Collective Transitions (Collective Coordinates)

Nuclei far from the closed shell region exhibit collective properties such that (a) the energy of the lowest 2^+ state in an even-A nucleus is considerably lower than typical particle excitation energy ($2\triangle \sim 1500\ keV$) and that (b) the $B(E2; 2^+ \rightarrow 0^+)$ is greatly enhanced over the single-particle estimate. These features can only be understood if a number of nucleons contribute coherently to the radiation. Here, the concept of independent-particle motion may lose its original meaning. The experimental observations are much better described by the motion of a charged liquid drop [Boh52, Boh53] which may produce small surface oscillations around the spherical equilibrium shape or may rotate if the nucleus has a stable ground state deformation.

Before we can investigate these oscillations and rotations, we have to parameterize the nuclear surface in some way. One possibility is to describe it by the length of the radius vector pointing from the origin to the surface

$$R = R(\theta, \phi) = R_0(1 + \alpha_{00} + \sum_{\lambda=1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha^*_{\lambda\mu} Y_{\lambda\mu}(\theta, \phi)) \tag{1}$$

where R_0 is the radius of the sphere with the same volume. The constant α_{00} describes changes of the nuclear volume. Since we know that the incompressibility of the nuclear fluid is rather high, we require that the volume be kept fixed for all deformations as

$$V = \frac{4}{3}\pi R_0^3 \tag{2}$$

This defines the constant α_{00}. Up to second order, we get

$$\alpha_{00} = -\frac{1}{4\pi} \sum_{\lambda \geq 1}^{\infty} \sum_{\mu=-\lambda}^{\lambda} |\alpha_{\lambda\mu}|^2 \tag{3}$$

In the case of only quadrupole deformations (λ=2), the nucleus looks like a pure ellipsoid. We have five parameters $\alpha_{\lambda\mu}$, but not all of them describe the nuclear shape. Three determine only the orientation of the nucleus in space, and correspond to the three Euler angles. By a suitable rotation, we can transform to the body-fixed system characterized by three axis 1, 2, 3, which coincide with the principal axes of the mass distribution of the nucleus. The five coefficients $\alpha_{2\mu}$ reduce to two real independent variables a_{20} and $a_{22} = a_{2-2}$ ($a_{21} = a_{2-1} = 0$), which, together with the three Euler angles, give a complete description of the system. It is convenient to introduce instead of a_{20} and a_{22} the so-called Hill-Wheeler coordinates $\beta, \gamma (\beta > 0)$ through the relation

$$a_{20} = \beta cos\gamma \tag{4}$$

$$a_{22} = \frac{1}{\sqrt{2}}\beta sin\gamma \tag{5}$$

from which we have

$$\sum_{\mu} |\alpha_{2\mu}|^2 = a_{20}^2 + 2a_{22}^2 = \beta^2 \tag{6}$$

and

$$R(\theta,\phi) = R_0\{1 + \beta\sqrt{\frac{5}{16\pi}}[cos\gamma(3cos^2\theta - 1) + \sqrt{3}sin\gamma sin^2\theta cos2\phi]\} \tag{7}$$

We can calculate the increments of the three semi-axes in the body-fixed frame as functions of β and γ:

$$R_1(\frac{\pi}{2},0) = R_0\{1 + \sqrt{\frac{5}{4\pi}}\beta cos(\gamma - \frac{2\pi}{3})\} \tag{8}$$

$$R_2(\frac{\pi}{2},\frac{\pi}{2}) = R_0\{1 + \sqrt{\frac{5}{4\pi}}\beta cos(\gamma + \frac{2\pi}{3})\} \tag{9}$$

$$R_3(0,0) = R_0\{1 + \sqrt{\frac{5}{4\pi}}\beta cos\gamma\} \tag{10}$$

or

$$R_i = R_0\{1 + \sqrt{\frac{5}{4\pi}}\beta cos(\gamma - \frac{2\pi}{3}i)\} \quad i = 1,2,3 \tag{11}$$

Bibliography

[Boh52] A. Bohr: Kgl. Dan. Vid. Sels. Mat. Fys. Medd. **26** No.14 (1952)

[Boh53] A. Bohr and B.R. Mottelson: Kgl. Dan. Vid. Sels. Mat. Fys. Medd. **27** No.16 (1953)