Rotations for Deformed Shapes

The pure liquid drop model has a stable equilibrium only for spherical surfaces. As a consequence of quantum mechanics - i.e. shell effects - it can happen that the nucleus has a permanent deformation. We shall restrict ourselves to **axial symmetric deformations** of an even-mass nucleus (see fig. 1) which should also be invariant with resept to a rotation of 180^0 about the 2-axis. In this case the nucleus can only rotate around an axis perpendicular to the symmetry axis.

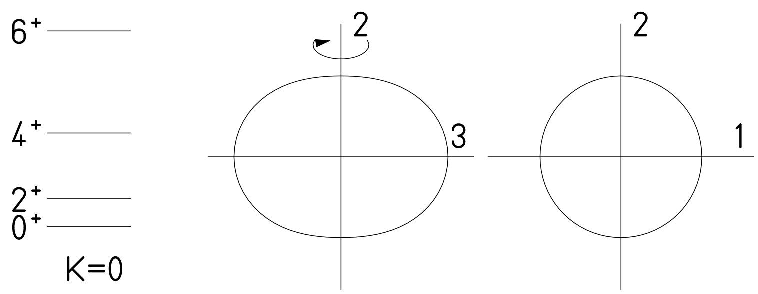

Figure 1: Energy spectrum (left) and shape (right) of an axial symmetric rotator.

Before we will investigate these rotations, we have to parametrize the deformed nuclear surface. One possibility is

$$R = R(\theta, \phi) = R_0(1 + \beta_0 + \beta_2 Y_{20}(\theta, \phi) + \beta_4 Y_{40}(\theta, \phi) + ...) \tag{1}$$

where R_0 is the radius of the sphere with the same volume and $Y_{\lambda 0}(\theta, \phi)$ are the spherical harmonics. The expansion (Eq. 1) contains only even values of λ, with β_2 and β_4 being the quadrupole and hexadecapole deformation parameters, respectively. Since we require that the nuclear volume is kept fixed for all deformations, we get

$$\beta_0 = -\frac{1}{4\pi}(|\beta_2|^2 + |\beta_4|^2 + ...) \tag{2}$$

The rotational spectrum is given by

$$E_I = \frac{\hbar^2}{2\mathcal{J}} I(I+1) + E_{K=0} \tag{3}$$

with the band head $E_{K=0}$ and the moment of inertia $\mathcal{J}$. If we consider the motion of an irrotational liquid that is held in a nonspherical shape, which is rotated uniformly, the moment of inertia is given by

$$\mathcal{J}_{irr} = \frac{9}{8\pi} AMR_0^2(\beta_2^2 + \frac{5}{3}\beta_4^2) \tag{4}$$

It differs from the moment of inertia of a rigid body with the same deformation

$$\mathcal{J}_{rig} = \frac{2}{5} AMR_0^2 (1 + \sqrt{\frac{5}{16\pi}}\beta_2 + \sqrt{\frac{9}{16\pi}}\beta_4) \tag{5}$$

For the reduced transition matrix elements (Eq. ??) to the first excited states we find

$$< 2^+||M(E2)||0^+ >= \sqrt{\frac{5}{16\pi}} Q_{20}\ e \tag{6}$$

and

$$< 4^+||M(E4)||0^+ >= \sqrt{\frac{9}{16\pi}} Q_{40}\ e \tag{7}$$

where the intrinsic quadrupole moment Q_{20} and intrinsic hexadecapole moment Q_{40} is calculated for a uniform charge distribution in the liquid-drop model up to second order

$$Q_{20} = \frac{3ZR_0^2}{\sqrt{5\pi}} (\beta_2 + 0.360\beta_2^2 + 0.967\beta_2\beta_4 + 0.328\beta_4^2) \tag{8}$$

and

$$Q_{40} = \frac{ZR_0^4}{\sqrt{\pi}} (\beta_4 + 0.411\beta_4^2 + 0.983\beta_2\beta_4 + 0.725\beta_2^2) \tag{9}$$

Energy-B(E2) Product In the irrotational flow model the energy-B(E2) product for a pure quadrupole deformation is given by

$$E_{2^+}\ B(E2; 2^+ \to 0^+) = \frac{1}{5} (\frac{3ZeR^2}{4\pi})^2 \beta_2^2\ \frac{\hbar^2}{2\mathcal{J}_{irr}} 6 \tag{10}$$

with

$$\mathcal{J}_{irr} = \frac{9}{8\pi} AMR_0^2 \beta_2^2 \tag{11}$$

Thus, we get for the rotational energy-B(E2) product,

$$E_{2^+}\ B(E2; 2^+ \to 0^+) = 5.75\ 10^{-4} \frac{Z^2}{A^{1/3}}\ MeV\ e^2 barn^2 \tag{12}$$

which is a factor of 2.5 smaller than the vibrational energy-B(E2) product (Eq. ??). However, the experimental product of the energy and B(E2)-value reaches only 17-19% of the rotational limit.

For the other intraband E2-transitions we find

$$< I+2, K=0||M(E2)||I, K=0 >= \sqrt{\frac{3(I+2)(I+1)}{2(2I+3)}} \sqrt{\frac{5}{16\pi}} Q_{20}\ e \tag{13}$$

$$< I, K = 0||M(E2)||I, K = 0 >= -\sqrt{\frac{I(I+1)(2I+1)}{(2I-1)(2I+3)}}\sqrt{\frac{5}{16\pi}}Q_{20}\ e \tag{14}$$

$$< I-2, K = 0||M(E2)||I, K = 0 >= \sqrt{\frac{3I(I-1)}{2(2I-1)}}\sqrt{\frac{5}{16\pi}}Q_{20}\ e \tag{15}$$

For the static M1 and E2 moments we obtain

$$\mu(I) = g_R \mu_N I \tag{16}$$

$$Q(I) = -\frac{I(I+1)}{(I+1)(2I+3)}Q_{20} \tag{17}$$

The quantity g_R is the effective g-factor for the collective motion; it is again expected to be of the order of $g_R \sim Z/A$ ($\mu_N = \frac{e\hbar}{2Mc}$).