# Asymmetric Rotator Model

Rotational levels in even nuclei were investigated by Davydov [Dav58, Dav59a, Dav59b, Dav60a] under the assumption that in first approximation the equilibrium shape of the nucleus can be represented by a triaxial ellipsoid (see fig. 1).

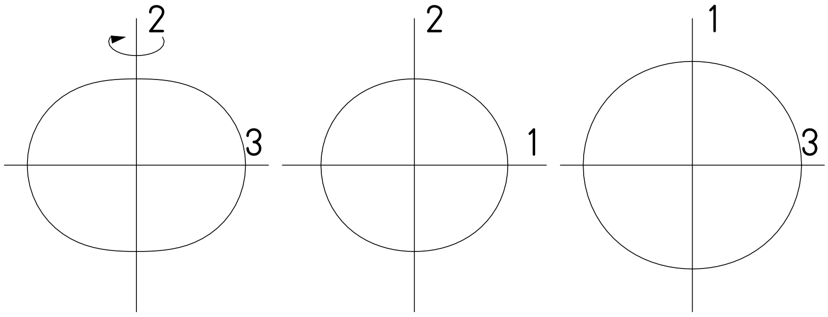


Figure 1: Shape of a triaxial ellipsoid with $\beta = 0.3$, $\gamma = 30^0$ and $R_0 = 6.9\ fm$.

The Hamiltonian of the axially asymmetric nuclei is written as

$$H = \sum_{i=1}^{3} \frac{R_i^2}{2\Theta_i} \tag{1}$$

where we assume that the energy states of the even mass system correspond to the rotation of the nucleus as a whole without changing the internal state. Here $R_i$ (i=1,2,3) are the components of the angular momentum operator of the rotor in the body fixed system and $\Theta_i$ expressed as

$$\Theta_i = 4B\beta^2 \sin^2(\gamma - \frac{2\pi}{3}i) \qquad i = 1,2,3 \tag{2}$$

are the corresponding moments of inertia obtained by considering the nucleus as irrotational. The parameter $\gamma$ varies between 0 and $\pi/3$ and determines the deviation of the shape of the nucleus from axial symmetry. If $\gamma = 0^0$ the nucleus is an elongated ellipsoid (American football) which is symmetric around the 3-axis. If $\gamma = 60^0$ the nucleus is an oblate ellipsoid (pancake) which is symmetric around the 2-axis.

The wave function corresponding to the state with total angular momentum I, and fulfilling the conditions of symmetry found by Bohr, can be represented in the form

$$\Psi_{IM} = \sum_{K \geq 0} A_K |IK> \tag{3}$$

where

$$|IK> = \sqrt{\frac{2I+1}{16\pi^2(1+\delta_{K0})}} [D^I_{MK} + (-1)^I D^I_{M-K}] \tag{4}$$

The functions $D^I_{MK}$ are functions of the Euler angles that determine the orientation of the principal axes of the nucleus in space.

# 1 Energy States in Non-Axial Nuclei

If the energy is expressed in units of $A = \hbar^2/4B\beta^2$ the two energy levels for spin I=2 are defined by the expression [Dav58]

$$\epsilon_1(2) = \frac{9(1 - \sqrt{1 - \frac{8}{9}\sin^2(3\gamma)})}{\sin^2(3\gamma)} \qquad \epsilon_2(2) = \frac{9(1 + \sqrt{1 - \frac{8}{9}\sin^2(3\gamma)})}{\sin^2(3\gamma)} \tag{5}$$

The parameter $\gamma$ is determined in an unique way in the interval $0^0 \leq \gamma \leq 30^0$ from the ratio of the energies of two levels with spin I=2 by means of the formula

$$\frac{E_2(2)}{E_1(2)} = \frac{3 + \sqrt{9 - 8\sin^2(3\gamma)}}{3 - \sqrt{9 - 8\sin^2(3\gamma)}} \tag{6}$$

For different $\gamma$-values the corresponding energy ratios are given in table 1.

| $\gamma$ | $0^0$ | $5^0$ | $10^0$ | $15^0$ | $20^0$ | $25^0$ | $30^0$ |
|---|---|---|---|---|---|---|---|
| $\frac{\epsilon_2(2)}{\epsilon_1(2)}$ | $\infty$ | 64.2 | 15.9 | 6.85 | 3.73 | 2.41 | 2.00 |

Table 1: Dependence of the energy ratio $\frac{\epsilon_2(2)}{\epsilon_1(2)}$ of even nucleus on parameter $\gamma$.

Having thus determined the value of $\gamma$, we can, with the help of fig. 2, find the position of the remaining rotational states with different values of the spin.

It can be seen that as a result of deviation of the nuclear shape from axial symmetry the interval rule 1 : 3.3 : 7 : 12 : 18.3 which is observed for $\gamma = 0^0$ is violated in the ground state rotational band. Thus, for example, for $\gamma = 30^0$ the levels of the ground state rotational band should satisfy the interval rule 1 : 2.67 : 5 : 8 : 11.67.

