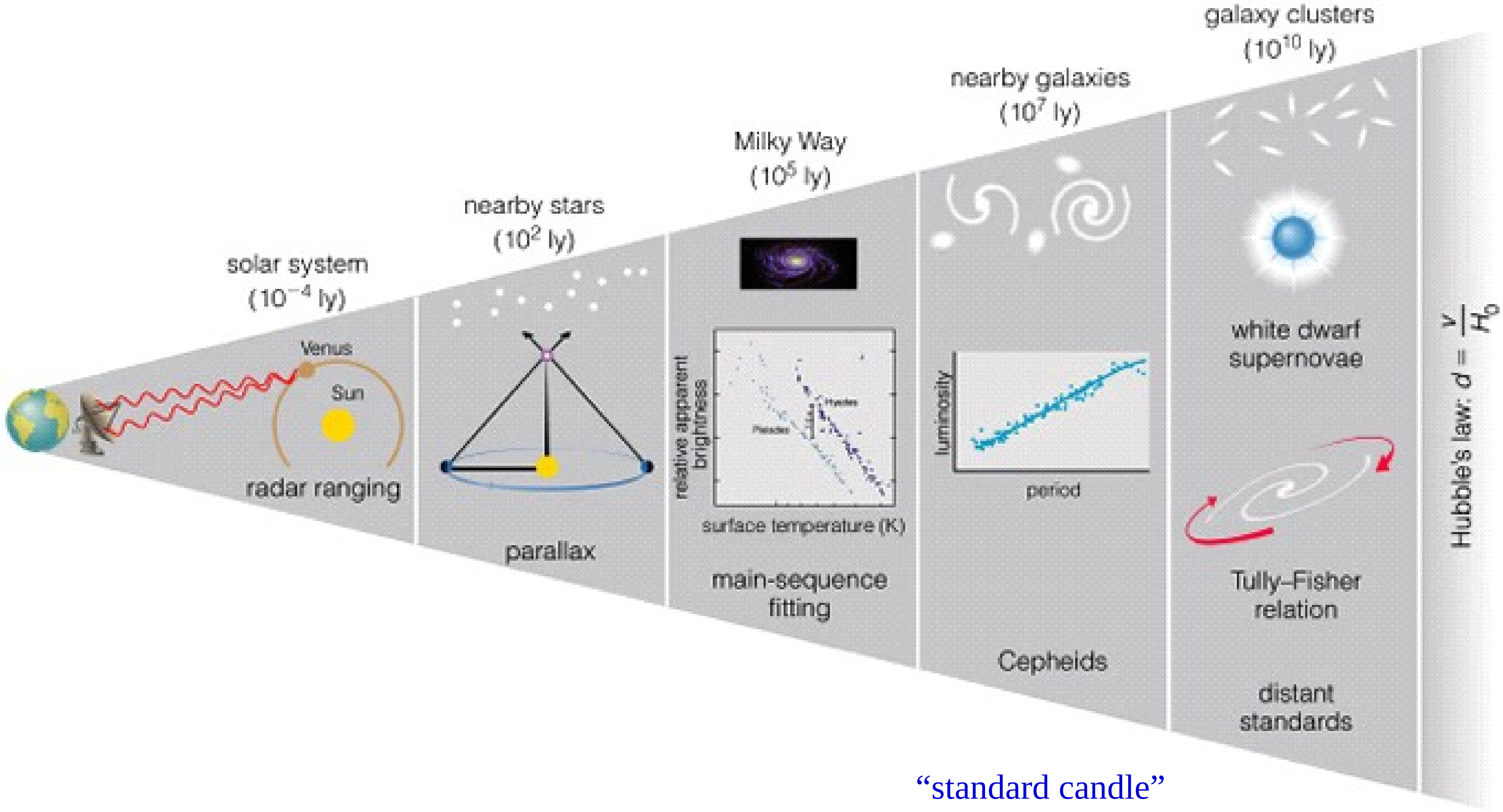
Supernovae Ia

1 astronomical unit (AU) = $1.496 \cdot 10^{11}$ m

1 light year (ly) = $9.461 \cdot 10^{15}$ m = 63.240 AU = 0.3066 pc

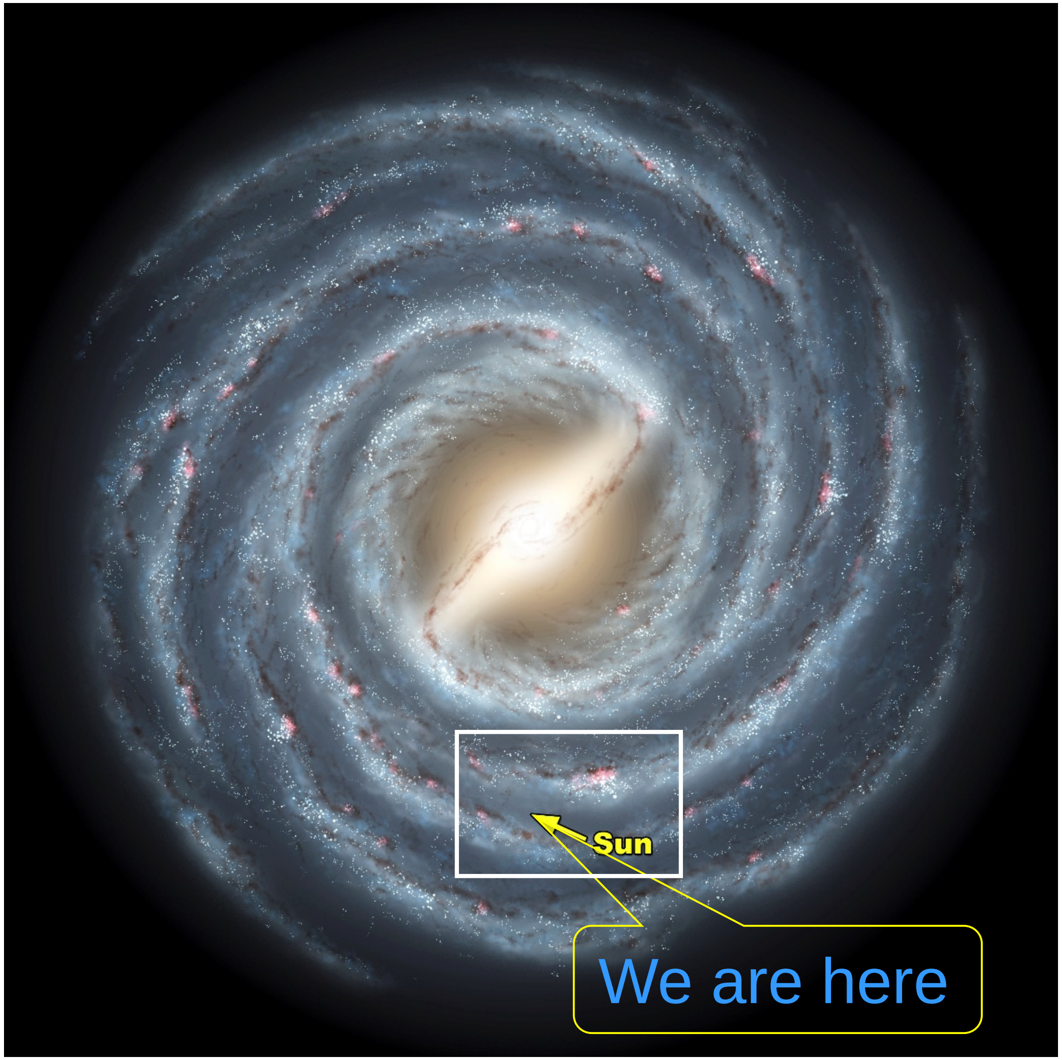
1 Parsec (pc) = $3.086 \cdot 10^{16}$ m =2.06 ·105 AU = 3.262 ly

cosmic distance ladder



"standard candle"

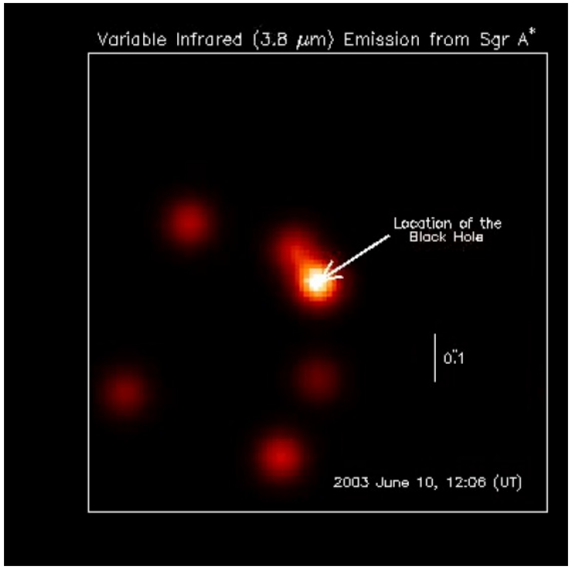
50 000 light years (Milky Way)



http://www.news.wisc.edu/newsphotos/images/Milky_Way_galaxy_sun05.jpg

Milky Way Galaxy

- The radio source Sagittarius A* (+) is the supermassive black hole at the Galactic Center of the Milky Way



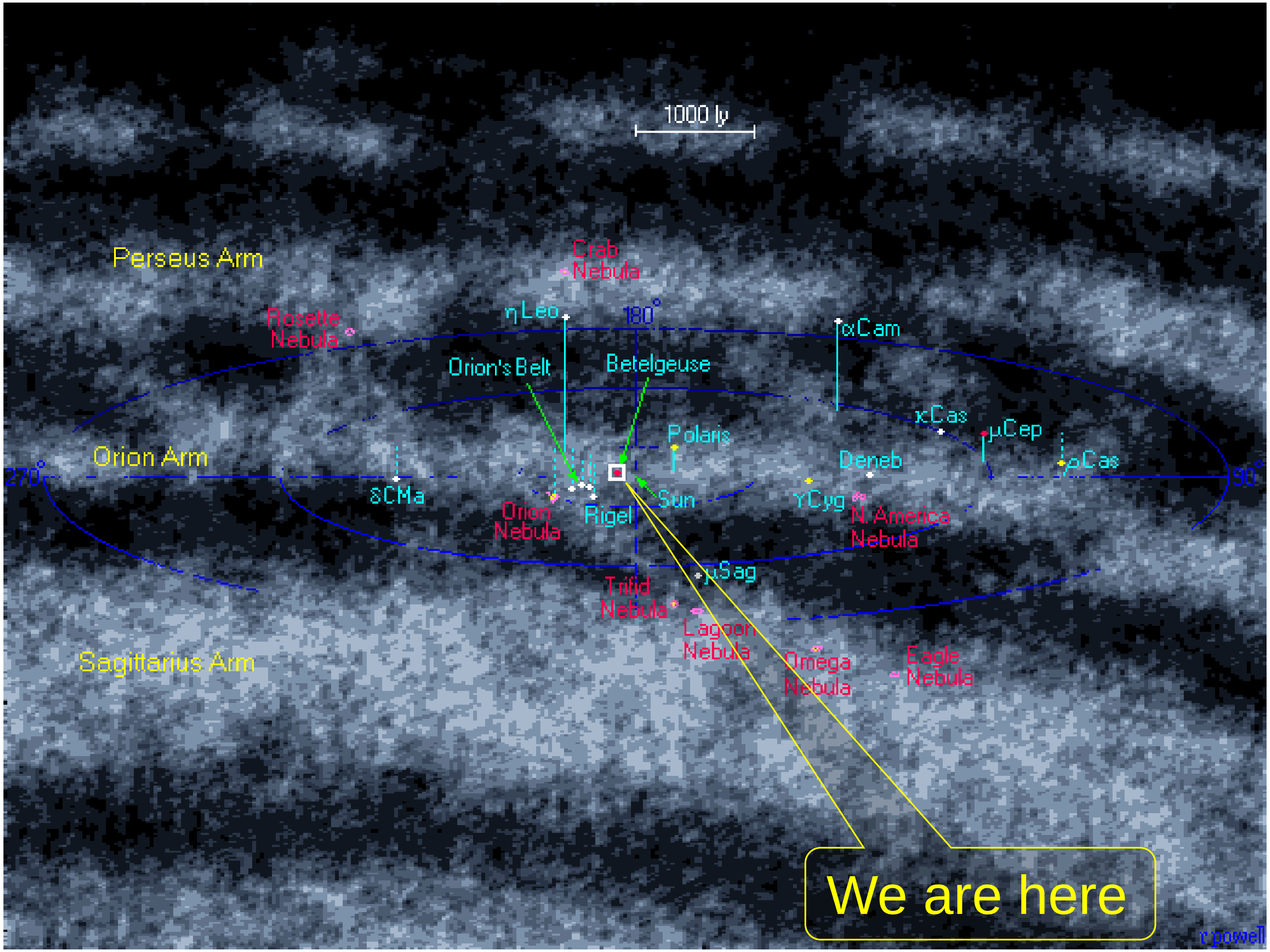
Intrinsic motion of the stars near Sgr A*