The energy of a level with spin I=3 is given by

$$\epsilon(3) = \frac{18}{\sin^2(3\gamma)} \tag{7}$$

One finds the following simple relation between the spin I=2 and spin I=3 energy levels

$$\epsilon_1(2) + \epsilon_2(2) = \epsilon(3) \tag{8}$$

The three spin I=4 energy levels are the roots of the third degree equation

$$\epsilon^3 - \frac{90}{\sin^2(3\gamma)}\epsilon^2 + \frac{48}{\sin^4(3\gamma)}[27 + 26\sin^2(3\gamma)]\epsilon - \frac{640}{\sin^4(3\gamma)}[27 + 7\sin^2(3\gamma)] = 0 \tag{9}$$

The two spin I=5 energy levels are given by the formula

$$\epsilon_{1,2}(5) = \frac{45 \pm 9\sqrt{9 - 8\sin^2(3\gamma)}}{\sin^2(3\gamma)} \tag{10}$$

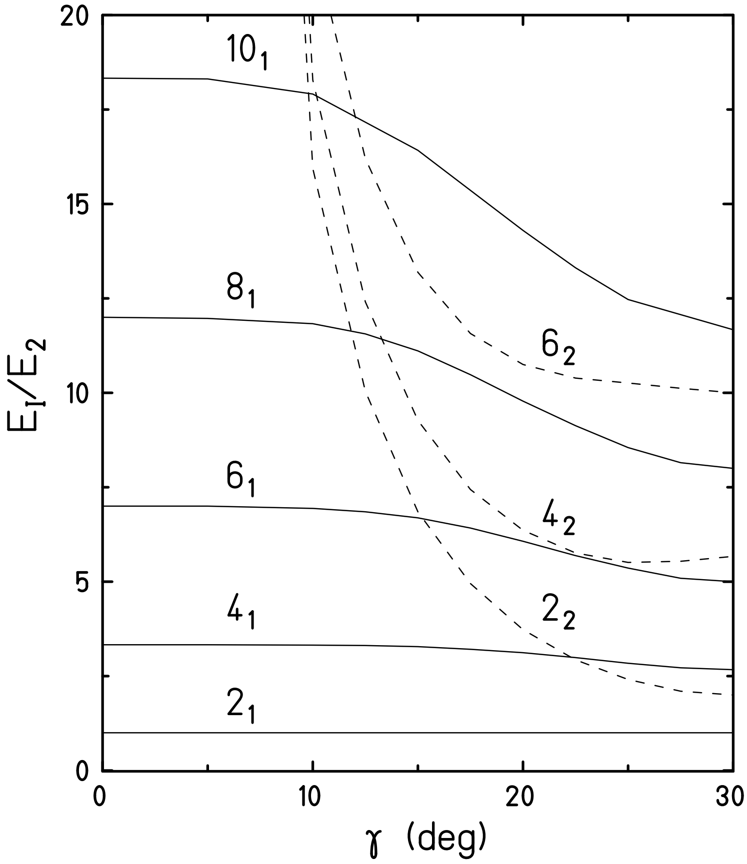


Figure 2: Dependence of rotational energy (normalised to I=2 level) of even nucleus on parameter $\gamma$.

## 2 Rotation-Vibration Interaction in Non-Axial Even Nuclei

In the previous section it was assumed that rotation of the nucleus takes place without a change of its intrinsic state. In the general case Coriolis interaction between the angular momentum and single-nucleon states or centrifugal interaction can lead to a change of the shape of the nucleus during rotation and hence to a change of the moments of inertia of the nucleus.

The energy of the collective nuclear excitations including the rotation-vibration interaction is given by the formula [Dav60b, Dav66, Dav68]

$$E_{K\nu}(I) = \hbar\omega_0\{(\nu+\frac{1}{2})\sqrt{1+\frac{3}{2}(\frac{\mu}{p})^4\epsilon_K(I)}+\frac{1}{4}(\frac{\mu}{p})^2\epsilon_K(I)+\frac{(p-1)^2}{2\mu^2}\} \tag{11}$$

where the quantum number $\nu$ (which in general is not an integer) is the root of the transcendental equation

$$H_\nu(\zeta) = [2\Gamma(-\nu)]^{-1}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\Gamma(\frac{k-\nu}{2})(2\zeta)^k = 0 \tag{12}$$

with $\zeta = -p/\mu_1$ and $\Gamma(x)$ is the gamma function. The parameters p and $\mu_1$ can be expressed with the aid of the following formulas

$$(p-1)p^3 = \frac{1}{2}\epsilon_K(I)\mu^4 \tag{13}$$

$$\mu_1^4 = \mu^4[1 + \frac{3\epsilon_K(I)\mu^4}{2p^4}]^{-1} \tag{14}$$

in terms of $\mu$ and $\epsilon_K(I)$ which depends only on $\gamma$. For each value of the angular momentum I several values of $\epsilon_K(I)$ may exist (e.g. for I=2, K=1,2 [see eq. 5]; for I=3, K=1; for I=4, K=1,2,3 etc). One finds that for certain values of K and I the energy of collective excited states will be expressed only through two parameters, the "non-axiality" parameter $\gamma$ and the "non-adiabaticity" parameter $\mu$. Some energy levels normalized to the first excited state are given in the following tables as a function of $\gamma$ and $\mu$.