→ Mass $(4{,}154 \pm 0{,}014)*10^{6}\ M_{\odot}$

minimal distance of a star 17 light hours

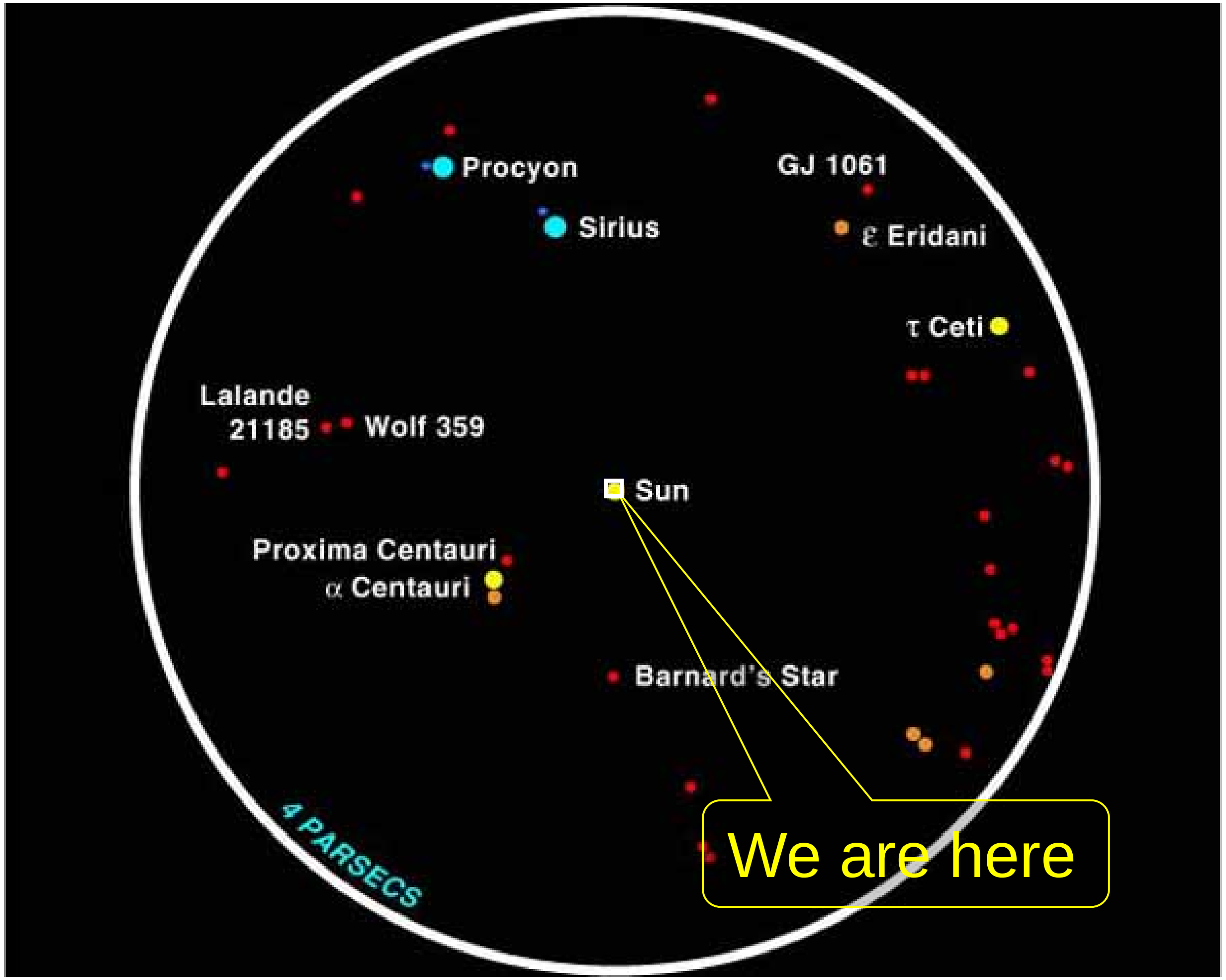
Genzel et al. (2003)

The universe within 5000 Ly – the Orion arm



http://www.atlasoftheuniverse.com

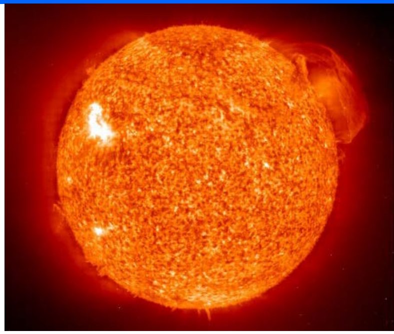
The universe within 5000 Ly – the Orion arm



All stars within 13 light years (4 parsecs) around the Sun. There are 25 other stars, many are weak shining red dwarves, which can be seen from Earth with the naked eye.

http://stardate.org/

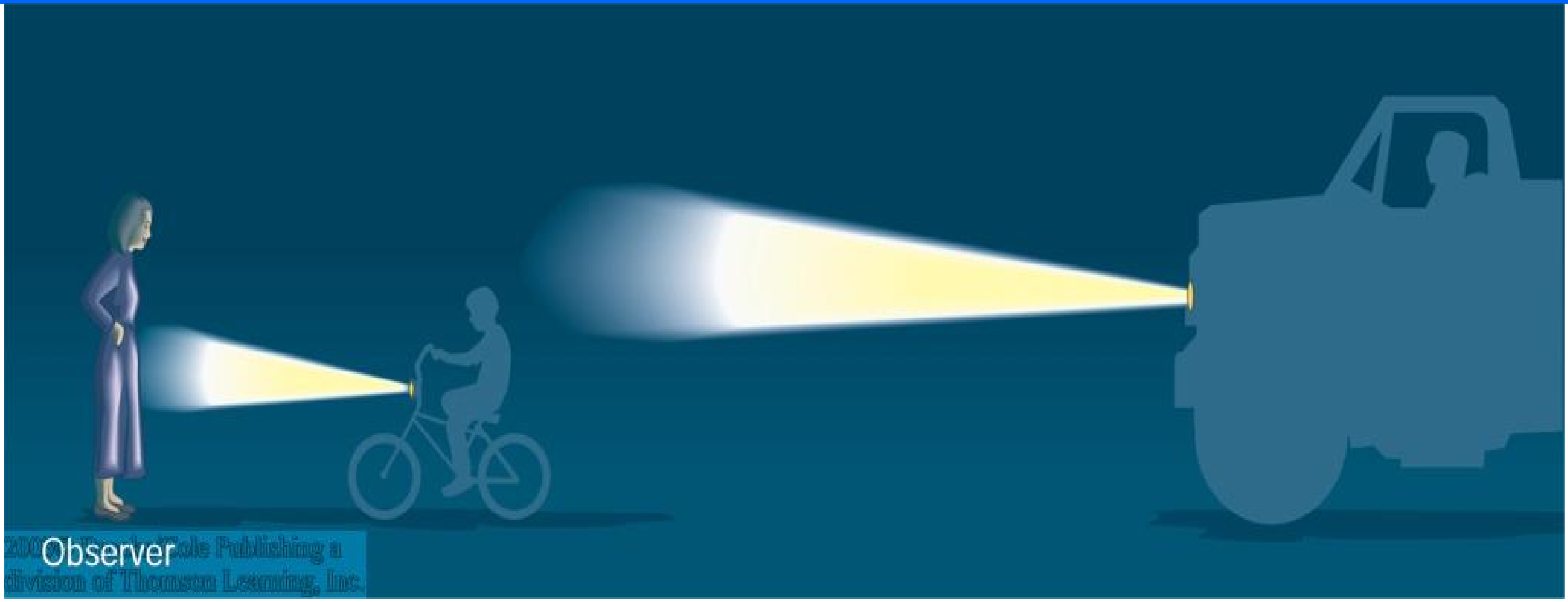
From the Sun we have learned:



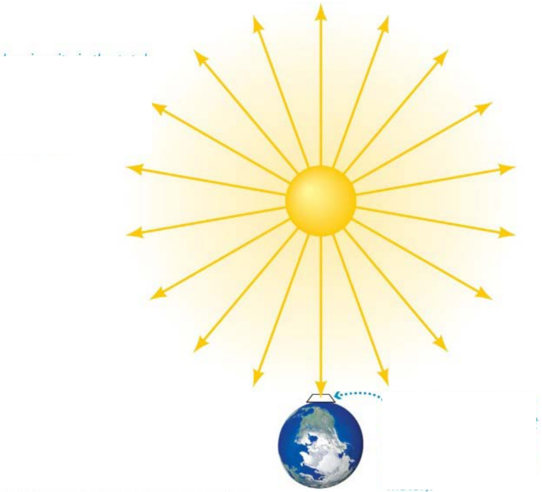
- stars are far away
- stars are bright
- stars are hot
- stars are massive

- How FAR AWAY? (DISTANCE)
- How BRIGHT? (LUMINOSITY)
- How HOT? (SPECTRAL TYPE)
- How MASSIVE? (MASS)

Stellar brightness



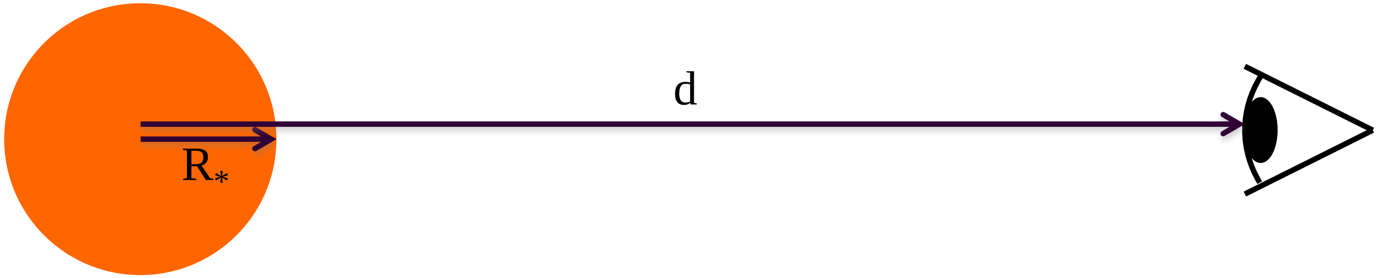
The magnitude or brightness of an object depends on both **distance** and **energy output**.



Amount of energy output a star radiates is called the **Luminosity L**: the energy per second

Amount of starlight that reaches Earth is called the **apparent magnitude (m)**

Stars show spectra very close to black-body radiation



J. Stefan & L. Boltzmann

flux at the surface:

$$F = \sigma_{SB} \cdot T_*^4$$ with $\sigma_{SB} = 5.67 \cdot 10^{-8}$ W m^{-2} K^{-4}

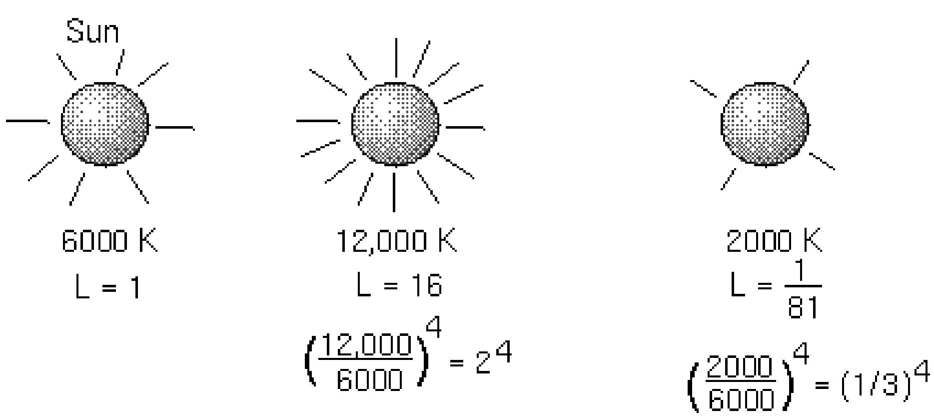
measured flux:

$$F = \left(\frac{R_*}{d}\right)^2 \cdot \sigma_{SB} \cdot T_*^4$$

luminosity (Stefan-Boltzmann law) is the flux multiplied by entire spherical surface:

$$L = 4\pi \cdot R_*^2 \cdot \sigma_{SB} \cdot T_*^4$$

Luminosity is proportional to *fourth* power of temperature.



Stellar magnitudes

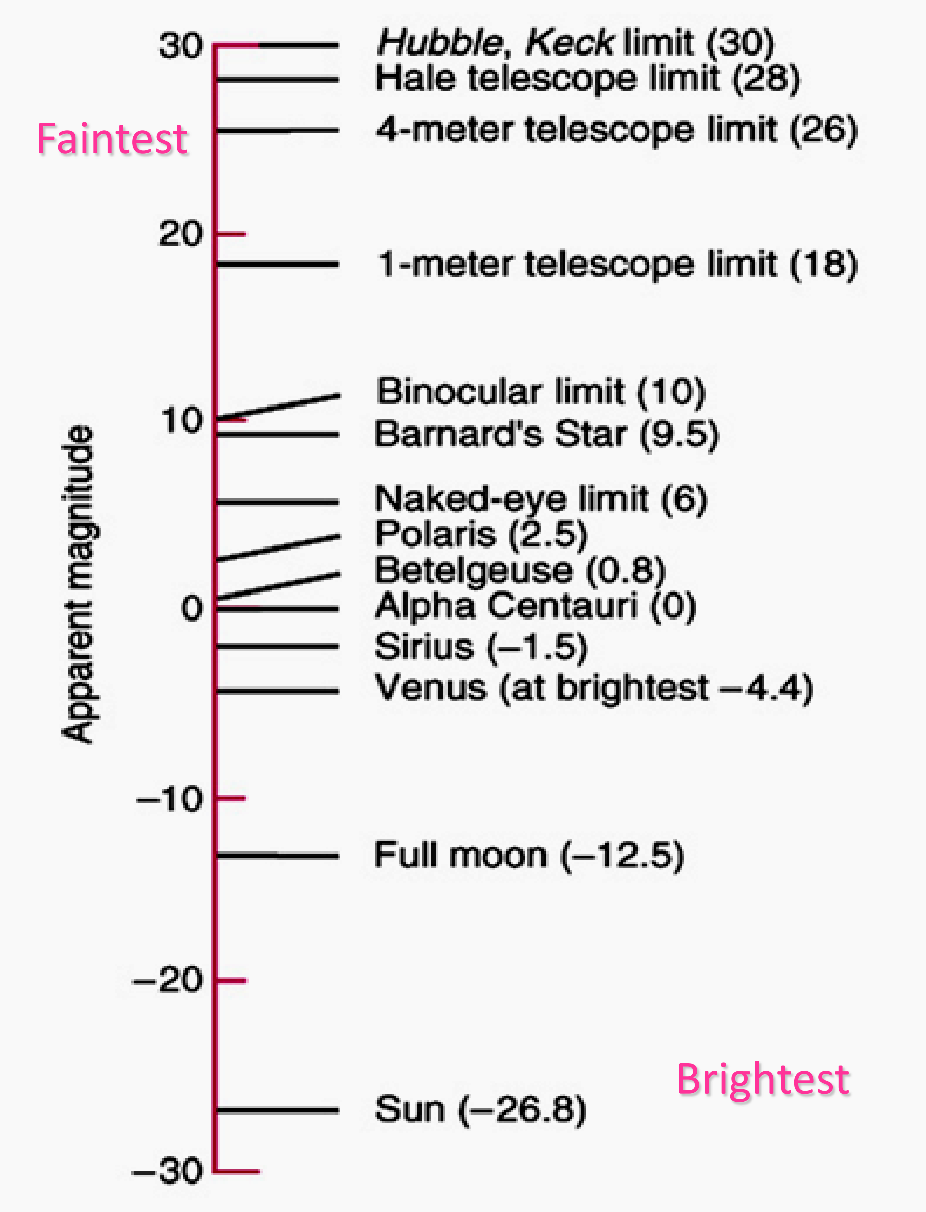
2nd century BC, Hipparchus ranked all visible stars – brightest = magnitude 1 faintest = magnitude 6.

To our eyes, a change of one magnitude = a factor of 2.5 in flux.

Hence

The magnitudes scale is logarithmic.

A change of 5 magnitudes means the flux 100 x greater!



Luminosity and apparent magnitude

- Modern definition: If two stars have fluxes F_1 and F_2, then their **apparent magnitudes** m_1 and m_2 are given by

$$m_2 - m_1 = 2.5 \log_{10} \frac{F_1}{F_2}$$

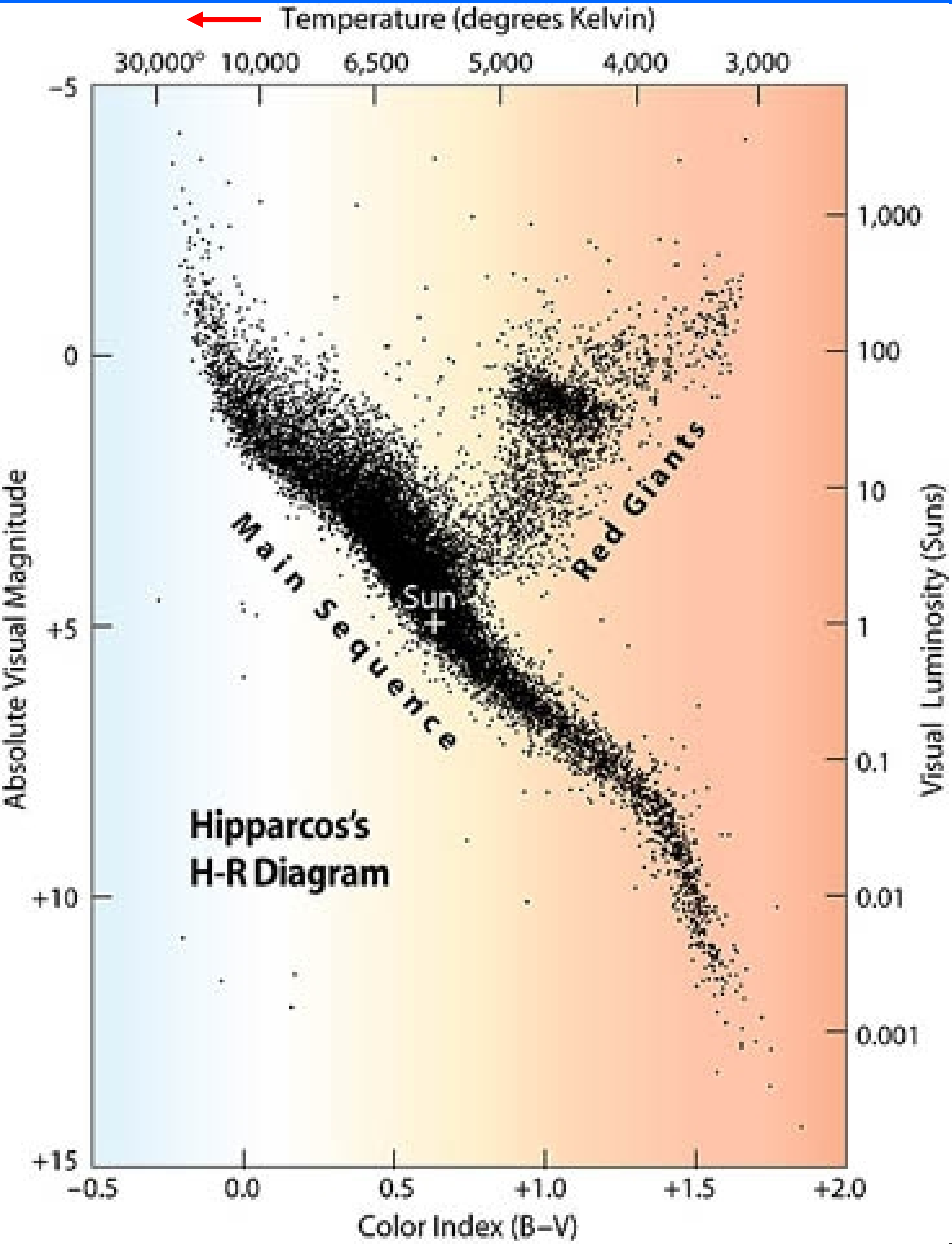
- Notes
  - The star **Vega** is defined to have an apparent magnitude of **zero**! This allows one to talk about the apparent magnitude of a given star rather than just differences in apparent magnitudes

$$m = k - 2.5 \log_{10} F$$

Luminosity and apparent magnitude

- Higher apparent magnitudes, are fainter stars!
- A difference of 5 magnitudes corresponds to a factor of 100 in flux
- Brightest star (Sirius) has m=-1.44
- Faintest stars visible to human eye have m=6.5
- Sun has m=-26.7
- Full Moon has m=-12.6
- Venus at its brightest m=-4.7
- Pluto has m=13.65
- Faintest object visible by Hubble Space Telescope is m=30

Hertzsprung-Russell (HR) diagram of all stars at a range of 300 light years



Ejnar Hertzsprung, Henry Norris Russell

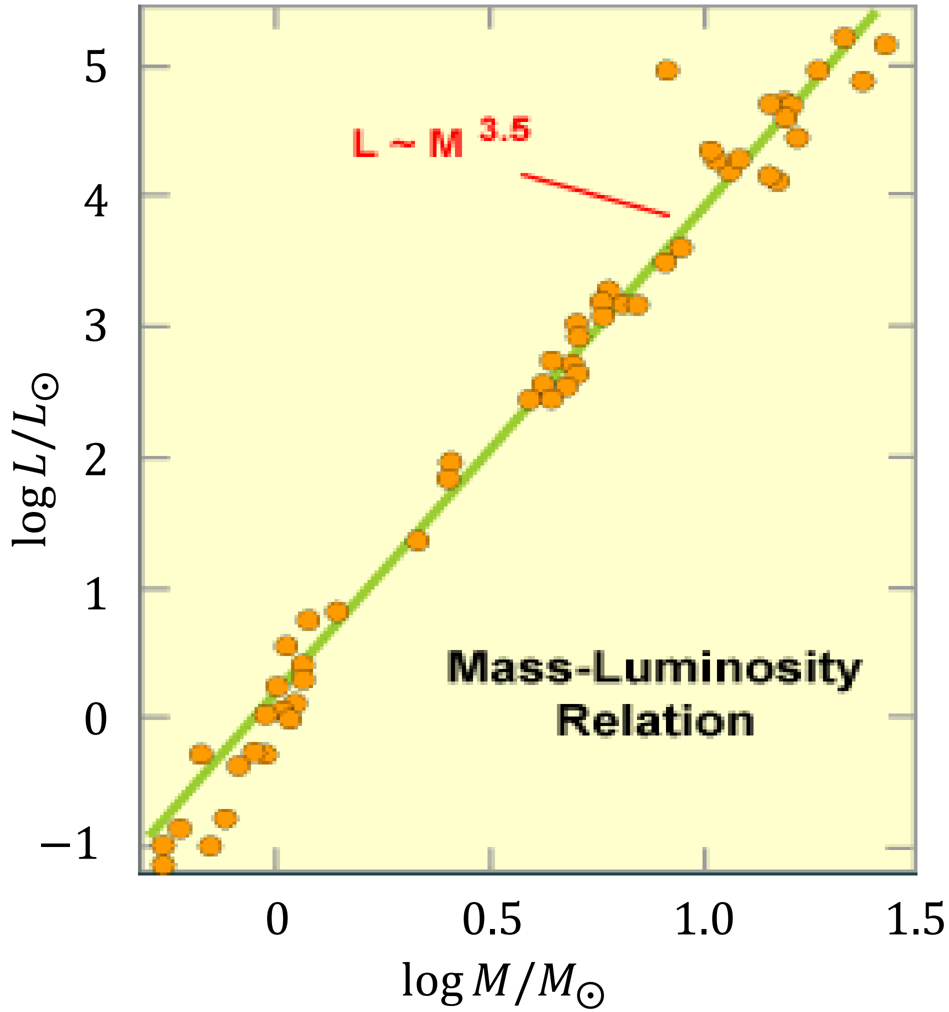
absolute luminosity
versus
temperature

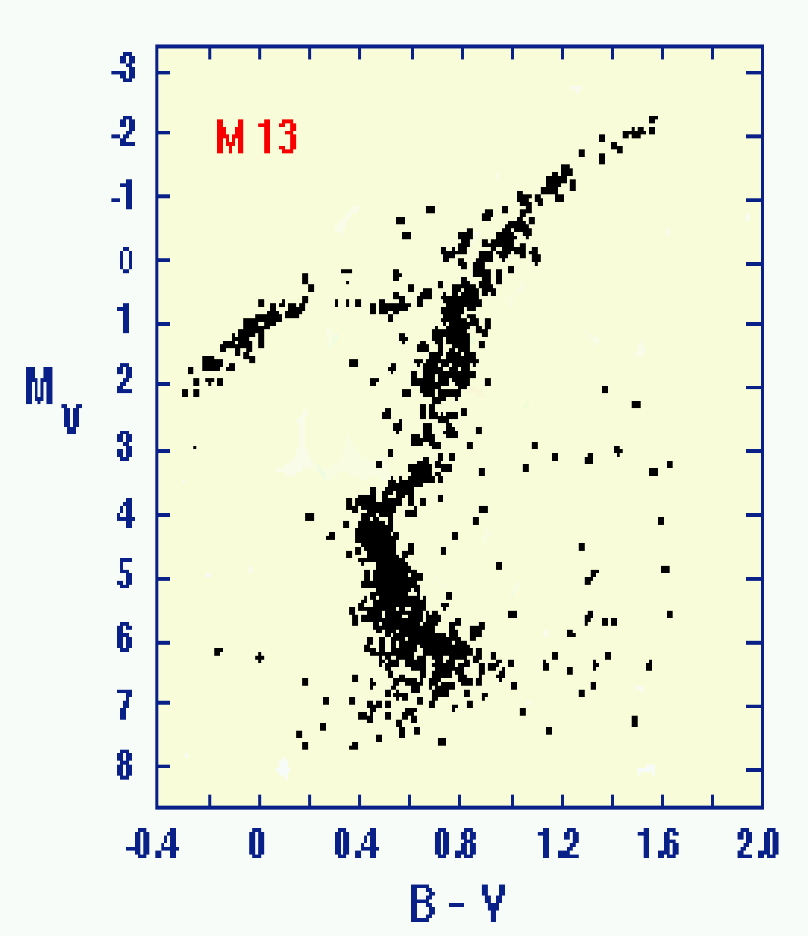
The stars are stationary on the
´main sequence´,
as long as the fusion of protons
to helium persists