| $\gamma = 10.0^0$ | $\mu$=0.0 | $\mu$=0.1 | $\mu$=0.2 | $\mu$=0.3 | $\mu$=0.4 | $\mu$=0.6 | $\mu$=0.8 | $\mu$=1.0 |
|---|---|---|---|---|---|---|---|---|
| $E_4/E_2$ | 3.32 | 3.32 | 3.29 | 3.17 | 2.95 | 2.59 | 2.41 | 2.32 |
| $E_6/E_2$ | 6.94 | 6.93 | 6.79 | 6.26 | 5.48 | 4.42 | 3.97 | 3.75 |
| $E_8/E_2$ | 11.82 | 11.79 | 11.36 | 10.02 | 8.36 | 6.38 | 5.60 | 5.23 |
| $E_{2\gamma}/E_2$ | 15.94 | 15.88 | 15.12 | 12.95 | 10.52 | 7.80 | 6.76 | 6.27 |
| $E_{4\gamma}/E_2$ | 18.28 | 18.21 | 17.22 | 14.55 | 11.67 | 8.54 | 7.37 | 6.82 |
| $E_{6\gamma}/E_2$ | 22.00 | 21.90 | 20.49 | 16.97 | 13.39 | 9.64 | 8.26 | 7.61 |

| $\gamma = 12.5^0$ | $\mu$=0.0 | $\mu$=0.1 | $\mu$=0.2 | $\mu$=0.3 | $\mu$=0.4 | $\mu$=0.6 | $\mu$=0.8 | $\mu$=1.0 |
|---|---|---|---|---|---|---|---|---|
| $E_4/E_2$ | 3.31 | 3.30 | 3.28 | 3.15 | 2.93 | 2.57 | 2.40 | 2.31 |
| $E_6/E_2$ | 6.85 | 6.85 | 6.70 | 6.17 | 5.40 | 4.36 | 3.92 | 3.71 |
| $E_8/E_2$ | 11.56 | 11.53 | 11.10 | 9.79 | 8.17 | 6.25 | 5.50 | 5.14 |
| $E_{2\gamma}/E_2$ | 10.04 | 10.02 | 9.70 | 8.66 | 7.32 | 5.68 | 5.03 | 4.71 |
| $E_{4\gamma}/E_2$ | 12.41 | 12.38 | 11.89 | 10.42 | 8.64 | 6.56 | 5.75 | 5.37 |
| $E_{6\gamma}/E_2$ | 16.20 | 16.14 | 15.33 | 13.08 | 10.59 | 7.84 | 6.80 | 6.31 |

| $\gamma = 15.0^0$ | $\mu$=0.0 | $\mu$=0.1 | $\mu$=0.2 | $\mu$=0.3 | $\mu$=0.4 | $\mu$=0.6 | $\mu$=0.8 | $\mu$=1.0 |
|---|---|---|---|---|---|---|---|---|
| $E_4/E_2$ | 3.27 | 3.27 | 3.24 | 3.12 | 2.90 | 2.54 | 2.37 | 2.29 |
| $E_6/E_2$ | 6.69 | 6.68 | 6.53 | 6.01 | 5.26 | 4.27 | 3.84 | 3.64 |
| $E_8/E_2$ | 11.10 | 11.07 | 10.67 | 9.41 | 7.87 | 6.05 | 5.33 | 4.99 |
| $E_{2\gamma}/E_2$ | 6.85 | 6.84 | 6.69 | 6.15 | 5.37 | 4.34 | 3.91 | 3.70 |
| $E_{4\gamma}/E_2$ | 9.27 | 9.25 | 8.97 | 8.04 | 6.83 | 5.35 | 4.75 | 4.47 |
| $E_{6\gamma}/E_2$ | 13.19 | 13.15 | 12.58 | 10.92 | 8.99 | 6.79 | 5.95 | 5.55 |

| $\gamma = 17.5^0$ | $\mu$=0.0 | $\mu$=0.1 | $\mu$=0.2 | $\mu$=0.3 | $\mu$=0.4 | $\mu$=0.6 | $\mu$=0.8 | $\mu$=1.0 |
|---|---|---|---|---|---|---|---|---|
| $E_4/E_2$ | 3.21 | 3.21 | 3.18 | 3.06 | 2.84 | 2.50 | 2.34 | 2.25 |
| $E_6/E_2$ | 6.42 | 6.41 | 6.27 | 5.78 | 5.06 | 4.12 | 3.72 | 3.53 |
| $E_8/E_2$ | 10.48 | 10.45 | 10.07 | 8.91 | 7.47 | 5.77 | 5.11 | 4.79 |
| $E_{2\gamma}/E_2$ | 4.95 | 4.94 | 4.86 | 4.56 | 4.09 | 3.42 | 3.13 | 2.99 |
| $E_{4\gamma}/E_2$ | 7.44 | 7.43 | 7.24 | 6.59 | 5.70 | 4.57 | 4.10 | 3.88 |
| $E_{6\gamma}/E_2$ | 11.57 | 11.54 | 11.08 | 9.71 | 8.07 | 6.17 | 5.44 | 5.09 |

| $\gamma = 20.0^0$ | $\mu$=0.0 | $\mu$=0.1 | $\mu$=0.2 | $\mu$=0.3 | $\mu$=0.4 | $\mu$=0.6 | $\mu$=0.8 | $\mu$=1.0 |
|---|---|---|---|---|---|---|---|---|
| $E_4/E_2$ | 3.12 | 3.12 | 3.09 | 2.97 | 2.76 | 2.43 | 2.28 | 2.20 |
| $E_6/E_2$ | 6.07 | 6.06 | 5.93 | 5.47 | 4.81 | 3.93 | 3.57 | 3.39 |
| $E_8/E_2$ | 9.79 | 9.76 | 9.42 | 8.35 | 7.03 | 5.47 | 4.86 | 4.57 |
| $E_{2\gamma}/E_2$ | 3.73 | 3.73 | 3.68 | 3.51 | 3.22 | 2.78 | 2.58 | 2.48 |
| $E_{4\gamma}/E_2$ | 6.36 | 6.35 | 6.21 | 5.71 | 4.99 | 4.07 | 3.68 | 3.50 |
| $E_{6\gamma}/E_2$ | 10.75 | 10.72 | 10.31 | 9.05 | 7.56 | 5.83 | 5.16 | 4.86 |

| $\gamma = 22.5^0$ | $\mu$=0.0 | $\mu$=0.1 | $\mu$=0.2 | $\mu$=0.3 | $\mu$=0.4 | $\mu$=0.6 | $\mu$=0.8 | $\mu$=1.0 |
|---|---|---|---|---|---|---|---|---|
| $E_4/E_2$ | 2.98 | 2.98 | 2.96 | 2.84 | 2.65 | 2.35 | 2.20 | 2.13 |
| $E_6/E_2$ | 5.69 | 5.68 | 5.56 | 5.14 | 4.53 | 3.73 | 3.40 | 3.24 |
| $E_8/E_2$ | 9.12 | 9.10 | 8.79 | 7.81 | 6.60 | 5.17 | 4.62 | 4.35 |
| $E_{2\gamma}/E_2$ | 2.93 | 2.93 | 2.90 | 2.80 | 2.61 | 2.31 | 2.18 | 2.11 |
| $E_{4\gamma}/E_2$ | 5.76 | 5.76 | 5.63 | 5.20 | 4.58 | 3.77 | 3.43 | 3.27 |
| $E_{6\gamma}/E_2$ | 10.39 | 10.36 | 9.96 | 8.74 | 7.30 | 5.65 | 5.01 | 4.71 |

| $\gamma = 25.0^0$ | $\mu$=0.0 | $\mu$=0.1 | $\mu$=0.2 | $\mu$=0.3 | $\mu$=0.4 | $\mu$=0.6 | $\mu$=0.8 | $\mu$=1.0 |
|---|---|---|---|---|---|---|---|---|
| $E_4/E_2$ | 2.84 | 2.83 | 2.81 | 2.71 | 2.53 | 2.25 | 2.12 | 2.06 |
| $E_6/E_2$ | 5.34 | 5.34 | 5.23 | 4.84 | 4.28 | 3.55 | 3.25 | 3.10 |
| $E_8/E_2$ | 8.55 | 8.53 | 8.24 | 7.34 | 6.23 | 4.92 | 4.40 | 4.16 |
| $E_{2\gamma}/E_2$ | 2.41 | 2.41 | 2.39 | 2.32 | 2.19 | 1.99 | 1.89 | 1.84 |
| $E_{4\gamma}/E_2$ | 5.51 | 5.51 | 5.39 | 4.98 | 4.39 | 3.63 | 3.32 | 3.16 |
| $E_{6\gamma}/E_2$ | 10.26 | 10.23 | 9.81 | 8.59 | 7.17 | 5.56 | 4.94 | 4.65 |