This time depends very sensitively
on the mass of the individual star

$L = 4\pi \cdot R_*^2 \cdot \sigma_{SB} \cdot T_*^4$

Age of GC from ´kink´ at main sequence





$T_{MS} = M/L \propto M^{-2.5}$

mass (M_{sun})	*lifetime (years)*
1	~10^{10}
5	~10^{8}
10	~10^{7}

$T_{GC}/T_\odot = [M_\odot/M_{GC}]^{2.5}$

How does mass effect how long a star will live

Lifetime $\propto$ Fuel available / How fast fuel is burned

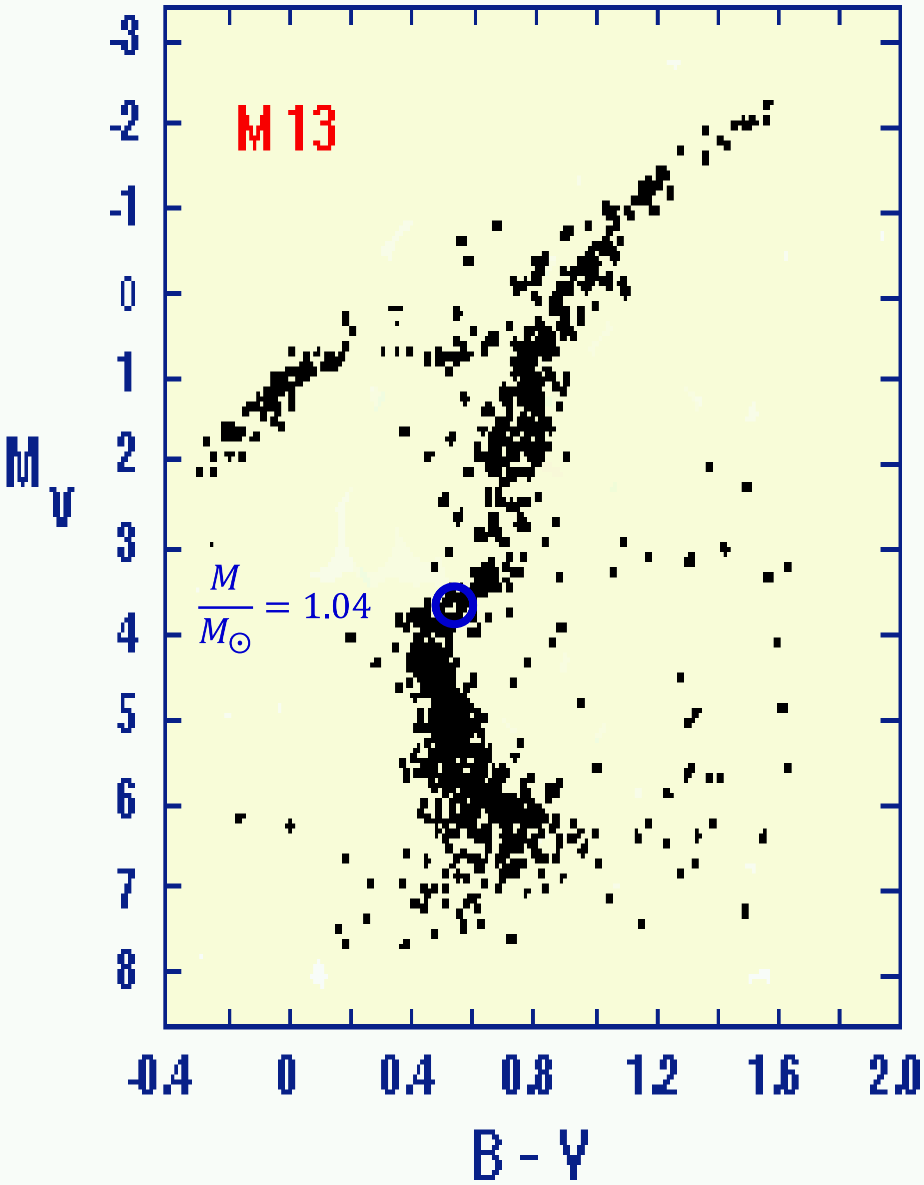
So for a star

Lifetime $\propto$ Mass / Luminosity

Or, since Luminosity $\propto$ Mass^{3,5} For main sequence stars

Lifetime $\propto$ Mass / Mass^{3,5} = 1 / Mass^{2,5}

Big stars live shorter lives, burn their fuel faster …



For our Sun this time
is about 9 billion years (Gyr)
for lighter stars longer,
for heavier ones shorter

$$T_{main\ sequence} = 9\ Ga \cdot [M_\odot / M]^{2.5}$$

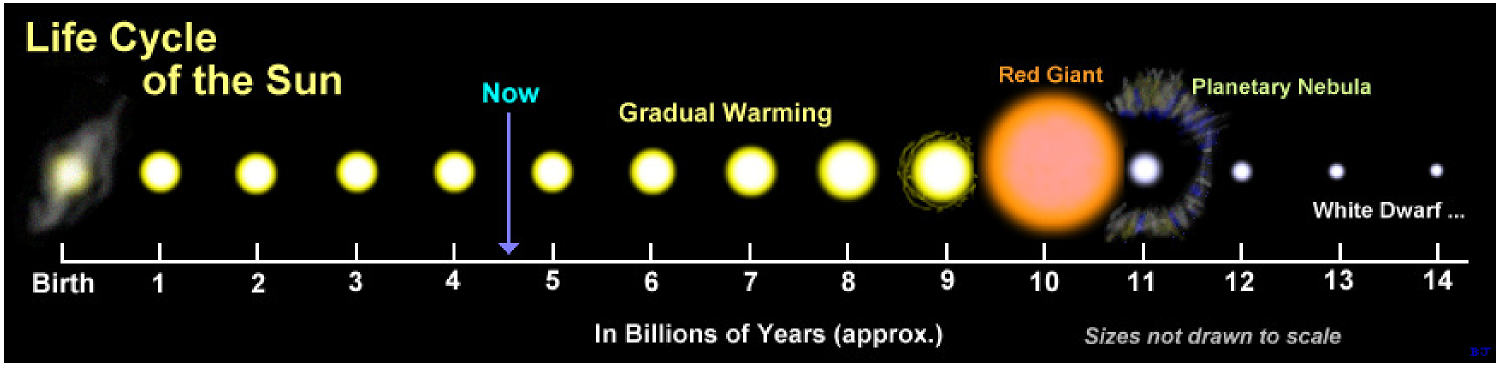
when observing at which mass M the stars of M13 are leaving the main sequence

one can determine the age of M13 - and therewith the minimum age T_G of our galaxy

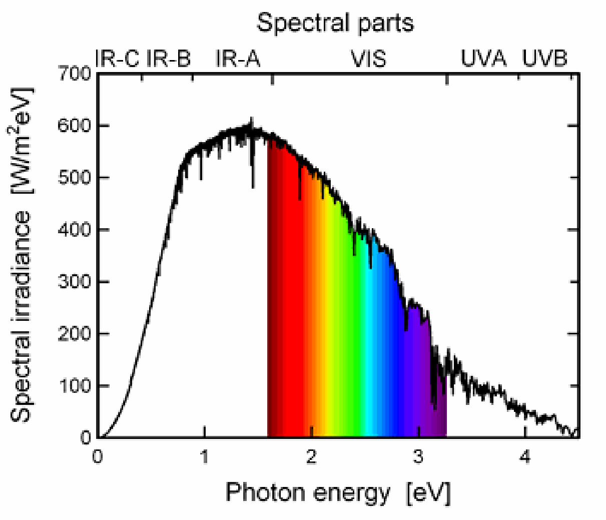
from $M = 1.04 \cdot M_\odot$

$$\rightarrow T_G > 8\ Ga$$

Life cycle of the Sun low-mass stars



The sun can be thought of as simply a source of **blackbody radiation**

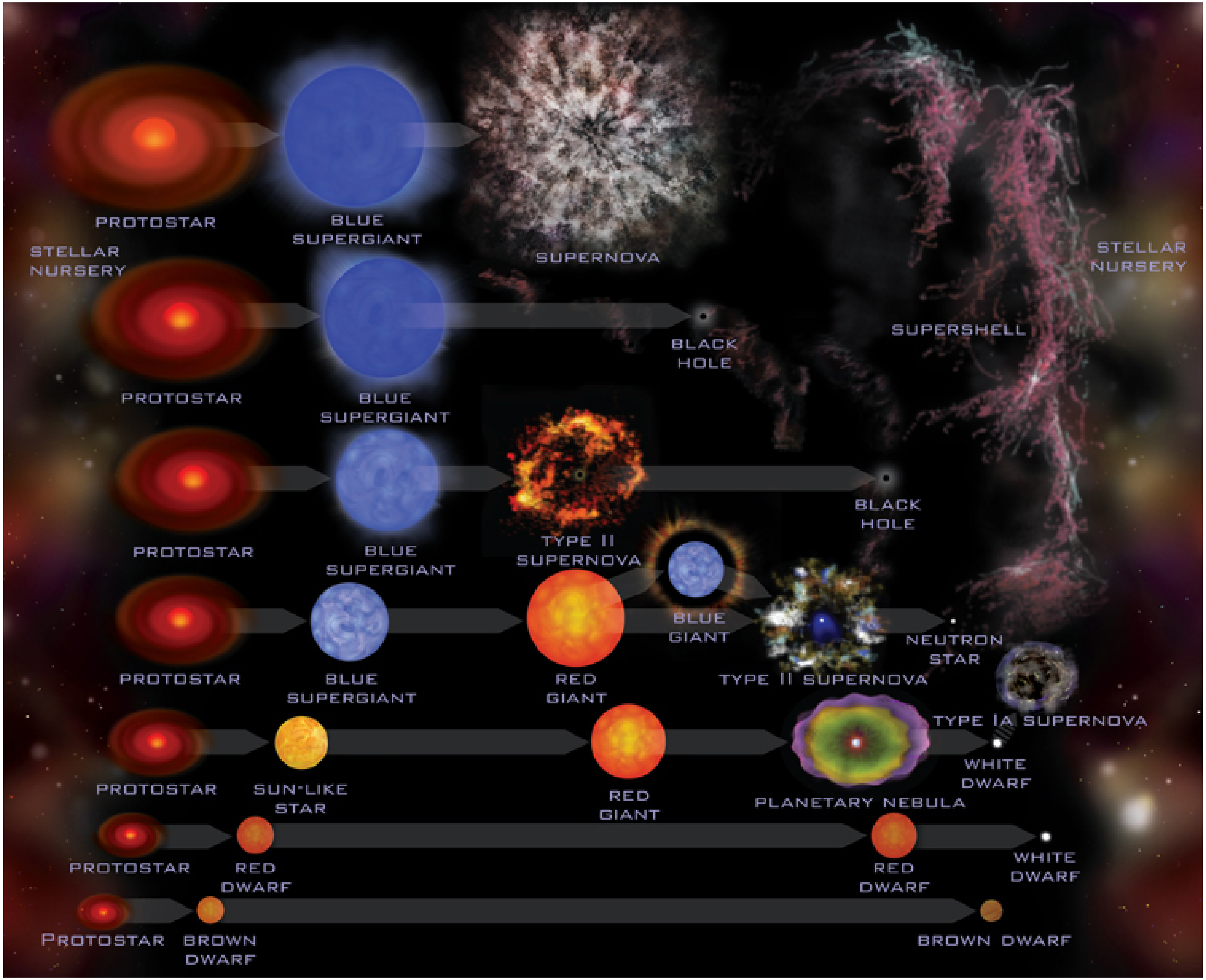


Planck´s law:

$$I_\lambda d\lambda = \frac{2\pi hc^2}{\lambda^5}\frac{1}{(e^{hc/\lambda kT}-1)}d\lambda$$

$$I_E dE = \frac{2\pi\nu^3}{c^2}\frac{1}{(e^{h\nu/kT}-1)}dE$$

Stellar evolution



Solar irradiation

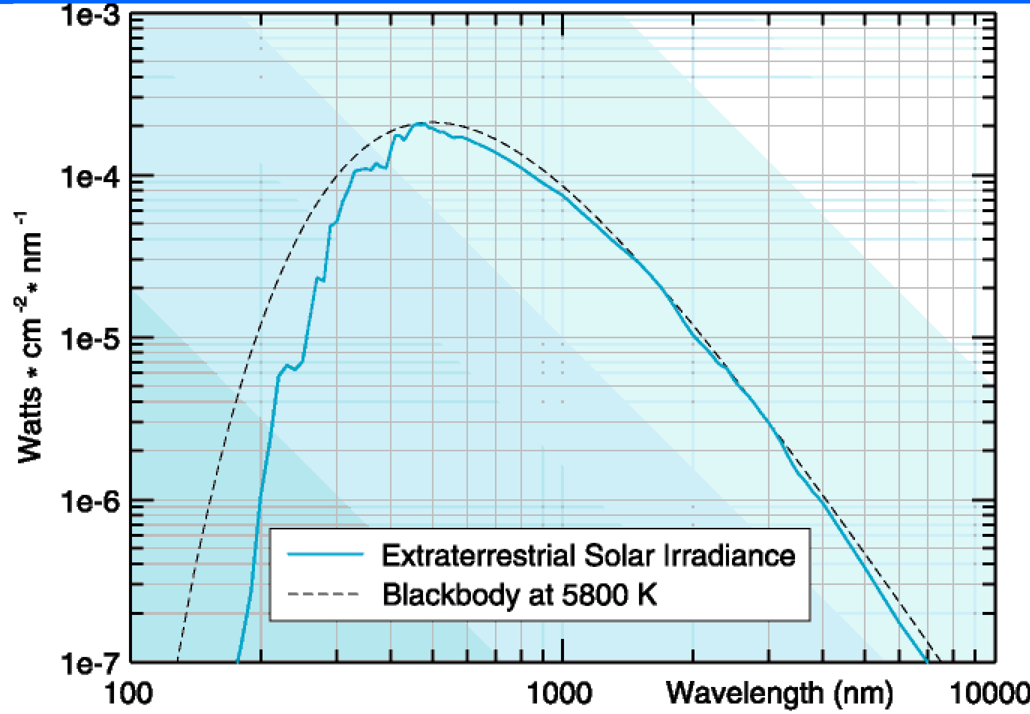
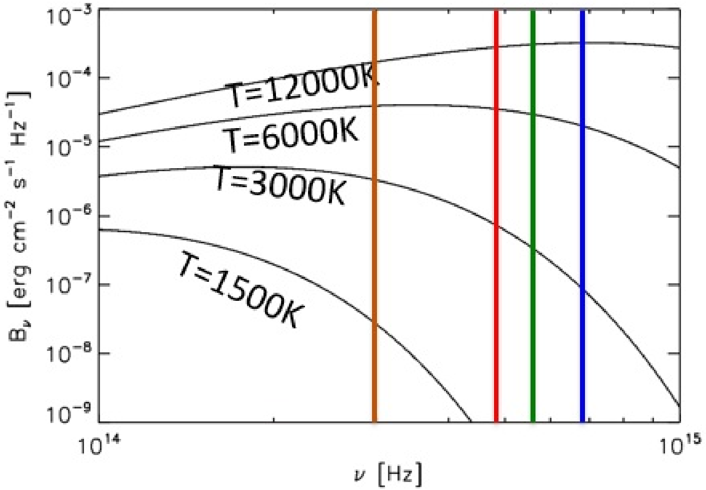


Fig. 5.7 Extraterrestrial solar irradiance compared to a blackbody.

spectrum of a black-body:

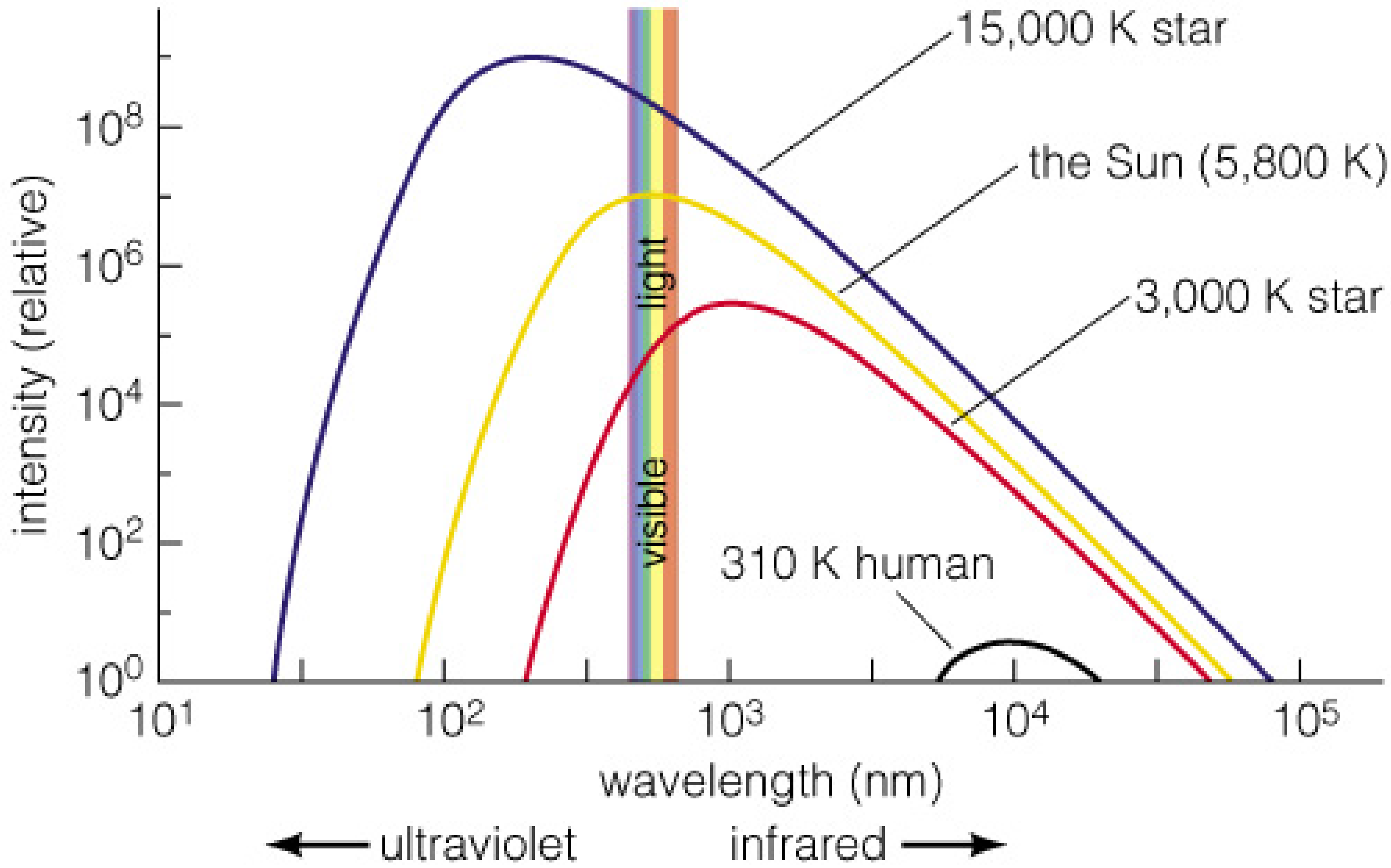
$$F_\nu = \pi \cdot B_\nu$$

$$B_\nu = 2 \cdot \frac{\nu^2}{c^2} \cdot h\nu \cdot \frac{1}{exp(h\nu/k_B T) - 1}$$



Hotter blackbodies are brighter and *bluer“

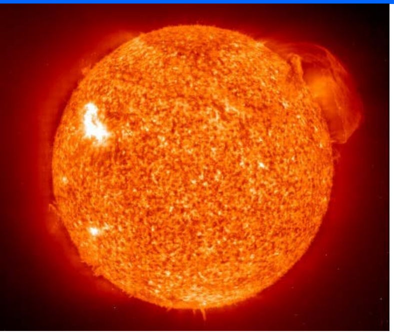
Wien´s displacement law



❖ „Hotter bodies radiate more strongly at shorter wavelengths (i.e. they are bluer)“

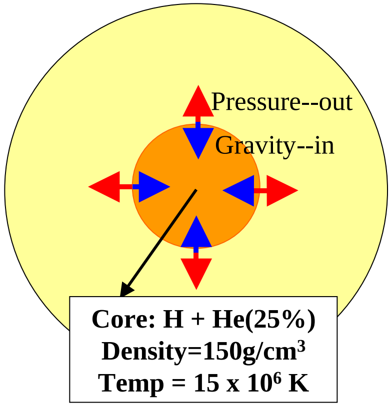
$$\lambda_{max} = \frac{0.29\ cm}{T(K)}$$

Nature of stellar evolution



Stellar structure and evolution controlled by:

1) gravity → collapse
2) internal pressure → expansion



Star composed of many particles (~10^{57} in the sun)

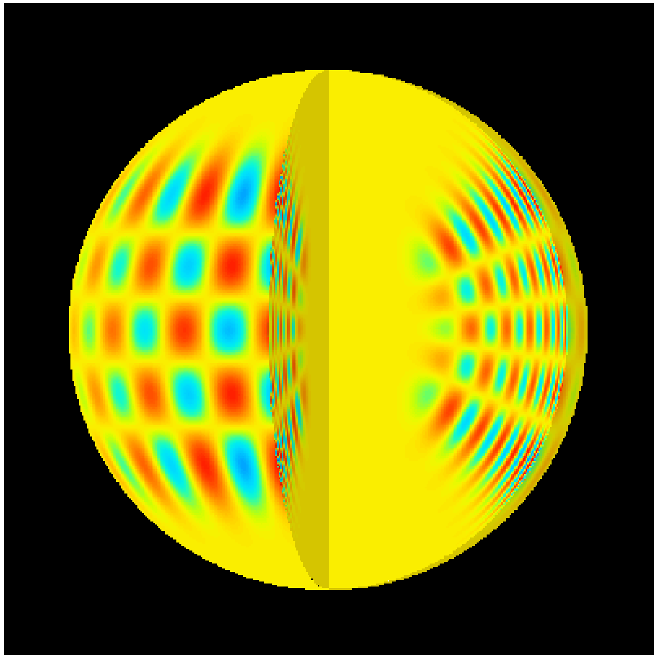
Total energy:

a) mutual gravitational energy of particles (Ω)
b) internal (kinetic) energy of particles (including photons) (U)

For an ideal gas in hydrostatic equilibrium: $2U + \Omega = 0$ (virial theorem)

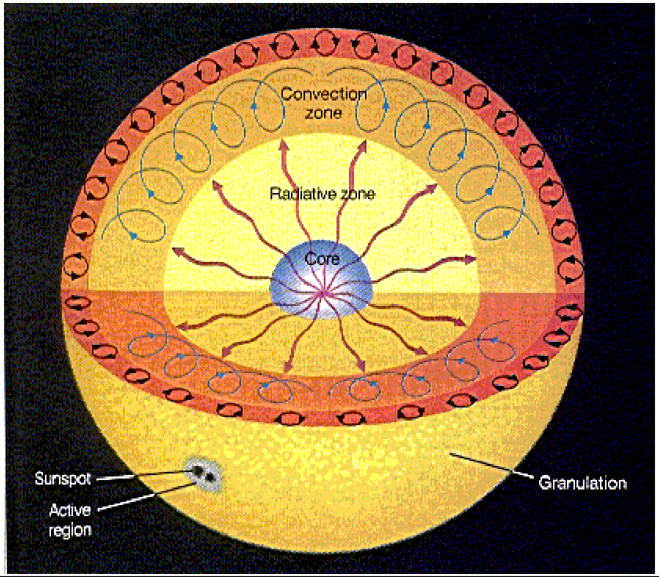
Assume pressure imbalance
→ gravitational contraction sets in
→ amount of energy released $-\Delta\Omega$
→ internal energy change to restore equilibrium $\Delta U = -\frac{1}{2}\Delta\Omega$
→ gas temperature increases
→ energy excess $-\frac{1}{2}\Delta\Omega$ lost from star in form of radiation

Helioseismology



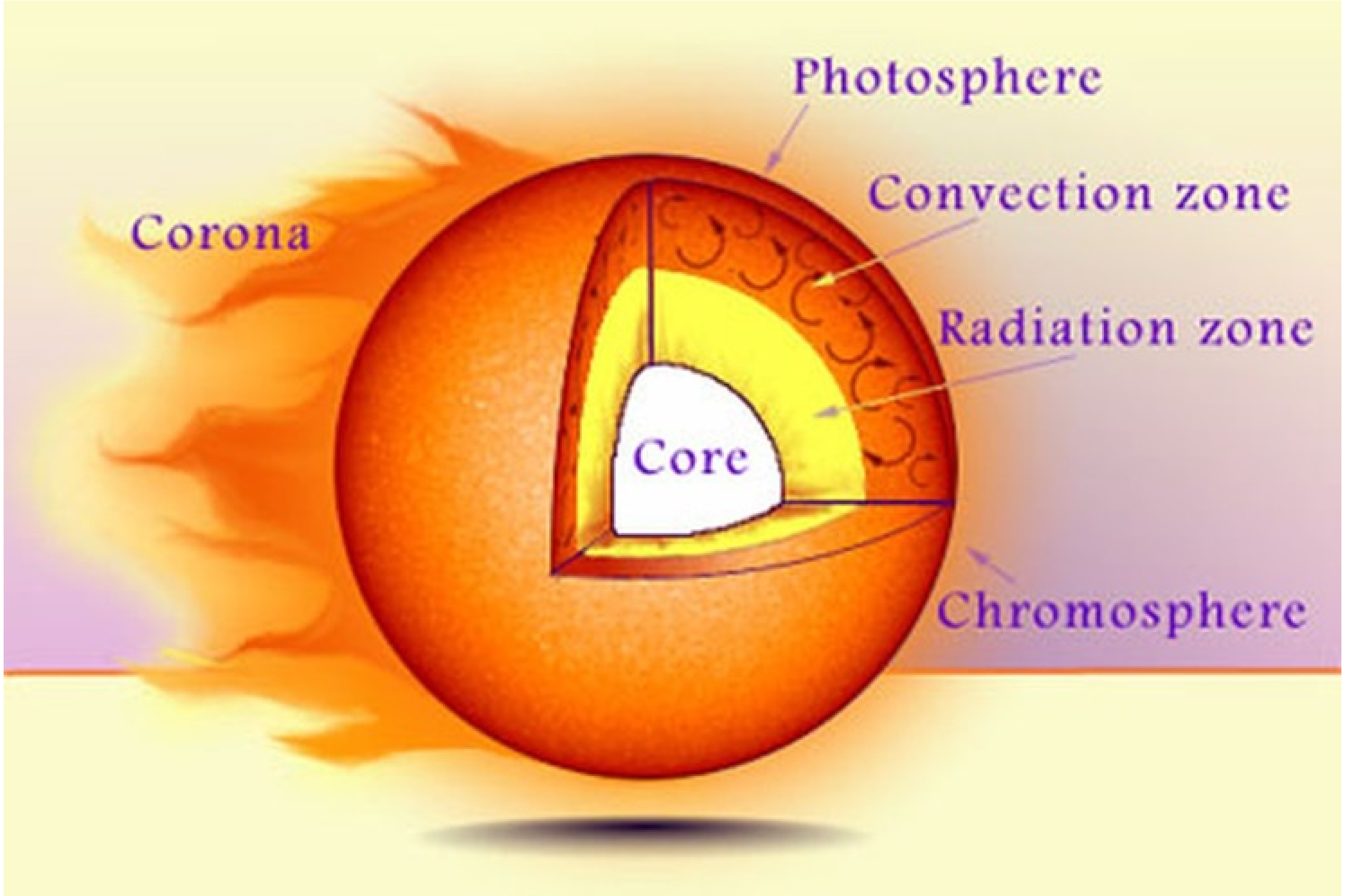
surface oscillations with periods of 1-20 minutes

max. 0.1 m/s

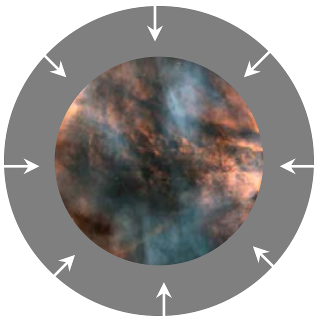


Stellar convective zone decoupled from core by radiative heat transport

The Sun


Photosphere
Corona
Convection zone
Radiation zone
Core
Chromosphere

Principle of stellar structure and evolution



gravitational contraction of gas (mainly H) → increase of central temperature T
T high enough → "nuclear burning" takes place

Hydrogen burning (1st equilibrium)

$$4H \;\rightarrow\; {}^{4}He + 2\beta^{+} + 2\nu + 26\ MeV$$

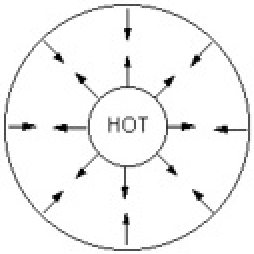
↑ ash ↑ energy source

of nuclear burning

gravitational collapse is halted → star undergoes phase of hydrostatic equilibrium

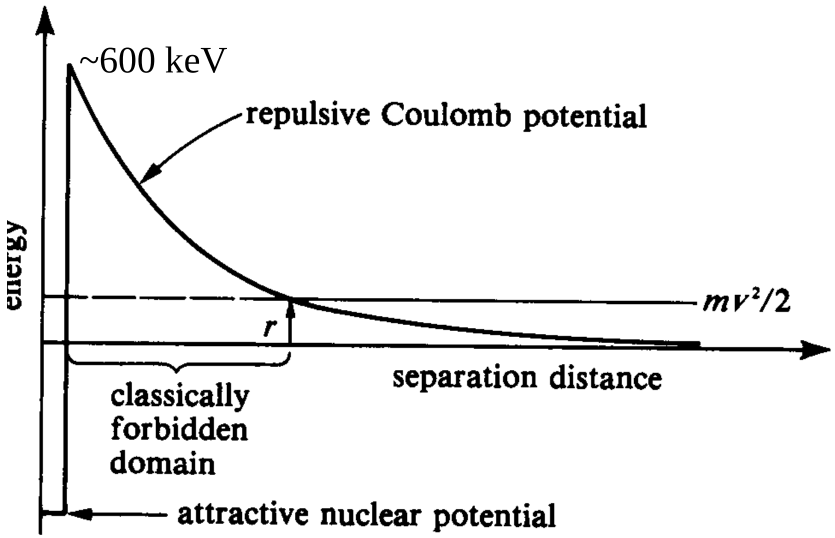
⇓

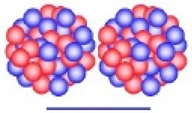
main sequence stars



Here: T ~ 10 -15·10^6 K and ρ ~ 10^2 g/cm^3 are required → M > 0.1 $M_\odot$ (Jupiter ($10^{-3} M_\odot$) = failed star)

Fusion reaction in a gas





R

The difficultly arises from the Coulomb repulsion between positively charged nuclei.