| $\gamma = 27.5^0$ | $\mu$=0.0 | $\mu$=0.1 | $\mu$=0.2 | $\mu$=0.3 | $\mu$=0.4 | $\mu$=0.6 | $\mu$=0.8 | $\mu$=1.0 |
|---|---|---|---|---|---|---|---|---|
| $E_4/E_2$ | 2.71 | 2.71 | 2.69 | 2.59 | 2.43 | 2.17 | 2.05 | 1.99 |
| $E_6/E_2$ | 5.09 | 5.09 | 4.98 | 4.62 | 4.10 | 3.42 | 3.14 | 3.00 |
| $E_8/E_2$ | 8.15 | 8.13 | 7.86 | 7.00 | 5.96 | 4.73 | 4.25 | 4.02 |
| $E_{2\gamma}/E_2$ | 2.10 | 2.10 | 2.09 | 2.04 | 1.95 | 1.79 | 1.72 | 1.68 |
| $E_{4\gamma}/E_2$ | 5.54 | 5.53 | 5.41 | 4.98 | 4.39 | 3.62 | 3.31 | 3.16 |
| $E_{6\gamma}/E_2$ | 10.12 | 10.09 | 9.67 | 8.44 | 7.05 | 5.47 | 4.87 | 4.59 |

| $\gamma = 30.0^0$ | $\mu$=0.0 | $\mu$=0.1 | $\mu$=0.2 | $\mu$=0.3 | $\mu$=0.4 | $\mu$=0.6 | $\mu$=0.8 | $\mu$=1.0 |
|---|---|---|---|---|---|---|---|---|
| $E_4/E_2$ | 2.67 | 2.67 | 2.64 | 2.55 | 2.39 | 2.14 | 2.03 | 1.97 |
| $E_6/E_2$ | 5.00 | 4.99 | 4.89 | 4.54 | 4.03 | 3.37 | 3.09 | 2.96 |
| $E_8/E_2$ | 8.00 | 7.98 | 7.72 | 6.89 | 5.86 | 4.66 | 4.19 | 3.97 |
| $E_{2\gamma}/E_2$ | 2.00 | 2.00 | 1.99 | 1.94 | 1.86 | 1.72 | 1.66 | 1.62 |
| $E_{4\gamma}/E_2$ | 5.67 | 5.66 | 5.53 | 5.03 | 4.46 | 3.68 | 3.36 | 3.20 |
| $E_{6\gamma}/E_2$ | 10.00 | 9.97 | 9.56 | 8.35 | 6.97 | 5.42 | 4.83 | 4.55 |

The value of $E_{2\gamma}/E_2$ for $\gamma = 30^0$ was a minimum, equal to 2 in the adiabatic approximation theory ($\mu$=0). If the coupling between the rotation and vibration of the nuclear surface are taken into account, $E_{2\gamma}/E_2$ may assume values below 2. For example for $\mu$=1 the minimum value of $E_{2\gamma}/E_2$ is 1.6.

It should also be mentioned that a second band of excited states with a spin sequence 0,2,4,... ($\beta$-band) is predicted which correspond to the second root $\nu = \nu_2$ of the transcendental equation.

# 3 Electric Quadrupole Transitions between Rotational States

Since we assume that the shape of the nucleus does not change, the reduced electric quadrupole transition probability between two rotational levels can be expressed through the mean value of $\beta$ and $\gamma$. For transitions between rotational levels of spin I=2 and I=0, we find the following values expressed in $e^2Q_0^2/16\pi$ units [Dav58]

$$b(E2; 2_1 \rightarrow 0_1) = \frac{B(E2; 2_1 \rightarrow 0_1)}{\frac{e^2Q_0^2}{16\pi}} = \frac{1}{2}\left[1 + \frac{3 - 2\sin^2(3\gamma)}{\sqrt{9 - 8\sin^2(3\gamma)}}\right] \tag{15}$$

where

$$Q_0 = \frac{3ZR^2\beta}{\sqrt{5\pi}} \tag{16}$$

is the intrinsic electric quadrupole moment of an axial nucleus with a deformation parameter $\beta$.

$$b(E2; 2_2 \rightarrow 0_1) = \frac{1}{2}\left[1 - \frac{3 - 2\sin^2(3\gamma)}{\sqrt{9 - 8\sin^2(3\gamma)}}\right] \tag{17}$$

$$b(E2; 2_2 \rightarrow 2_1) = \frac{10}{7}\frac{\sin^2(3\gamma)}{9 - 8\sin^2(3\gamma)} \tag{18}$$

It is interesting to note that

$$b(E2; 2_1 \rightarrow 0_1) + b(E2; 2_2 \rightarrow 0_1) = 1 \tag{19}$$

The relative reduced transition probabilities and the ratio of the reduced electric quadrupole transition probabilities $b(E2; 2_2 \rightarrow 2_1)/b(E2; 2_2 \rightarrow 0_1)$ are listed in table 2 for several values of $\gamma$.

| $\gamma$ | $b(E2; 2_1 \rightarrow 0_1)$ | $b(E2; 2_2 \rightarrow 0_1)$ | $b(E2; 2_2 \rightarrow 2_1)$ | $\frac{b(E2;2_2\rightarrow 2_1)}{b(E2;2_2\rightarrow 0_1)}$ |
|---|---|---|---|---|
| $0^0$ | 1.000 | 0. | 0. | 1.43 |
| $5^0$ | 0.993 | 0.0074 | 0.011 | 1.49 |
| $10^0$ | 0.972 | 0.028 | 0.051 | 1.70 |
| $15^0$ | 0.947 | 0.053 | 0.143 | 2.70 |
| $20^0$ | 0.933 | 0.067 | 0.357 | 5.35 |
| $25^0$ | 0.955 | 0.0425 | 0.865 | 20.6 |
| $30^0$ | 1.000 | 0. | 1.43 | $\infty$ |

Table 2: Reduced transition probabilities for several values of $\gamma$.

From the data in table 2 it follows that the reduced transition probability $b(E2; 2_1 \rightarrow 0_1)$ from the first excited state to the ground state changes only slightly when axial symmetry of the nuclear shape is violated. One may use this quantity to determine the deformation parameter $\beta$.

In table 3 the B(E2) values are shown as a function of the deformation parameter $\gamma$ [Tok76]. Here one sees the transitions in the ground band do not change so much as a function of the $\gamma$-deformation as for the so-called $\gamma$-band.

| $I_i \rightarrow I_f$ | $\gamma = 0^0$ | $10^0$ | $15^0$ | $20^0$ | $22.5^0$ | $25^0$ | $27.5^0$ | $30^0$ |
|---|---|---|---|---|---|---|---|---|
| $2_1 \rightarrow 0_1$ | 1.00 | 0.97 | 0.95 | 0.93 | 0.94 | 0.96 | 0.99 | 1.00 |
| $4_1 \rightarrow 2_1$ | 1.43 | 1.39 | 1.38 | 1.37 | 1.37 | 1.36 | 1.38 | 1.39 |
| $6_1 \rightarrow 4_1$ | 1.57 | 1.55 | 1.56 | 1.62 | 1.67 | 1.71 | 1.73 | 1.73 |
| $8_1 \rightarrow 6_1$ | 1.65 | 1.64 | 1.70 | 1.82 | 1.86 | 1.89 | 1.91 | 1.91 |
| $10_1 \rightarrow 8_1$ | 1.69 | 1.70 | 1.81 | 1.94 | 1.98 | 2.00 | 2.02 | 2.02 |
| $2_2 \rightarrow 0_1$ | - | 0.03 | 0.05 | 0.07 | 0.06 | 0.04 | 0.01 | 0.00 |
| $2_2 \rightarrow 2_1$ | - | 0.00 | 0.14 | 0.36 | 0.56 | 0.87 | 1.24 | 1.43 |
| $3_1 \rightarrow 2_1$ | - | 0.05 | 0.09 | 0.12 | 0.11 | 0.08 | 0.03 | 0.00 |
| $3_1 \rightarrow 2_2$ | - | 1.74 | 1.69 | 1.67 | 1.68 | 1.71 | 1.76 | 1.79 |
| $4_2 \rightarrow 2_1$ | - | 0.01 | 0.01 | 0.00 | 0.01 | 0.02 | 0.02 | 0.00 |
| $4_2 \rightarrow 2_2$ | - | 0.58 | 0.54 | 0.48 | 0.45 | 0.43 | 0.48 | 0.60 |
| $4_2 \rightarrow 3_1$ | - | 1.28 | 1.17 | 0.90 | 0.68 | 0.45 | 0.21 | 0.00 |
| $4_2 \rightarrow 4_1$ | - | 0.06 | 0.17 | 0.31 | 0.34 | 0.31 | 0.27 | 0.27 |
| $5_1 \rightarrow 3_1$ | - | 0.93 | 0.90 | 0.89 | 0.90 | 0.91 | 0.94 | 0.95 |
| $5_1 \rightarrow 4_1$ | - | 0.04 | 0.05 | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 |
| $5_1 \rightarrow 4_2$ | - | 0.94 | 0.94 | 0.96 | 0.96 | 0.93 | 0.90 | 0.95 |
| $6_2 \rightarrow 4_1$ | - | 0.01 | 0.00 | 0.01 | 0.01 | 0.00 | 0.00 | 0.00 |
| $6_2 \rightarrow 4_2$ | - | 1.13 | 1.03 | 0.88 | 0.84 | 0.81 | 0.74 | 0.73 |
| $6_2 \rightarrow 5_1$ | - | 0.66 | 0.53 | 0.27 | 0.14 | 0.05 | 0.00 | 0.00 |
| $6_2 \rightarrow 6_1$ | - | 0.06 | 0.15 | 0.17 | 0.14 | 0.13 | 0.14 | 0.14 |
| $7_1 \rightarrow 5_1$ | - | 1.28 | 1.26 | 1.26 | 1.27 | 1.29 | 1.30 | 1.31 |
| $7_1 \rightarrow 6_1$ | - | 0.03 | 0.02 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $7_1 \rightarrow 6_2$ | - | 0.53 | 0.56 | 0.60 | 0.62 | 0.69 | 0.81 | 0.85 |
| $8_2 \rightarrow 6_1$ | - | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $8_2 \rightarrow 6_2$ | - | 1.35 | 1.19 | 1.05 | 1.01 | 0.99 | 1.03 | 1.05 |
| $8_2 \rightarrow 7_1$ | - | 0.38 | 0.24 | 0.07 | 0.02 | 0.00 | 0.00 | 0.00 |
| $8_2 \rightarrow 8_1$ | - | 0.06 | 0.11 | 0.09 | 0.08 | 0.09 | 0.09 | 0.09 |
| $9_1 \rightarrow 7_1$ | - | 1.45 | 1.44 | 1.46 | 1.48 | 1.51 | 1.53 | 1.54 |
| $9_1 \rightarrow 8_1$ | - | 0.02 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $9_1 \rightarrow 8_2$ | - | 0.35 | 0.39 | 0.45 | 0.54 | 0.66 | 0.73 | 0.75 |
| $10_2 \rightarrow 8_1$ | - | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $10_2 \rightarrow 8_2$ | - | 1.45 | 1.27 | 1.16 | 1.16 | 1.23 | 1.27 | 1.28 |
| $10_2 \rightarrow 9_1$ | - | 0.23 | 0.10 | 0.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| $10_2 \rightarrow 10_1$ | - | 0.06 | 0.08 | 0.06 | 0.06 | 0.06 | 0.06 | 0.06 |