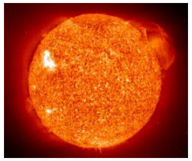
$$U = \frac{q_1 \cdot q_2}{4\pi\varepsilon_0 \cdot r}$$

We can classically calculate the point of closest approach if the initial velocity of approach is *v*

$$r_{close} = \frac{2q_1q_2}{4\pi\varepsilon_o \cdot mv^2}$$

Naively setting $r \sim 10^{-15}\ m$ and $mv^2 = 3kT$ would require $\boldsymbol{T \sim 10^{10}\ K}$ to get the nuclei close enough to fuse.

What is wrong?



$T \sim 15 \cdot 10^6 K$

kT ~ 8.6 x 10^{-8} T[K] keV $\Longrightarrow$ $k \cdot T \sim 1\ keV$

We have forgotten two crucial effects

1. The broad distribution of velocities of the nuclei at a given T

The velocities in the center of mass frame of two particles with reduced $m = m_1 \cdot m_2/(m_1 + m_2)$ will be given by Maxwell distribution:

$$f(v)dv \propto \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 \cdot exp\left(-\frac{mv^2}{2kT}\right) dv$$

Note that, at a given T, the number of particles at high *velocity v drops exponentially with* v^2

2. Quantum tunneling through a potential barrier

The probability of quantum tunneling through a distance r is given in terms of the de Broglie wavelength λ:

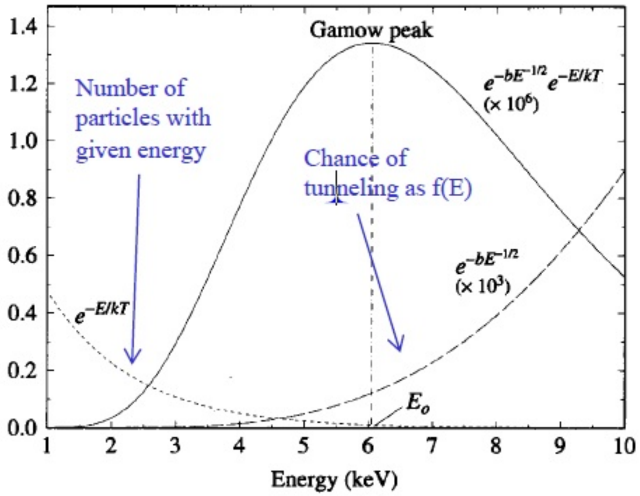
$$P \propto exp\left(-\frac{2\pi^2 r_{close}}{\lambda}\right) \propto exp\left(-\frac{4\pi^2 q_1 q_2}{4\pi\varepsilon_0 hv}\right)$$

Note that the probability of quantum tunneling therefore *increases exponentially with v as* $e^{-1/v}$

Gamow peak

The probability of a fusion reaction happening to a given pair of particles with a certain *v* will therefore be proportional to the product of these two competing terms

$$dN \propto n_1 n_2 \cdot \sigma \cdot v \cdot exp\left(-\frac{mv^2}{2kT} - \frac{\pi q_1 q_2}{\varepsilon_0 h v}\right) dvdt \propto exp\left(-\alpha E - \beta E^{-1/2}\right) dEdt$$



This is a <u>highly peaked function</u> with a maximum $(dN/dE) = 0$ when the two terms are equal (plus a factor of 2 from differentiating)

$$v_{max} = \left(\frac{\pi q_1 q_2 kT}{\varepsilon_0 hm}\right)^{1/3}$$

- most of the reactions will occur with kinetic energies close to the **Gamow peak**
- the overall rate reactions strongly *increases with temperature*

$$E_0 \cong 0.12204(Z_1^2 Z_2^2 \cdot A)^{1/3} T_9^{2/3}\ MeV$$

$$\Delta E \cong 0.23682(Z_1^2 Z_2^2 \cdot A)^{1/6} T_9^{5/6}\ MeV$$

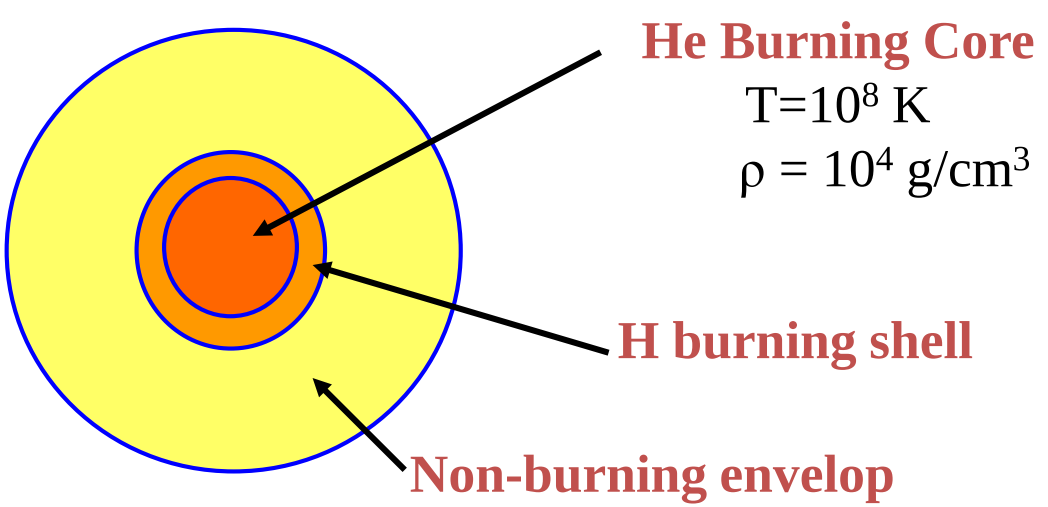
How the sun evolves

Core hydrogen burning ends

- Consumed central 10% of sun
- No heat source, pressure decreases, gravity wins
- Core collapses, releases gravitational energy which heats the core

Core helium burning starts

- Core hot-allows fusion of two a's (Z=2)
- Helium fuses to ^{12}C, ^{16}O
- Hydrogen burns in shell



What's next for the sun?

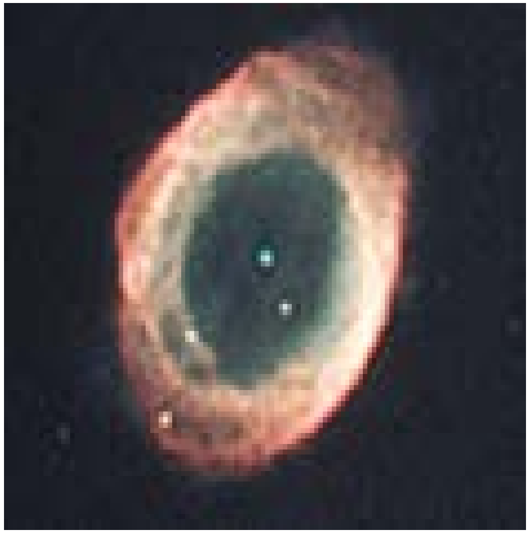
It's the end of the line

- Helium burning ends after 10^8 years, C and O core
- Gravitational collapse, BUT, never reach sufficient T to fuse C + C.
- Collapse continues to 10^7 g/cm^3-- electron pressure stops collapse

- Shells still burning, unstable, blow off planetary nebula

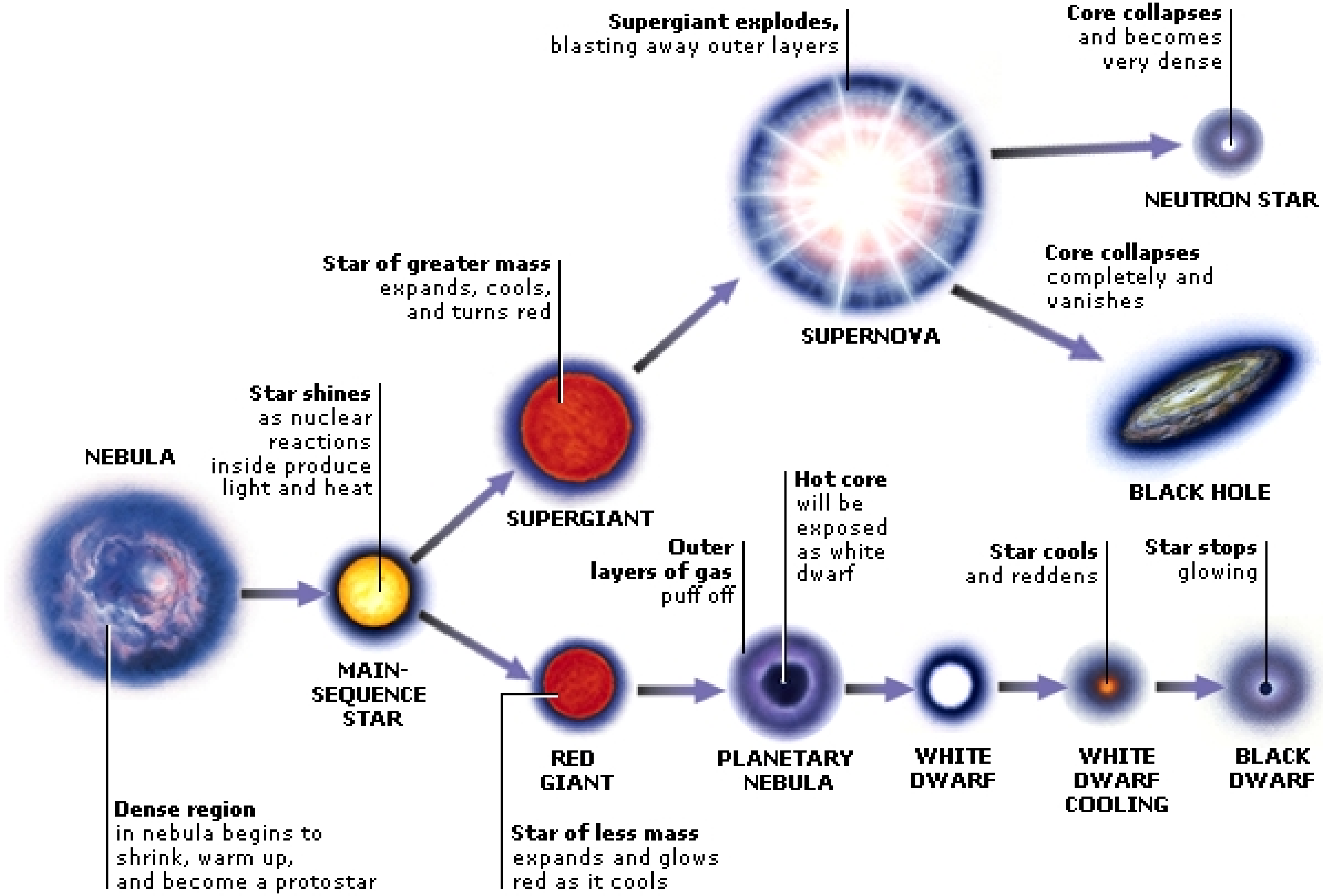
Star becomes a white dwarf (e.g. Sirius B).