Table 3: Reduced transition probabilities in unit of $e^2Q_0^2/16\pi$ as a function of the asymmetry parameter $\gamma$.

It should be mentioned that one cannot decide on the basis of measured level energies or reduced electromagnetic transition probabilities between them whether the nucleus is an elongated or oblate ellipsoid. The only way to answer this question is to measure the spectroscopic quadrupole moment $Q_S(I)$. In even nuclei these moments are zero in the ground state. In the first excited state with spin I=2 the spectroscopic quadrupole moment is

$$Q_s(2_1) = -\frac{6\cos(3\gamma)}{7\sqrt{9 - 8\sin^2(3\gamma)}} \tag{20}$$

The spectroscopic quadrupole moments of prolate $0^0 \leq \gamma \leq 30^0$ and oblate $30^0 \leq \gamma \leq 60^0$ nuclei differ in sign. In fig. 3 the spectroscopic quadrupole moments $Q_s(2_1)$ in units of the intrinsic quadrupole moment $Q_0$ are plotted versus the energy ratio $E(2_2)/E(2_1)$ for nuclei in the $A \simeq 190$ mass region.

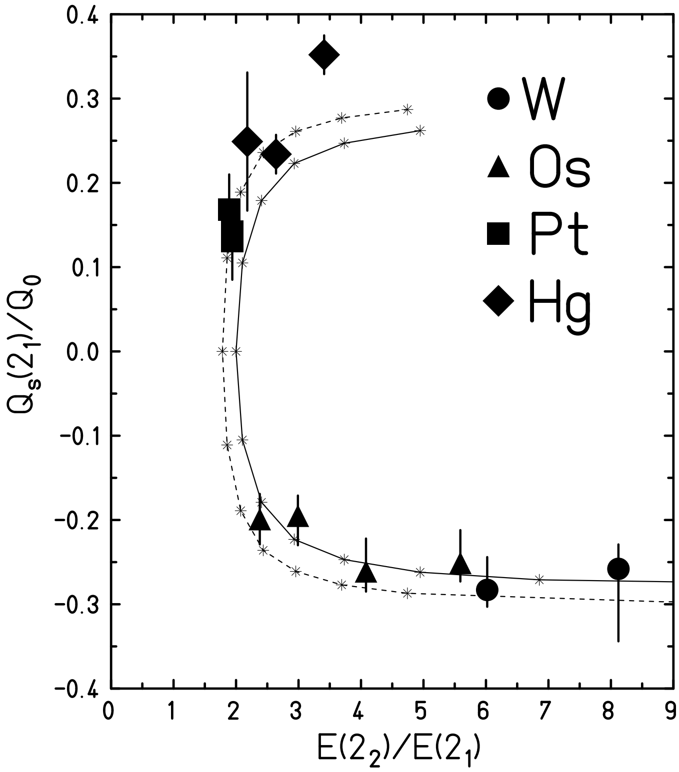


Figure 3: Spectroscopic quadrupole moment $Q_s(2_1)$ in units of $Q_0$ as a function of the energy ratio $E(2_2)/E(2_1)$ for nuclei in the $A \simeq 190$ mass region. The data are compared with predictions of the rigid (solid line) and soft (dashed line) asymmetric rotator model.

The experimental data are compared with predictions of the rigid (solid line) and soft (dashed line) asymmetric rotator model. In the rigid asymmetric rotor model the energy ratio $E(2_2)/E(2_1)$ (Eq. 6) and the spectroscopic quadrupole moment $Q_s(2_1)$ (in units of $Q_0$) of the first excited I=2 state (Eq. 16,Eq. 20) depend only on the $\gamma$-asymmetry parameter. In fig. 3 the asterisk denote the theoretical results for $\gamma$-values between $15^0$ and $42.5^0$ which have been calculated in steps of $2.5^0$.