Property	Earth	Sirius B	Sun
Mass (M_{sun})	3×10^{-6}	0.94	1.00
Radius(R_{sun})	0.009	0.008	1.00
Luminosity(L_{sun})	0.0	0.0028	1.00
Surface T (K)	287	27,000	5770
Mean ρ (g/cm^3)	5.5	2.8×10^6	1.41
Central T (K)	4200	2.2×10^7	1.6×10^7
Central ρ (g/cm^3)	9.6	3.3×10^7	160

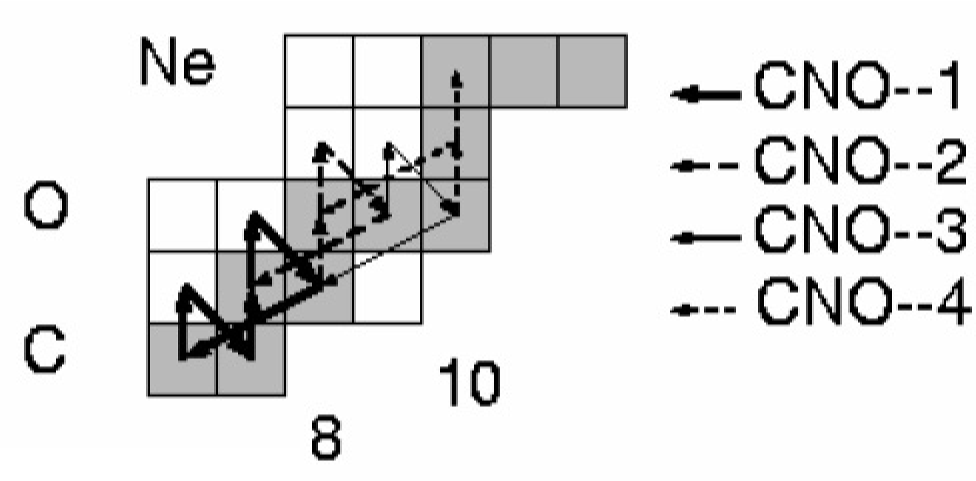


Ring nebula in Lyra-NGC 6720—a planetary nebula

Stellar evolution

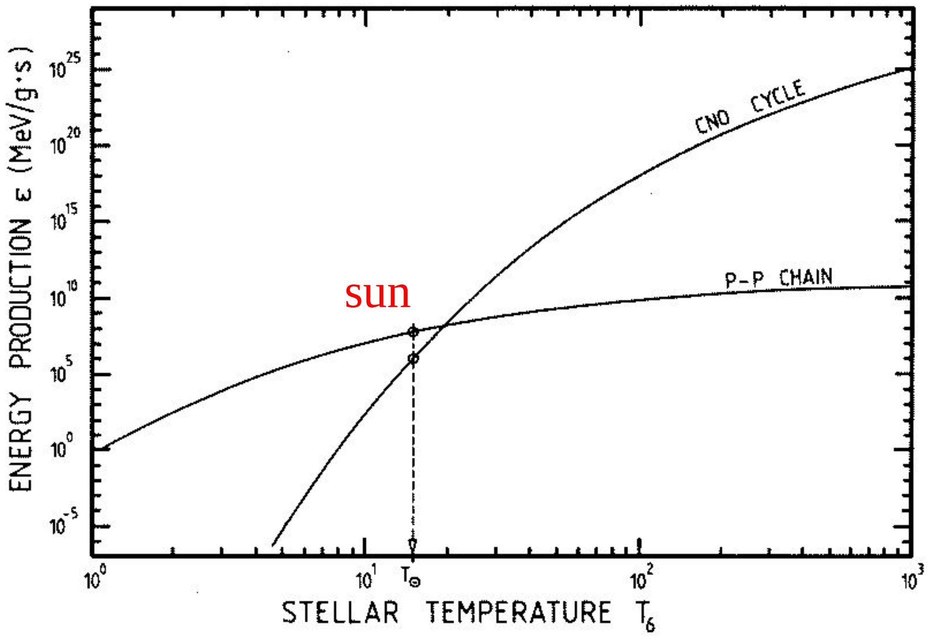


Hydrogen burning in massive stars



Requires existing CNO abundances as catalyzing isotopes for He production through consecutive four proton capture and two beta-decay processes

energy production rate:



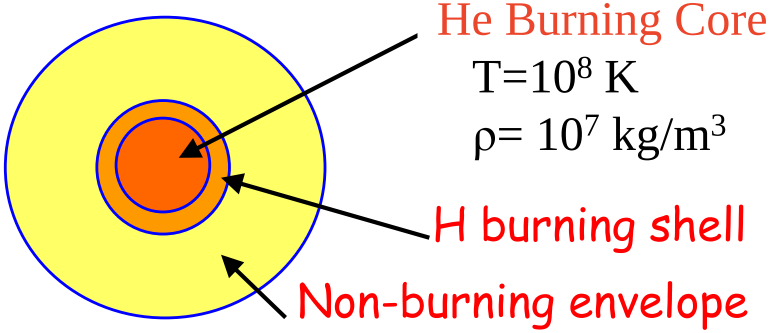
Competition between the pp chain and the CNO cycle

$$M \geq 1.5\ M_\odot \;\Rightarrow\; T_6 > 30$$

CNO burning is necessary for massive star evolution to stabilize stellar core against gravitational contraction!

Heavy-mass stars – the stellar onion

Starts like the sun:

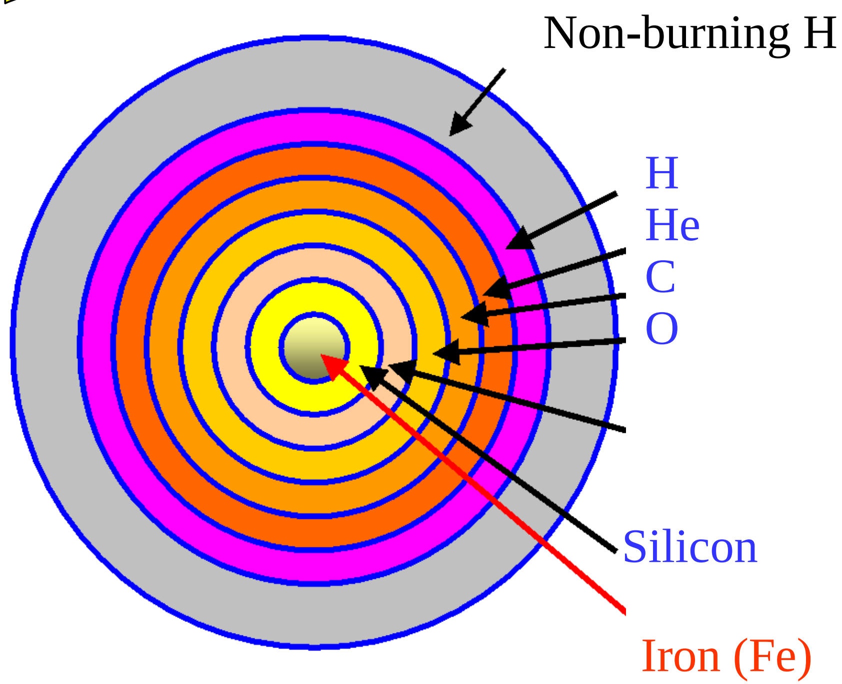


But now, when He is exhausted in the core and the core collapses, it does get hot enough to burn carbon and oxygen.

The successive stages in the core are H → He, gravity, He → C,O, gravity, → C,O → Mg, Si, gravity, Si →Fe.

The Result

The Stellar Onion



Attention!!!

Fusion of more massive nuclei

Fusion of more massive nuclei will require higher temperatures because of larger Z·e nuclear charges produce higher Coulomb barrier

H to He	$1 \cdot 10^7$ K
He to C,O	$1 \cdot 10^8$ K
C to O, Ne, Na, Mg	$5 \cdot 10^8$ K
Ne to O, Mg	$1 \cdot 10^8$ K
O to Mg – S	$2 \cdot 10^8$ K
Si to around Fe	$3 \cdot 10^9$ K

Also, the flattening of the binding energy curve *per nucleon means* that less energy is released per reaction at higher nuclear masses as we approach ^{56}Fe

Evolutionary stages of a 25 M_{sun} star Weaver et al, 80

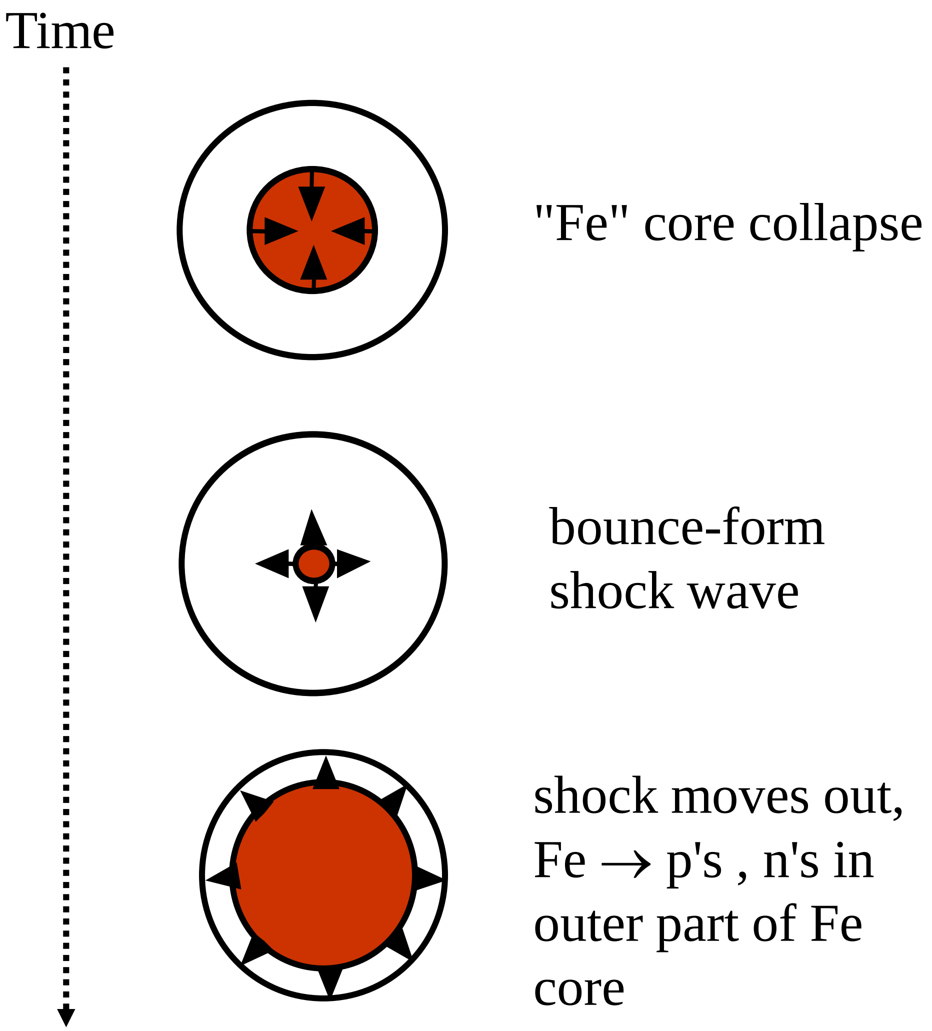
Burning Stage	**Time Scale**	**T(K) x 10^9**	**ρ (g/cm³)**
H	7 x 10^6 y	0.006	5
He	5 x 10^5 y	0.23	700
C	600 y	0.93	2 x 10^5
Ne	1 y	1.7	4 x 10^6
O	0.5 y	2.3	1 x 10^7
Si	1 d	4.1	3 x 10^7
Core collapse	Seconds	8.1	3 x 10^9
Core Bounce	Millisec	34.8	3 x 10^{14}
Explosive	0-1-10 sec	1.2-7.0	

Supernovae core collapse

Fe (Iron) is special core of our stellar onion is “Fe”, most tightly bound nucleus. Result of fusing two “Fe's” is heavier than two “Fe's”; costs energy to fuse them. No more fusion energy is available.

Core collapses, keeps on collapsing, until reach nuclear density. Then nuclei repel, outer core bounces.

Outgoing shock wave forms



What next?

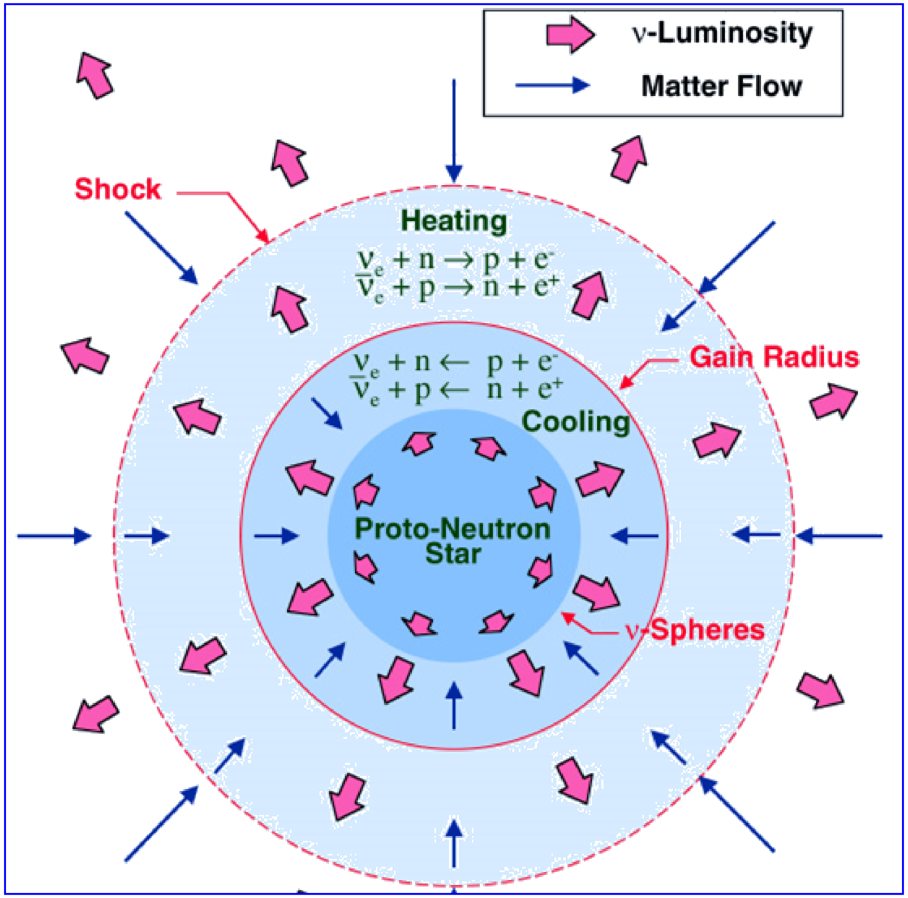
1-D model (T. Mezzacappa)