It is interesting to note that for $\gamma = 30^0$ the theory predicts an equidistant spacing of the energy levels $E(2_1)$ and $E(2_2)$.

In the second excited state with spin I=2 the spectroscopic quadrupole moment has a different sign

$$Q_s(2_2) = -Q_s(2_1) \tag{21}$$

In table 4 the spectroscopic quadrupole moments $Q_s(I)$ for different spin states are shown as a function of the $\gamma$-deformation [Tok76].

| $I_i$ | $\gamma = 0^0$ | $10^0$ | $15^0$ | $20^0$ | $22.5^0$ | $25^0$ | $27.5^0$ | $30^0$ |
|---|---|---|---|---|---|---|---|---|
| $2_1$ | -0.28 | -0.28 | -0.27 | -0.25 | -0.22 | -0.18 | -0.10 | 0.00 |
| $4_1$ | -0.36 | -0.35 | -0.32 | -0.24 | -0.18 | -0.11 | -0.05 | 0.00 |
| $6_1$ | -0.40 | -0.38 | -0.32 | -0.21 | -0.15 | -0.10 | -0.05 | 0.00 |
| $8_1$ | -0.42 | -0.39 | -0.31 | -0.20 | -0.15 | -0.10 | -0.05 | 0.00 |
| $10_1$ | -0.43 | -0.40 | -0.30 | -0.19 | -0.14 | -0.09 | -0.05 | 0.00 |
| $2_2$ | - | 0.28 | 0.27 | 0.25 | 0.22 | 0.18 | 0.10 | 0.00 |
| $3_1$ | - | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| $4_2$ | - | -0.15 | -0.17 | -0.23 | -0.28 | -0.32 | -0.30 | 0.00 |
| $5_1$ | - | -0.23 | -0.22 | -0.20 | -0.18 | -0.14 | -0.08 | 0.00 |
| $6_2$ | - | -0.29 | -0.33 | -0.40 | -0.41 | -0.34 | -0.15 | 0.00 |
| $7_1$ | - | -0.32 | -0.30 | -0.26 | -0.21 | -0.15 | -0.08 | 0.00 |
| $8_2$ | - | -0.36 | -0.42 | -0.45 | -0.38 | -0.23 | -0.10 | 0.00 |
| $9_1$ | - | -0.36 | -0.34 | -0.27 | -0.21 | -0.14 | -0.07 | 0.00 |
| $10_2$ | - | -0.41 | -0.47 | -0.42 | -0.30 | -0.18 | -0.09 | 0.00 |

Table 4: Spectroscopic quadrupole moments $Q_s$ in units of the intrinsic quadrupole moment $Q_0$ as a function of the asymmetry parameter $\gamma$.

The rotation-vibration coupling has also an influence on the electric quadrupole moments and the E2 transition probabilities. The results for the spectroscopic quadrupole moment $Q_s$ (in units of $Q_0$) of the first excited $2^+$ state are given in the table 5 (for $0^0 \leq \gamma \leq 30^0$ and $\mu \leq 1$) that enables a comparison between theory and experimental findings.

| $\mu$ | $\gamma = 10^0$ | $\gamma = 15^0$ | $\gamma = 20^0$ | $\gamma = 25^0$ | $\gamma = 30^0$ |
|---|---|---|---|---|---|
| 0.0 | -0.281 | -0.271 | -0.247 | -0.179 | 0.0 |
| 0.1 | -0.281 | -0.271 | -0.247 | -0.179 | 0.0 |
| 0.2 | -0.281 | -0.271 | -0.247 | -0.179 | 0.0 |
| 0.3 | -0.287 | -0.277 | -0.252 | -0.183 | 0.0 |
| 0.4 | -0.295 | -0.285 | -0.262 | -0.190 | 0.0 |
| 0.6 | -0.334 | -0.325 | -0.299 | -0.218 | 0.0 |
| 0.8 | -0.383 | -0.382 | -0.363 | -0.260 | 0.0 |
| 1.0 | -0.458 | -0.447 | -0.413 | -0.302 | 0.0 |

Table 5: Dependence of the spectroscopic quadrupole moment $Q_s/Q_0$ in units of the intrinsic quadrupole moment $Q_0$ for the first excited state on the parameters $\gamma$ and $\mu$.

When $\gamma$ lies in the range of $30^0$ and $60^0$, the values of $Q_s$ are obtained by means of $Q_s(\gamma) = -Q_s(60^0 - \gamma)$. With increasing "softness" of the nucleus, the effect of "stretching" of the nucleus in the $2^+$ state increases, this being connected with the increase of the absolute value of the ratio $Q_s/Q_0$. On the other hand, when $\gamma$ tends to $30^0$ the boundary between the prolate and oblate nuclei becomes more and more obliterated. This effect causes a decrease in the absolute value of the ratio $Q_s/Q_0$.

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