We know that

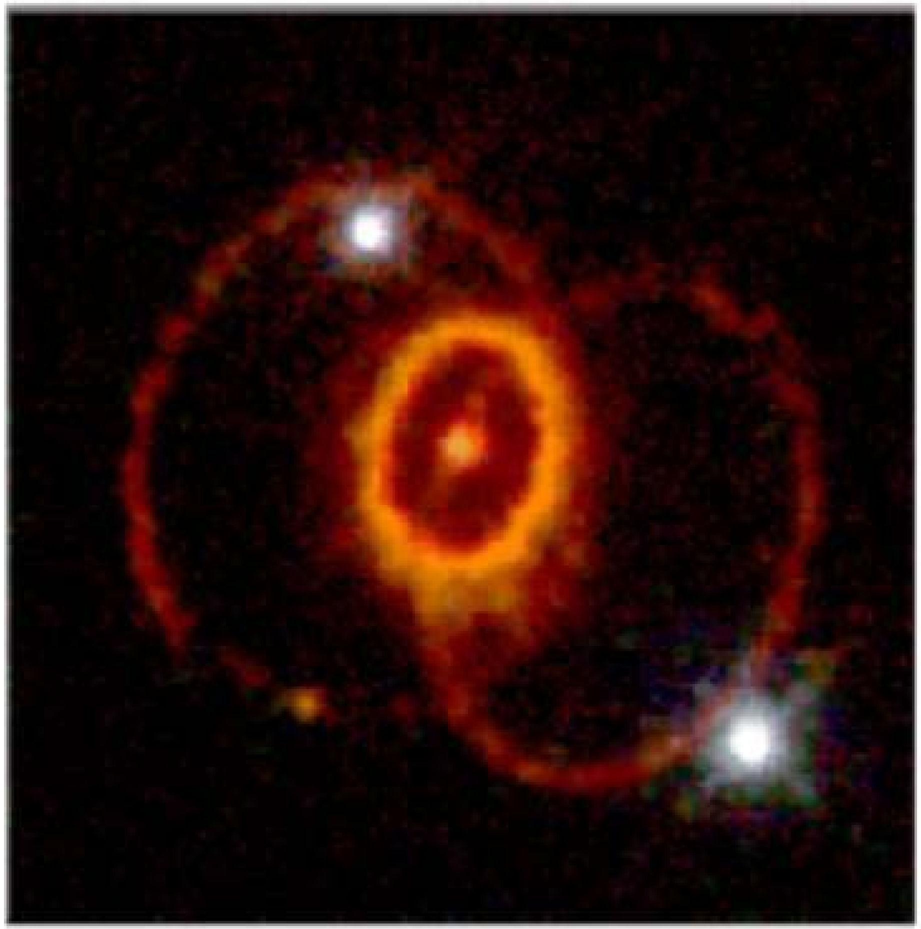
- Shock blows off outer layers of star, a supernova
- 10^{51} ergs (1foe) visible energy released (total gravitational energy of 10^{53} ergs mostly emitted as neutrinos).

Theoretically

- Spherical SN don't explode
- Shock uses its energy dissociating "Fe", stalls
- Later, ν's from proto-neutron star deposit energy, restart the shock. Still no explosion.



The question – how do we get from her to an explosion?



SN 1987a in Large Magallanic Cloud

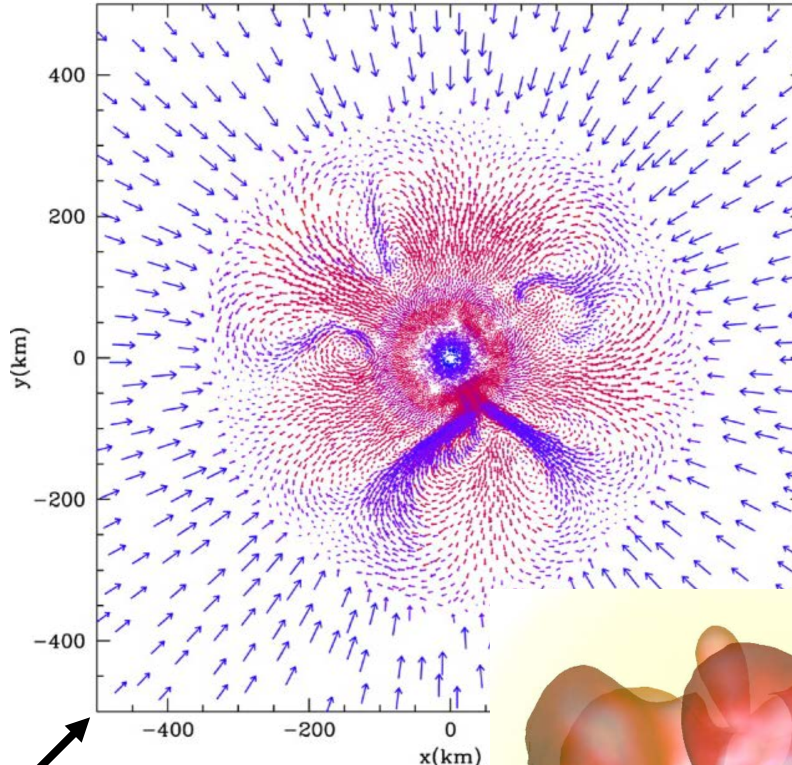
Non-spherical calculations

Two views

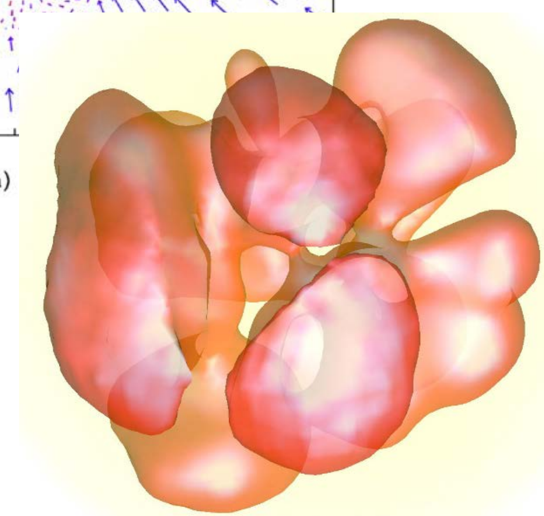
Is sphericity the problem?

- Now have 3-D calculations which explode, but have only a part of the detailed microphysics. Their stability against such changes is not known-we return to this later.

- See, e.g. C. Fryer and M. Warren, Astrophysical Journal, 574:L65-L68
- Find 2-D, 3-D similar



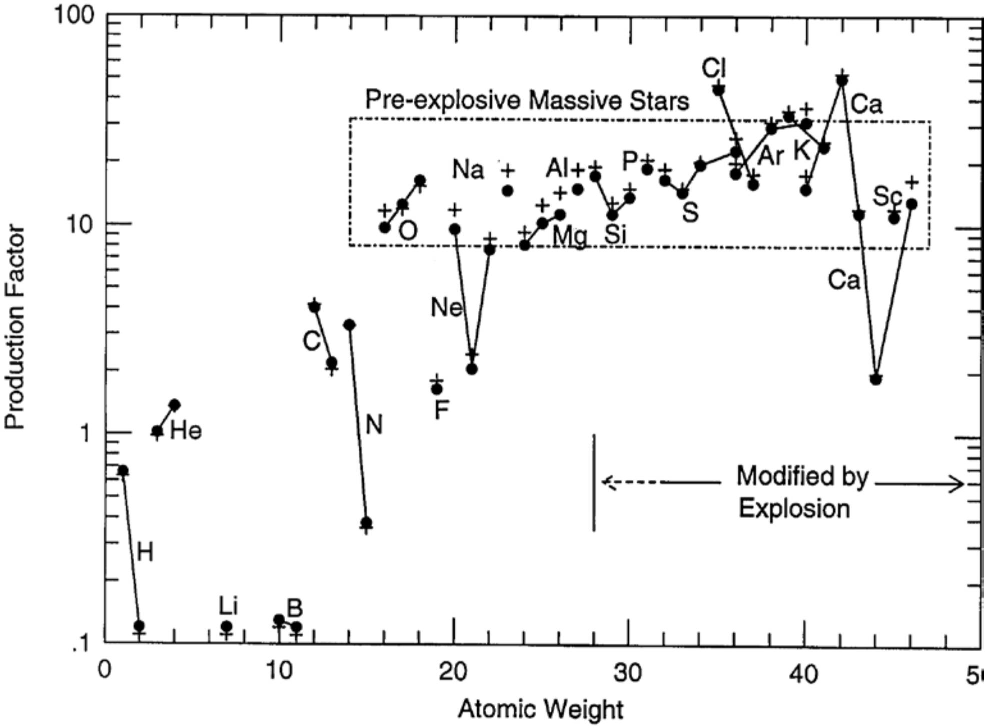
Red upwelling
Blue sinking



What is produced in a supernova?

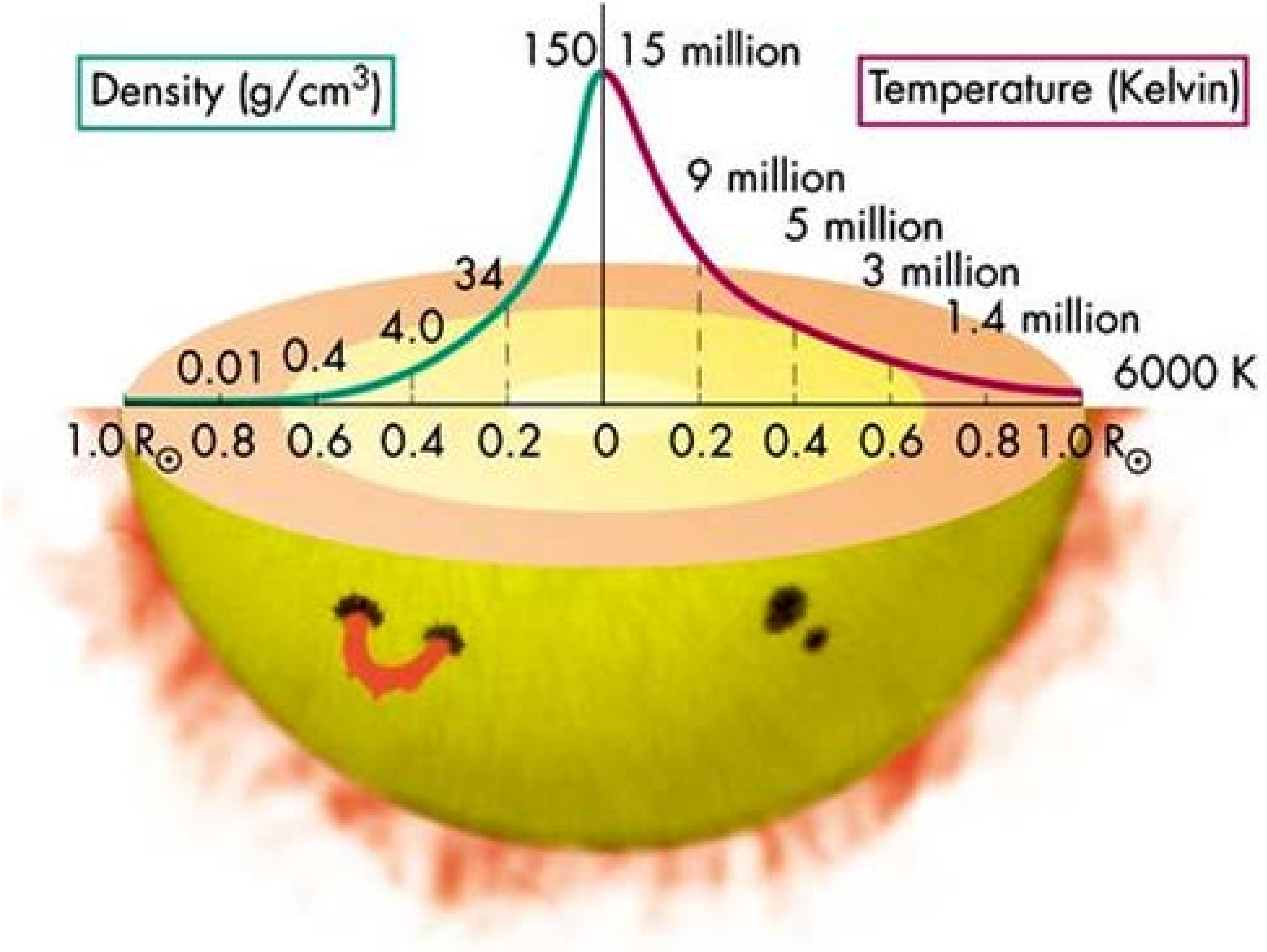
Model

- Evolve the Pre-SN star
- Put in a piston that gives the right energy to the ejecta (Don't know how explosion really works).
- Calculate what is ejected
- Calculate explosive processes as hot shock passes.
- Example: Wallace and Weaver, Phys. Rep. 227,65(93)



Find

- Elements, mass 20-50, generally reproduced at same ratio to solar.
- Modifications by explosive processes are small



Density (g/cm3)
Temperature (Kelvin)
150
15 million
9 million
5 million
3 million
1.4 million
6000 K
34
4.0
0.4
0.01
1.0 R⊙ 0.8 0.6 0.4 0.2 0 0.2 0.4 0.6 0.8 1.0 R⊙

The Sun: a few numbers

- mass = $1.99 \cdot 10^{30}$ kg
- average density = 1.4 g/cm^3
- luminosity = $3.84 \cdot 10^{26}$ W
- effective temperature = 5777 K
- core temperature = $15 \cdot 10^6$ K
- surface gravitational acceleration g = 274 m/s^2
- age = $4.55 \cdot 10^9$ years
- radius = $6.96 \cdot 10^5$ km
- distance = 1 AU = 1.496 $(\pm 0.025) \cdot 10^8$ km
- 1 arc sec =722±12 km on solar surface
- rotation period = 27 days at equator